Visualizing Mathematical Models, 2022 Mississippi Governor's School

Dr. Jim Newton

January 26, 2022

1 Objectives

You are a scholar participating in the 2022 Mississippi Governor's School major course Advanced Mathematical Concepts, and Functional Programming. There are several objectives of this course:

Mathematics to develop or ameliorate their love for Mathematics

Computer Science to learn about functional programming by applying programming principles to mathematical principles which you'll learn along the way,

Communication to reenforce your ability and confidence to explain and defend your ideas

In this course you will learn many cursory things along the fast and hectic journey. It is critical that the leaders of tomorrow, especially the leaders in the technology sector, understand the impact on humans of technological development, especially decisions made in haste under project pressure. These aspects include knowing how to apply abstract concepts to real computation problems, and how to explain their ideas and defend their decisions to their peers. Scientific discussions can sometimes become heated and experts might disagree about the best course of action. We'll learn to keep disagreements objective and use science to test ideas, rather than relying on emotions.

It is my belief that you don't understand something until you can explain it. Additionally, many ideas are lost because the developer neglected to share his ideas. After each unit, some students will present their solutions to their peers. This means defending your choices, recognizing strengths and shortcomings, and being subjected to peer review (important in the scientific process). This exercise is intended not only to learn to accept constructive criticism but also to build confidence.

2 Overview

The scholars will proceed through up to 6 units, perhaps fewer if we decide to dig deeper into particular ones. Each unit will be organized into three stages

- Theory — Learn mathematics and computer science concepts.
- Lab — Implement code relating to the theory and link it to previous units.
- Presentation — Present your work to your classmates, and defend your decisions.

3 Syllabus

3.1 Sets and Functions

Sets, functions, sets of functions, and functions on sets of functions. This includes discussions about finite and infinite sets. We will talk about programming languages and functional languages in particular, contrasting them with imperative languages.

3.1.1 Coding

Getting acquainted with the coding environment, implementing hello-world in a functional way, basic types, Double, Int, String, Set, List, Vector.

3.2 Abstract Algebra – Sets with mathematical structure

Monoids, fields, operators and algebraic axioms, commutativity, associativity, invertability, nullification. Test structural axioms (commutativity etc), implement exponentiation on a monoid, implement matrix multiplication. Implement the polynomial ring, with addition, scaling, subtraction multiplication, and exponentiation.

3.3 Convergence

The limit is the most fundamental concept in Calculus. We will discuss limits in terms of sequences and convergence, distinguishing between Cauchy sequences and convergent sequences. Coding strategies such as tail recursion, iteration, and folding.

3.3.1 Coding

Implement the limit function as a function which manipulates functions. Detect convergence of sequences. Contrast implementations of recursion, iteration, and folding.

3.4 Infinite sums

Contrast convergence and summability.

3.4.1 Coding

Implement the sin, cos, and exp functions in terms of the infinite Taylor series, understanding conditions of convergence, avoiding infinite loops. Improve convergence using shifting and scaling. Compute sin and cos of square matrices. Complex numbers as special case of 2x2 matrix.

3.5 Differential Calculus

Definition of derivative in terms of limit, linearity, derivative of polynomial using mathematical induction.

3.5.1 Coding

compute derivative by successive approximation using the previous implementation of limit. Compute exact derivative of a polynomial, and compare two approaches numerically. Compute sin and cos in terms of derivative, and compare to Taylor series computation.

3.6 Integral Calculus

Understanding the Riemann in terms of infinite sums of rectangle and trapezoid areas.

3.6.1 Coding

Implement integral (function of function) using limit. Compute explicit integral of polynomial