2. samostatná práce
IMA
Zadání 9

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1. príklad

1. Riešenie
$t: f(x_0) + f'(x_0)(x - x_0)$
$n: f(x_0) - \frac{1}{f'(x_0)}(x - x_0)$

$t: f(x_0) + f'(x_0)(x - x_0) = x_0^2 - 1 + 2x_0(x - x_0) = x_0^2 - 1 + 2x_0x - 2x_0^2 = -x_0^2 + 2x_0x - 1$
$n: f(x_0) - \frac{1}{f'(x_0)}(x - x_0) = x_0^2 - 1 - \frac{1}{2x_0}(x - x_0) = x_0^2 - 1 - \frac{x}{2x_0} + \frac{1}{2} = x_0^2 - \frac{1}{2x_0}x - \frac{1}{2}$

$\int_0^{x_0}(x_0^2 - \frac{1}{2x_0}x - \frac{1}{2} - (-x_0^2 + 2x_0x - 1))dx = \int_0^{x_0}(x_0^2 - \frac{1}{2x_0}x - \frac{1}{2} + x_0^2 - 2x_0x + 1)dx = \int_0^{x_0}(2x_0^2 - \frac{1}{2x_0}x - 2x_0x + \frac{1}{2})dx = 2x_0^2\int_0^{x_0} 1dx - \frac{1}{2x_0}\int_0^{x_0} xdx - 2x_0\int_0^{x_0} xdx + \frac{1}{2}\int_0^{x_0} 1dx = \left[2x_0^2x - \frac{1}{2x_0}\frac{x^2}{2} - 2x_0\frac{x^2}{2} + \frac{1}{2}x\right]_0^{x_0} = \left(2x_0^3 - \frac{x_0^2}{4x_0} - \frac{2x_0^3}{2} + \frac{1}{2}x_0\right) - 0 = 2x_0^3 - \frac{x_0}{4} - x_0^3 + \frac{1}{2}x_0 = x_0^3 - \frac{x_0}{4} + \frac{x_0}{2} = x_0^3 + \frac{1}{4}x_0$

$x_0^3 + \frac{1}{4}x_0 = \frac{17}{2}$
$4x_0^3 + x_0 = 34$
$x_0(4x_0^2 + 1) - 34 = 0$
$2(4 * 2^2 + 1) - 34 = 0$
$x_0 = 2$

2. Riešenie

$t: f(x_0) + f'(x_0)(x - x_0)$
$n: f(x_0) - \frac{1}{f'(x_0)}(x - x_0)$

$t: f(x_0) + f'(x_0)(x - x_0) = x_0^2 - 1 + 2x_0(x - x_0) = x_0^2 - 1 + 2x_0x - 2x_0^2 = -x_0^2 + 2x_0x - 1$
$n: f(x_0) - \frac{1}{f'(x_0)}(x - x_0) = x_0^2 - 1 - \frac{1}{2x_0}(x - x_0) = x_0^2 - 1 - \frac{x}{2x_0} + \frac{1}{2} = x_0^2 - \frac{1}{2x_0}x - \frac{1}{2}$

$\int_{-x_0}^{0}(x_0^2 - \frac{1}{2x_0}x - \frac{1}{2} - (-x_0^2 + 2x_0x - 1))dx = \int_{-x_0}^{0}(x_0^2 - \frac{1}{2x_0}x - \frac{1}{2} + x_0^2 - 2x_0x + 1)dx = \int_{-x_0}^{0}(2x_0^2 - \frac{1}{2x_0}x - 2x_0x + \frac{1}{2})dx = 2x_0^2\int_{-x_0}^{0} 1dx - \frac{1}{2x_0}\int_{-x_0}^{0} xdx - 2x_0\int_{-x_0}^{0} xdx + \frac{1}{2}\int_{-x_0}^{0} 1dx = \left[2x_0^2x - \frac{1}{2x_0}\frac{x^2}{2} - 2x_0\frac{x^2}{2} + \frac{1}{2}x\right]_{-x_0}^{0} = 0 - \left(2x_0^3 - \frac{x_0^2}{4x_0} - \frac{2x_0^3}{2} + \frac{1}{2}x_0\right) = -2x_0^3 + \frac{x_0}{4} + x_0^3 - \frac{1}{2}x_0 = -x_0^3 - \frac{1}{4}x_0$
$-x_0^3 - \frac{1}{4}x_0 = \frac{17}{2}$
$-4x_0^3 - x_0 = 34$
$-x_0(4x_0^2 + 1) - 34 = 0$
$-(-2)(4 * (-2)^2 + 1) - 34 = 0$
$x_0 = -2$

2. príklad

$$\int_0^{\infty}\left(\frac{1}{(x+3)^2} - \frac{2}{x+3} - \frac{2}{x+1} + \frac{4x}{x^2+1}\right)dx$$

$$\lim_{a\to\infty}\int_0^{a}\left(\frac{1}{(x+3)^2} - \frac{2}{x+3} - \frac{2}{x+1} + \frac{4x}{x^2+1}\right)dx$$

$$\lim_{a\to\infty}\left[-\frac{1}{x+3} - 2ln|x+3| - 2ln|x+1| + 2ln|x^2+1|\right]_0^a$$

$$\lim_{a\to\infty}\left[ln\left|\frac{(x^2+1)^2}{(x+3)^2(x+1)^2}\right|-\frac{1}{x+3}\right]_0^a$$

$$\lim_{a\to\infty}\left[ln\left(\frac{(x^2+1)}{(x+3)(x+1)}\right)^2-\frac{1}{x+3}\right]_0^a$$

$$\lim_{a\to\infty}\left[ln\left(\frac{1+\frac{1}{x^2}}{1+\frac{4}{x}+\frac{3}{x^2}}\right)^2-\frac{1}{x+3}\right]_0^a$$

$$\left[(ln1+0)-\left(ln\left(\frac{1}{9}\right)+\frac{1}{3}\right)\right]=\frac{1}{3}+2ln(3)$$

3. príklad

$$\int_0^1 \frac{1}{x^5+32}dx = \frac{1}{32}\int_0^1 \frac{1}{1+\frac{x^5}{2^5}}dx = \frac{1}{32}\int_0^1 \frac{1}{1-\left(\frac{-x}{2}\right)^5}dx$$

$$\frac{1}{32}\int_0^1 \sum_{n=0}^{\infty}(-1)^n(\frac{x}{2})^{5n}dx$$

$$\frac{1}{32}\sum_{n=0}^{\infty}\int_0^1 (-1)^n(\frac{x}{2})^{5n}dx$$

$$\frac{1}{32}\sum_{n=0}^{\infty}\int_0^1 \left(1-\frac{x^5}{2^5}+\frac{x^{10}}{x^{10}}-\frac{x^{15}}{2^{15}}+...+(-1)^n\left(\frac{x}{2}\right)^{5n}+...\right)dx$$

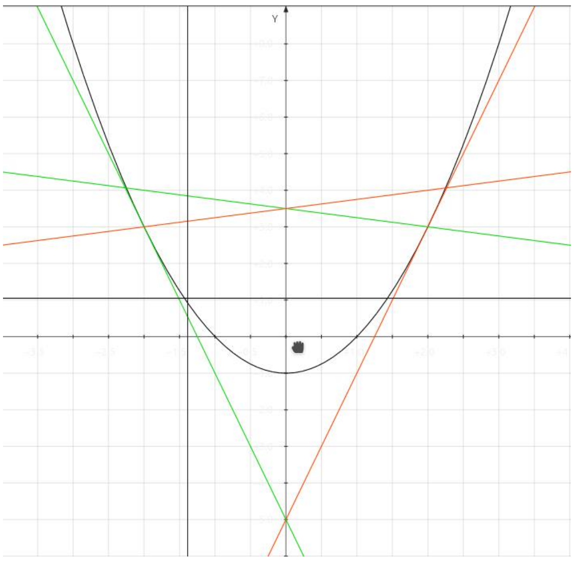
$$\frac{1}{32}\left[x-\frac{x^6}{6*2^5}+\frac{x^{11}}{11*2^{10}}-\frac{x^{16}}{16*2^{15}}+...+(-1)^n\frac{x^{n+1}}{(x+1)2^{5n}}+...\right]_0^1$$

$$\frac{1}{32}\frac{17212895}{17301504} \doteq \frac{0.9948853772}{32} \doteq 0.03109016803$$

4. príklad

$$f(x,y) = \frac{1}{1-4x^2-y^2} - \ln(x(4y^2-x^2-1))$$

$$\begin{array}{lclcl}
1-4x^2-y^2 \neq 0 & \wedge & x(4y^2-x^2-1) > 0 & & \\
4x^2+y^2 \neq 1 & & (x>0 \wedge 4y^2-x^2-1>0) & \vee & (x<0 \wedge 4y^2-x^2-1<0) \\
 & & 4y^2-x^2>1 & & 4y^2-x^2<1
\end{array}$$



5. príklad

$$f(x,y) = e^{3x+2y}(3x^2 - 6xy + 8y^2)$$

$$\begin{aligned} f'x &= 3e^{3x+2y}(3x^2 - 6xy + 8y^2) + e^{3x+2y}(6x - 6y) \\ f'y &= 2e^{3x+2y}(3x^2 - 6xy + 8y^2) + e^{3x+2y}(-6x + 16y) \end{aligned}$$

$$\begin{aligned} e^{3x+2y} * 3(3x^2 - 6xy + 8y^2 + 2x - 2y) &= 0 / * 2 \\ e^{3x+2y} * 2(3x^2 - 6xy + 8y^2 - 3x + 8y) &= 0 / * 3 \end{aligned}$$

$$\begin{aligned} 5x - 10y &= 0 \\ 5x &= 10y \\ x &= 2y \end{aligned}$$

$$\begin{aligned} 3(2y)^2 - 6(2y) * y + 8y^2 + 2(2y) - 2y &= 0 \\ 12y^2 - 12y^2 + 8y^2 + 4y - 2y &= 0 \\ 8y^2 + 2y &= 0 \\ 2y(4y+1) &= 0 \\ 2y &= 0 \\ y_1 &= 0 \\ 4y + 1 &= 0 \\ 4y &= -1 \\ y_2 &= -\frac{1}{4} \end{aligned}$$

$$x_1 = 0 \qquad x_2 = -\frac{1}{2}$$

$$P_1\,[0;0] \qquad P_2\left[-\frac{1}{2};-\frac{1}{4}\right]$$

$$\begin{aligned} f''_{xx} &= 9e^{3x+2y} * (3x^2 - 6xy + 8y^2) + 6e^{3x+2y} \\ f''_{xy} &= 6e^{3x+2y} * (3x^2 - 6xy + 8y^2) - 6e^{3x+2y} \\ f''_{yy} &= 4e^{3x+2y} * (3x^2 - 6xy + 8y^2) + 16e^{3x+2y} \end{aligned}$$

Hessova matica:

$$\begin{vmatrix} f''xx & f''xy \\ f''yx & f''yy \end{vmatrix} = \begin{vmatrix} 6 & -6 \\ -6 & 16 \end{vmatrix} = (6*16) - (-6)(-6) = 60 > 0$$

V bode P_1, P_2 má lokálne minimum.

$$\begin{vmatrix} \frac{21}{2e^2} & -\frac{3}{e^2} \\ -\frac{3}{e^2} & \frac{18}{e^2} \end{vmatrix} = \left(\frac{21}{2e^2} * \frac{18}{e^2}\right) - \left(\left(-\frac{3}{e^2}\right)\left(-\frac{3}{e^2}\right) = \frac{1}{e^4}\left(21*9-9\right) = \frac{180}{e^4} > 0$$

$$9e^{3(-\frac{1}{2})+2(-\frac{1}{4})} * \left(3\left(-\frac{1}{2}\right)^2 - 6\left(-\frac{1}{2}\right) - \left(-\frac{1}{4}\right) + 8\left(-\frac{1}{4}\right)^2\right) + 6e^{3(-\frac{1}{2})+2(-\frac{1}{4})} =$$

$$= 9e^{-\frac{3}{2}-\frac{1}{2}} * \left(\frac{3}{4} - \frac{3}{4} + \frac{1}{2}\right) + 6e^{-2} = \frac{9}{e^2} * \frac{1}{2} * \frac{6}{e^2} = \frac{9}{2e^2} + \frac{6}{e^2} = \frac{9+12}{2e^2} = \frac{21}{2e^2}$$

$$\frac{6}{e^2} * \frac{1}{2} - \frac{6}{e^2} = \frac{6}{2e^2} - \frac{6}{e^2} = \frac{6-12}{2e^2} = -\frac{3}{e^2}$$

$$\frac{4}{e^2} * \frac{1}{2} + \frac{16}{e^2} = \frac{4}{2e^2} + \frac{16}{e^2} = \frac{4+32}{2e^2} = \frac{18}{e^2}$$