Home assignment number 1, 2021, in SF2863 Systems Engineering.

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The home assignments are a mandatory part of the course. In total, there are two home assignments and you need to collect 4.0 out of 6.0 points in order to pass this part of the course. Home assignment 1 can give up to 2.0 points.

The home assignments can, in addition, give bonus points on the exam. If a home assignment is handed in before the deadline, then the points awarded on the assignment will also count as bonus points on the exam (hence, home assignment 1 can give up to 2.0 bonus points on the exam). In order to get bonus points on the exam the report should be submitted to canvas before 23:59, Tuesday, November 16, 2021.

This home assignment should be done in groups of 2 persons, and one report per group is to be submitted to canvas. See the information on canvas on how to form groups and submit your results. State your name, and email address on the front of the report.

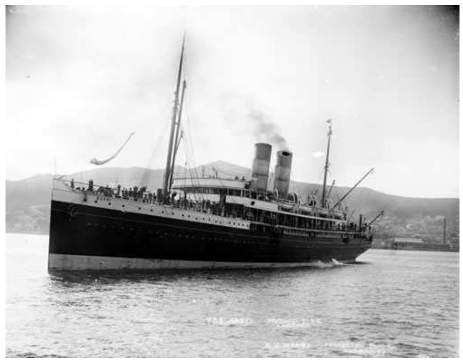
In the report you should describe in your own words how the problem was solved. In particular, carefully describe how you defined your states, modelled the problem and implemented the simulations. Do not explain the code, but rather the mathematical ideas. You should not need more than 4 pages for the report. The answers to the questions in the assignment should be given in the main report and numbered as in this text. In particular, it should not be necessary to look at your code to understand how you define and calculate things. Relevant print-outs and plots should be included in the report. You may use any computer program to facilitate the computations, but you have to include your computer code in the submission. We use automated plagiarism-control tools and will also make random checks that you have not copied parts of another groups report or computer code!

Questions should be posed in the discussion group in canvas.

System and data

We consider a big ferry with two engines that each drives one propeller. If one engine breaks down the ferry can still move, but it runs a bit slower, and if both breaks down the ferry stops.

We assume that the operation of the ferry is well described by a simple Markov process in continuous time.



Assume that engine number 1 breaks down after some time that is modelled as a stochastic variable with $\mathrm{Exp}(\lambda_1)$ distribution. Similarly, engine number 2 breaks down after some time that is $\mathrm{Exp}(\lambda_2)$ distributed. Three repairmen are working on the ferry for repairing the engines. The repair time for engine k is $\mathrm{Exp}(n_k\mu_k)$ distributed, where n_k is the number of repairmen working on engine k.

Let $V(t)$ denote the speed of the ferry at time t. If both the engines are working the ferry is running at a speed which is v, if only engine 1 is working the speed is v_1 and if only engine 2 is working the speed is v_2.

Parameter values

- $v = 18$, $v_1 = 10$, and $v_2 = 15$.
- $\lambda_1 = 14$, $\lambda_2 = 20$.
- $\mu_1 = 10, \mu_2 = 8$
- $h = 0.001$

Now, the system is a Markov process that will depend on how many workers are assigned to repair the engines if they break down. If only one engine is broken it is obvious that it is optimal to let all three repairmen work on it. If both engines are broken it is not so clear how to distribute the workers. To decide this you should for each different repair strategy determine the average speed of the ferry at steady state and this way choose the best strategy.

You should do this both analytically and by simulation.

Analytic solution
First, you should determine the average speed using the theory of Markov processes. For this approach you need to (for all possible repair strategies)

1. define the states of the Markov process,

2. determine the intensity matrix,

3. motivate why there should exist a stationary distribution

4. determine the stationary distributions (for all repair strategies using the intensity matrices defined above.)

5. determine the average speed of the ferry from this stationary distribution.

You may use a computer program to solve the equation systems.

Next you should simulate the system numerically and determine the average speed by ergodic estimates.
This will be done in two different ways; in (a) using a direct simulation of the continuous time process, and in (b) using a time discretization approach.

Continuous time approach
Here we simulate the continuous time process by randomly generating dwell times. Start the process at time 0 with both engines of the ferry functioning. Then you should use that the jump times are exponentially distributed.
Hint: If you use Matlab an exponentially distributed variable can be generated using the command `exprnd`, where you specify the mean of the variable (which is inversely proportional to the intensity).
You can use several exponential distributions, or one exponential distribution and a uniform distribution, to determine how long you remain in a state and which will be the next state.

6a. Describe in detail how you determine the time to the next jump and where you jump.

7a. Run the simulation for a time interval $[0, T]$.
By simulating the process we mean that you start the process in some state and then you determine one realization of the process by using the probability description of the process to determine how long you remain in a state and where you transition.

If you run the simulation twice you would get different results. Comparing the two different results can give an indication on if the averages have converged or not. Choose (the magnitude of) T large enough so that the errors are less than 1%.

8a. Take averages to determine estimates of the expected speed of the ferry.

Hint: Since a large number of time steps have to be made to get reasonable ergodic estimates you may note that it is enough to keep track on how long time the Markov process has spent so far in each state to determine the estimates for the stationary probabilities and the average speed.

Discretization approach

Now a discretization of the continuous time process will be made.

Start the process at time 0 with both engines of the ferry functioning. Then make a discretization of the time axis so that we only consider the process at times $t_k = kh$ for $k = 0, 1, 2, \cdots, N$. Then the probability of a jump to another state during the time interval $[t_k, t_{k+1}]$ can be determined (approximatively) using the intensity matrix for the continuous time Markov process. (You could use the matrix exponential function to determine the transition probabilities, but we would like you to use the approximative expressions that hold for small h used in the lectures.)

If the time step h is small enough this approximation is good.

6b. Determine the transition matrix of a discrete time Markov chain that will approximate the continuous time process.
Hint: Check if the transition matrix you obtain has row sums equal to one.

7b. Now use this discretization to simulate the process.
By simulating the process we mean that you start the process in some state and then you determine one realization of the process by using the transition probabilities to determine which transitions occur.

If you run the simulation twice you would get different results. Comparing the two different results can give an indication on if the averages have converged or not. Choose (the magnitude of) N large enough so that the errors are less than 1%.

8b. Take averages to determine estimates of the expected speed of the ferry.

Hint: If you use Matlab, a stochastic variable with uniform probability in [0, 1] can be generated using the command `rand`.

The approach just described has also the purpose of showing the connection between discrete time and continuous time Markov chains.

Comparison

9. How does the computation times between the three different methods compare? How does the simulation lengths compare, *i.e.*, hN and T?

Hint: In Matlab the computation time can be determined using `tic, toc` or `profile`.

Reliability - average time to full break down

10. Use a method of your choice to determine the average time it takes until the first time that the ferry comes to a halt due to a total engine break down. Assuming that the ferry starts with both engines working.

Good luck!