

5) How many binary tree shapes of n nodes are there with height $n-1$?

Base case	$n=1$	$h=0$	•	1 shape
	$n=2$	$h=1$	/ \	2 shapes
	$n=3$	$h=2$	↙ ↘	4 shapes 2^{n-1} possible shapes

I. H. the number of binary tree shapes of with k nodes to a height of $k-1$ is 2^{k-1} for $1 \leq k < n$

The number of shapes of a tree with n nodes can be broken into 2 parts the tree before the n th node of size $k=n-1$ & the one node of size 1 at height $n-1$. The number of shapes of the tree can be assumed to be given by 2^{k-1} by I. H. The possible number of places the n th node can go at the end of the k node tree is 2, the left child or right child.

\therefore The number of possible shapes at n is given by $2(2^{k-1})$

$$= 2^k \quad k=n-1$$

\therefore The number of possible shapes of the binary tree with n nodes to a height of $n-1$ is

$$\underline{\underline{2^{n-1}}}$$