Stereo Vision

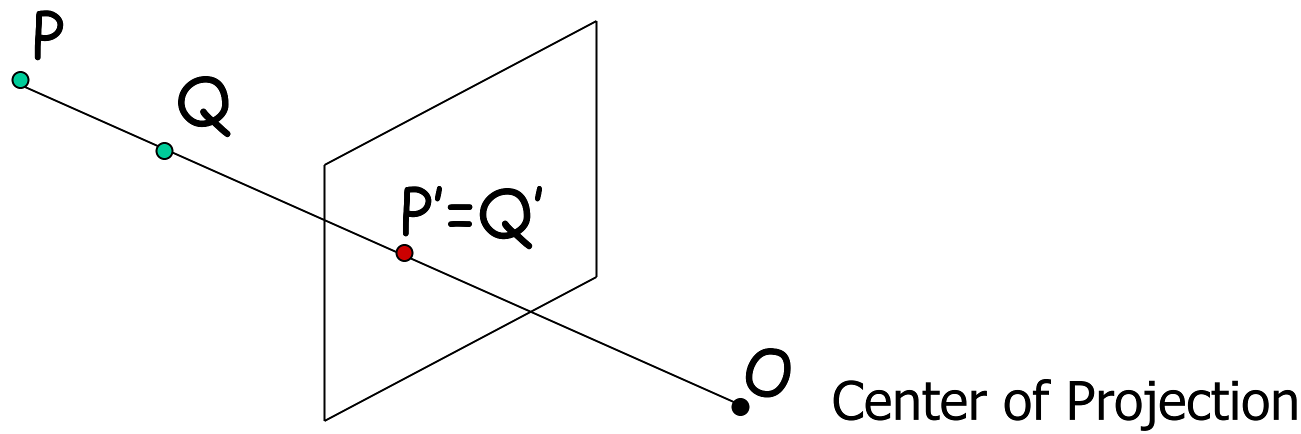
Outline

Basic Equations

Correspondence

Epipolar Geometry

Why Stereo Vision?

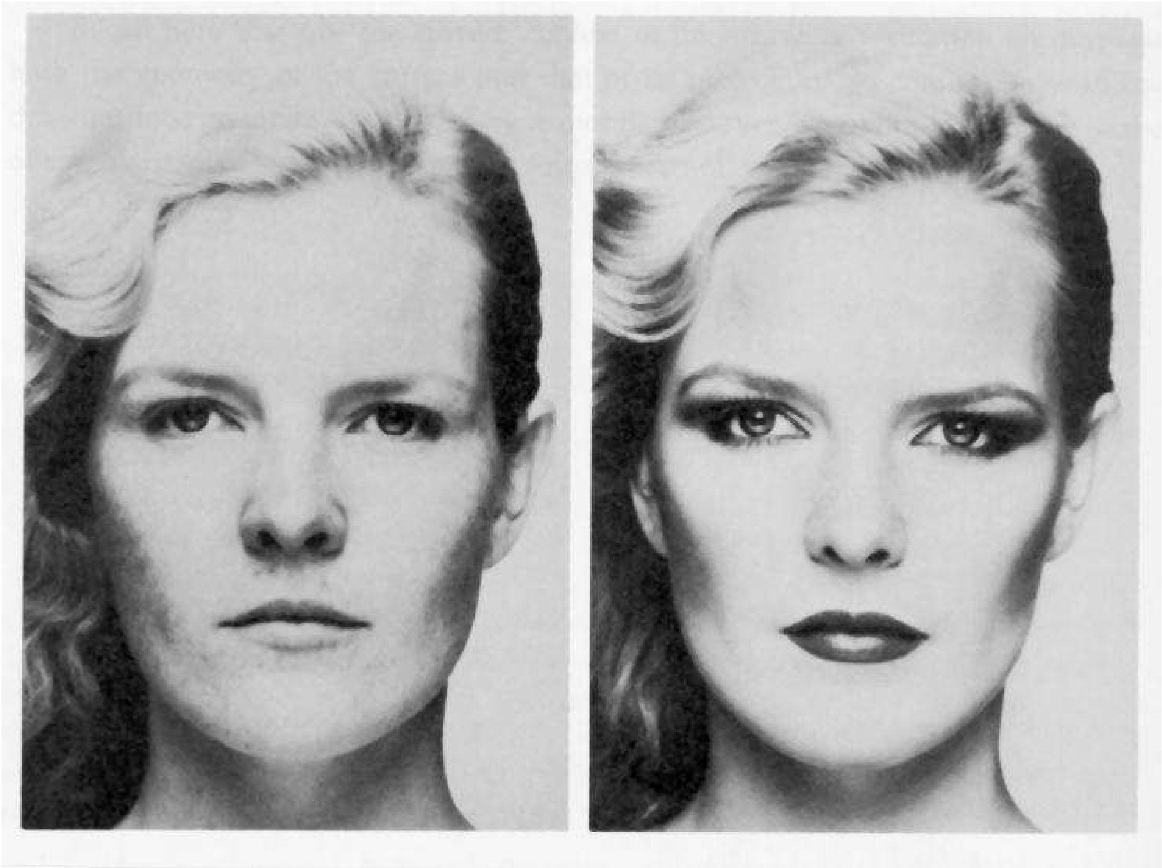


- 2D images project 3D points into 2D

- Recovering 3D from Images
  - How can we automatically compute 3D geometry from images?
  - What cues in the image provide 3D information?
  - ماهي المعلومات الموجودة في الصور و تعطي معلومات عن البعد الثالث

Visual Cues for 3D

Shading

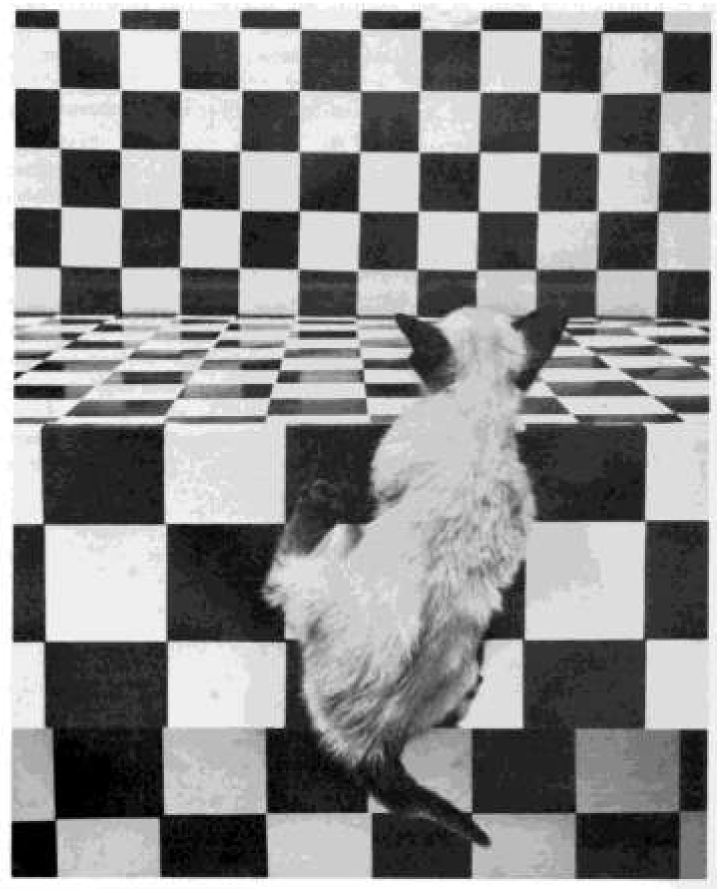


Merle Norman Cosmetics, Los Angeles

Visual Cues for 3D

Shading

Texture



***The Visual Cliff*, by William Vandivert, 1960**

Visual Cues for 3D

Shading

Texture

Focus

From *The Art of Photography*, Canon

Visual Cues for 3D

Shading

Texture

Focus

Motion

Visual Cues for 3D

Shading

Texture

Focus

Motion

- Others:
  - Highlights
  - Shadows
  - Silhouettes
  - Inter-reflections
  - Symmetry
  - Light Polarization
  - ...

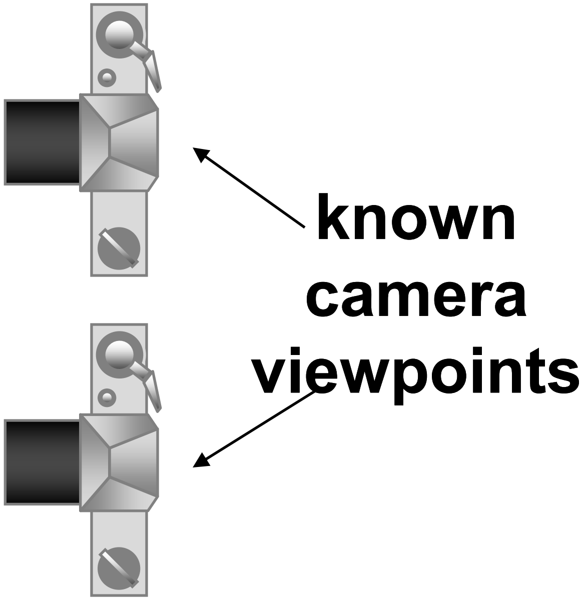
Shape From X

- X = shading, texture, focus, motion, ...

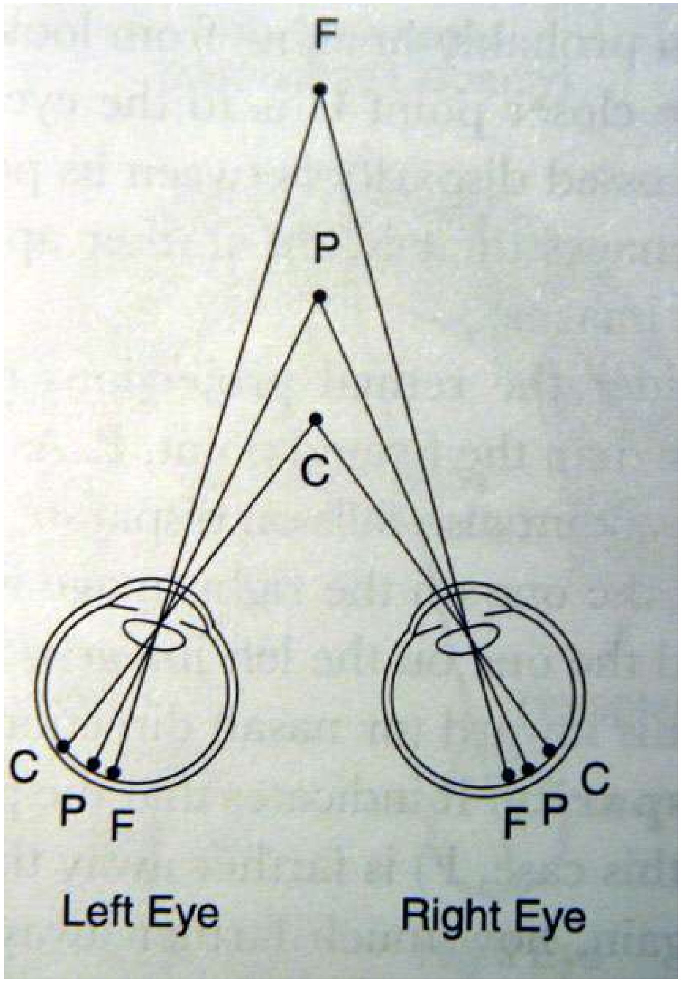
Stereo Reconstruction

The Stereo Problem

- Shape from two (or more) images
- Biological motivation



3. Depth from binocular disparity



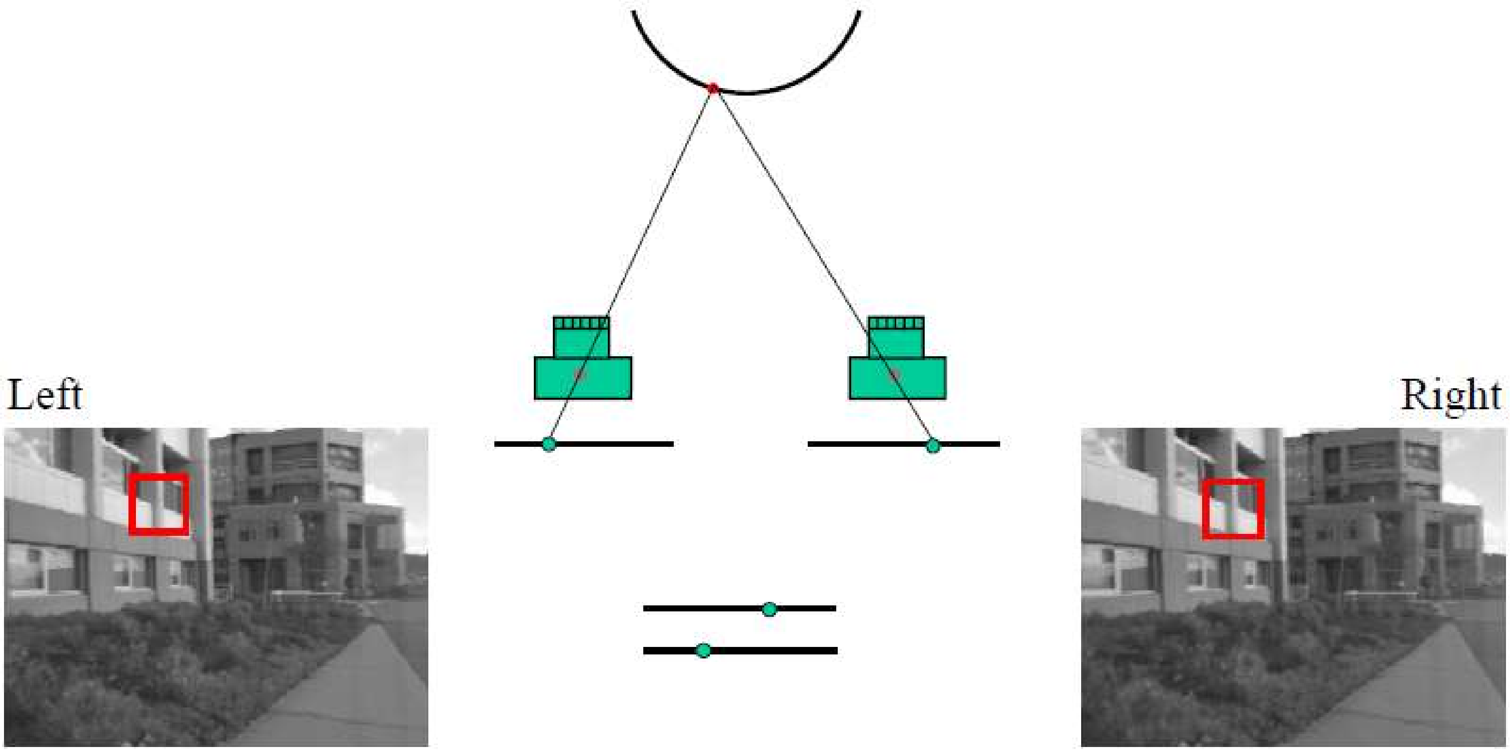
P: converging point

C: object nearer projects to the outside of the P, disparity = +

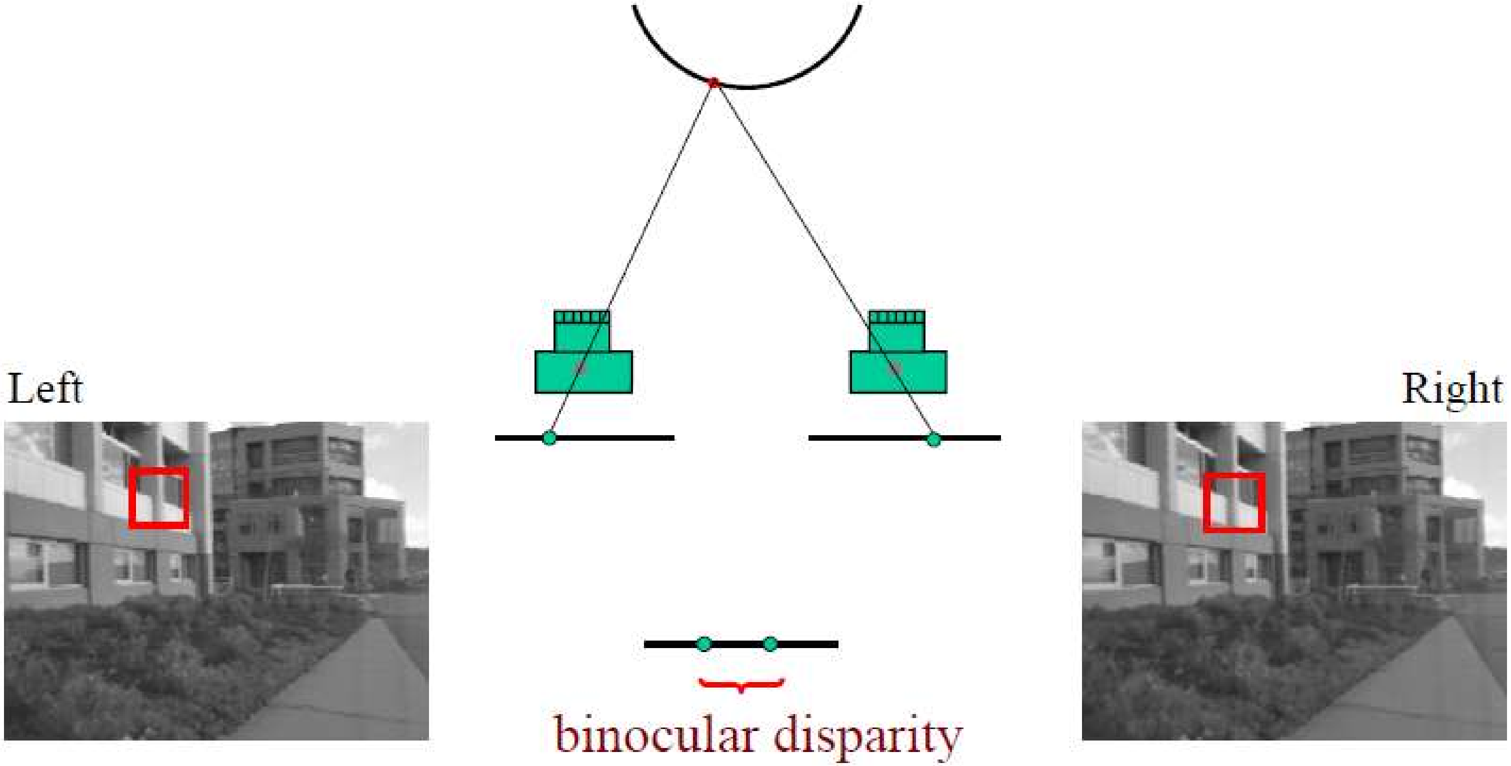
F: object farther projects to the inside of the P, disparity = -

Sign and magnitude of disparity

Binocular Stereo

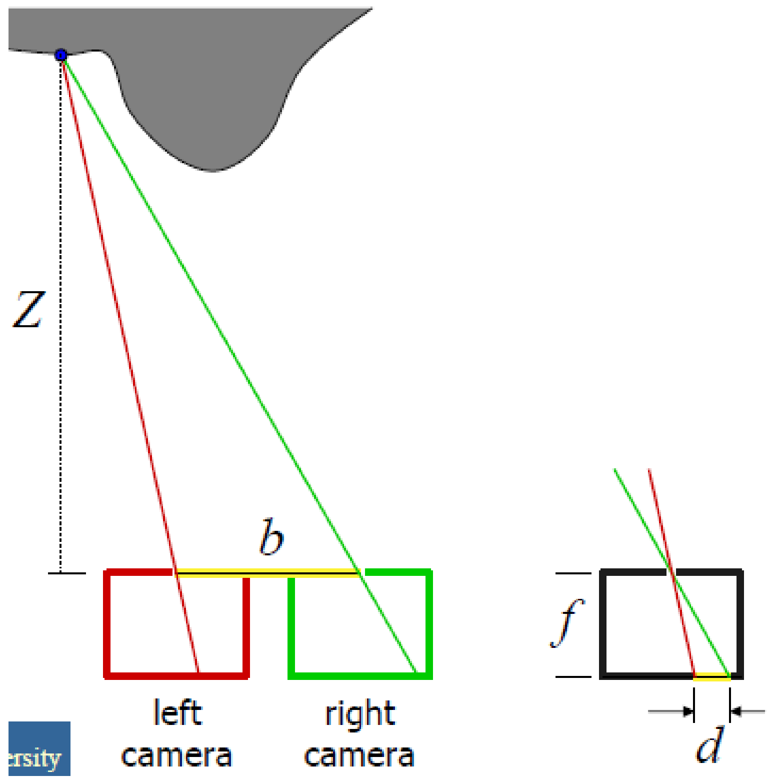


Binocular Stereo



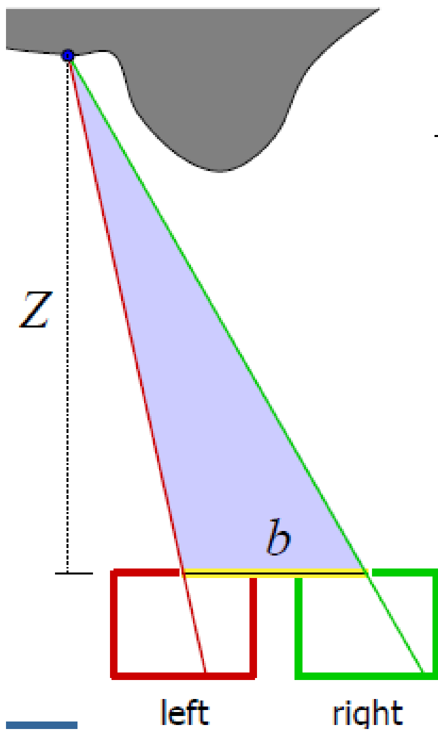
From known geometry of the cameras and estimated disparity, recover depth in the scene

Stereo Geometry



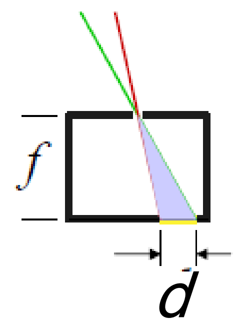
Disparity *d* = difference in image position

Stereo Geometry

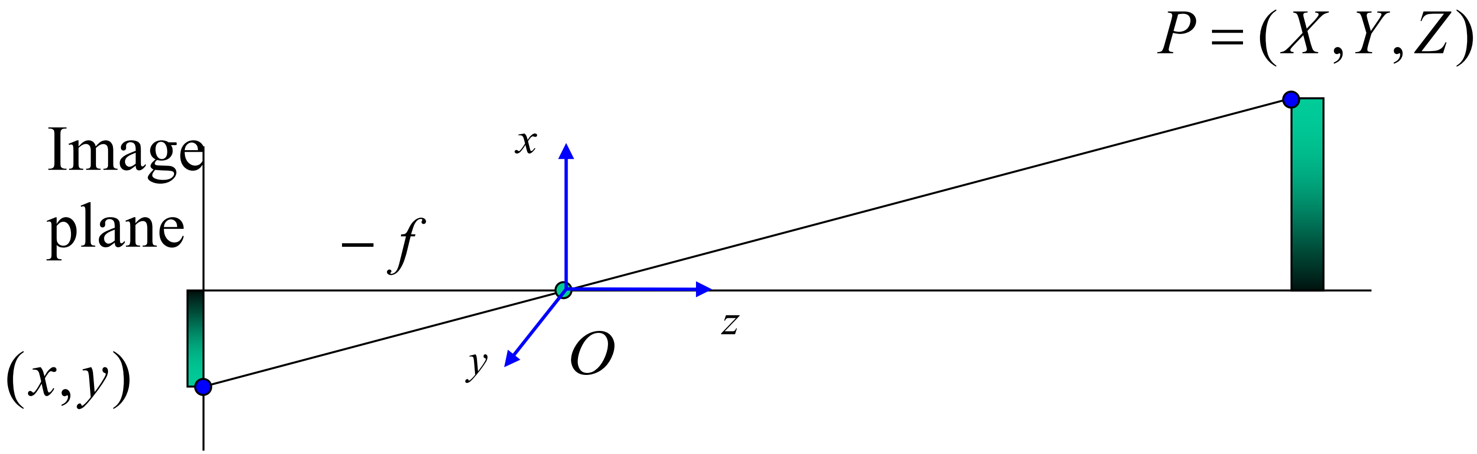


$$\frac{d}{f} = \frac{b}{Z}$$

Disparity $d = bf\,\frac{1}{Z}$

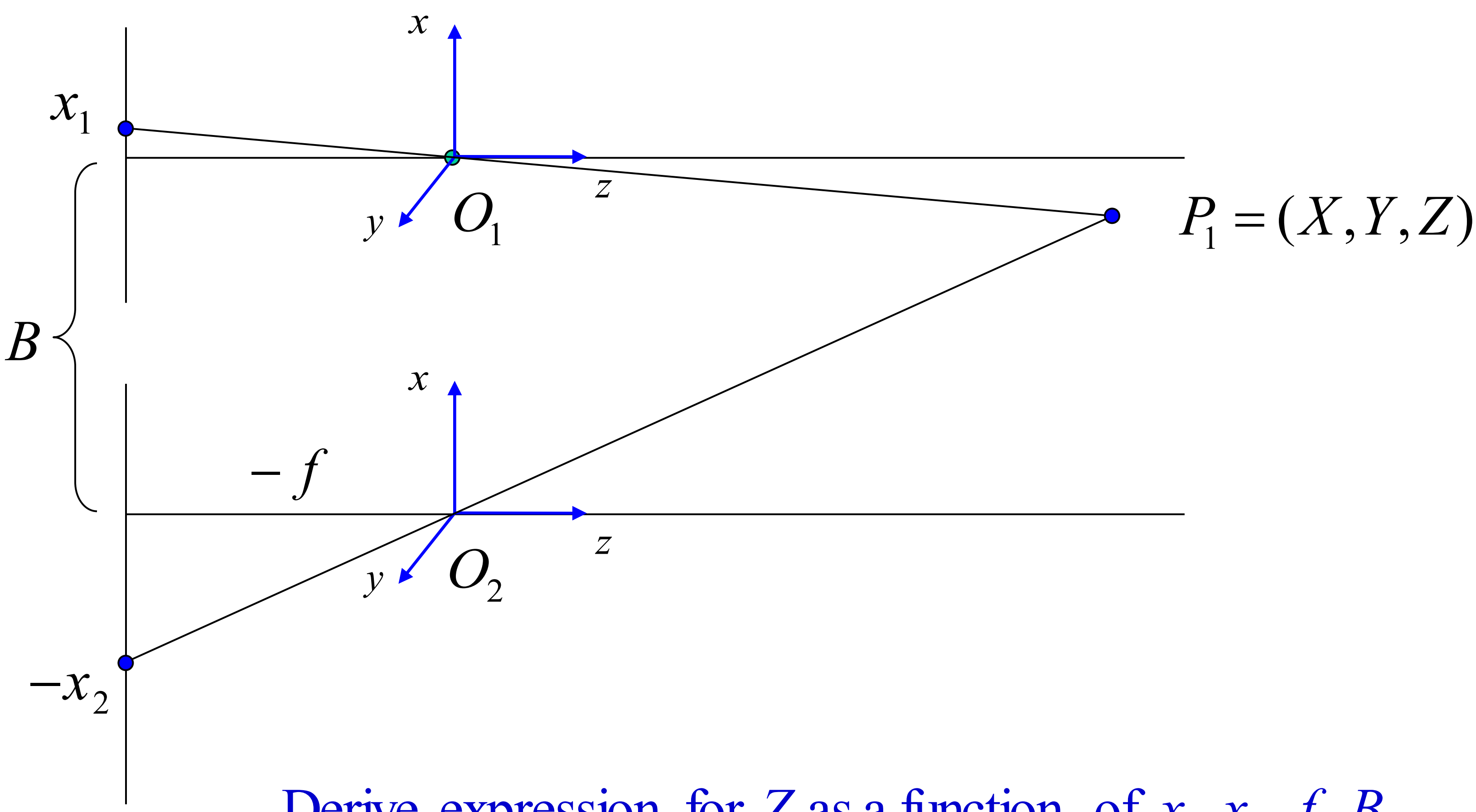


Pinhole Camera Model



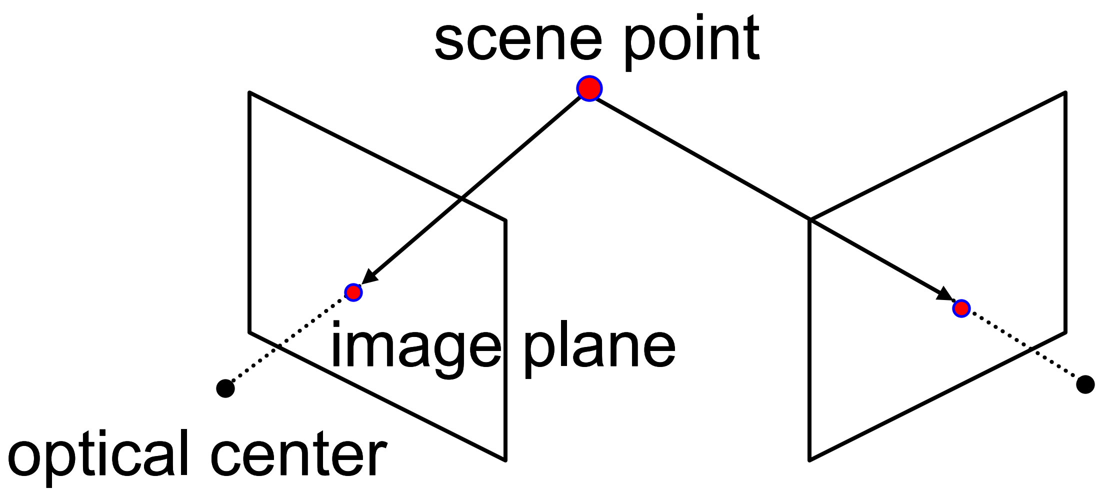
$$(x, y) = (f\frac{X}{Z}, f\frac{Y}{Z})$$

Basic Stereo Derivations

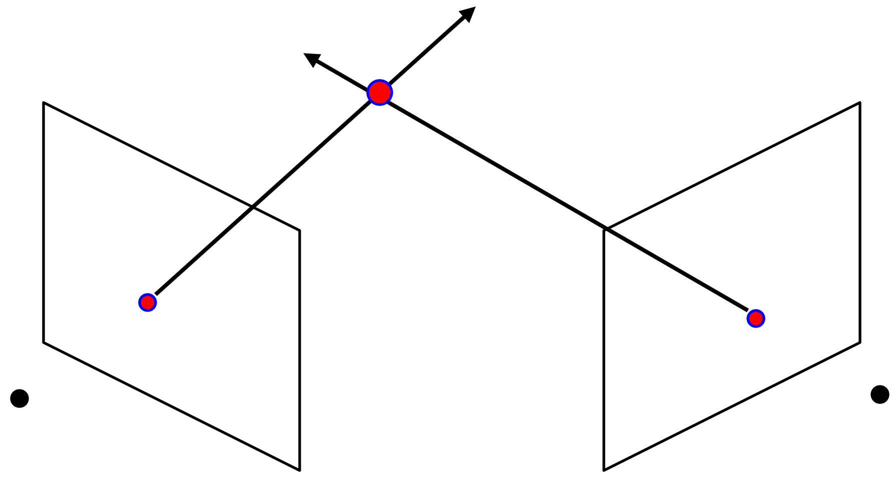


Derive expression for Z as a function of x_1, x_2, f, B

Binocular Stereo

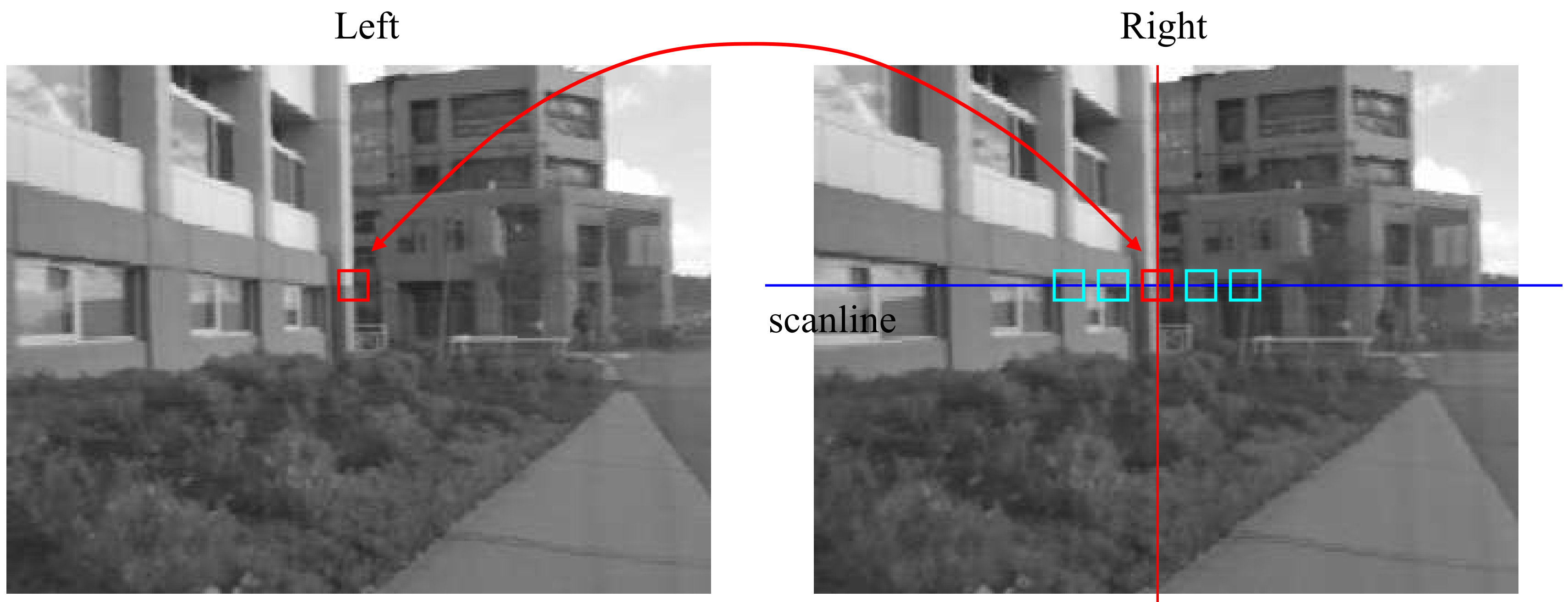


Binocular Stereo

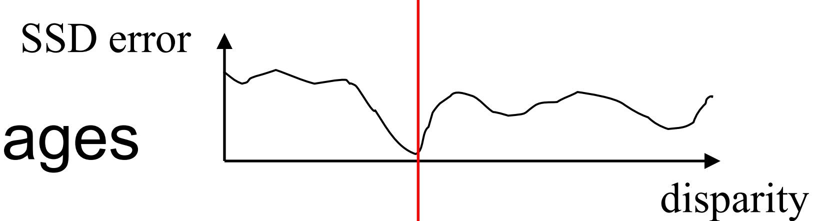


- Basic Principle: Triangulation
  - Gives reconstruction as intersection of two rays
- Requires:
  - Select features
  - Calibration
  - **Point correspondence**

Correspondence via Correlation

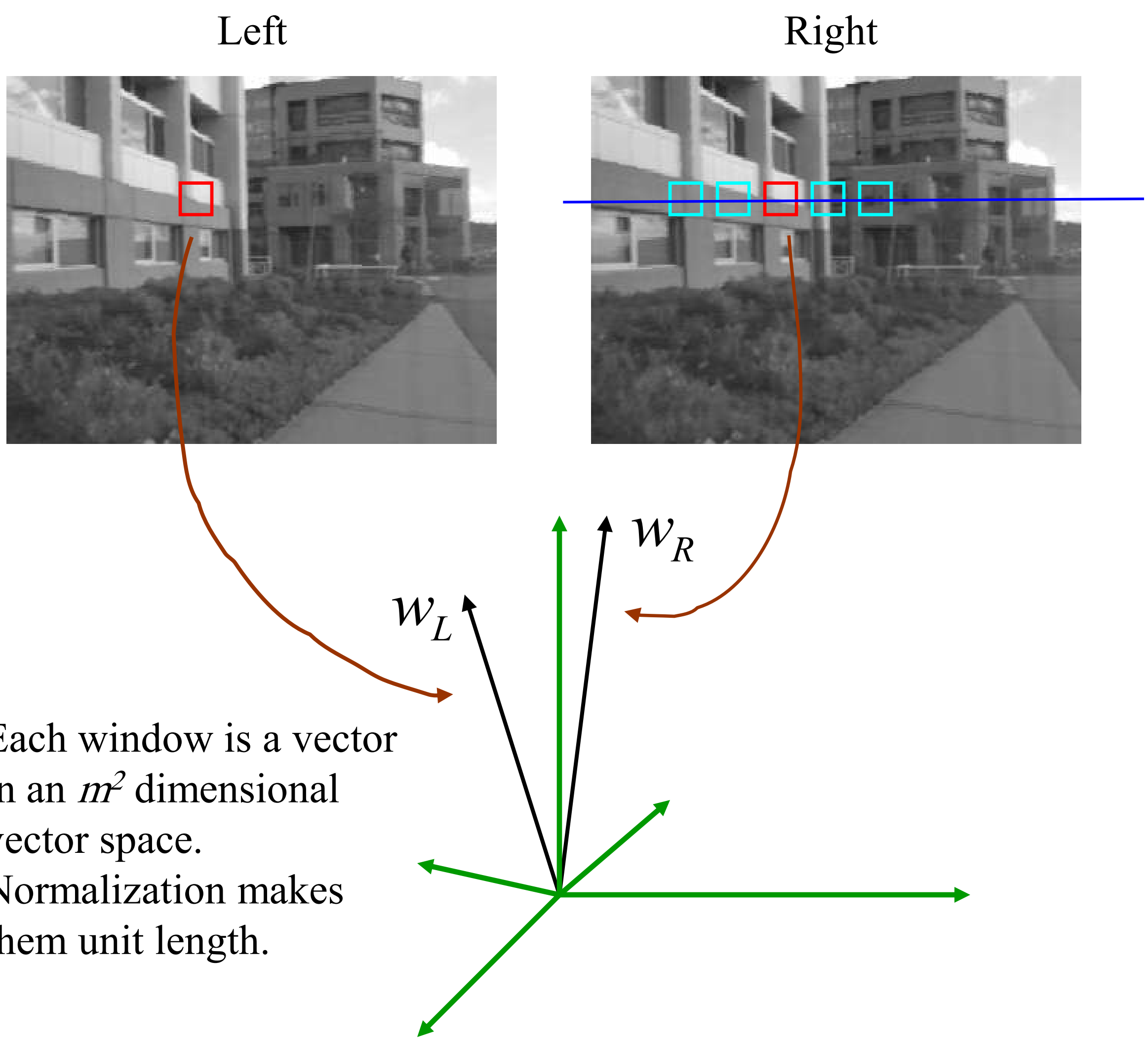


Rectified images



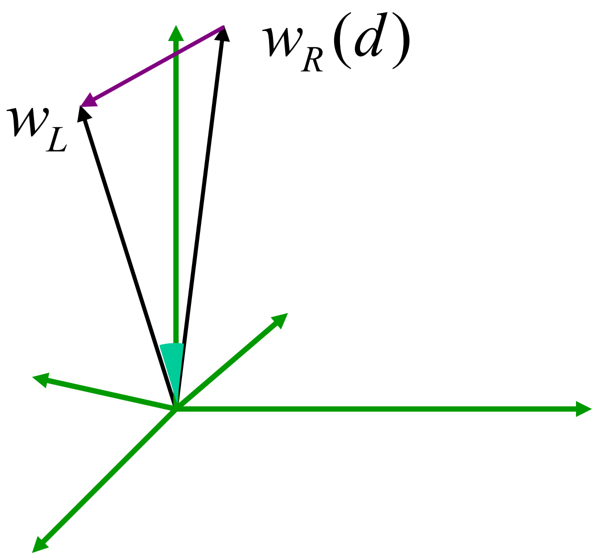
(Same as max-correlation / max-cosine for normalized image patch)

Images as Vectors



Each window is a vector in an m^2 dimensional vector space. Normalization makes them unit length.

Correspondence Metrics



(Normalized) Sum of Squared Differences

$$C_{\mathrm{SSD}}(d) = \sum_{(u,v)\in W_m(x,y)} [\hat{I}_L(u,v) - \hat{I}_R(u-d,v)]^2$$

$$= \left\| w_L - w_R(d) \right\|^2$$

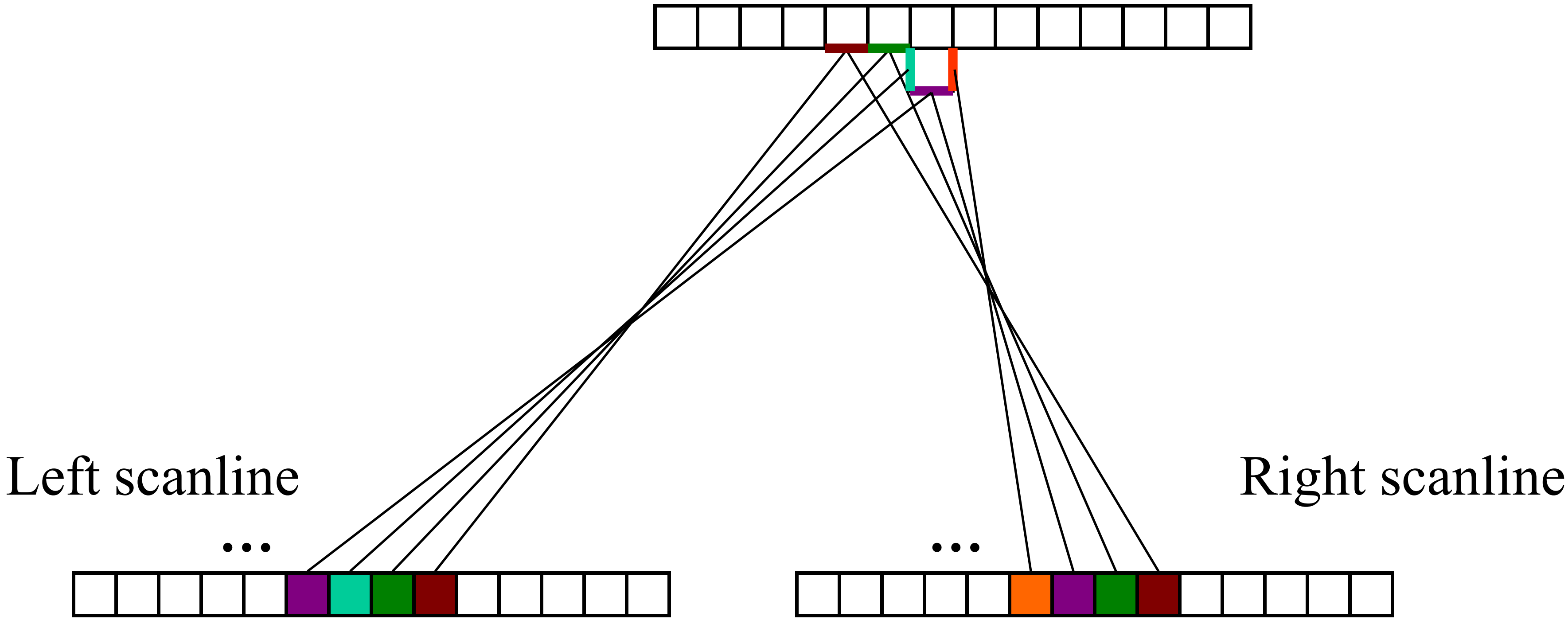
Normalized Correlation

$$C_{\mathrm{NC}}(d) = \sum_{(u,v)\in W_m(x,y)} \hat{I}_L(u,v)\hat{I}_R(u-d,v)$$

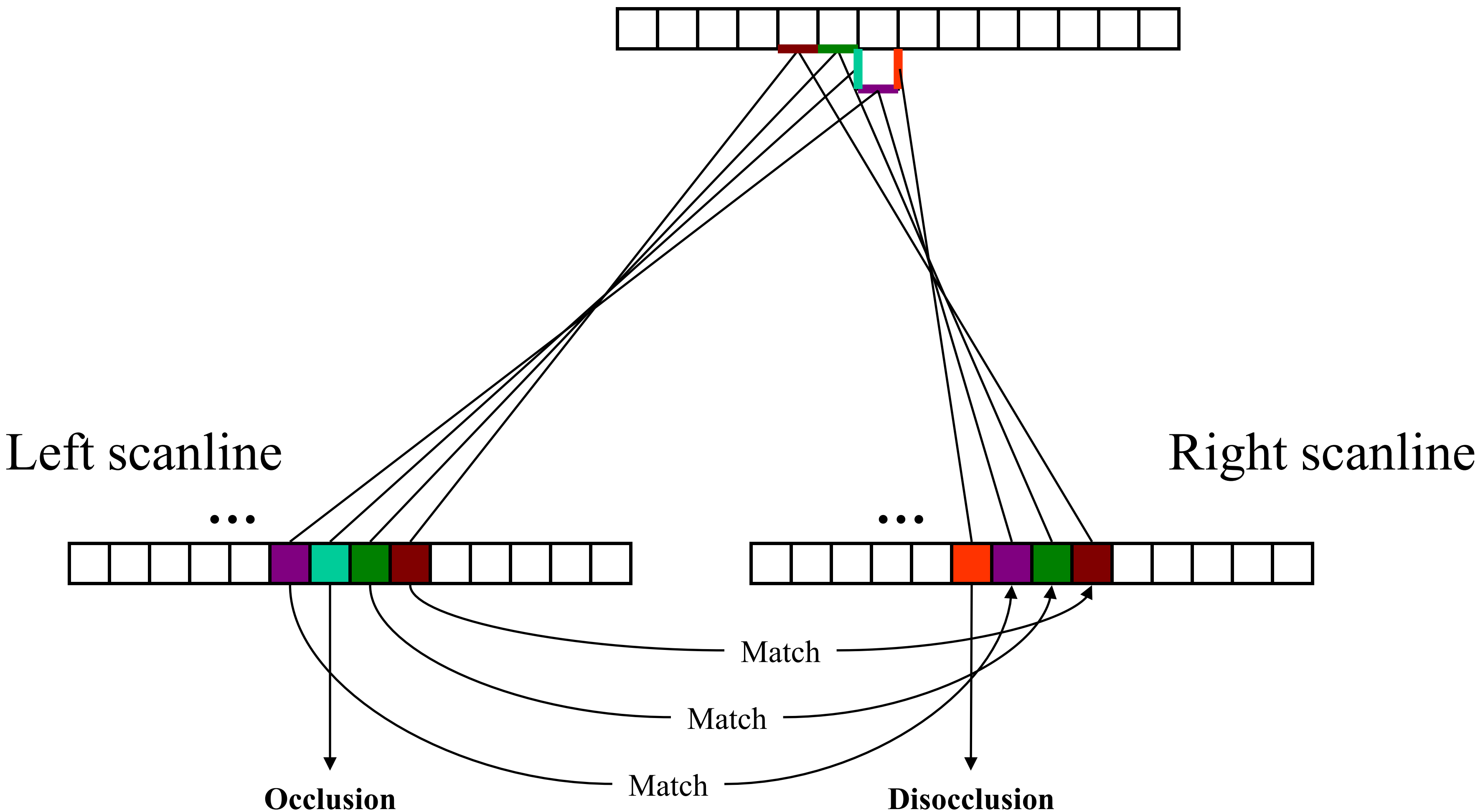
$$= w_L \cdot w_R(d) = \cos\theta$$

$$d^* = \arg\min_d \left\| w_L - w_R(d) \right\|^2 = \arg\max_d \, w_L \cdot w_R(d)$$

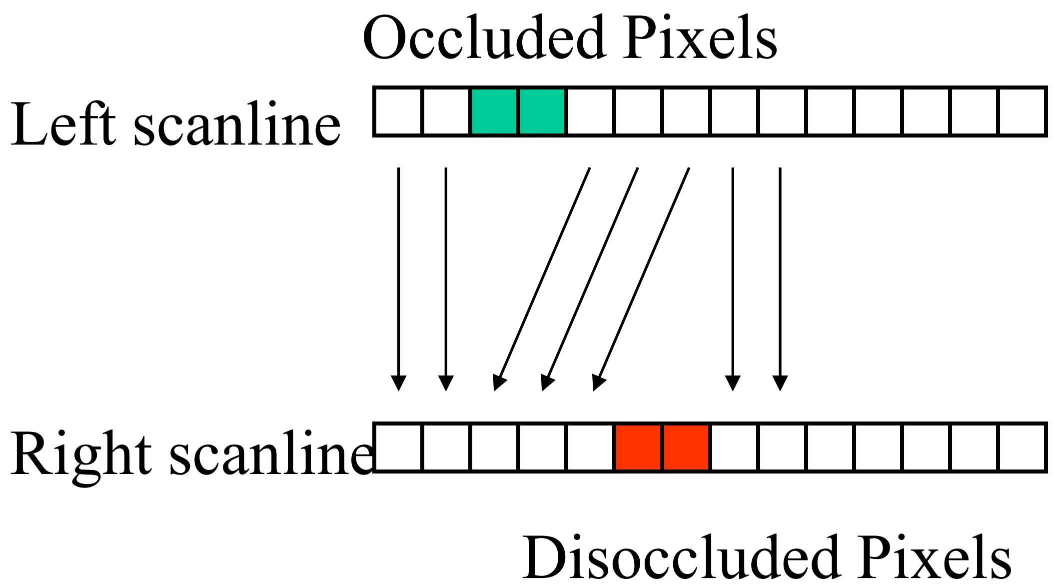
Stereo Correspondences



Stereo Correspondences



Search Over Correspondences



Three cases:

- Sequential – cost of match
- Occluded – cost of no match
- Disoccluded – cost of no match

Correspondence Problem

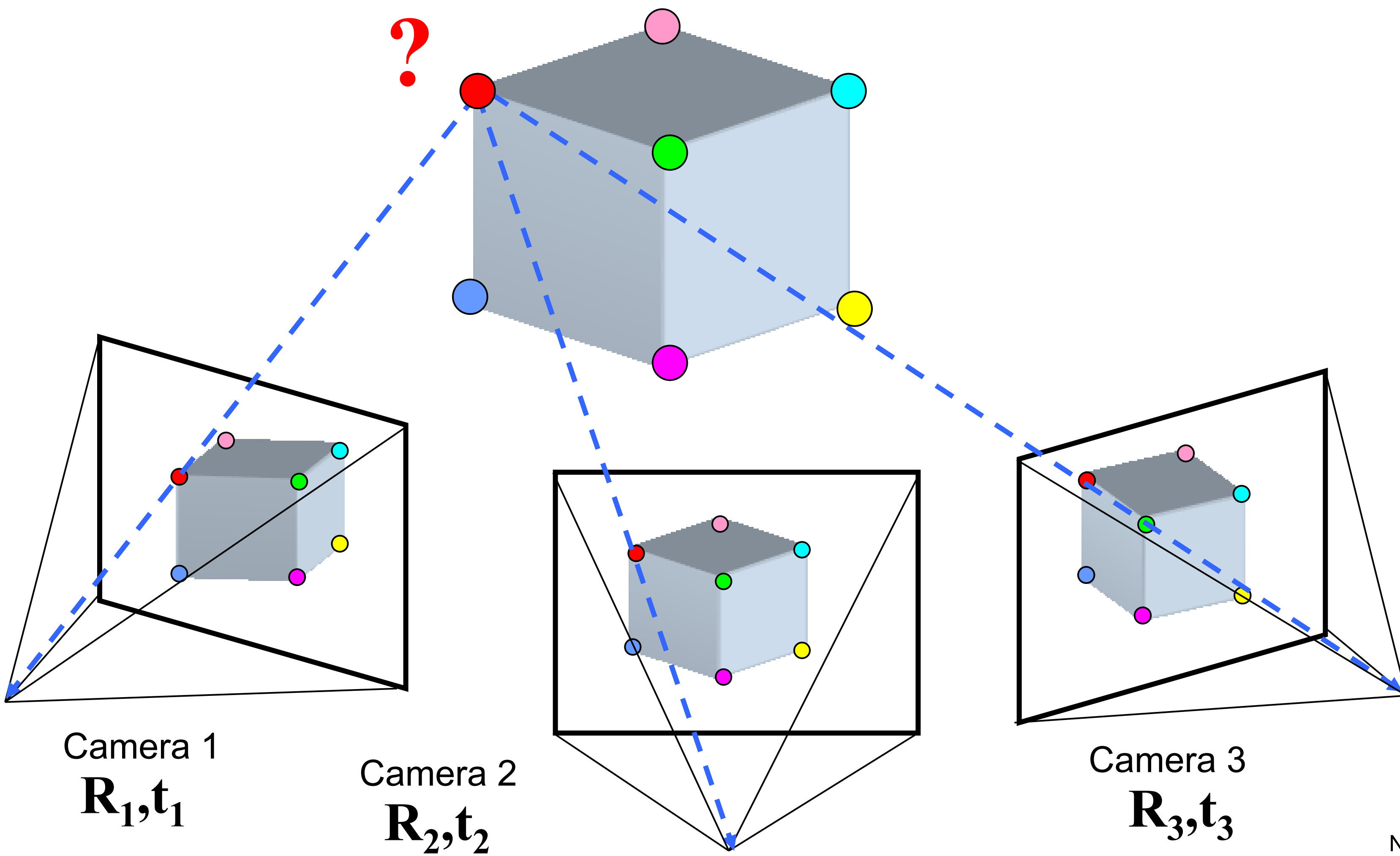
Regions without texture

Highly Specular surfaces

Translucent objects

Multi-view geometry problems

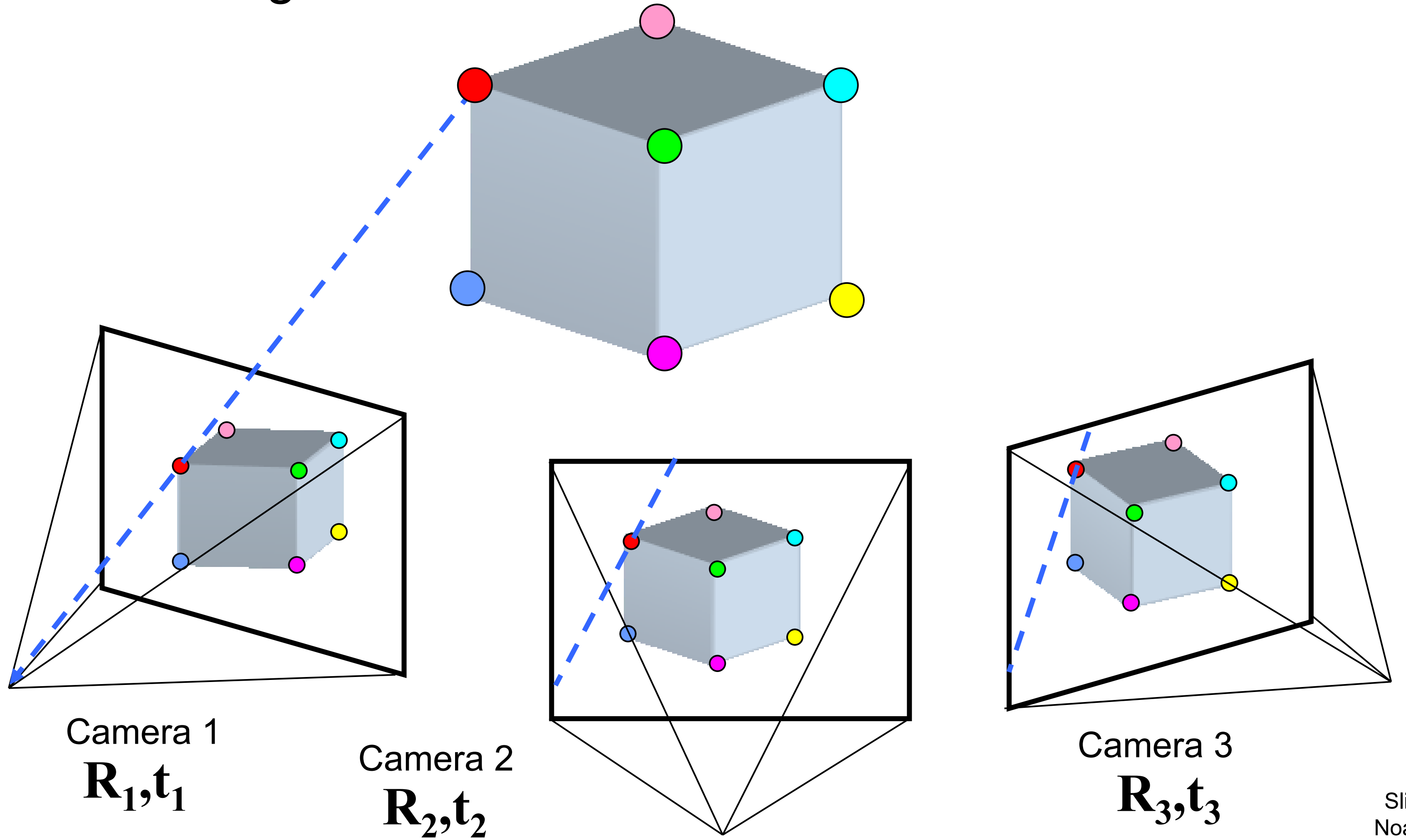
- **Structure:** Given projections of the same 3D point in two or more images, compute the 3D coordinates of that point



Slide credit: Noah Snavely

Multi-view geometry problems

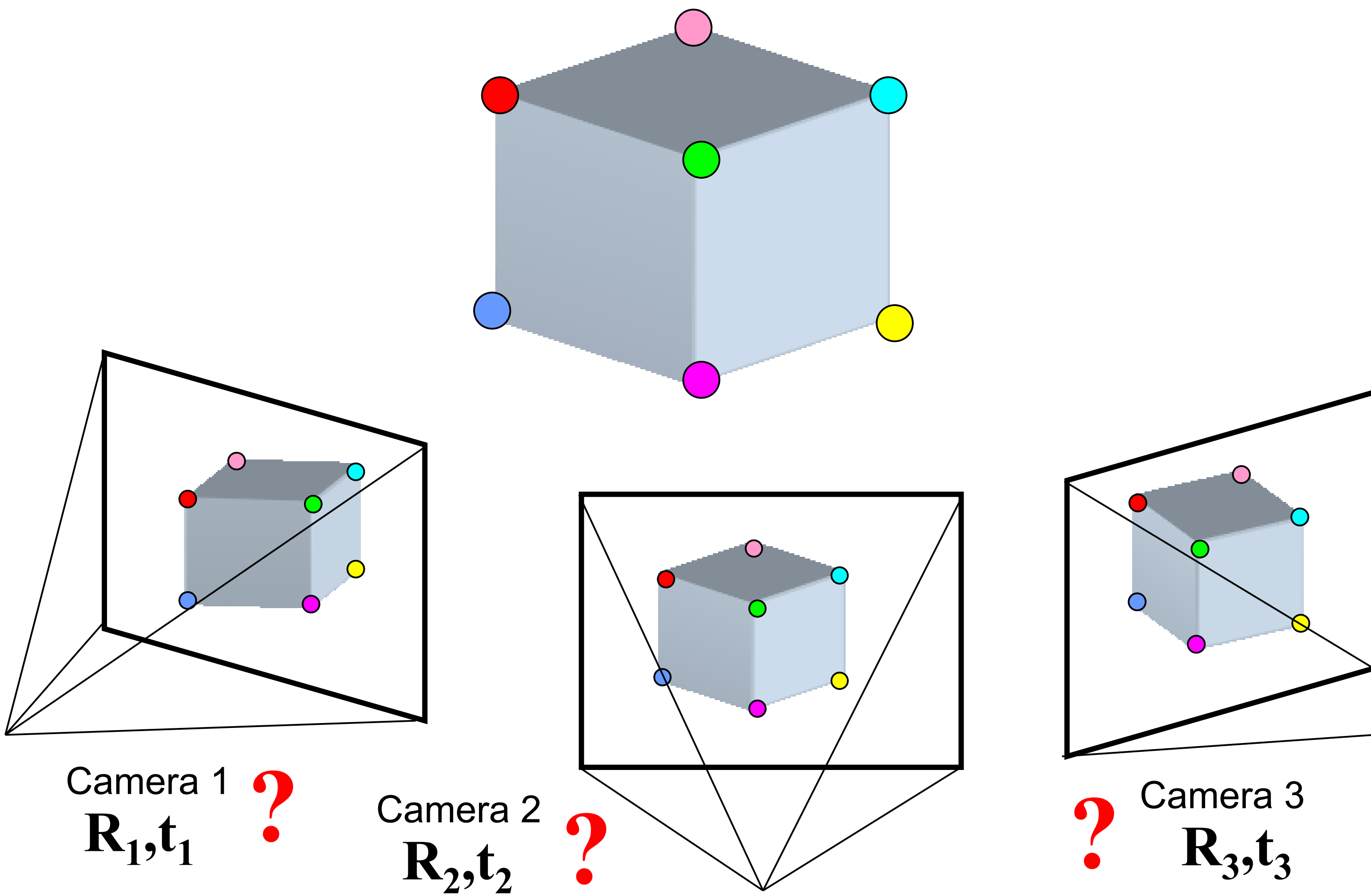
- **Stereo correspondence:** Given a point in one of the images, where could its corresponding points be in the other images?



Slide credit: Noah Snavely

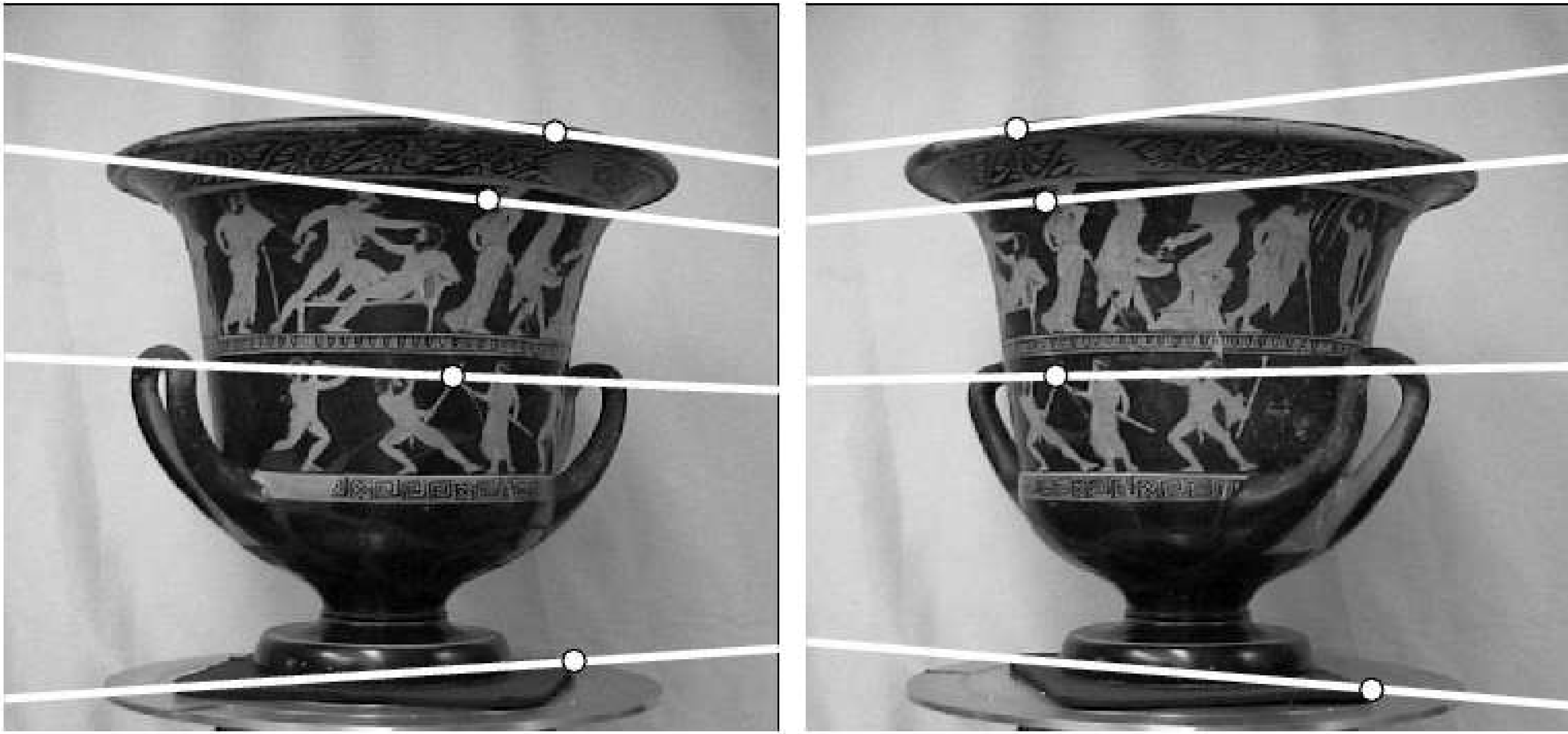
Multi-view geometry problems

- **Motion:** Given a set of corresponding points in two or more images, compute the camera parameters



Slide credit: Noah Snavely

Two-view geometry



Outline

Basic Equations

Correspondence

Epipolar Geometry

Image Rectification

Epipolar Geometry

Epipolar plane: plane going through point P and the Centers Of Projection (COPs) of the two cameras

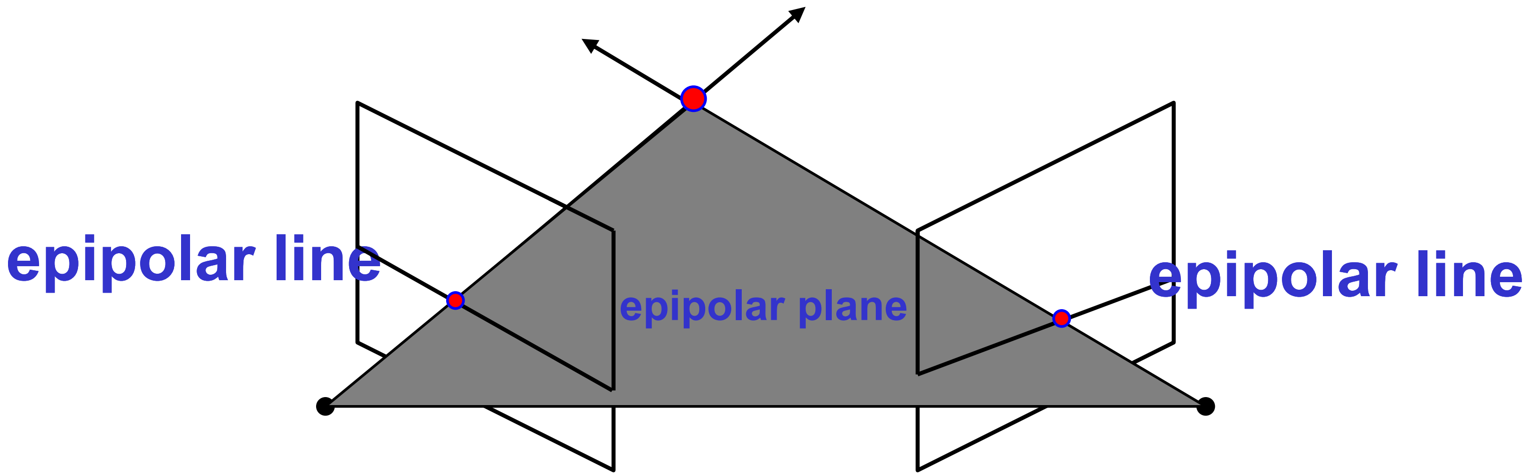
Epipoles: The image in one camera of the COP of the other.

Epipolar Constraint: Corresponding points must lie on epipolar lines.

Stereo Correspondence: Epipolar Geometry

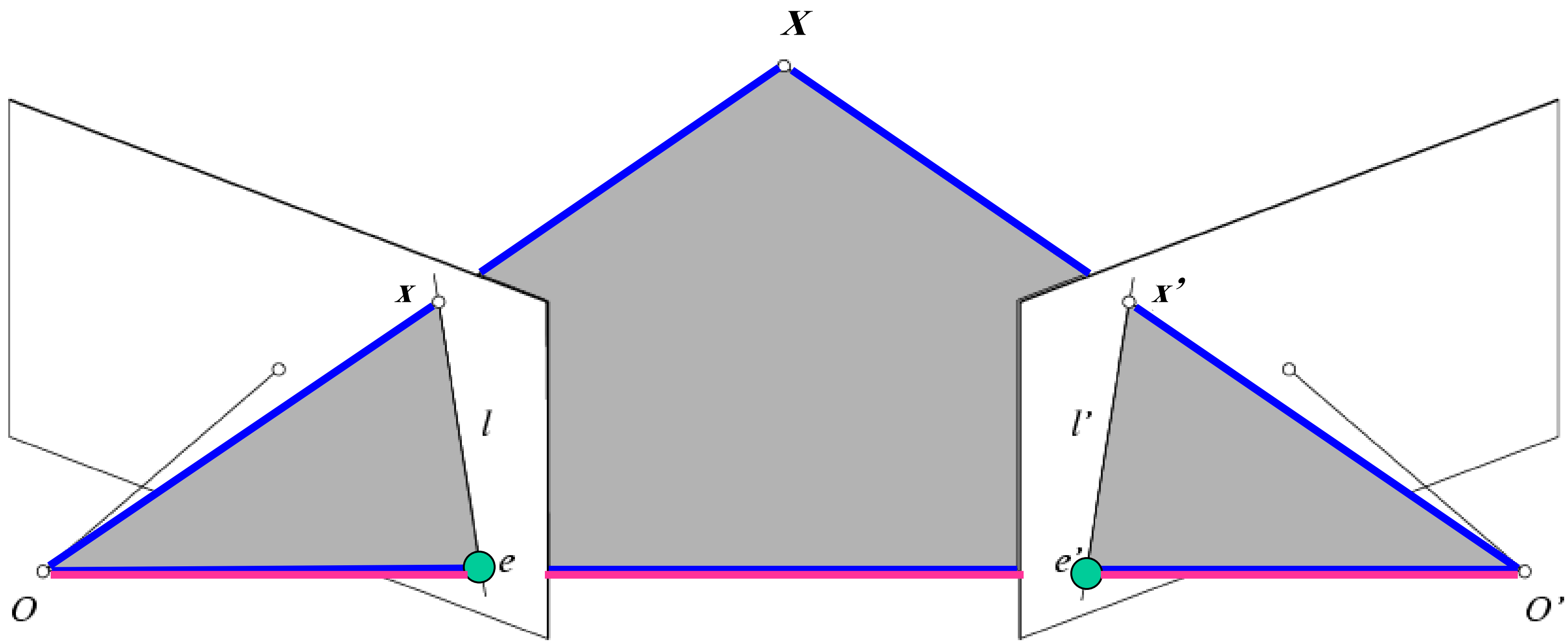
Determine Pixel Correspondence

- Pairs of points that correspond to same scene point



- Epipolar Constraint
  - Reduces correspondence problem to 1D search along *conjugate epipolar lines*

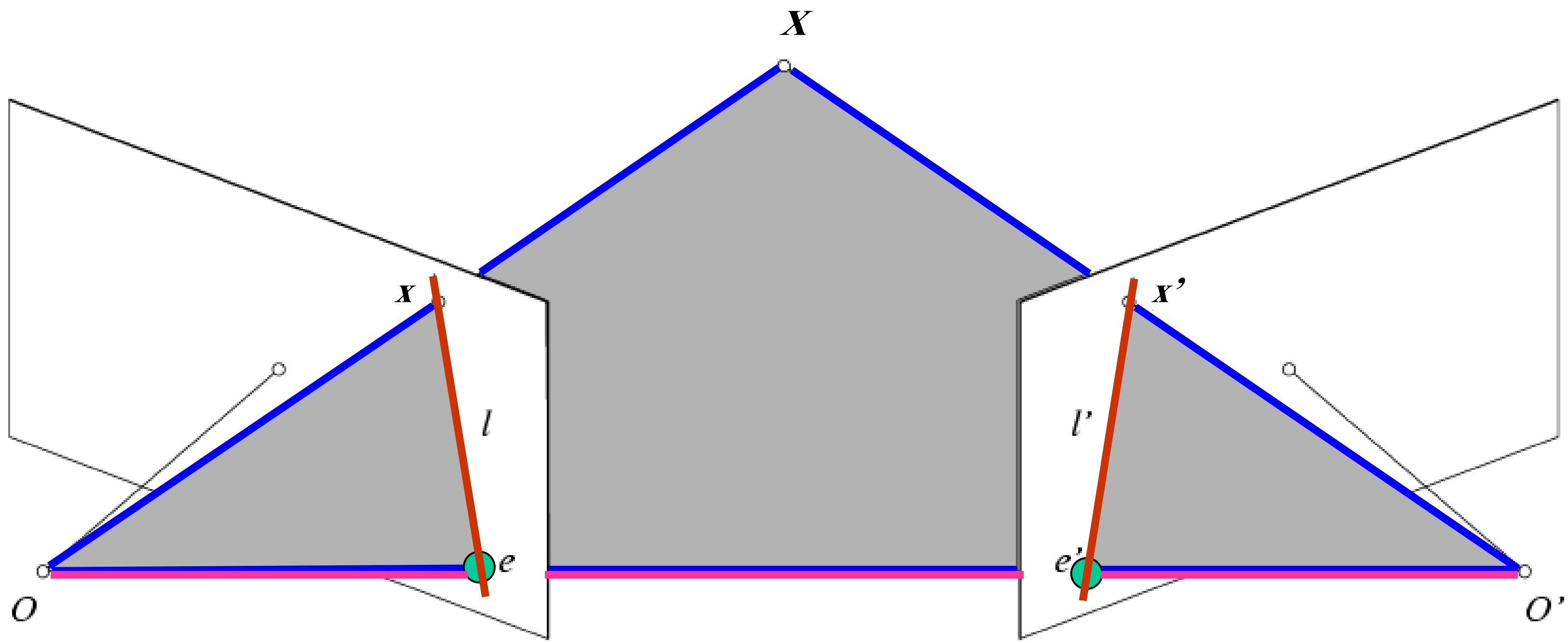
Epipolar geometry



- **Baseline** – line connecting the two camera centers
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipoles**

= intersections of baseline with image planes
= projections of the other camera center
= vanishing points of the motion direction

Epipolar geometry



- **Baseline** – line connecting the two camera centers
- **Epipolar Plane** – plane containing baseline (1D family)
- **Epipoles**

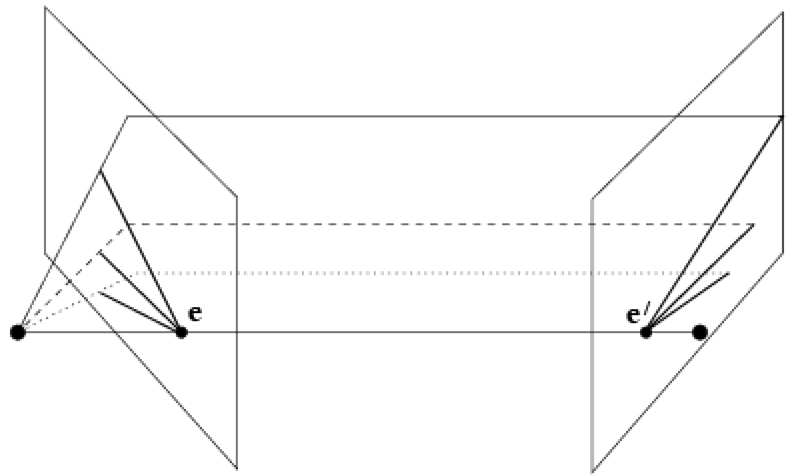
= intersections of baseline with image planes

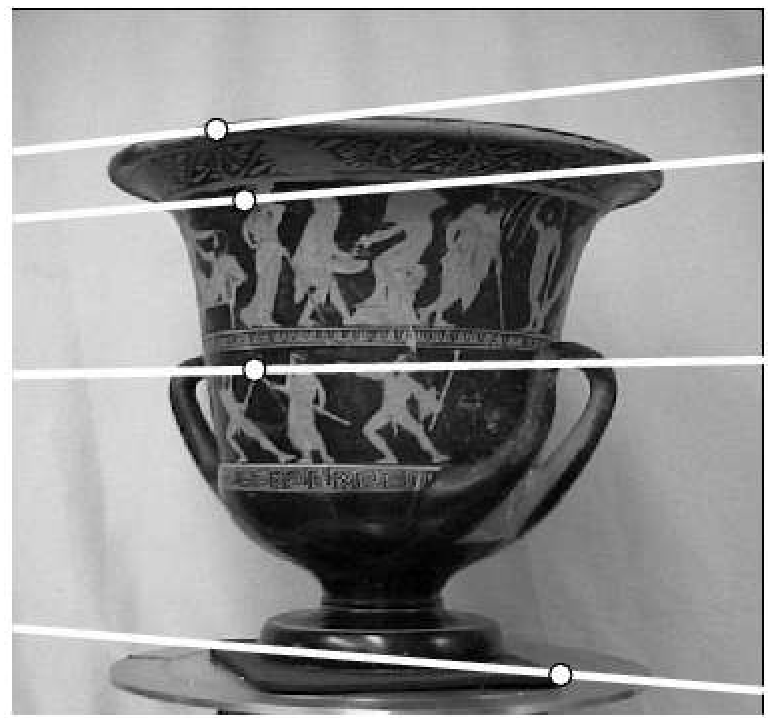
= projections of the other camera center

= vanishing points of the motion direction

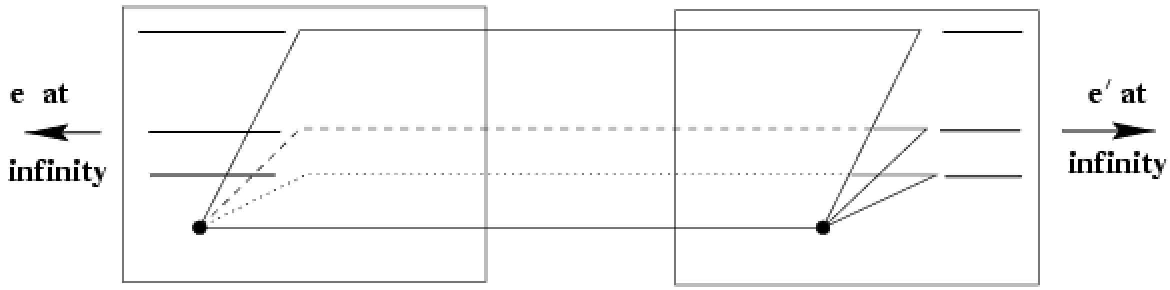
- **Epipolar Lines** - intersections of epipolar plane with image planes (always come in corresponding pairs)

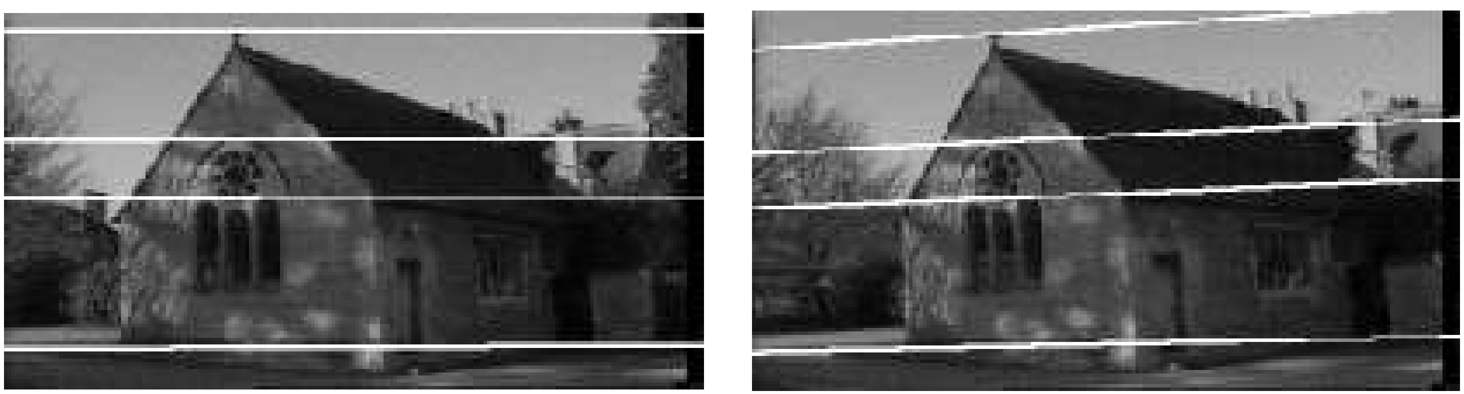
Example: Converging cameras



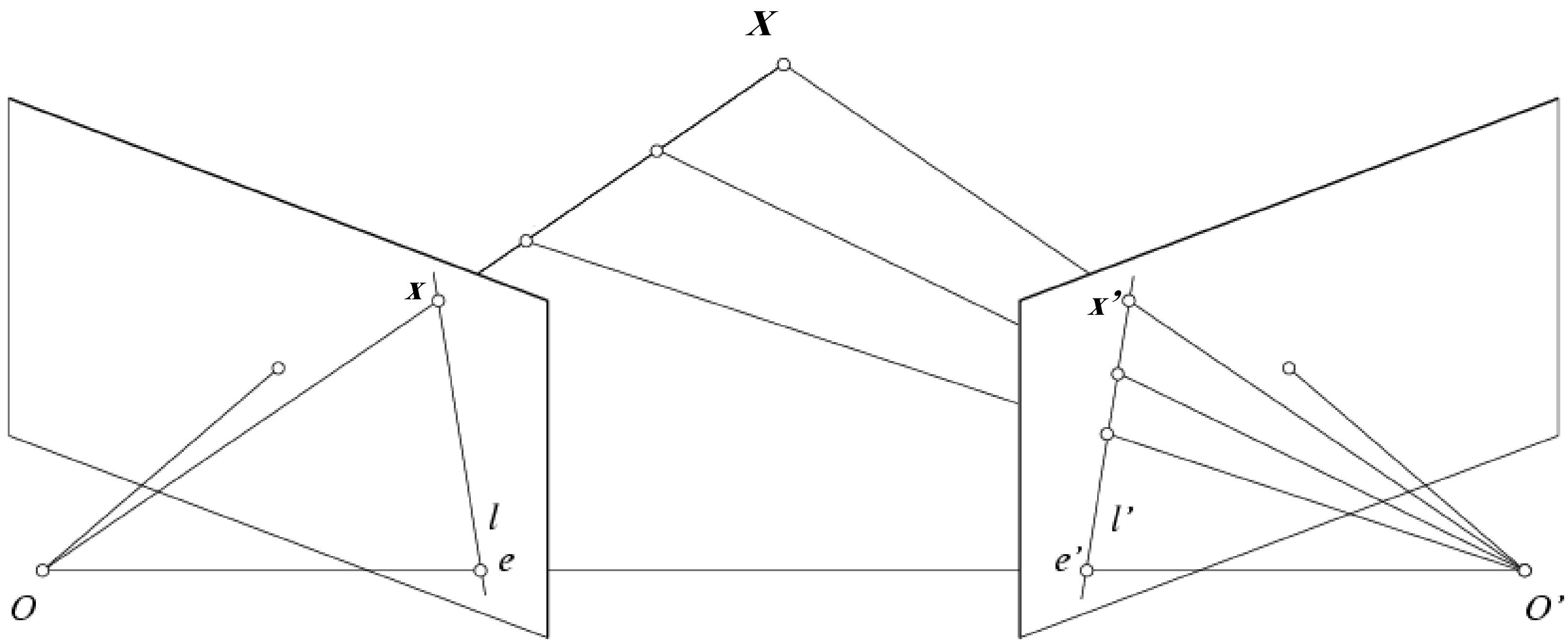


Example: Motion parallel to image plane



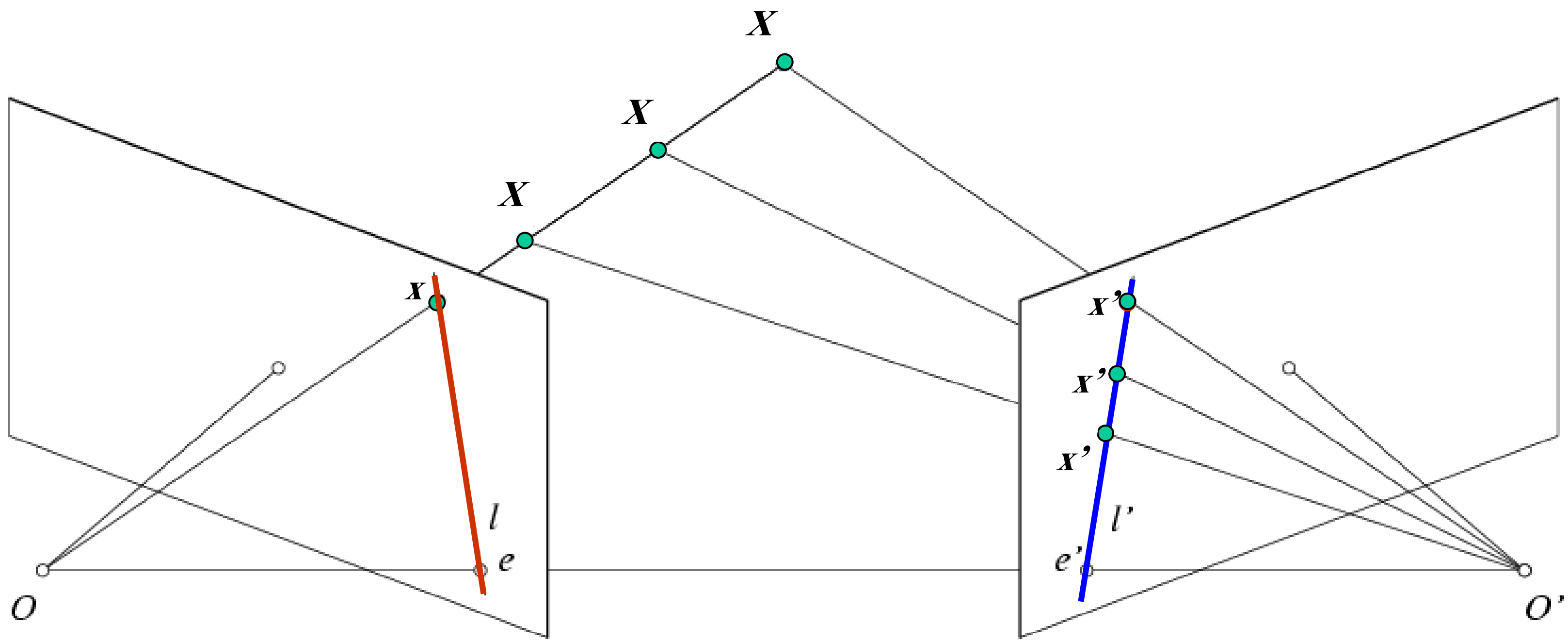


Epipolar constraint



- If we observe a point $\boldsymbol{x}$ in one image, where can the corresponding point $\boldsymbol{x'}$ be in the other image?

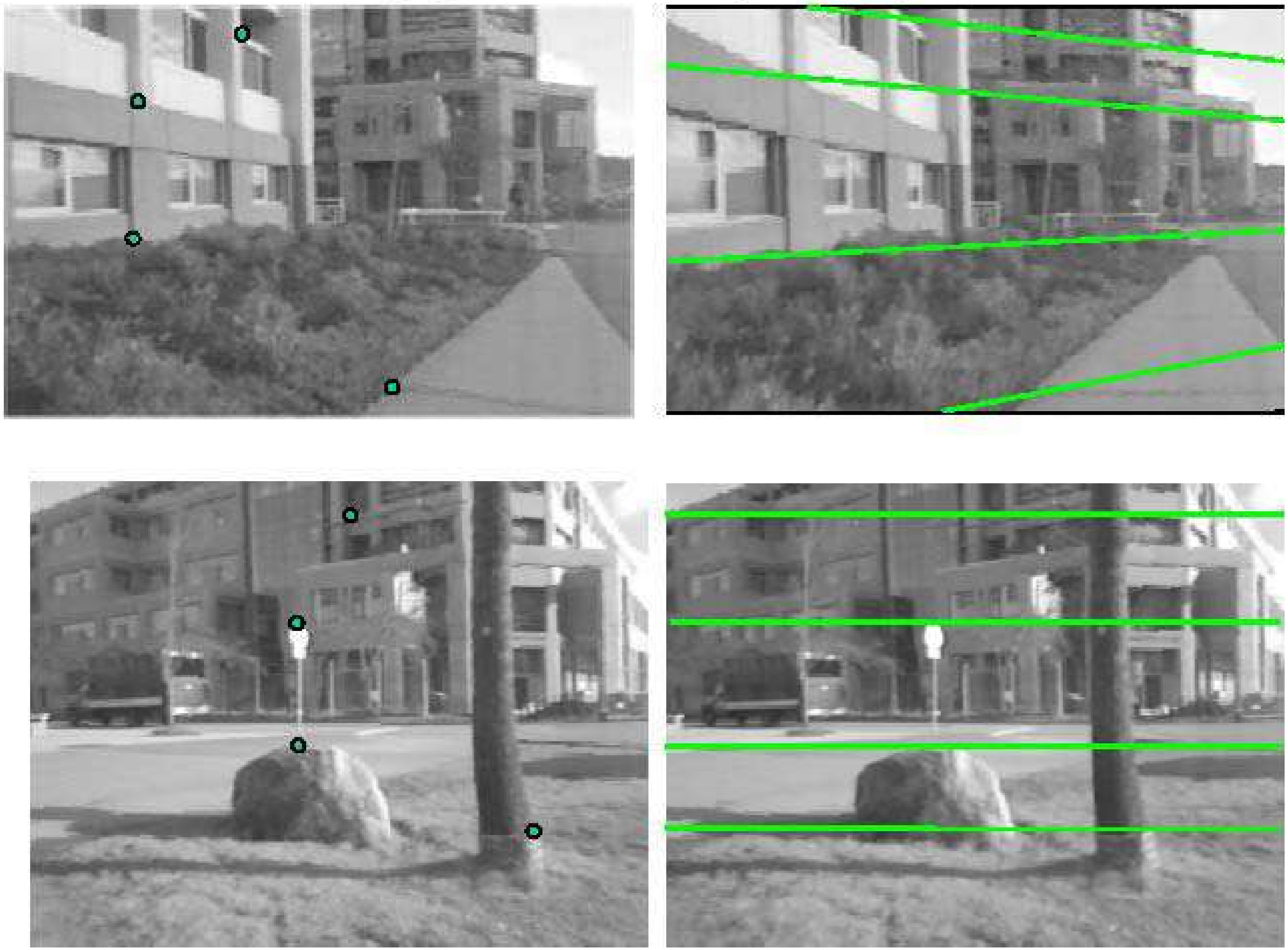
Epipolar constraint

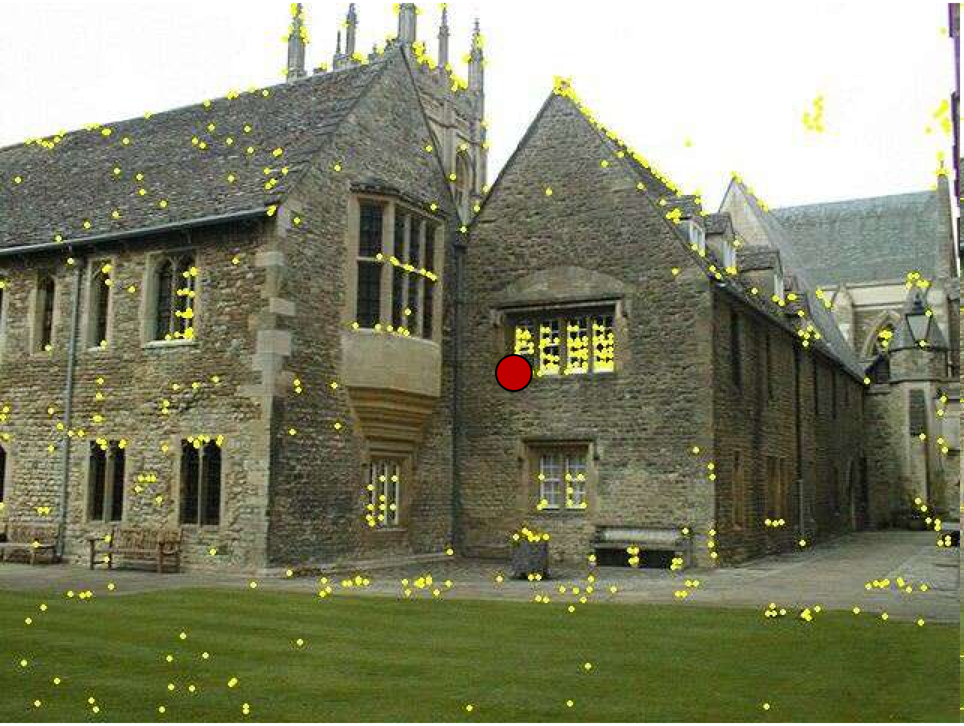


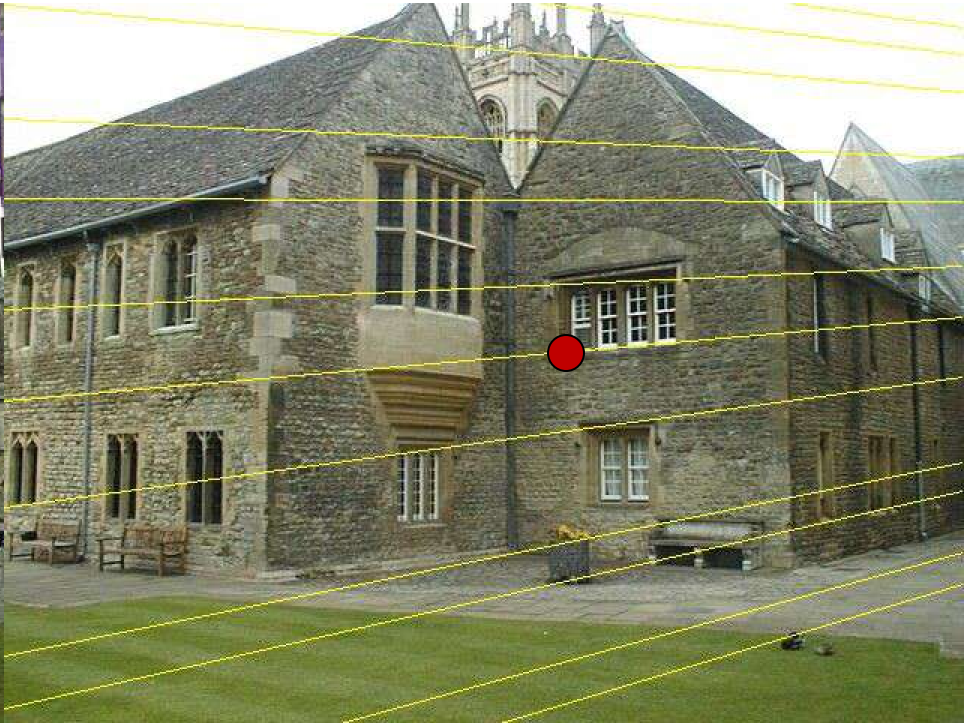
Potential matches for ***x*** have to lie on the corresponding epipolar line ***l'***.

Potential matches for ***x'*** have to lie on the corresponding epipolar line ***l***.

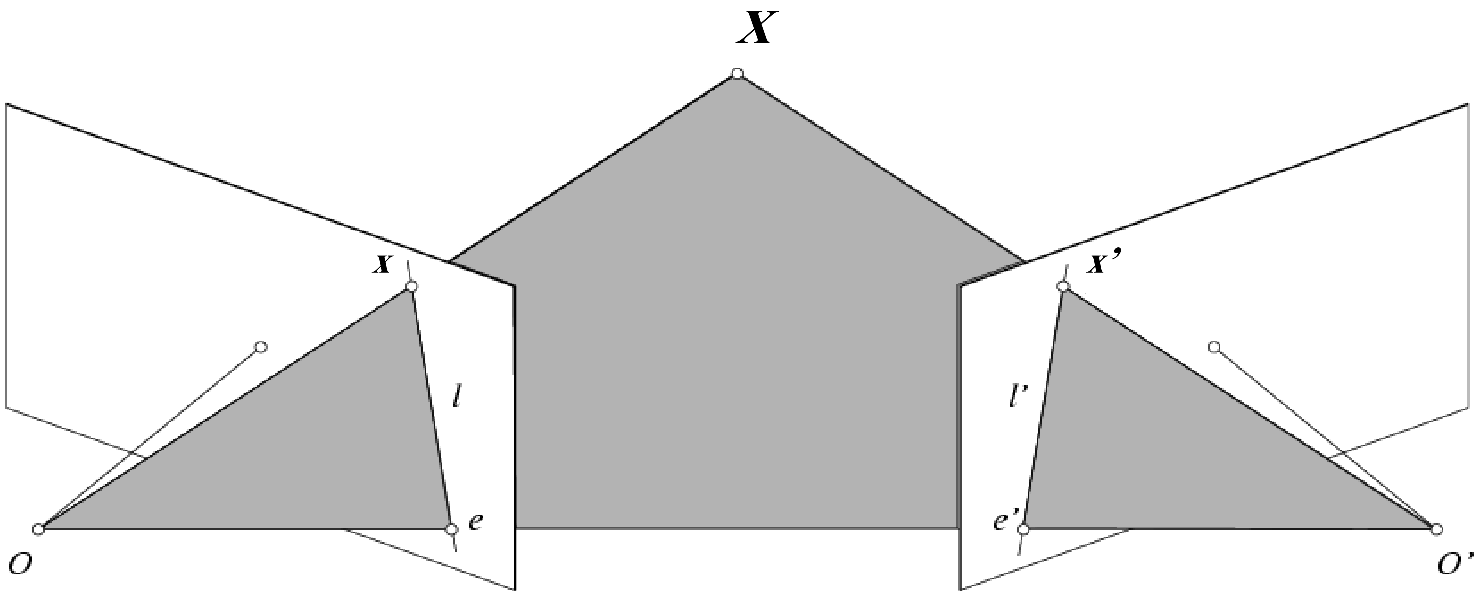
Epipolar constraint example







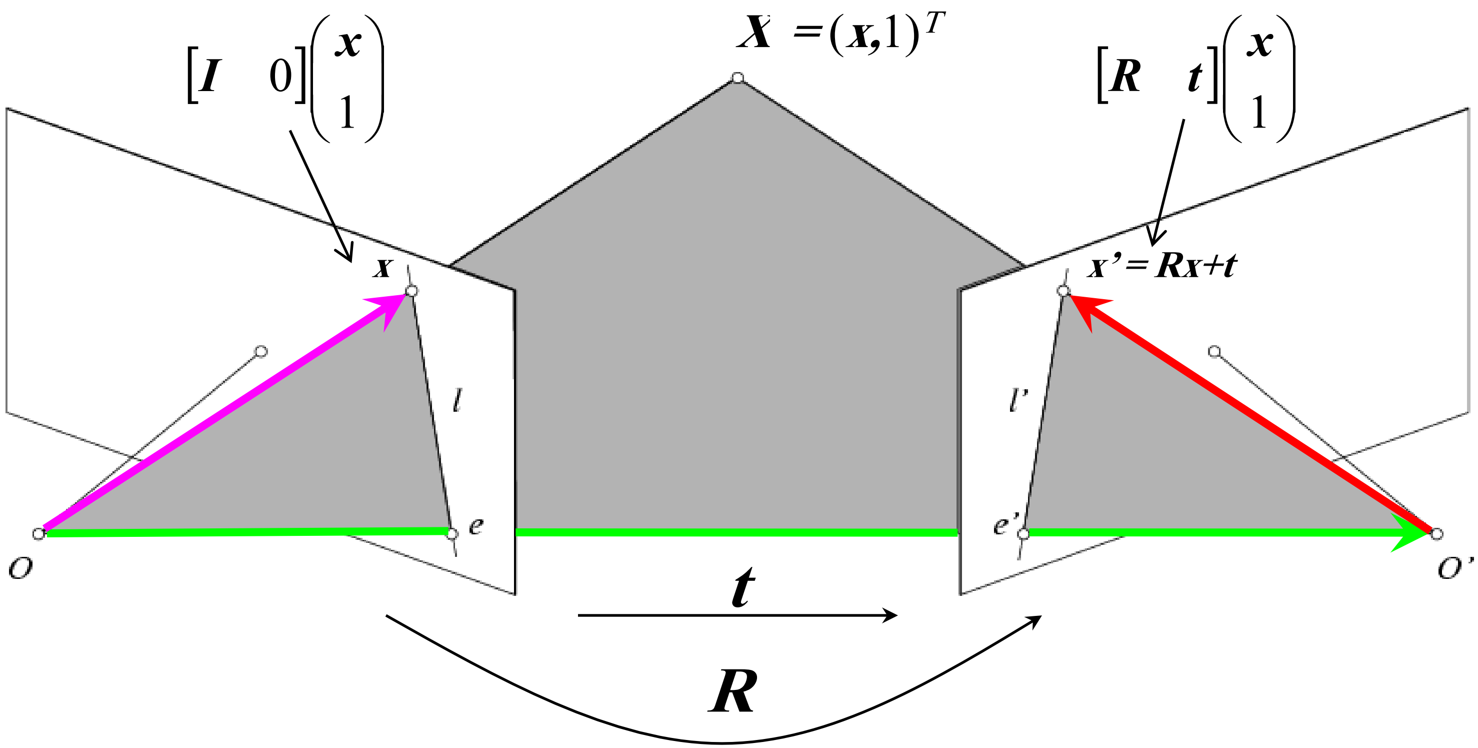
Epipolar constraint: Calibrated case



- Intrinsic and extrinsic parameters of the cameras are known, world coordinate system is set to that of the first camera
- Then the projection matrices are given by $\boldsymbol{K}[\boldsymbol{I} \mid \mathbf{0}]$ and $\boldsymbol{K}'[\boldsymbol{R} \mid \boldsymbol{t}]$
- We can multiply the projection matrices (and the image points) by the inverse of the calibration matrices to get *normalized* image coordinates:

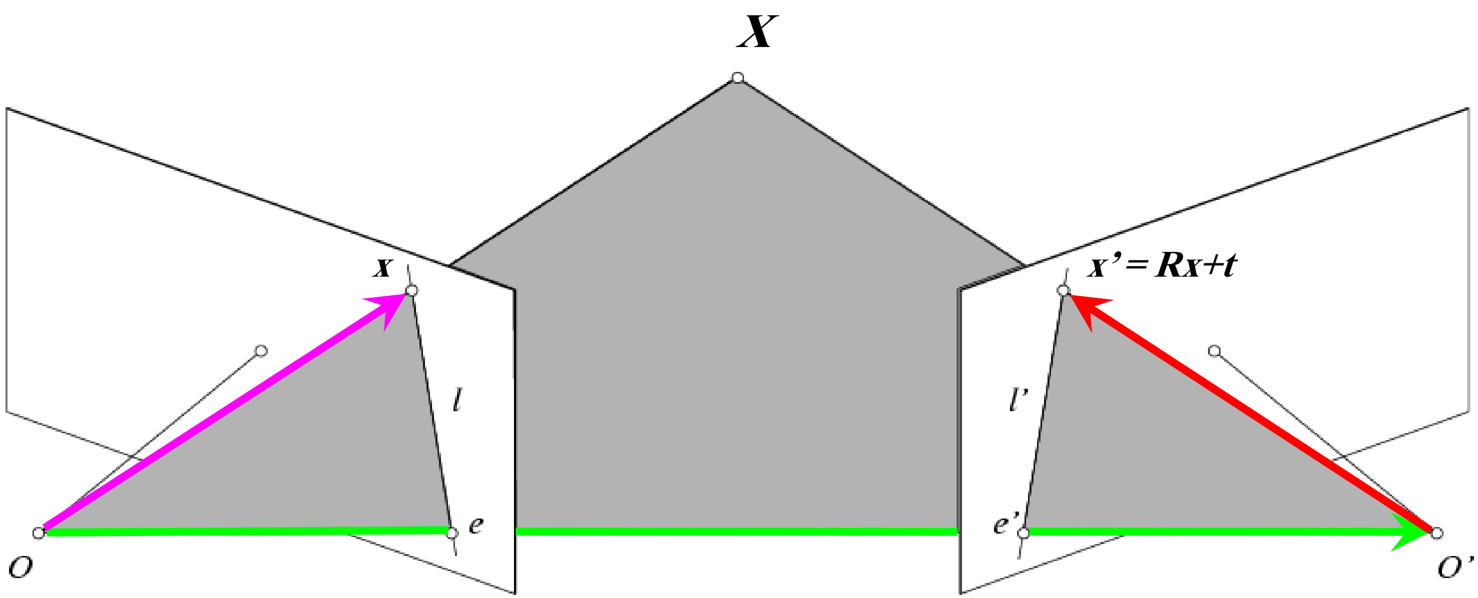
$$\boldsymbol{x}_{\text{norm}} = \boldsymbol{K}^{-1}\boldsymbol{x}_{\text{pixel}} = [\boldsymbol{I}\ \ 0]\boldsymbol{X}, \qquad \boldsymbol{x}'_{\text{norm}} = \boldsymbol{K}'^{-1}\boldsymbol{x}'_{\text{pixel}} = [\boldsymbol{R}\ \ \boldsymbol{t}]\boldsymbol{X}$$

Epipolar constraint: Calibrated case



The vectors ***Rx***, ***t***, and ***x'*** are coplanar

Epipolar constraint: Calibrated case



$$\boldsymbol{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\boldsymbol{x})] = 0 \quad \Rightarrow \quad \boldsymbol{x}'^{T}[\boldsymbol{t}_{\mathrm{x}}]\boldsymbol{R}\boldsymbol{x} = 0$$

Recall: $\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_x]\mathbf{b}$

The vectors ***Rx***, ***t***, and ***x'*** are coplanar

Cross product (Reminder)

- The cross product is one way of taking the product of two vectors. This method yields a third vector perpendicular to both. $\mathbf{a}\times\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\sin(\theta)\,\mathbf{n}$

  where **n** is the unit vector perpendicular to both **a** and **b.**

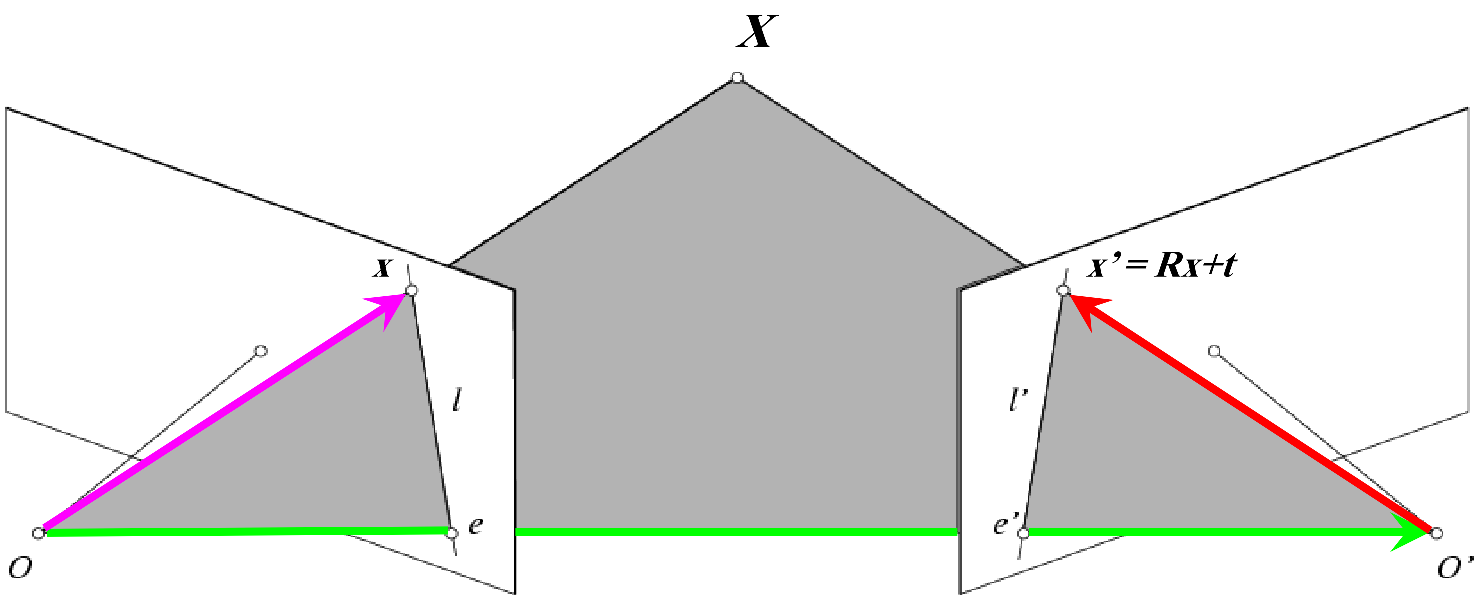
  a x **a** = 0

- It can be computed other ways as well:

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\mathbf{i} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\mathbf{j} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\mathbf{k}$$

$$= (a_y b_z - a_z b_y)\mathbf{i} + (a_z b_x - a_x b_z)\mathbf{j} + (a_x b_y - a_y b_x)\mathbf{k}$$

Epipolar constraint: Calibrated case

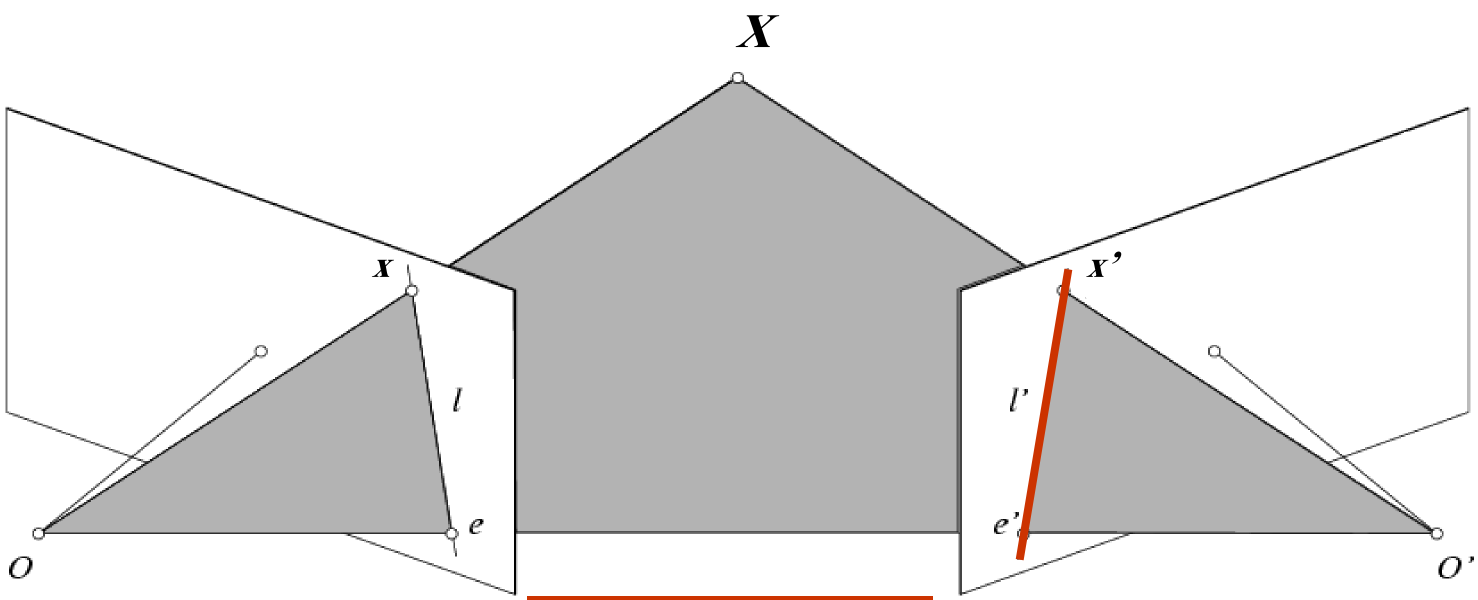


$$\boldsymbol{x}' \cdot [\boldsymbol{t} \times (\boldsymbol{R}\boldsymbol{x})] = 0 \quad \Rightarrow \quad \boldsymbol{x}'^T [\boldsymbol{t}_\times] \boldsymbol{R}\boldsymbol{x} = 0 \quad \Rightarrow \quad \boldsymbol{x}'^T E \boldsymbol{x} = 0$$

Essential Matrix
(Longuet-Higgins, 1981)

The vectors $\boldsymbol{Rx}$, $\boldsymbol{t}$, and $\boldsymbol{x'}$ are coplanar

Epipolar constraint: Calibrated case

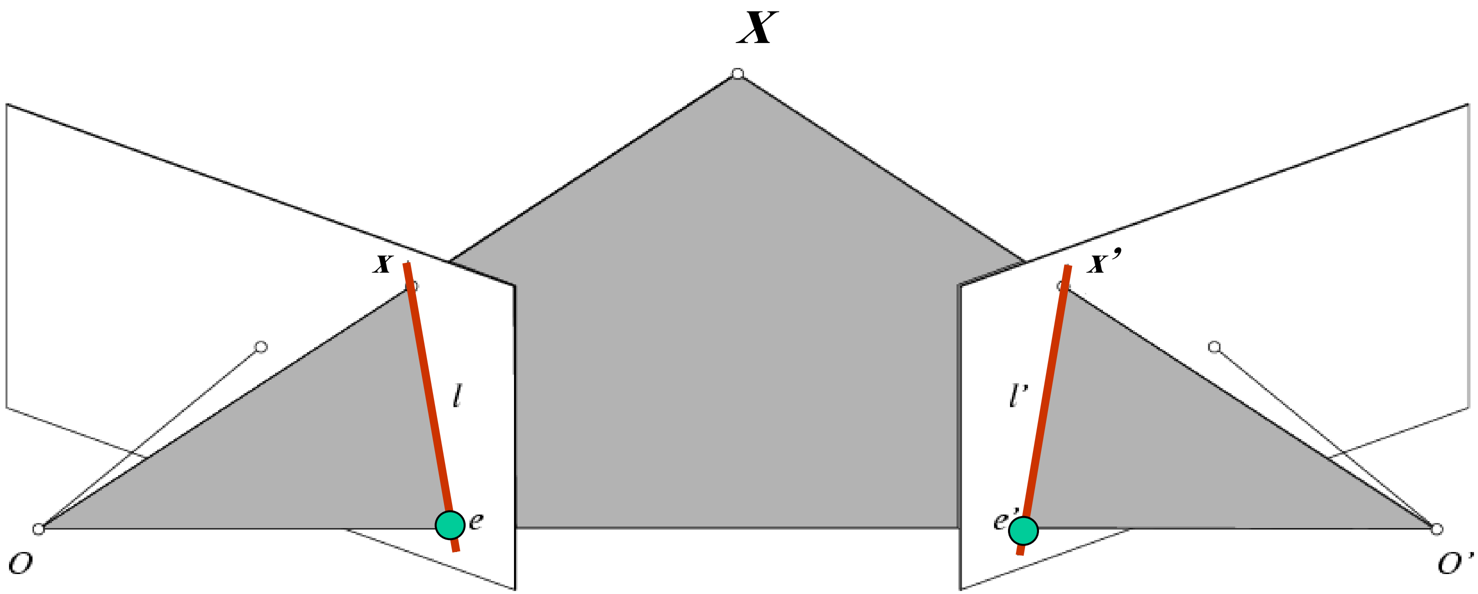


$$\boxed{\boldsymbol{x}'^{T} E \boldsymbol{x} = 0}$$

- ***E x*** is the epipolar line associated with ***x*** (***l'*** = ***E x***)
  - Recall: a line is given by $ax + by + c = 0$ or

$$\mathbf{l}^T \mathbf{x} = 0 \quad \text{where} \quad \mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Epipolar constraint: Calibrated case



$$\boxed{\mathbf{x}'^{T} E \mathbf{x} = 0}$$

- ***E x*** is the epipolar line associated with ***x*** (***l'*** = ***E x***)
- ***E***^{T}***x'*** is the epipolar line associated with ***x'*** (***l*** = ***E***^{T}***x'***)
- ***E e*** = 0 and ***E***^{T}***e'*** = *0*
- ***E*** is singular (rank two)
- ***E*** has five degrees of freedom