Equations For Organ Matching Optimization Model

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1 Maximal Bipartite Matching between People and Organs

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$$\begin{gathered}\text{Set } A \\ \text{Set } B \\ E \in (A \times B)\end{gathered}$$

$$\max_{(u,v)\in E} \Sigma_{u,v} X_{u,v}$$

$$X_{u,v} \in \{0,1\} \quad \forall (u,v) \in E \tag{1}$$

$$\forall u \in A \quad \Sigma_u X_{u,v} \leq 1 \quad such \;\; that \;\; (u,v) \in E \tag{2}$$

$$\forall v \in B \quad \Sigma_v X_{u,v} \leq 1 \quad such \;\; that \;\; (u,v) \in E \tag{3}$$

We have two sets A and B which contain the nodes of our bipartite graph. Constraint (1) indicates $X_(u,v)$ exists if there is an edge between u and v is a binary variable that is either 0 or 1 . This variable here can be thought of indicating that the edge is in the matching if it is 1 and 0 if not. Constraint (2) enforces that for every node in the A it can only be matched with atmost one node in B. (3) is just the converse but for every node in B. We then seek to maximize $X_{u,v}$ for our constraints.

2 Maximal Bipartite Matching between People and Organs that also minimizes distance

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$$\text{Set } A$$
$$\text{Set } B$$
$$E \in (A \times B)$$

$$\max_{S, W_{u,v}, (u,v) \in E} S * \Sigma_{u,v} X_{u,v} - \Sigma_{u,v} W_{u,v}$$

$$S = \Sigma e_{u,v} \quad \forall (u,v) \in E \tag{1}$$

$$W_{u,v} = e_{u,v} * X_{u,v} \quad \forall (u,v) \in E \tag{2}$$

$$X_{u,v} \in \{0,1\} \quad \forall (u,v) \in E \tag{3}$$

$$\forall u \in A \quad \Sigma_u X_{u,v} \leq 1 \quad such \ \ that \ \ (u,v) \in E \tag{4}$$

$$\forall v \in B \quad \Sigma_v X_{u,v} \leq 1 \quad such \ \ that \ \ (u,v) \in E \tag{5}$$

We have two sets A and B which contain the nodes of our bipartite graph. Constraint (1) declares a new variable S which is the sum of all of the edge weights in the graph. Constraint (2) declares a new variable $W_{u,v}$ that is equal to the edge between u and v if $X_{u,v}$ is 1 and 0 otherwise. Constraint (1) indicates $X_(u,v)$ exists if there is an edge between u and v is a binary variable that is either 0 or 1 . This variable here can be thought of indicating that the edge is in the matching if it is 1 and 0 if not. Constraint (2) enforces that for every node in the A it can only be matched with atmost one node in B. (3) is just the converse but for every node in B. We then seek to maximize $X_{u,v}$ for our constraints. Here we seek to maximize the weighted sum of $X_{u,v}$ and $W_{u,v}$. The constant S ensures that maximal matching is the first priority and only then to minimize distance.