Equations For Organ Matching Optimization Model

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1 Maximal Bipartite Matching between People and Organs

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$$\text{Set } A$$
$$\text{Set } B$$
$$E \in (A \times B)$$

$$\max_{(u,v)\in E} \Sigma_{u,v} X_{u,v}$$

$$X_{u,v} \in \{0,1\} \quad \forall (u,v) \in E \tag{1}$$

$$\forall u \in A \quad \Sigma_u X_{u,v} \leq 1 \quad such \;\; that \;\; (u,v) \in E \tag{2}$$

$$\forall v \in B \quad \Sigma_v X_{u,v} \leq 1 \quad such \;\; that \;\; (u,v) \in E \tag{3}$$

We have two sets A and B which contain the nodes of our bipartite graph. Constraint (1) indicates $X_(u,v)$ exists if there is an edge between u and v is a binary variable that is either 0 or 1 . This variable here can be thought of indicating that the edge is in the matching if it is 1 and 0 if not. Constraint (2) enforces that for every node in the A it can only be matched with atmost one node in B. (3) is just the converse but for every node in B. We then seek to maximize $X_{u,v}$ for our constraints.

2 Maximal Bipartite Matching between People and Organs that also minimizes distance

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$$\text{Set } A$$
$$\text{Set } B$$
$$E \in (A \times B)$$

$$\max_{(u,v)\in E} \Sigma_{u,v} X_{u,v}(S - e_{u,v})$$

$$S = \Sigma_{u,v} e_{u,v} \qquad \forall (u,v) \in E \tag{1}$$

$$X_{u,v} \in \{0,1\} \qquad \forall (u,v) \in E \tag{2}$$

$$\forall u \in A \qquad \Sigma_u X_{u,v} \leq 1 \qquad such \;\; that \;\; (u,v) \in E \tag{3}$$

$$\forall v \in B \qquad \Sigma_v X_{u,v} \leq 1 \qquad such \;\; that \;\; (u,v) \in E \tag{4}$$

We have two sets A and B which contain the nodes of our bipartite graph. Constraint (1) defines a new constant S which is the sum of all of the edge weights in the graph. Constraint (2) indicates $X_(u,v)$ exists if there is an edge between u and v is a binary variable that is either 0 or 1 . This variable here can be thought of indicating that the edge is in the matching if it is 1 and 0 if not. Constraint (2) enforces that for every node in the A it can only be matched with atmost one node in B. (3) is just the converse but for every node in B. We then seek to maximize $X_{u,v}$ for our constraints. Here we seek to maximize $\Sigma_{u,v} X_{u,v}(S - e_{u,v})$. This can be reduced from (5). The reasoning behind this is that we seek to prioritize maximal cardinality matching first and then pick the best weighted model from the set of maximal matchings.

$$\max_{(u,v)\in E} S * \Sigma_{u,v} X_{u,v} - \Sigma_{u,v} X_{u,v} e_{u,v} \tag{5}$$