大数据计算及应用

Recommendation Systems (2)

The \$1 Million Question



The Netflix Prize

- □ Training data
 - 100 million ratings, 480,000 users, 17,770 movies
 - 6 years of data: 2000-2005
- □ Test data
 - Last few ratings of each user (2.8 million)
 - Evaluation criterion: Root Mean Square Error (RMSE)

$$= \sqrt{\sum_{(i,x)\in R} (\hat{r}_{xi} - r_{xi})^2 / |R|}$$

- Netflix's system RMSE: 0.9514
- Competition
 - 2,700+ teams
 - **\$1 million** prize for 10% improvement on Netflix

The Netflix Utility Matrix R

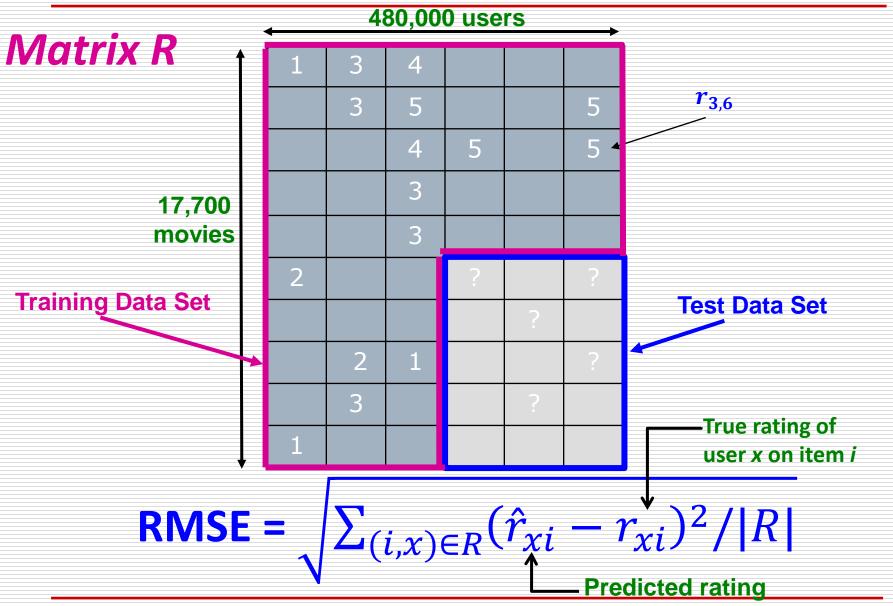
Matrix R

17,700 movies

4			•			\longrightarrow
	1	3	4			
		3	5			5 5
			4	5		5
			3			
			3			
	2			2		2
					5	
		2	1			1
		3			3	
	1					

480,000 users

Utility Matrix R: Evaluation

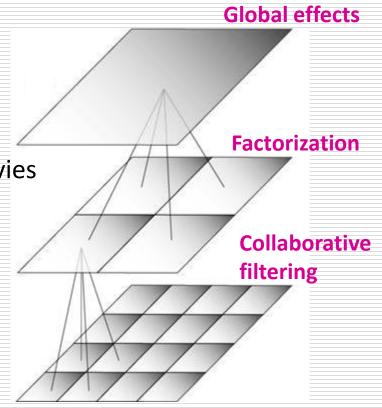


BellKor Recommender System

- □ The winner of the Netflix Challenge!
- Multi-scale modeling of the data:

Combine top level, "regional" modeling of the data, with a refined, local view:

- Global:
 - Overall deviations of users/movies
- Factorization:
 - Addressing "regional" effects
- Collaborative filtering:
 - Extract local patterns



Modeling Local & Global Effects

☐ Global:

- Mean movie rating: 3.7 stars
- The Sixth Sense is 0.5 stars above avg.
- Joe rates 0.2 stars below avg.
 - ⇒ Baseline estimation:

Joe will rate The Sixth Sense 4 stars



- Joe didn't like related movie Signs
- ⇒ Final estimate: Joe will rate The Sixth Sense 3.8 stars





Recap: Collaborative Filtering (CF)

- □ Earliest and most popular collaborative filtering method
- Derive unknown ratings from those of "similar" movies (item-item variant)
- \square Define **similarity measure** s_{ii} of items i and j
- \square Select k-nearest neighbors, compute the rating
 - N(i; x): items most similar to i that were rated by x

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij}... similarity of items i and j
r_{xj}... rating of user x on item j
N(i;x)... set of items similar to
item i that were rated by x

Modeling Local & Global Effects

□ In practice we get better estimates if we model deviations:

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} S_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} S_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

 μ = overall mean rating

 b_x = rating deviation of user x

= (avg. rating of user x) – μ

 $b_i = (avg. rating of movie i) - \mu$

Problems/Issues:

- 1) Similarity measures are "arbitrary"
- 2) Pairwise similarities neglect interdependencies among users
- 3) Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weights w_{ii}

☐ Use a weighted sum rather than weighted avg.:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj})$$

- ☐ A few notes:
 - N(i; x) ... set of movies rated by user x that are similar to movie i
 - lacksquare w_{ij} is the interpolation weight (some real number)
 - \square We allow: $\sum_{j \in N(i,x)} w_{ij} \neq 1$
 - \mathbf{w}_{ij} models interaction between pairs of movies (it does not depend on user \mathbf{x})

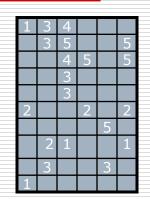
Idea: Interpolation Weights w_{ii}

- $\square \widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij} (r_{xj} b_{xj})$
- \square How to set w_{ii} ?
 - Remember, error metric is: $\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi}-r_{xi})^2}/|R|$ or equivalently SSE: $\sum_{(i,x)\in R}(\hat{r}_{xi}-r_{xi})^2$
 - Find w_{ij} that minimize SSE on training data!
 Models relationships between item i and its neighbors j
 - w_{ij} can be learned/estimated based on x and all other users that rated i

Why is this a good idea?

Recommendations via Optimization

- □ Goal: Make good recommendations
 - Quantify goodness using RMSE: Lower RMSE ⇒ better recommendations



- Want to make good recommendations on items that user has not yet seen. Can't really do this!
- Let's set build a system such that it works well on known (user, item) ratings And hope the system will also predict well the unknown ratings

Recommendations via Optimization

- □ Idea: Let's set values w such that they work well on known (user, item) ratings
- ☐ How to find such values w?
- Idea: Define an objective function and solve the optimization problem
- \square Find \mathbf{w}_{ij} that minimize **SSE** on training data!

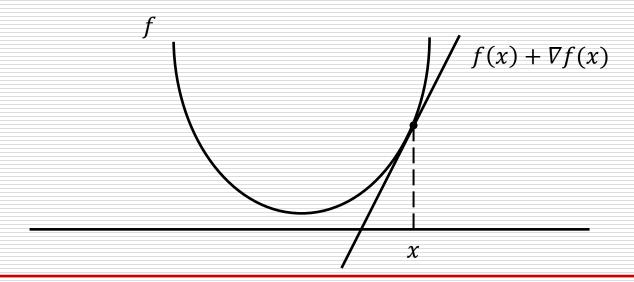
$$J(w) = \sum_{x,i} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$
Predicted rating

Predicted rating

Think of w as a vector of real numbers

Detour: Minimizing a function

- \square A simple way to minimize a function f(x):
 - lacksquare Compute the derivative ∇f
 - Start at some point x and evaluate $\nabla f(x)$
 - Make a step in the reverse direction of the gradient: $x = x \nabla f(x)$
 - Repeat until converged



Interpolation Weights

☐ We have the optimization problem, now what?

$$J(w) = \sum_{x} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$

□ Gradient decent:

 η ... learning rate

- Iterate until convergence: $w \leftarrow w \eta \nabla_w J$
- where $\nabla_w J$ is the gradient (derivative evaluated on data):

$$\nabla_{w}J = \left[\frac{\partial J(w)}{\partial w_{ij}}\right] = 2\sum_{x,i} \left(\left[b_{xi} + \sum_{k \in N(i;x)} w_{ik}(r_{xk} - b_{xk})\right] - r_{xi}\right) (r_{xj} - b_{xj})$$

$$\mathbf{for} \, \mathbf{j} \in \{\mathbf{N}(\mathbf{i}; \mathbf{x}), \forall \mathbf{i}, \forall \mathbf{x}\}$$

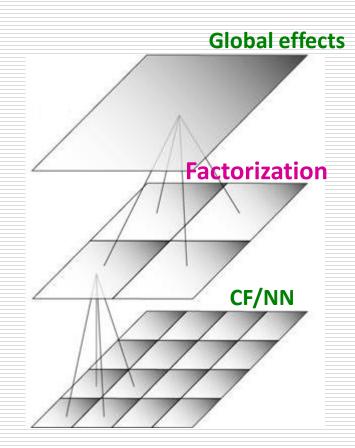
$$\mathbf{else} \, \frac{\partial J(w)}{\partial w_{ij}} = \mathbf{0}$$

Note: We fix movie i, go over all r_{xi} , for every movie $j \in N(i;x)$, we compute $\frac{\partial J(w)}{\partial w_{ij}}$ while $|w_{new} - w_{old}| > \varepsilon$: $w_{old} = w_{new}$

 $\mathbf{w}_{new} = \mathbf{w}_{old} - \boldsymbol{\eta} \cdot \boldsymbol{V} \mathbf{w}_{old}$

Interpolation Weights

- \square So far: $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} b_{xj})$
 - Weights w_{ij} derived based on their role; no use of an arbitrary similarity measure (w_{ij} ≠ s_{ij})
 - Explicitly account for interrelationships among the neighboring movies
- Next: Latent factor model
 - Extract "regional" correlations



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

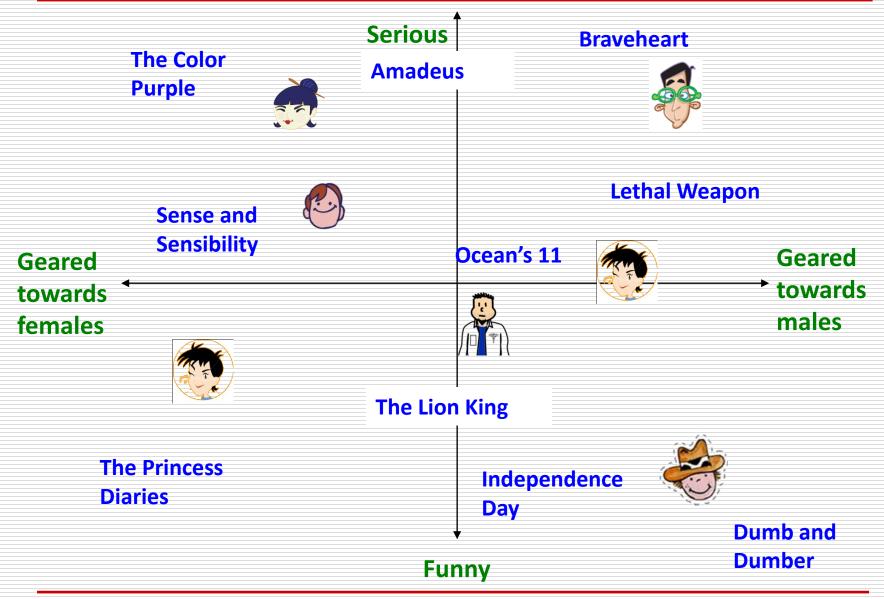
Netflix: 0.9514

Basic Collaborative filtering: 0.94

CF+Biases+learned weights: 0.91

Grand Prize: 0.8563

Latent Factor Models (e.g., SVD)



SVD: $A = U \Sigma V^T$

□ "SVD" on Netflix data: $\mathbf{R} \approx \mathbf{Q} \cdot \mathbf{P}^T$

users											
	3			5			5		4		
	5	4			4			2	1	3	
4		1	2		3		4	3	5		l.
2	4		5			4			2		ľ
	4	3	4	2					2	5	
	3		3			2			4		
		5 4 2 4 4	5 4 4 1 2 4 4 3	3	3	3 5 5 4 4 4 1 2 3 2 4 5 4 4 3 4 2	3 5 5 4 4 4 2 4 4 5 4 4 4 4 4 4 4 4	3 5 5 5 5 4 4 6 4 1 2 3 4 2 4 5 4 4 4 3 4 2 6 6	3 5 5 5 5 4 4 2 4 1 2 3 4 2 4 5 4 4 4 3 4 2 5	3 5 5 4 5 4 4 2 1 4 1 2 3 4 3 5 2 4 5 4 2 2 4 3 4 2 2	3 5 5 4 4 2 1 3 4 1 2 3 4 3 5 2 4 5 4 2 2 2 4 3 4 2 2 2 4 3 4 2 2 2



	users											
1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4		
8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1		
2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6		

PT

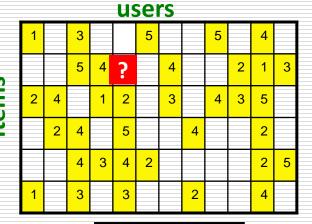
Λ

- ☐ For now let's assume we can approximate the rating matrix R as a product of "thin" $Q \cdot P^T$
 - R has missing entries but let's ignore that for now!
 - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

☐ How to estimate the missing rating of

user x for item i?



 \approx

\hat{r}_{xi}	$= q_i \cdot p_x$
= \frac{1}{2}	$\sum q_{if} \cdot p_{xf}$
	$ \frac{f}{q_i} = \text{row } i \text{ of } Q $ $ p_x = \text{column } x \text{ of } P^T $

	-1	.7	.3
Iţe	7	2.1	-2
	1.1	2.1	.3
tems	2	.3	.5
.	5	.6	.5
	.1	4	.2

ctors

ırs	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9
• ictc	8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
fa						.9						.1

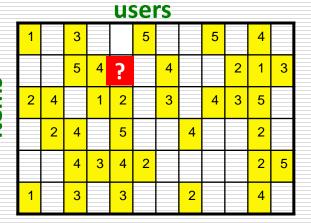
users

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Ratings as Products of Factors

☐ How to estimate the missing rating of

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 \approx

\hat{r}_{xi}	$= q_i \cdot p_x$
= \frac{1}{2}	$\mathbf{q}_{if} \cdot \mathbf{p}_{xf}$
	$ \frac{f}{q_i} = \text{row } i \text{ of } Q $ $ p_x = \text{column } x \text{ of } P^T $

	.1	4	.2
	5	.6	.5
items	2	.3	.5
ite	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3
	fo	ctore	

ırs	1.1	2	.3	.5	-2	5 -1	.8	4	.3	1.4	2.4	9
• icto	8	.7					1	l				1.3
fa	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1

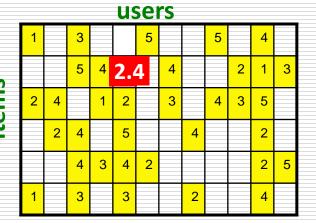
users

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Ratings as Products of Factors

☐ How to estimate the missing rating of

user x for item i?



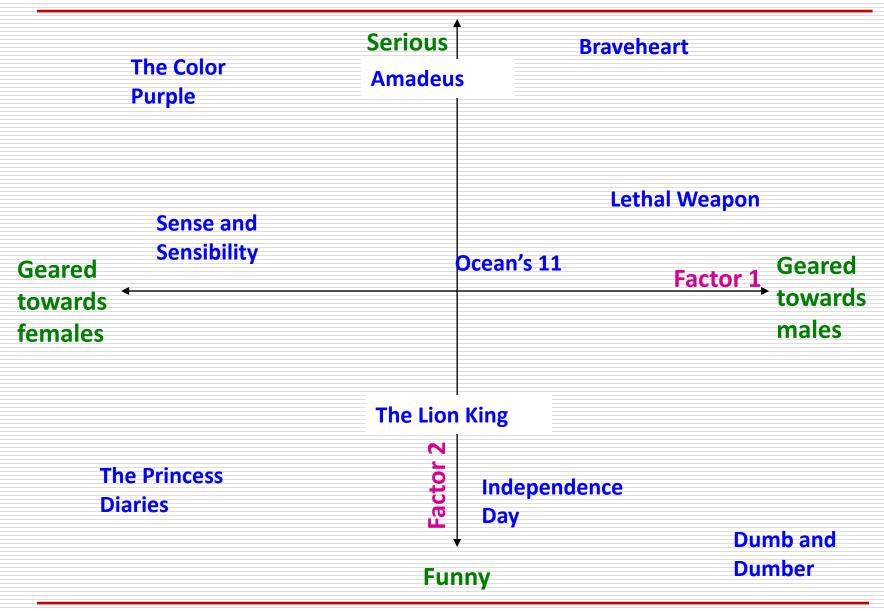
$\hat{r}_{xi} =$	$q_i \cdot p_x$
$=\sum$	$q_{if} \cdot p_{xf}$
	row <i>i</i> of <i>Q</i> column <i>x</i> of <i>P</i> ^T

	.1	4	.2
'	5	.6	.5
items	2	.3	.5
<u>판</u>	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3
	fa	ctors	

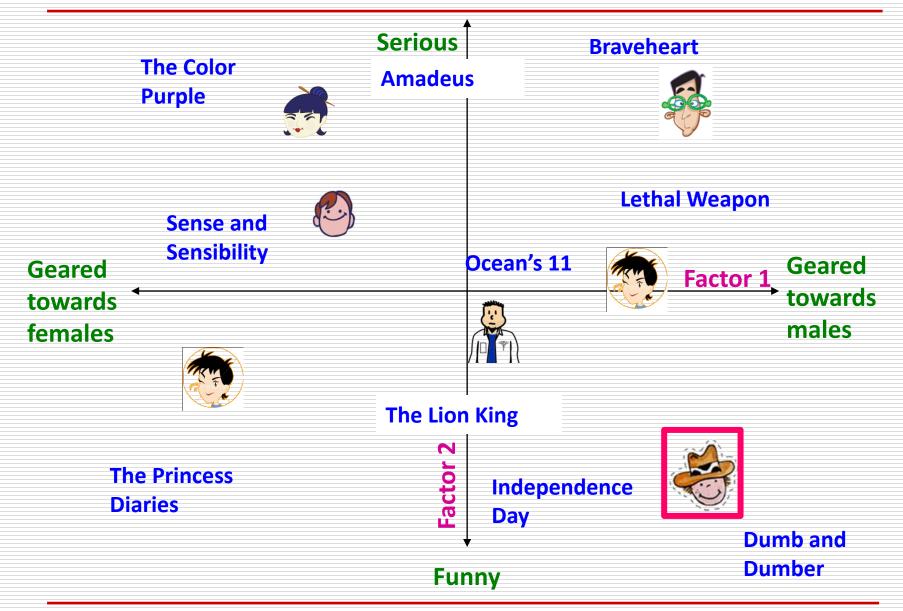
ırs	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9
• icto	8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
fa	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1

users

Latent Factor Models



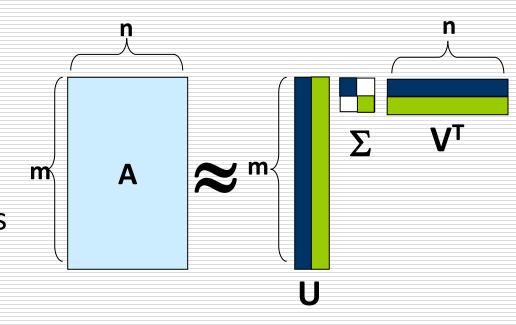
Latent Factor Models



Recap: SVD

□ Remember SVD:

- A: Input data matrix
- U: Left singular vecs
- V: Right singular vecs
- lacksquare Σ : Singular values



☐ So in our case:

"SVD" on Netflix data: $R \approx Q \cdot P^T$

$$A = R$$
, $Q = U$, $P^{T} = \sum V^{T}$

$$\hat{\boldsymbol{r}}_{xi} = \boldsymbol{q}_i \cdot \boldsymbol{p}_x$$

SVD: More good stuff

☐ We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):

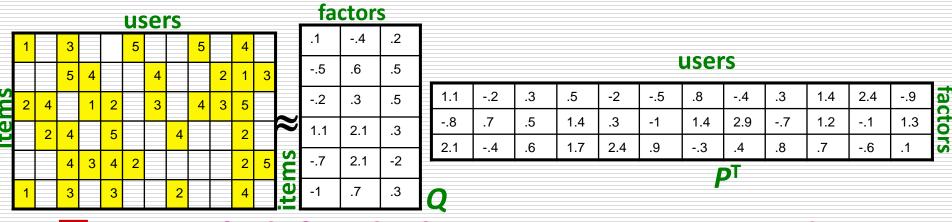
$$\min_{U,V,\Sigma} \sum_{ij \in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij} \right)^{2}$$

- Note two things:
 - **SSE** and **RMSE** are monotonically related:
 - $\square RMSE = \sqrt{SSE/|R|}$

Great news: SVD is minimizing RMSE

Complication: The sum in SVD error term is over all entries (no-rating in interpreted as zero-rating). But our R has missing entries!

Latent Factor Models



- ☐ SVD isn't defined when entries are missing!
- ☐ Use specialized methods to find *P*, *Q*

$$\hat{r}_{xi} = q_i \cdot p_x$$

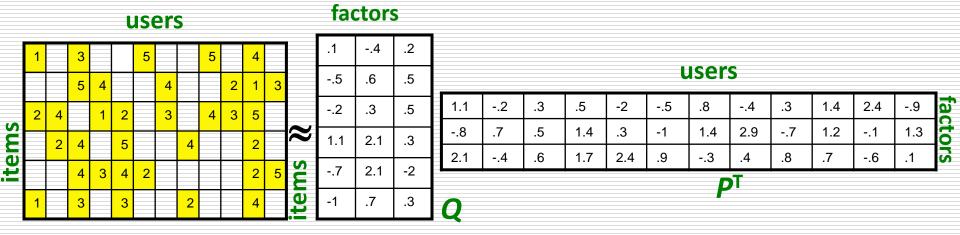
- Note:
 - ☐ We don't require cols of P, Q to be orthogonal/unit length
 - P, Q map users/movies to a latent space
 - ☐ The most popular model among Netflix contestants

Finding the Latent Factors

Latent Factor Models

☐ Our goal is to find P and Q such tat:

$$\min_{P,Q} \sum_{(i,x)\in R} (r_{xi} - q_i \cdot p_x)^2$$



Back to Our Problem

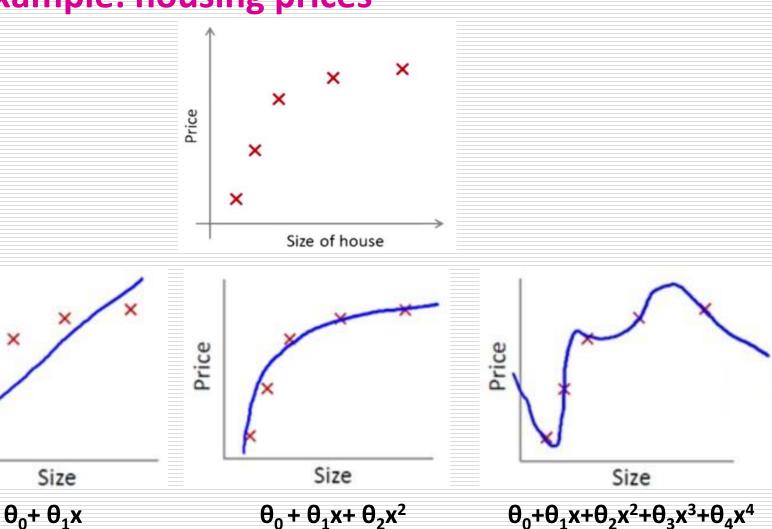
- Want to minimize SSE for unseen test data
- ☐ Idea: Minimize SSE on training data
 - Want large k (# of factors) to capture all the signals
 - But, **SSE** on $\underline{\text{test}}$ data begins to rise for k > 2
- This is a classical example of overfitting:
 - With too much freedom (too many free parameters)
 the model starts fitting noise
 - That is it fits too well the training data and thus not generalizing well to unseen test data

Overfitting

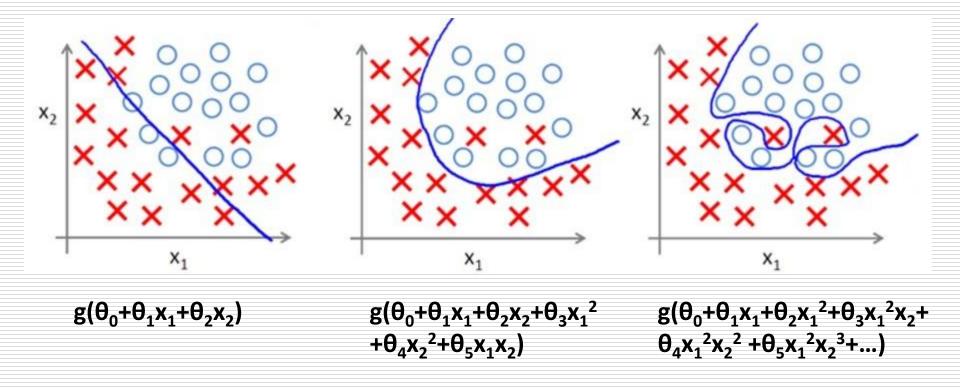
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Price

☐ Example: housing prices

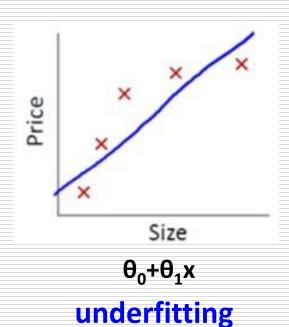


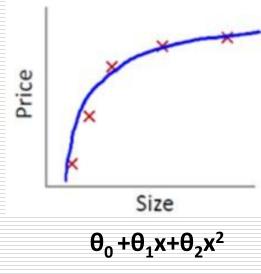
Overfitting

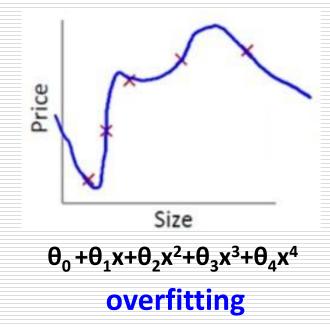


Overfitting

☐ Example: housing prices





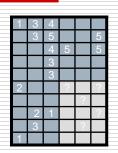


$$\min \sum_{i=1}^{n} (\widehat{y_{\theta}} - y)^2 + \sum_{j=1}^{n} \lambda_j \theta^2$$

Regularization (penalty)

Dealing with Missing Entries

To solve overfitting we introduce regularization:



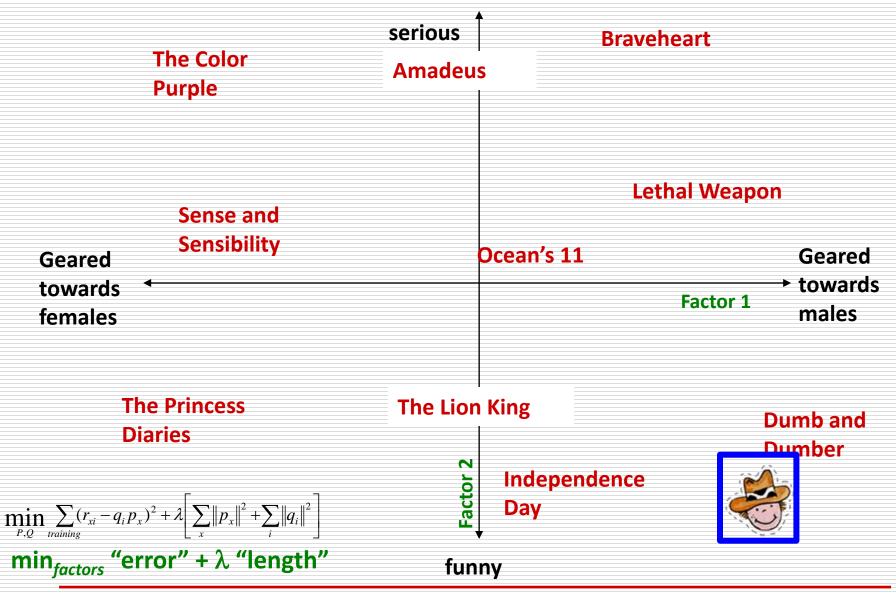
- Allow rich model where there are sufficient data
- Shrink aggressively where data are scarce

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} \|p_x\|^2 + \lambda_2 \sum_{i} \|q_i\|^2 \right]$$
"error"

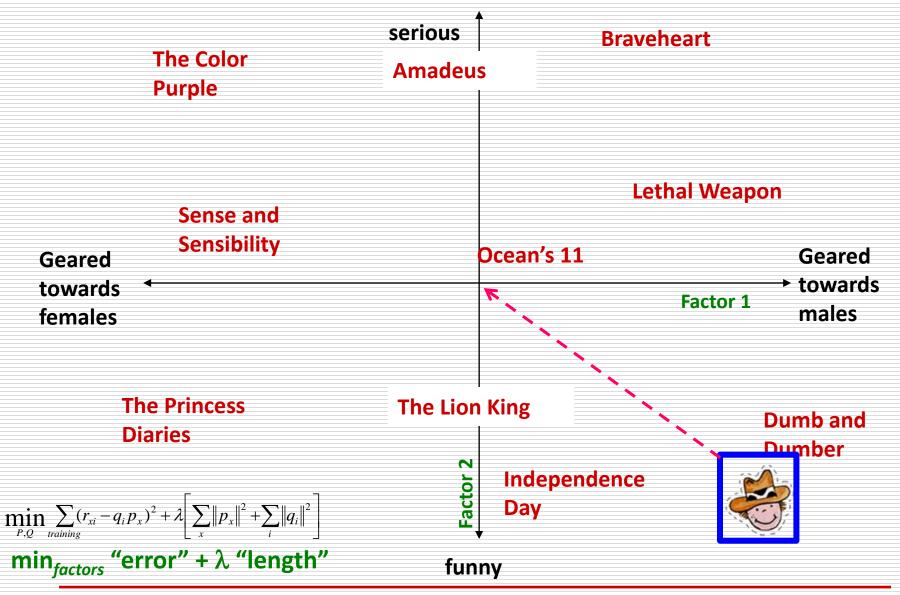
 λ_1 , λ_2 ... user set regularization parameters

Note: We do not care about the "raw" value of the objective function, but we care in P,Q that achieve the minimum of the objective

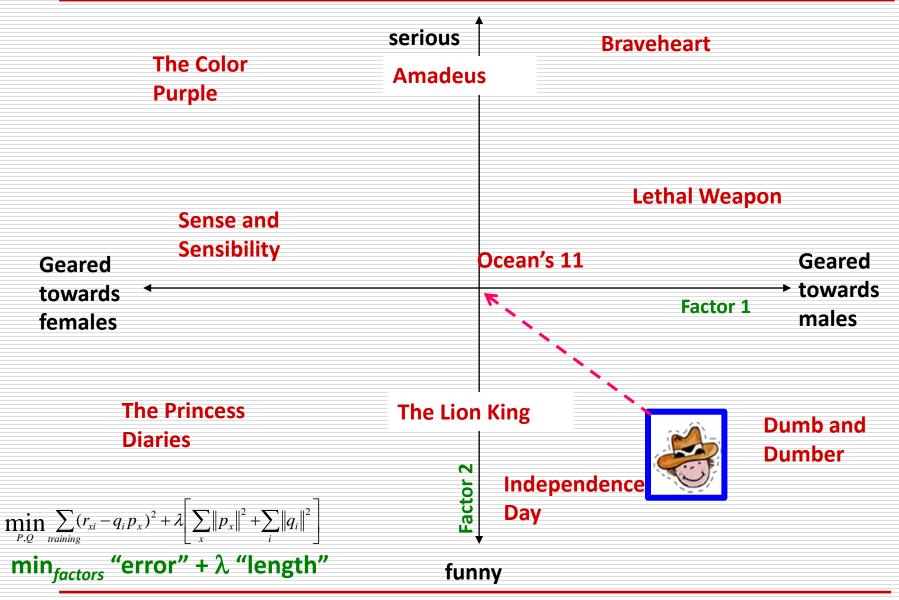
The Effect of Regularization



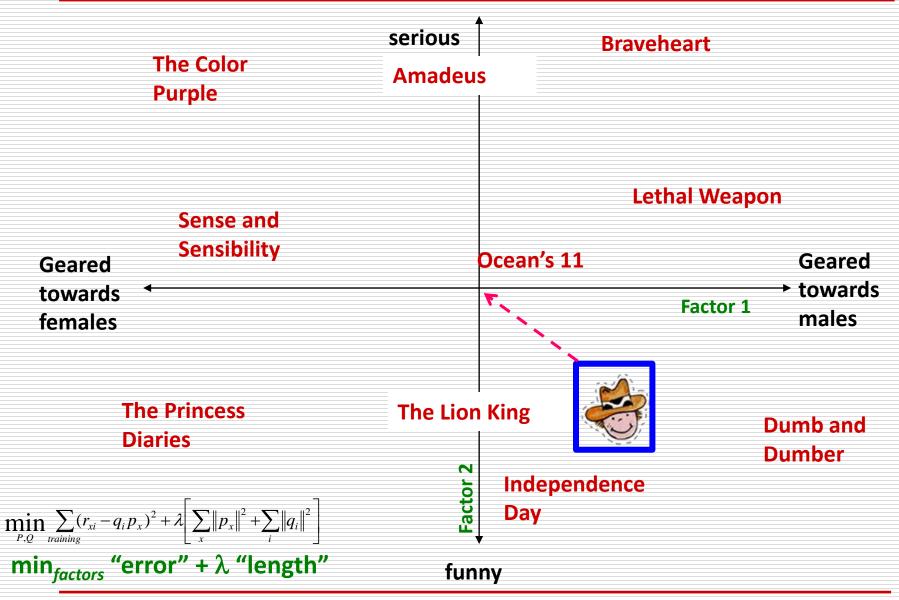
The Effect of Regularization



The Effect of Regularization



The Effect of Regularization



Gradient Descent

■ Want to find matrices P and Q:

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} ||p_x||^2 + \lambda_2 \sum_{i} ||q_i||^2 \right]$$

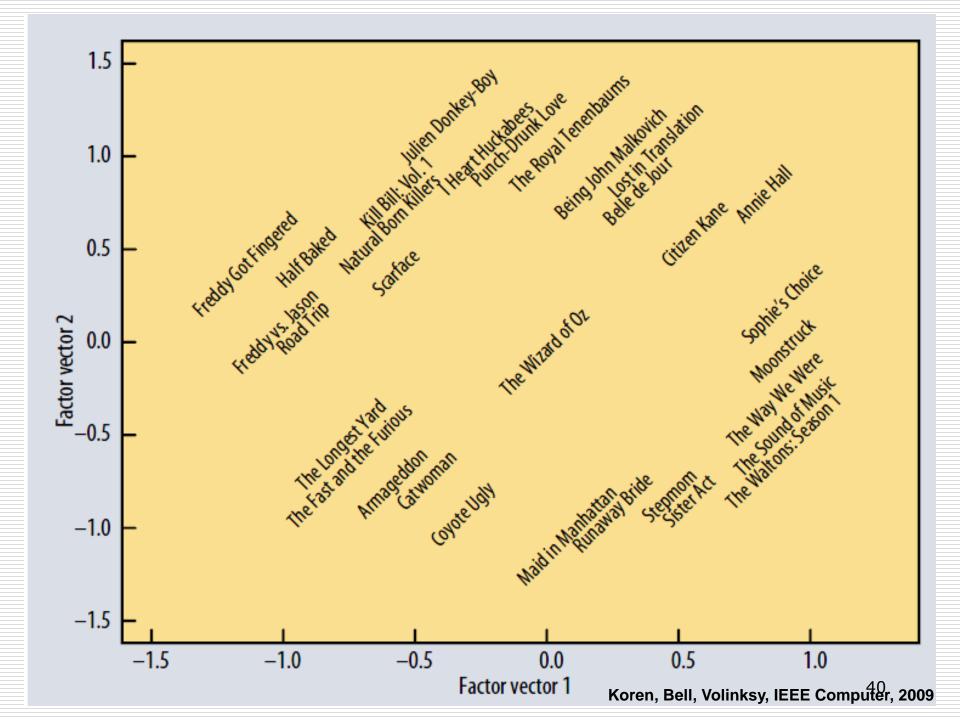
- ☐ Gradient decent:
 - Initialize P and Q (using SVD, pretend missing ratings are 0)
 - Do gradient descent:
 - $\square P \leftarrow P \eta \cdot \nabla P$
 - $\square Q \leftarrow Q \eta \cdot \nabla Q$
 - \square where ∇Q is gradient/derivative of matrix Q:

$$abla Q = [
abla q_{if}] \text{ and }
abla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x)p_{xf} + 2\lambda_2 q_{if}$$

Here q_{if} is entry f of row q_i of matrix Q

How to compute gradient of a matrix?

Compute gradient of every element independently!



Extending Latent Factor Model to Include Biases

Modeling Biases and Interactions

user bias



movie bias



user-movie interaction



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition

- **User-Movie interaction**
- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations
- μ = overall mean rating
- b_x = bias of user x b_i = bias of movie i

Baseline Predictor

□ We have expectations on the rating by user x of movie i, even without estimating x's attitude towards movies like i







- Rating scale of user x
- Values of other ratings user gave recently (day-specific mood, anchoring, multiuser accounts)

- (Recent) popularity of movie i
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting It All Together

$$r_{\chi i} = \mu + b_{\chi} + b_{i} + q_{i} \cdot p_{\chi}$$

Overall

Bias for

Bias for

Movie interaction

□ Example:

- Mean rating: $\mu = 3.7$
- You are a critical reviewer: your ratings are 1 star lower than the mean: $b_x = -1$
- Star Wars gets a mean rating of 0.5 higher than average movie: $b_i = + 0.5$
- Predicted rating for you on Star Wars:

$$= 3.7 - 1 + 0.5 = 3.2$$

Fitting the New Model

☐ Solve:

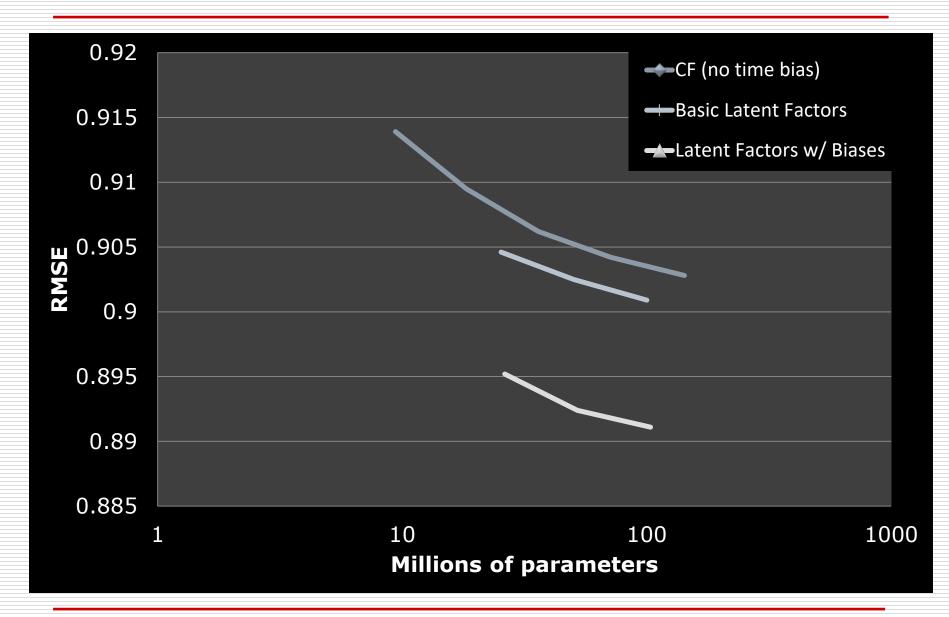
$$\min_{Q,P} \sum_{(x,i)\in R} (r_{xi} - (\mu + b_x + b_i + q_i p_x))^2$$
goodness of fit

$$+ \left(\frac{\lambda_{1} \sum_{i} \|q_{i}\|^{2} + \lambda_{2} \sum_{x} \|p_{x}\|^{2} + \lambda_{3} \sum_{x} \|b_{x}\|^{2} + \lambda_{4} \sum_{i} \|b_{i}\|^{2}}{\text{regularization}} \right)$$

λ is selected via grid-search on a validation set

- □ (Stochastic) gradient decent to find parameters
 - Note: Both biases b_x , b_i as well as interactions q_i , p_x are treated as parameters (we estimate them)

Performance of Various Methods



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

Latent factors+Biases: 0.89

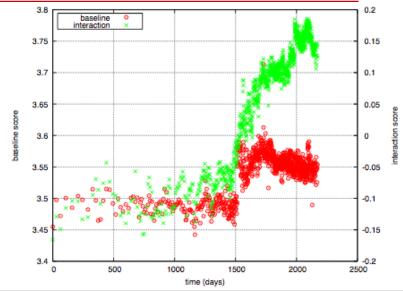
Grand Prize: 0.8563

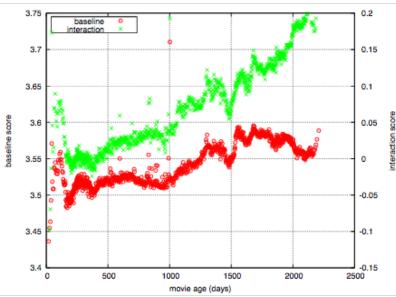
The Netflix Challenge: 2006-2009

Temporal Biases Of Users

- □ Sudden rise in the average movie rating (early 2004)
 - Improvements in Netflix
 - GUI improvements
 - Meaning of rating changed
- Movie age
 - Users prefer new movies without any reasons
 - Older movies are just inherently better than newer ones

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09





Temporal Biases & Factors

□ Original model:

$$r_{xi} = m + b_x + b_i + q_i \cdot p_x$$

□ Add time dependence to biases:

$$r_{xi} = m + b_x(t) + b_i(t) + q_i \cdot p_x$$

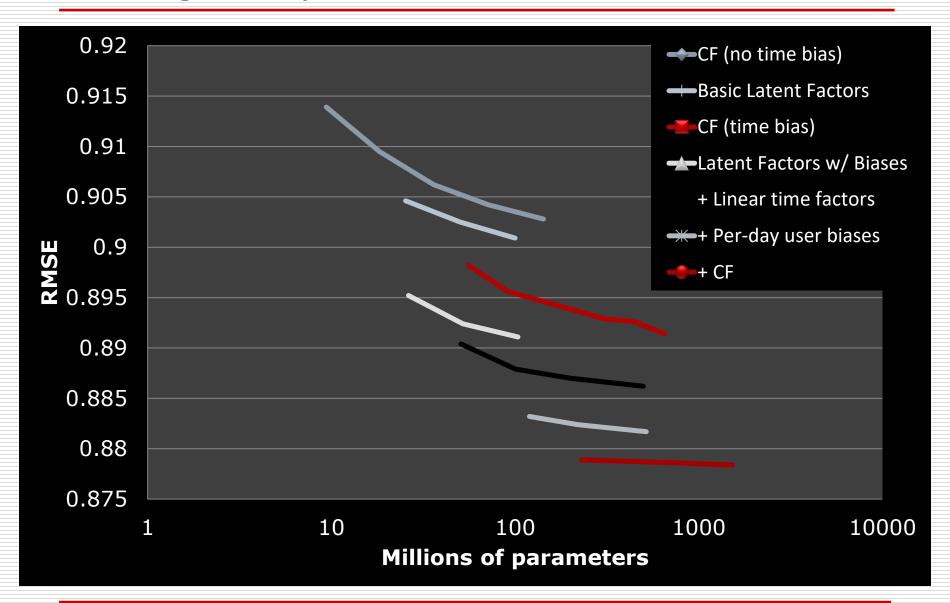
- Make parameters b_x and b_i to depend on time
- (1) Parameterize time-dependence by linear trends
 - (2) Each bin corresponds to 10 consecutive weeks

$$b_i(t) = b_i + b_{i,\operatorname{Bin}(t)}$$

- □ Add temporal dependence to factors
 - $p_{x}(t)$... user preference vector on day t

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09

Adding Temporal Effects



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

Latent factors+Biases: 0.89

Latent factors+Biases+Time: 0.876

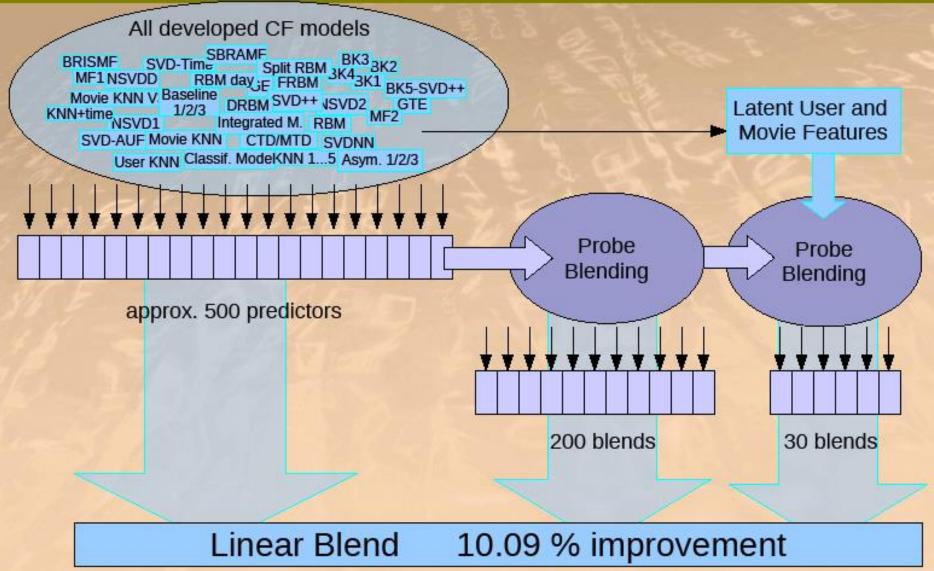
Still no prize!
Getting desperate.

Try a "kitchen sink" approach!

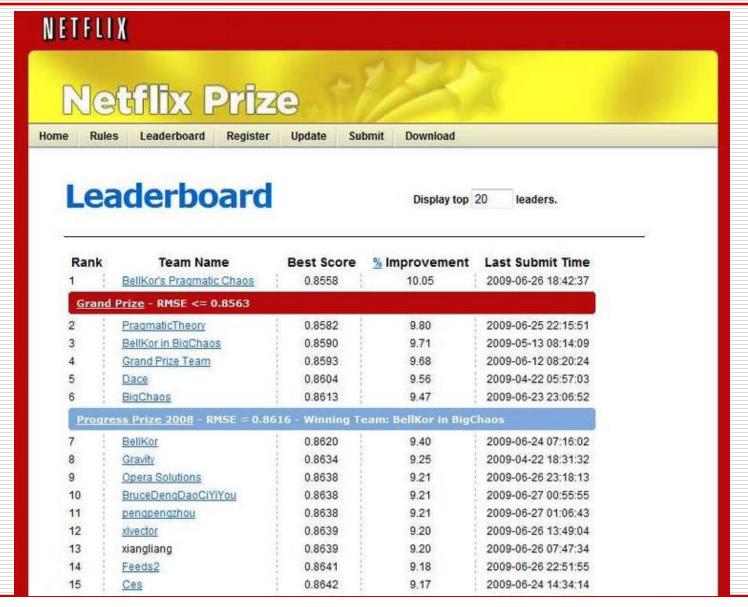
Grand Prize: 0.8563

The big picture

Solution of BellKor's Pragmatic Chaos



Standing on June 26th 2009



June 26th submission triggers 30-day "last call"

The Last 30 Days

■ Ensemble team formed

- Group of other teams on leaderboard forms a new team
- Relies on combining their models
- Quickly also get a qualifying score over 10%

□ BellKor

- Continue to get small improvements in their scores
- Realize that they are in direct competition with Ensemble

☐ Strategy

- Both teams carefully monitoring the leaderboard
- Only sure way to check for improvement is to submit a set of predictions
 - This alerts the other team of your latest score

24 Hours from the Deadline

- Submissions limited to 1 a day
 - Only 1 final submission could be made in the last 24h
- 24 hours before deadline...
 - BellKor team member in Australia notices (by chance) that Ensemble posts a score that is slightly better than BellKor's
- □ Frantic last 24 hours for both teams
 - Much computer time on final optimization
 - Carefully calibrated to end about an hour before deadline
- ☐ Final submissions
 - BellKor submits a little early (on purpose), 40 mins before deadline
 - **Ensemble** submits their final entry 20 mins later
 -and everyone waits....

Netflix Prize



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Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 ‡ leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	3.8002	J.9	00104:4.
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	<u>Dace</u>	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50

Million \$ Awarded Sept 21st 2009



Acknowledgments

- Some slides and plots borrowed from Yehuda Koren, Robert Bell and Padhraic Smyth
- ☐ Further reading:
 - Y. Koren, Collaborative filtering with temporal dynamics, KDD '09
 - http://www2.research.att.com/~volinsky/netflix/bpc.ht ml
 - http://www.the-ensemble.com/

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