Project Report

# 2. Root finding

## 2.1

Given that , when :

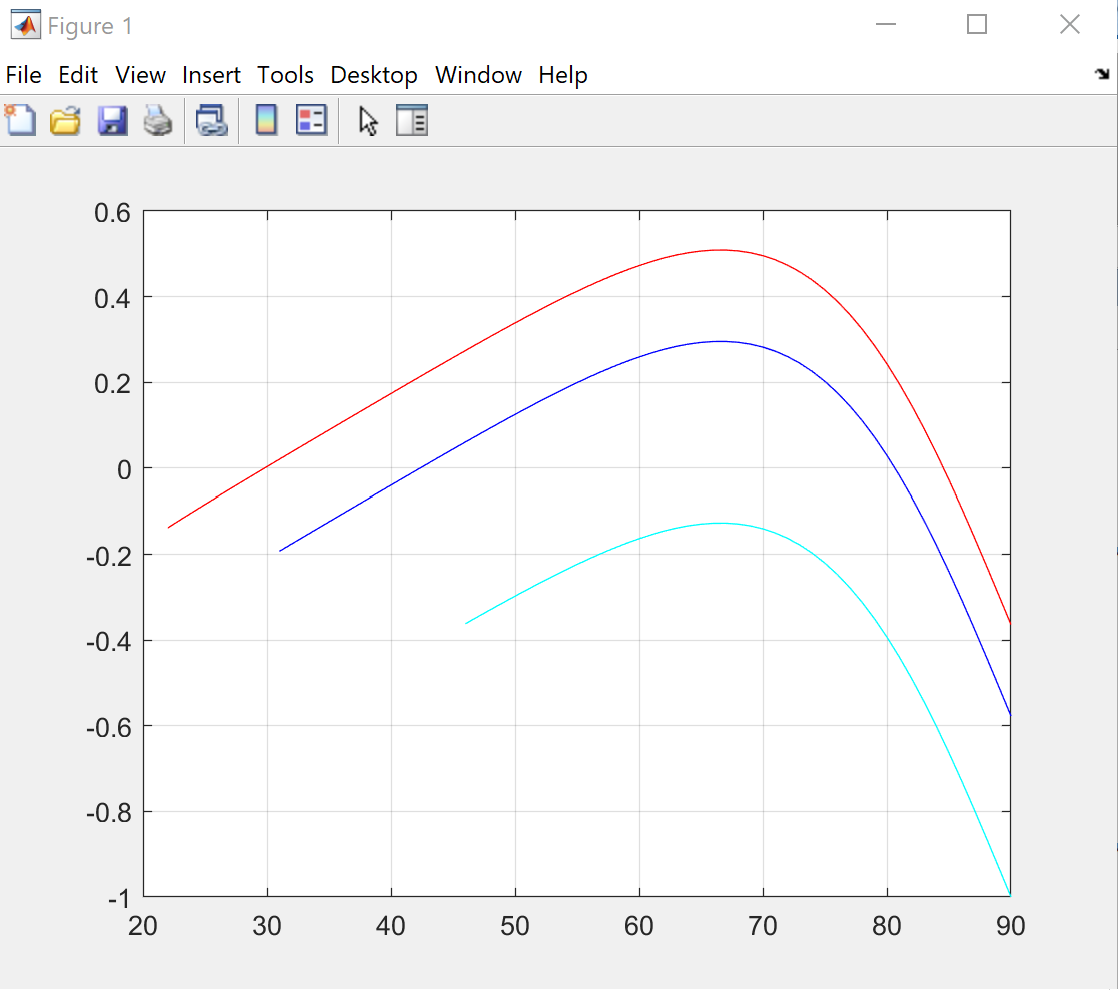
Hence, either or

1. , Since in this question, stands for the shock wave angle, thus

## 2.2

A close up of a map

Description automatically generated



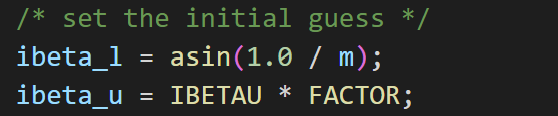
Observed from the graph, when **both and exist**, for the **same M**, when increases, decreases and increases.

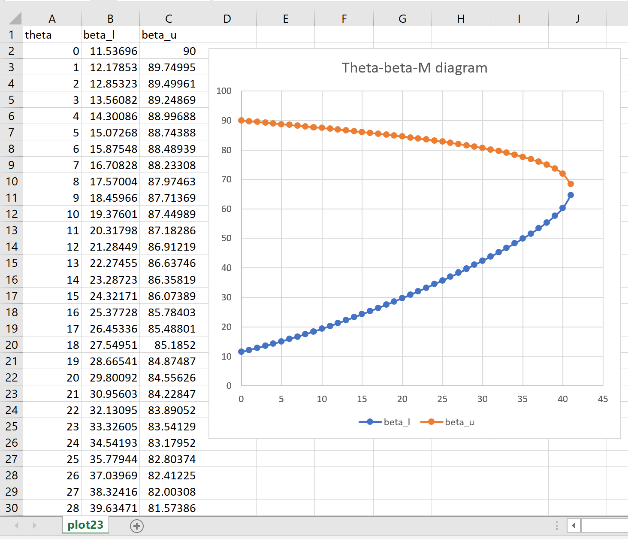
**In terms of ,** for M=1.5, when , there’s not a root, must be between and .

for M=5, when , there’s no root, must be above below .

## 2.3

1. For , I chose the two extremes: as the initial guesses for the roots.



1. 
2. A close up of a map

   Description automatically generated

M=8.0

M=7.0

M=6.0

M=4.0

M=3.0

M=2.0

# Regression

First, focus on the second row of matrix in each side, expand the equation:

Divide by N

Simplify the equation

Recall from lecture that:

To minimize S, find the derivative of S:

Substitute b from equation 1

Separate into two sums:

Since

So that we can obtain:

and

Add these two zero terms to both denominator and numerator:

Refine and combine the terms:

When , this linear regression will fail.

# Linear Algebraic Systems

**Start from the first row**, Multiply each term by

the new row becomes

then minus the second row by this new row, the second row now becomes

,

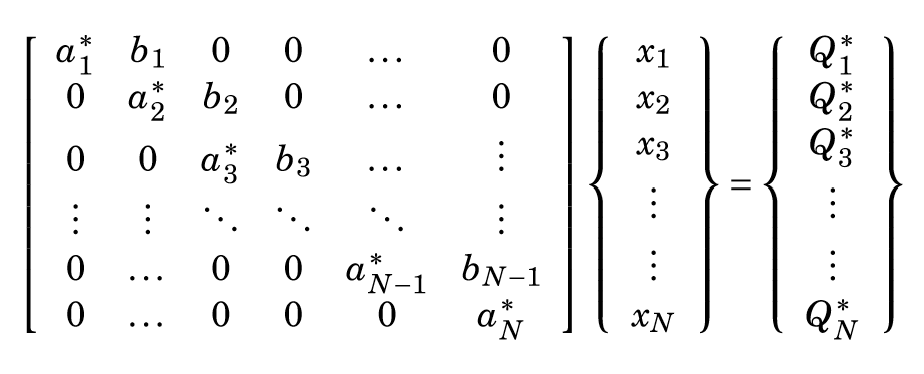
from now on each processed row only has **two terms left**.

for row , , continue applying **Gauss elimination**:

1. Multiply the row by , the first term in row is , the second term becomes , Q becomes

.

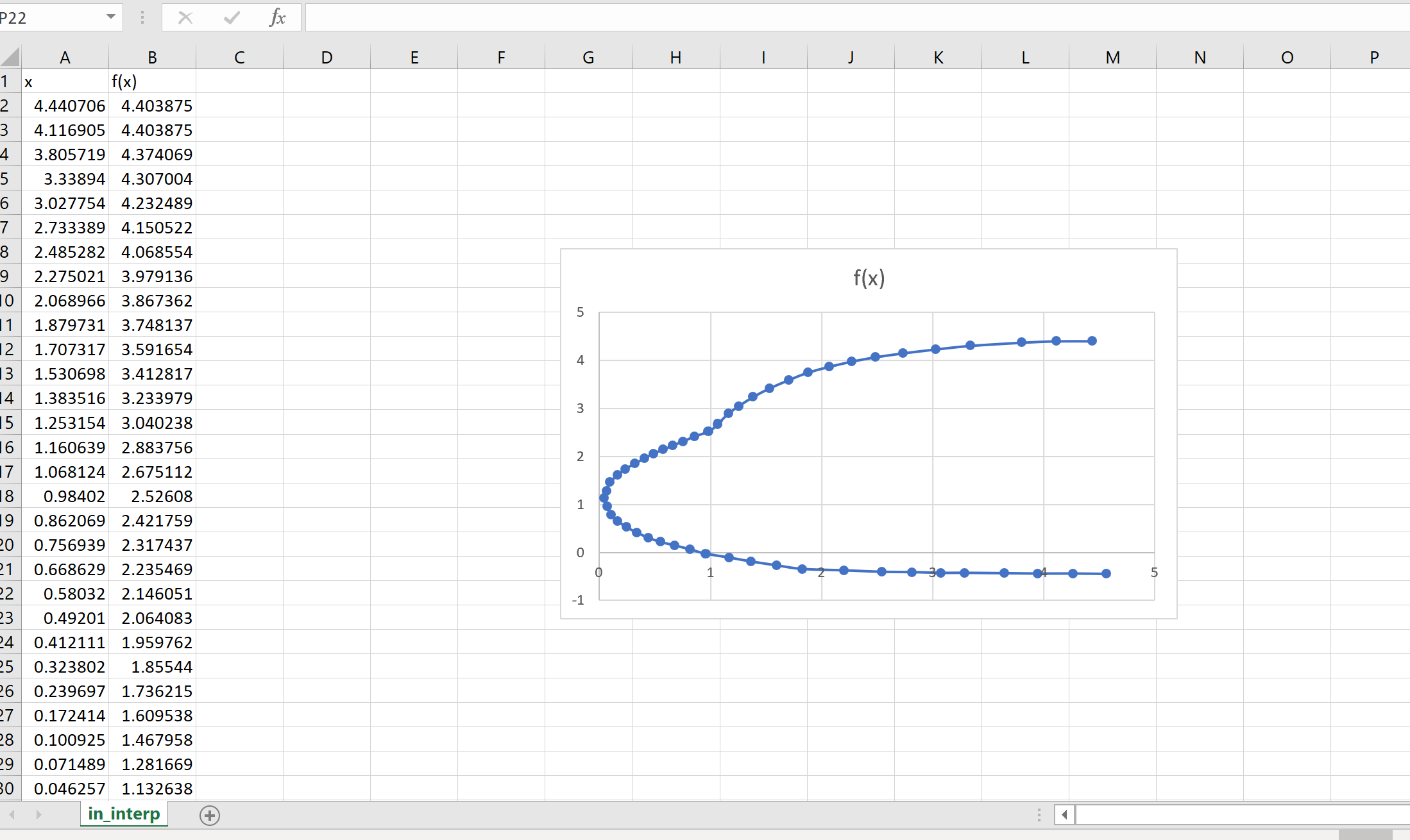
1. Minus the row by the new row, now the row become
2. Now the matrix becomes the form of the second given matrix.



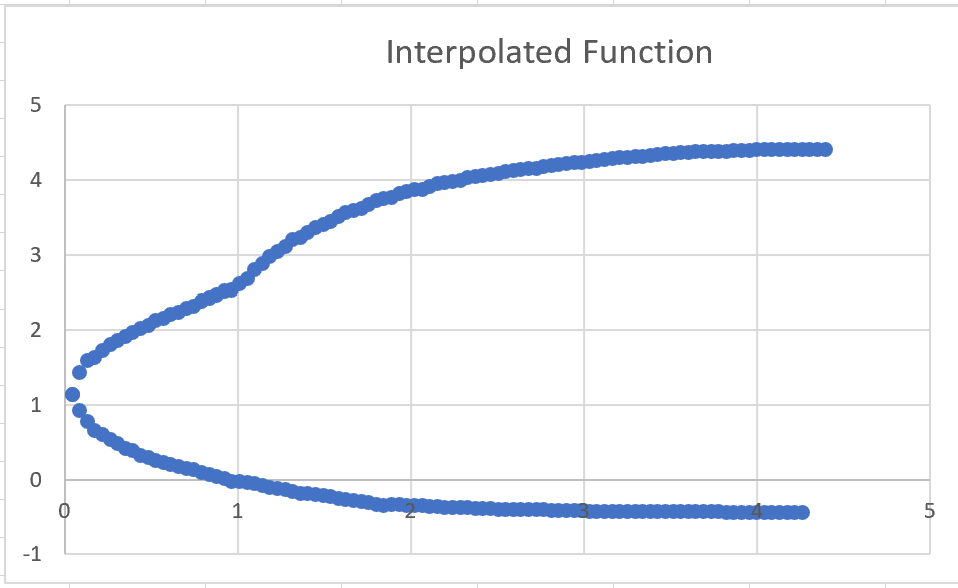
1. Hence, start from the last row, going from bottom to top, for , since only one term left,
2. For other rows, ,
3. Hence

# Interpolation

Plotted Data in Excel



The interpolated functions plotted:



In order to search the corresponded interval by x, two guarding mechanisms were introduced to check if x in within the interval that we desired:

* 

one is applied when the order of x variables is ascending in interpolated functions

* 

The other on is applied when the order of x variable is descending.

# Differentiation, differential equations

Below is the graph of the **exact solution**:

A close up of a map

Description automatically generated

It is easily seen that it shows a wave that shifts from left to right along x axis during a time. Compared with the exact solution, the **central scheme** gives a **more accurate prediction** than **upwind method**.

* **How does the agreement of the numerical prediction with the exact solution depend on ?**

Apply variable controlling method, keep CFL value unchanged, plot the curve with and in the same grid, here are the results:

* + A picture containing screenshot

    Description automatically generated**The Upwind scheme:**
  + A picture containing screenshot

    Description automatically generated

A screenshot of a social media post

Description automatically generated**A screenshot of a social media post

Description automatically generatedThe central scheme:**

A screenshot of a social media post

Description automatically generatedA screenshot of a map

Description automatically generated

**Discussion:** The grid resolution determines the accuracy of the numerical predicted solution. For all CFL values, the curves with , have closer peak of wave values with the exact solution than those with as well as smoother curves. This effect is more obvious in Upwind scheme. In addition, higher resolution also generates more stable curves in horizonal direction, observed from Central scheme graphs. Therefore, higher resolution provides more accurate prediction than lower resolution.

* **How does the agreement of the numerical prediction with the exact solution depend on the CFL number?**
  + **Upwind Scheme**

A screenshot of a social media post

Description automatically generated

A screenshot of a social media post

Description automatically generated

* + **Central Scheme**

A screenshot of a social media post

Description automatically generatedA screenshot of a social media post

Description automatically generated

**Discussion:** CFL value also determines the accuracy, not for but for , with lower CFL value comes higher time resolution. From the graphs, especially in Central scheme, the effect of different CFL values is significant, the curves with lower CFL value possess **smaller, more stable error area** before entering the wave. Hence, CFL number provides more accurate prediction, more agreement with the exact solution.

* **What happens if I chose CFL>1?**

Discussion: An appropriate CFL Number is meant to make the code run stably. Once the CFL number exceeds 1, the code can not generate stable curve predictions anymore. When CFL is higher, so as the influence. Below it the case **when CFL=1.002:**

A close up of a map

Description automatically generated

**When CFL = 2:**

