Systems Engineering HW

Group 13
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March 21, 2022

Problem formulation

A big ferry uses two engines which makes it go forward, both the engines can break down making the ferry move forward at a slower pace, if however both break down the ferry stops. The operation of the ferry is assumed to be a Markov process in continuous time.

Engine one breaks down according to a $\exp(\lambda_1)$ distribution, likewise is engine two, $\exp(\lambda_2)$. Moreover, the boat has three repairmen working on it whom can help in repairing a broken engine, this is vital since the repair time of a engine is modelled as $\exp(n_k\mu_k)$, where n_k denotes the amount of repairmen at ta given engine.

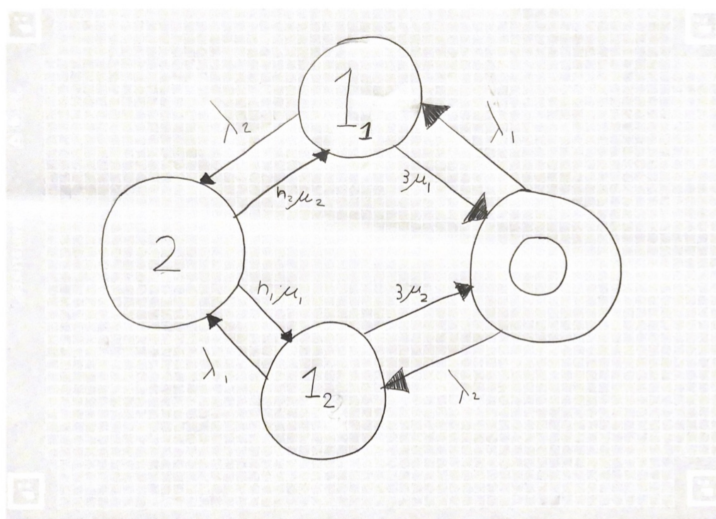
Lastly, let $V(t)$ denote the speed of the ferry, if both engines function properly it is going at a speed of v, if only engine one is working it is moving at v_1 and lastly if only engine 2 is working it is moving at a speed of v_2.

- $v = 18$, $v_1 = 10$, $v_2 = 15$
- $\lambda_1 = 14$, $\lambda_2 = 20$
- $u_1 = 10$, $u_2 = 8$
- $h = 0.001$

Analytic solution

1

We wish to determine the states of the Markov process in the assignment. It is readily seen that there are 4 states, both engines working, none working and two additional where only one is properly operating. Below listed is a drawn picture explaining the states.



First let $X(t) = \#Number\ of\ engines\ not\ working$, note here too that we have the range of $X(t) = \{0, 1_1, 1_2, 2\}$ where 1_1 and 1_2 denotes which unique engine is broken down.

Then it is seen that if both are working, the transition rate (seeing as they are exponentially distributed) are with a rate λ_1 to the engine 1_1 and at a rate of λ_2 to engine 1_2, this in turn explains the rate at which they move towards $X(t) = 2$ since the other is explained it has to be by the other engines rate. In terms of repairing, when only one is broken we assume all three repairmen are trying to restore it, making the various repair rates $3\mu_1$ and $3\mu_2$. However when both are broken we now consider a more nuanced case where n_1 and n_2 can be any divide of the three repairmen to the two broken engines. Making the rates $n_1\mu_1$ and $n_2\mu_2$.

As an additional comment, the case of repairing both at the same time, that is from going to 2 broken engines to none, or the possibility of repairing one while the other breaking at the exact same time is neglected here. Even though there is a possibility for this to happen it is 0 almost surely.

2

The intensity matrix can be determined by identifying the q_{ij} from range in $X(t)$, we should also note that in the intensity matrix it is necessary for the row sum to be zero. This is used in determining the negative diagonal entries.

$$Q = \begin{pmatrix} -(n_2\mu_2 + n_1\mu_1) & n_2\mu_2 & n_1\mu_1 & 0 \\ \lambda_2 & -(\lambda_1 + 3\mu_1) & 0 & 3\mu_1 \\ \lambda_1 & 0 & -(\lambda_1 + 3\mu_2) & 3\mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\lambda_1 + \lambda_2) \end{pmatrix}$$

3

To be able to determine if there is a stationary distribution we refer to the following theorem.

For a finite irreducible Markov process there always exists a unique steady state probabilities π that solve the steady state equations,

$$\pi Q = 0, \quad \sum_j \pi_j = 1.$$

Seeing as our process is finite, as we have a finite range for $X(t)$ and it is indeed a Markov process by the statement of the problem. Then the only thing left to show is that it is irreducible, but as we only have one class, that is, all states interact with one another (which can be seen in the picture or from the transition matrix) we know it is indeed irreducible, and satisfy the assumptions of the theorem. Thus we know it will indeed have a steady state distribution for our problem.

4

In solving the steady state distribution we must solve the following systems of equations,

$$\begin{pmatrix}\pi_1, \pi_2, \pi_3, \pi_4\end{pmatrix} \begin{pmatrix} -(n_2\mu_2 + n_1\mu_1) & n_2\mu_2 & n_1\mu_1 & 0 \\ \lambda_2 & -(\lambda_2 + 3\mu_1) & 0 & 3\mu_1 \\ \lambda_1 & 0 & -(\lambda_1 + 3\mu_2) & 3\mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\lambda_1 + \lambda_2) \end{pmatrix} = 0$$

and

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1.$$