

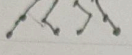


5) How many binary tree shapes of  $n$  nodes are there with height  $n-1$ ?

Base Case  $n=1$   $h=0$   1 shape  
 $n=2$   $h=1$   2 shapes  
 $n=3$   $h=2$   4 shapes  
 $2^{n-1}$  possible shapes

I.H. the number of binary tree shapes of with  $k$  nodes to a height of  $k-1$  is  $2^{k-1}$  for  $1 \leq k < n$

The number of shapes of a tree with  $n$  nodes can be broken into 2 parts the tree before the  $n$ th node of size  $k=n-1$  to the one node of size 1 at height  $n-1$ . The number of shapes of the tree can be assumed to be given by  $2^{k-1}$  by I.H. The possible number of places the  $n$ th node can go at the end of the  $k$  node tree is 2, the left child or right child.

$\therefore$  The number of possible shapes at  $n$  is given by  
 $2(2^{k-1})$   
 $= 2^k \quad k=n-1$

$\therefore$  The number of possible shapes of the binary tree with  $n$  nodes to a height of  $n-1$  is  
 $2^{n-1}$