Introduction to Computer Vision

Lecture 1. Course Introduction

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Skoltech
Skolkovo Institute of Science and Technology

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Credits

- Mikhail Belyaev's slides
  - Founder of medical imaging solutions startup
  - AI for radiology `https://ira-labs.com/`
- `https://github.com/Lavton/SkoltechLaTeXtemplates`

Course info

Term 2, 3 credit course.

- **Tuesday**, **Thursday**:9:00-12:00
  - Lectures: the first 90 minutes.
  - Practical exercises to work at home (provided after the lecture).
  - Online review of the exercises with QA after the next lecture: the remaining 60-90 minutes.
- 12 lectures + hands-ons.
- 1 invited lecture
- Final project.

Grading

- 3x20% - homeworks;
- 40% - final project.

Prerequisities

- **Basic knowledge of Python**
- Basic knowledge of linear algebra and statistics

Course outcomes

Knowledge

1. Statements of all major computer vision problems.
2. Mathematical details of the most important computer vision algorithms.

Skills

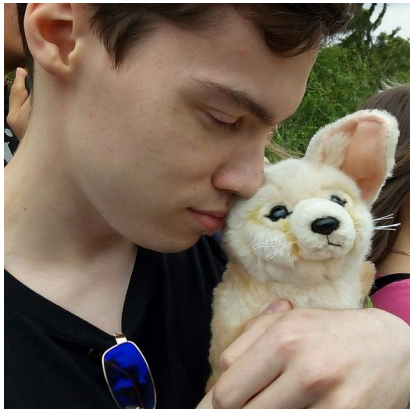
1. Select an appropriate method for solving particular computer vision problems.
2. Apply computer vision libraries.
3. Solve real-life computer vision problems.

Knowledge

1. Implementing computer vision basic algorithms and complex pipelines.

Computer Vision Course

Sergey Zagoruyko



Boris Shirokikh

Maxim Kurkin

Andrei Filatov

Course sturcture

- Introduction to the field.
- Convolutions and Template Matching. Edges: Sobel filter, Canny
- 3D Reconstruction 1: Introduction and Keypoints
- 3D Reconstruction 2: Descriptors, Transforms, RANSAC
- 3D Reconstruction 3: Dense Reconstruction, SfM, Panorama Stitching
- Face Recognition - Metric Learning (invited lecture)
- Optical Flow
- Face Recognition
- Visual Tracking and Motion Prediction
- Computer Vision in Autonomous Vehicles
- Multimodal Computer Vision
- Neural Network Compression, Knowledge Distillation, Quantization

Course evaluation

Why do we need classical computer vision methods in 2023?

1. Many ideas in modern deep learning methods are based on classical computer vision methods.
2. Deep learning methods are data hungry. Using classical CV method you can create a prototype faster. Also, some unsupervised approaches can be used as to generate annotation candidates

Plagiarism Policy

Rules are simple:

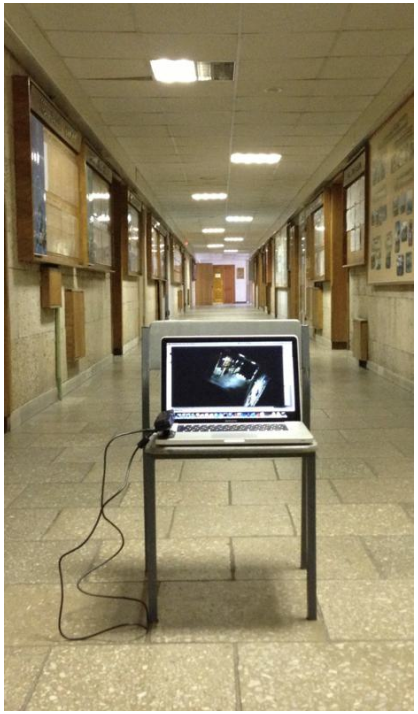
1. If you use external code, please add the link
2. Homeworks are individual assignments. If one homework is submitted twice, both students get 1/2 of the score.
3. Final projects are team-based. However, do not submit a random project from the internet! It will hurt your overall score!

About me

- Started working with neural networks in 2010 at **MIPT**
- Graduated from **BMSTU** in 2013 with a degree in mechatronics.
- Defended a PhD in deep learning from **Ecole des Ponts ParisTech** in 2018.
- Completed internships at **FAIR**, **Intel**, and **VisionLabs**.
- Completed postdoctoral research at **Inria** and **FAIR**.
- Worked on autonomous vehicles at **Lyft** and **Toyota**.
- Currently serving as the head of fundamental research at **MTS AI** and assistant professor at **Skoltech**.

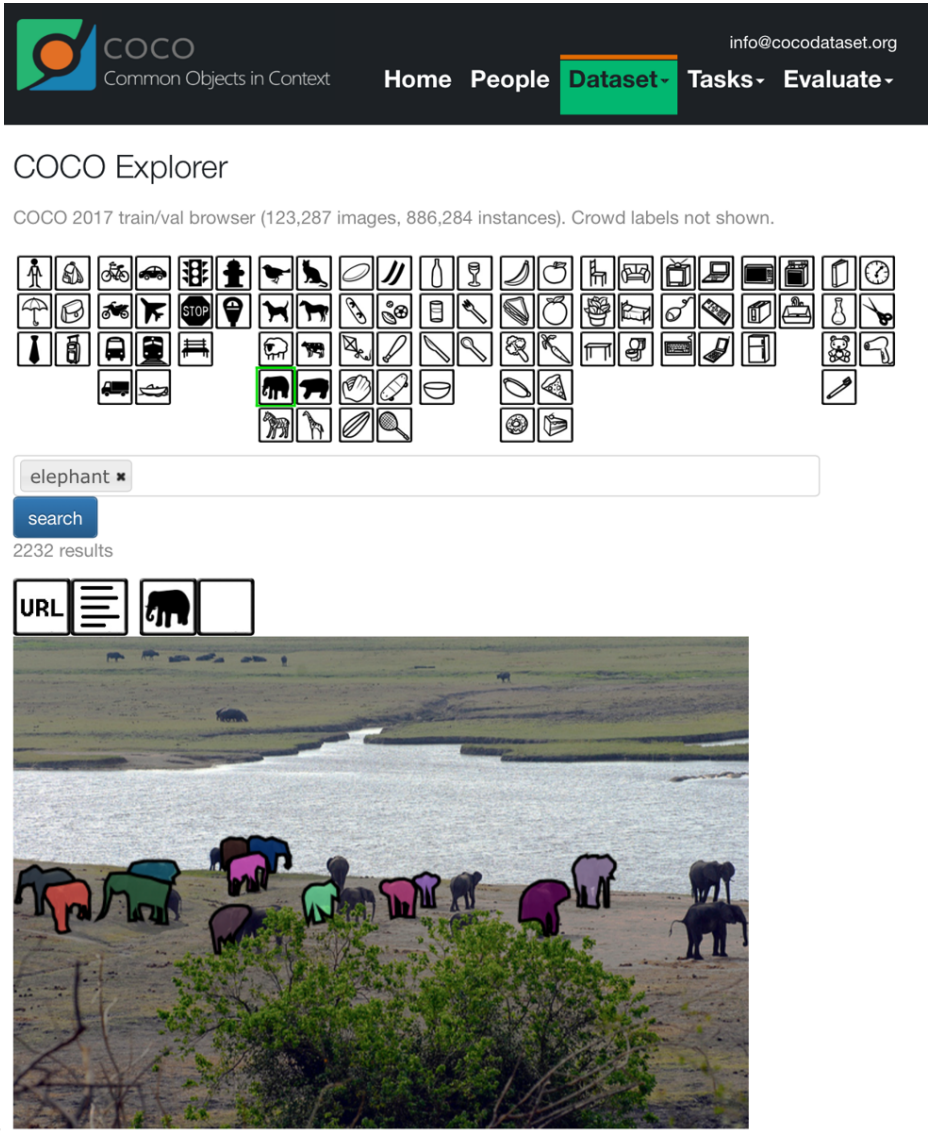
my page

Computer vision in 2013



PhD in France in 2014

COCO dataset: internship at FAIR in 2015



Try it out!

https://cocodataset.org

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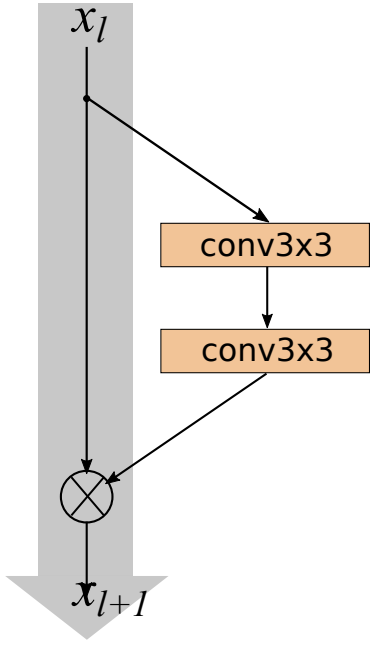
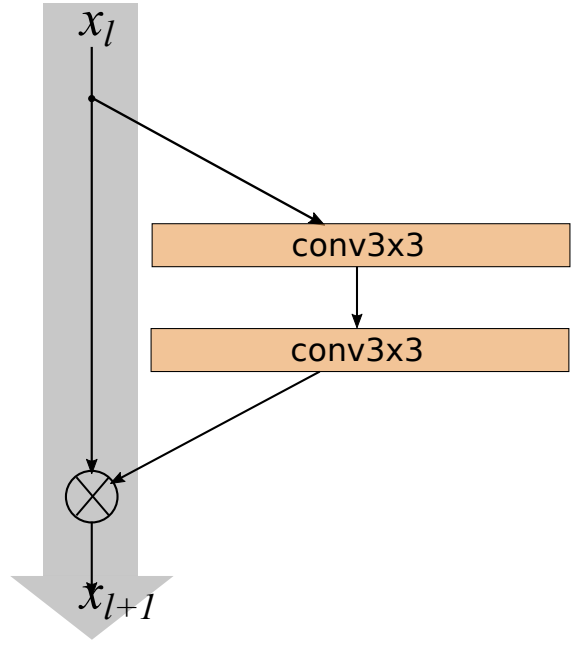
Wide Residual networks



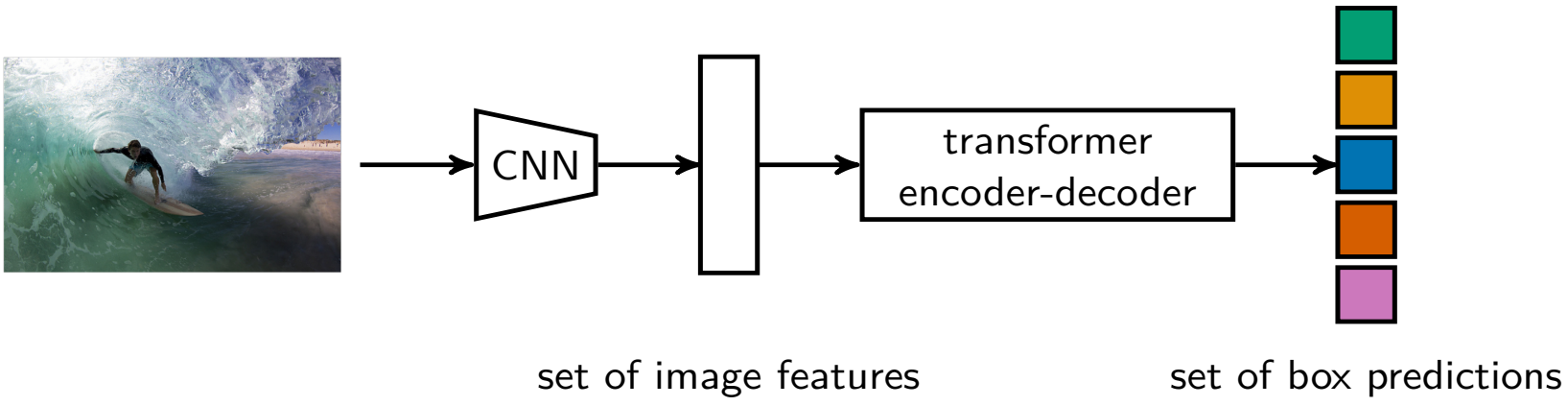
ResNet



Wide ResNet

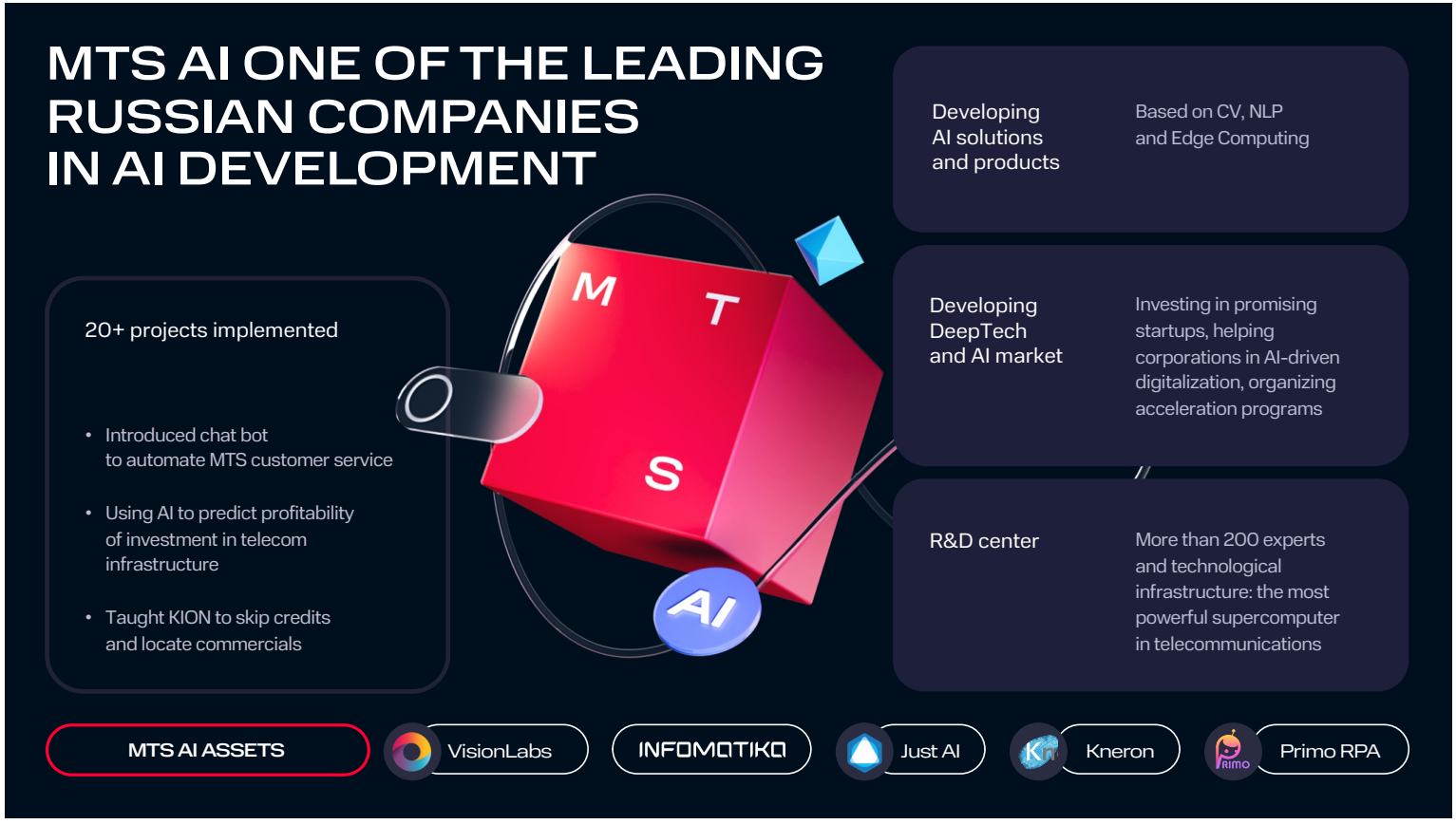
TITLE	CITED BY	YEAR
Wide residual networks S Zagoruyko, N Komodakis arXiv preprint arXiv:1605.07146	7679	2016

DETR



	TITLE	CITED BY	YEAR
☐	End-to-end object detection with transformers N Carion, F Massa, G Synnaeve, N Usunier, A Kirillov, S Zagoruyko European conference on computer vision, 213-229	8261	2020

About MTS AI


MTS AI ONE OF THE LEADING RUSSIAN COMPANIES IN AI DEVELOPMENT
Developing AI solutions and products
Based on CV, NLP and Edge Computing
Developing DeepTech and AI market
Investing in promising startups, helping corporations in AI-driven digitalization, organizing acceleration programs
R&D center
More than 200 experts and technological infrastructure: the most powerful supercomputer in telecommunications
20+ projects implemented
• Introduced chat bot to automate MTS customer service
• Using AI to predict profitability of investment in telecom infrastructure
• Taught KION to skip credits and locate commercials
M
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AI
MTS AI ASSETS
VisionLabs
INFOMATIKA
Just AI
Kneron
Primo RPA

About the fundamental research team at MTS AI

- Natural Language Processing (NLP) Research:
  - Large Language Models (LLM)
  - Generative Pre-trained Transformer (GPT)
- Computer Vision (CV) Research:
  - Graph Reasoning
  - Multi-modal approaches combining image or video with text

Why do a PhD?

- Why do a PhD?
  - Learn to formulate goals and plan research
  - Learn to write articles
  - Enjoy research freedom without product deadlines
  - Interact with other students and graduate students
  - Opportunity for a teaching career
- Why not do a PhD?
  - Less computational resources compared to industry
  - Lower salary during the period of the PhD
    - but possibly higher than the market after
  - Intense competition in publications

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Computer Vision vs Image Processing

Computer Vision:
Involves interpreting and understanding images or videos.

- Input: Image or Video
- Output: Interpretation or understanding

Computer Vision vs Image Processing

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Image Processing:
Focuses on manipulating images or sets of images to enhance or extract information.

- Input: Image or Images
- Output: Modified Image or Images

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- Input: Image or Images
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Main Difference:
Computer Vision is about interpreting images, while Image Processing is about manipulating images to achieve a specific goal.

CV problems: Image Classification

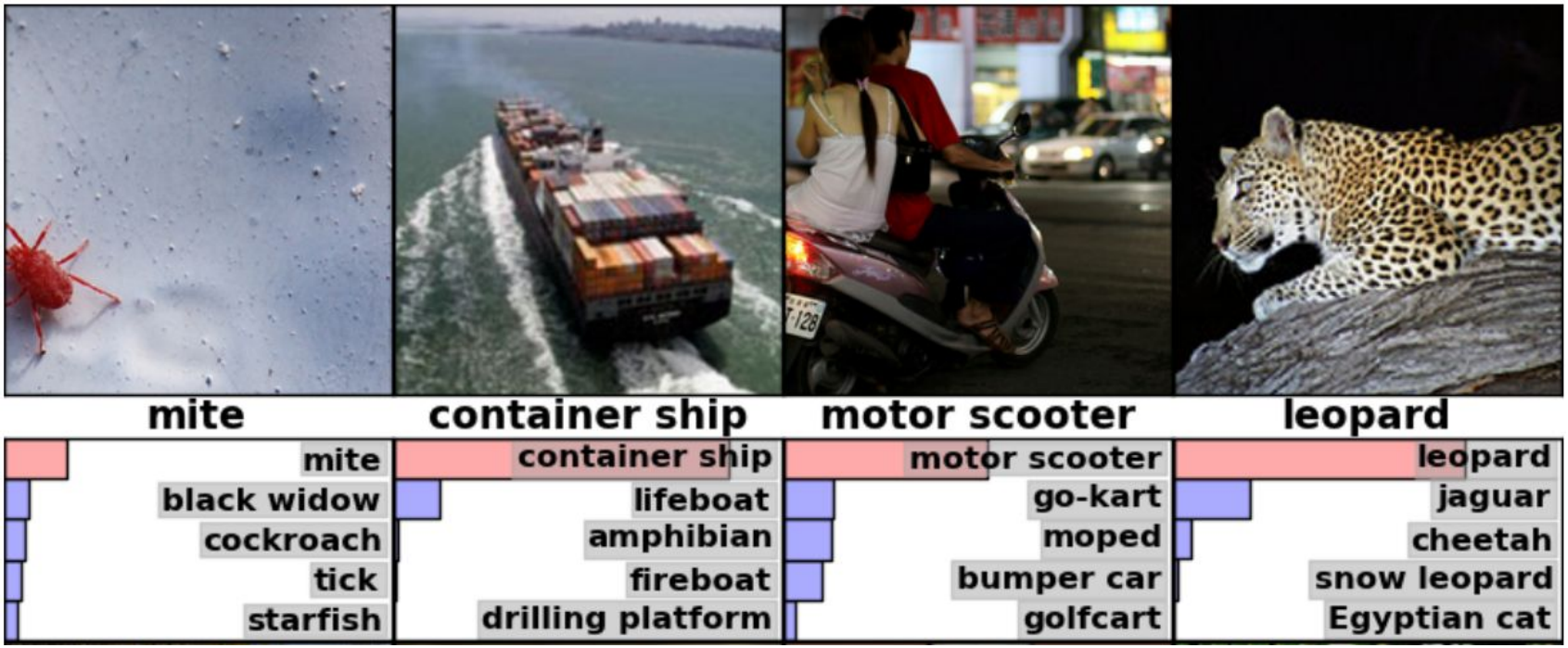
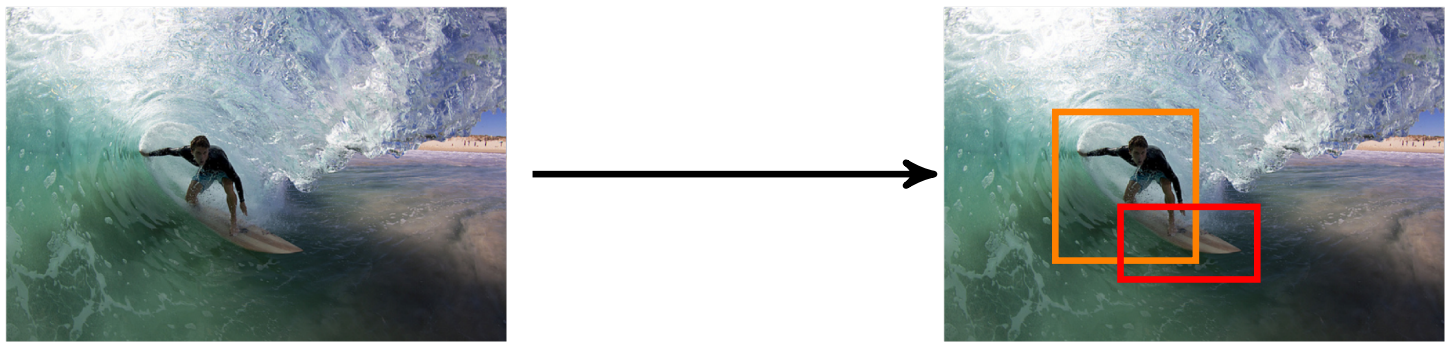


Image source: Krizhevsky et al. ImageNet Classification with Deep Convolutional Neural Networks

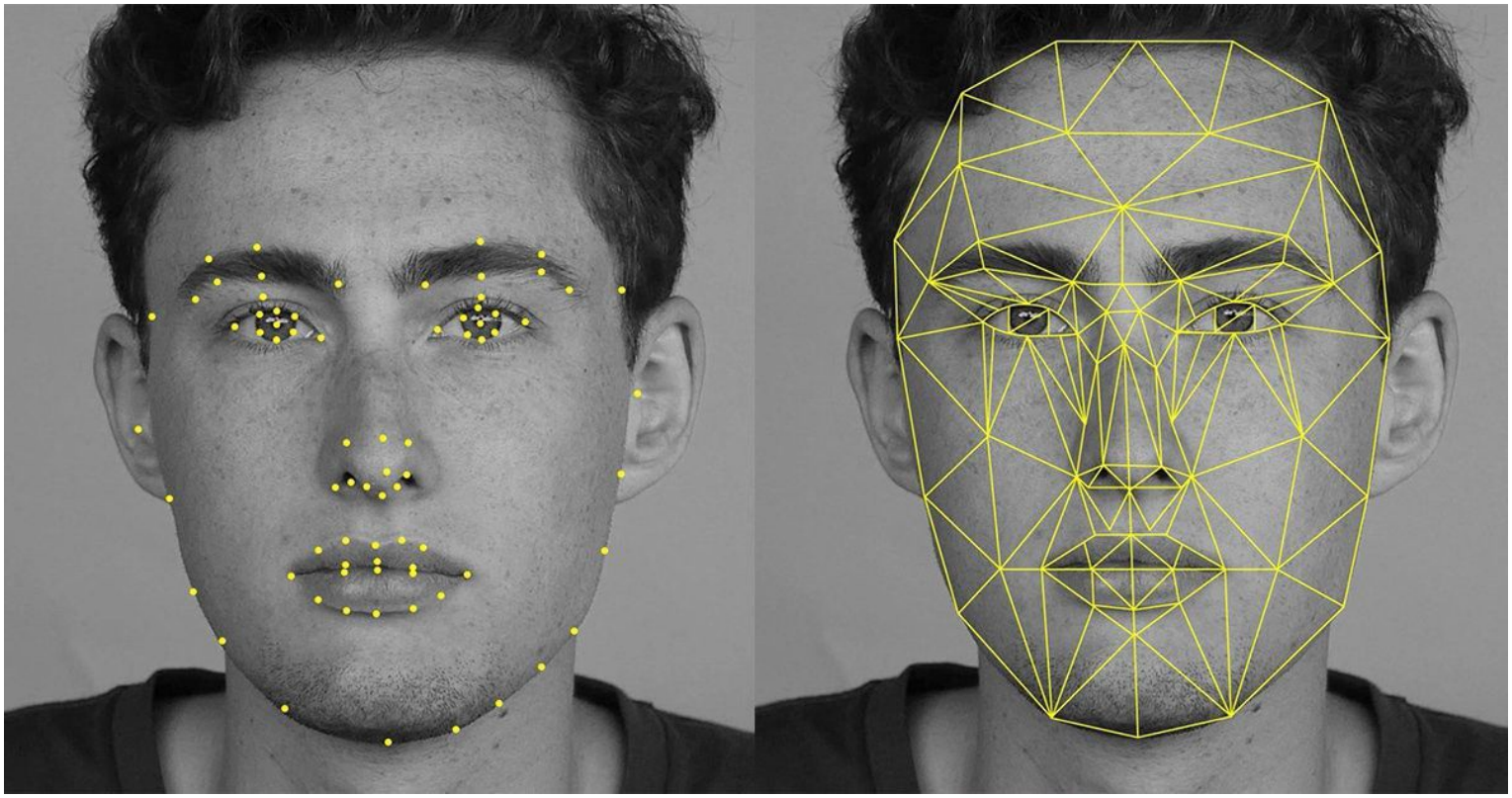
Object Detection Task

- Given an input image, predict **a set** of bounding boxes with corresponding categories

set(("person", bbox_1), ("surfboard", bbox_2))



CV problems: Keypoints Detection



CV problems: Segmentation

Image source: https://www2.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/resources.html

CV problems: Semantic Segmentation

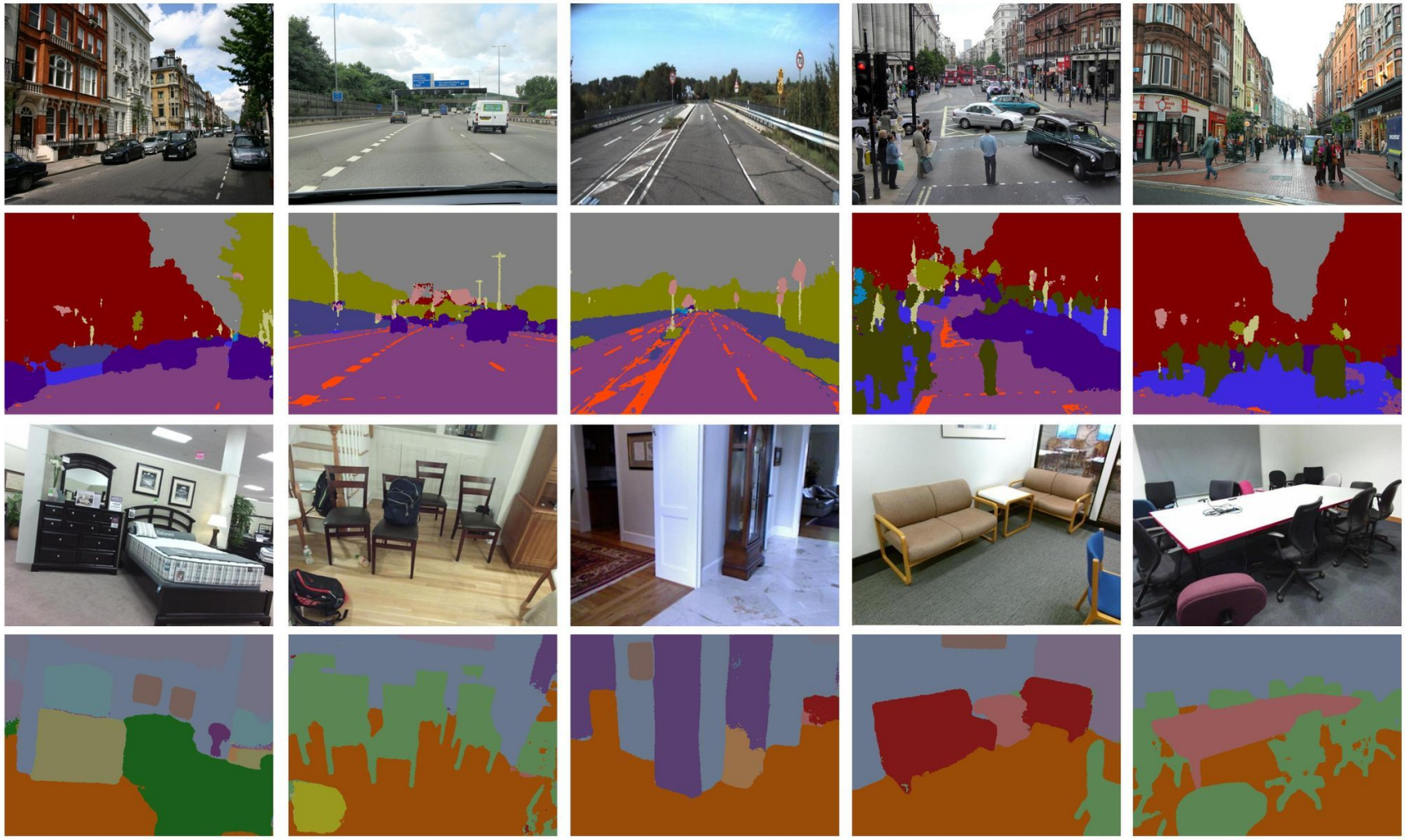


Image source: https://mi.eng.cam.ac.uk/projects/segnet/

CV problems: Instance Segmentation

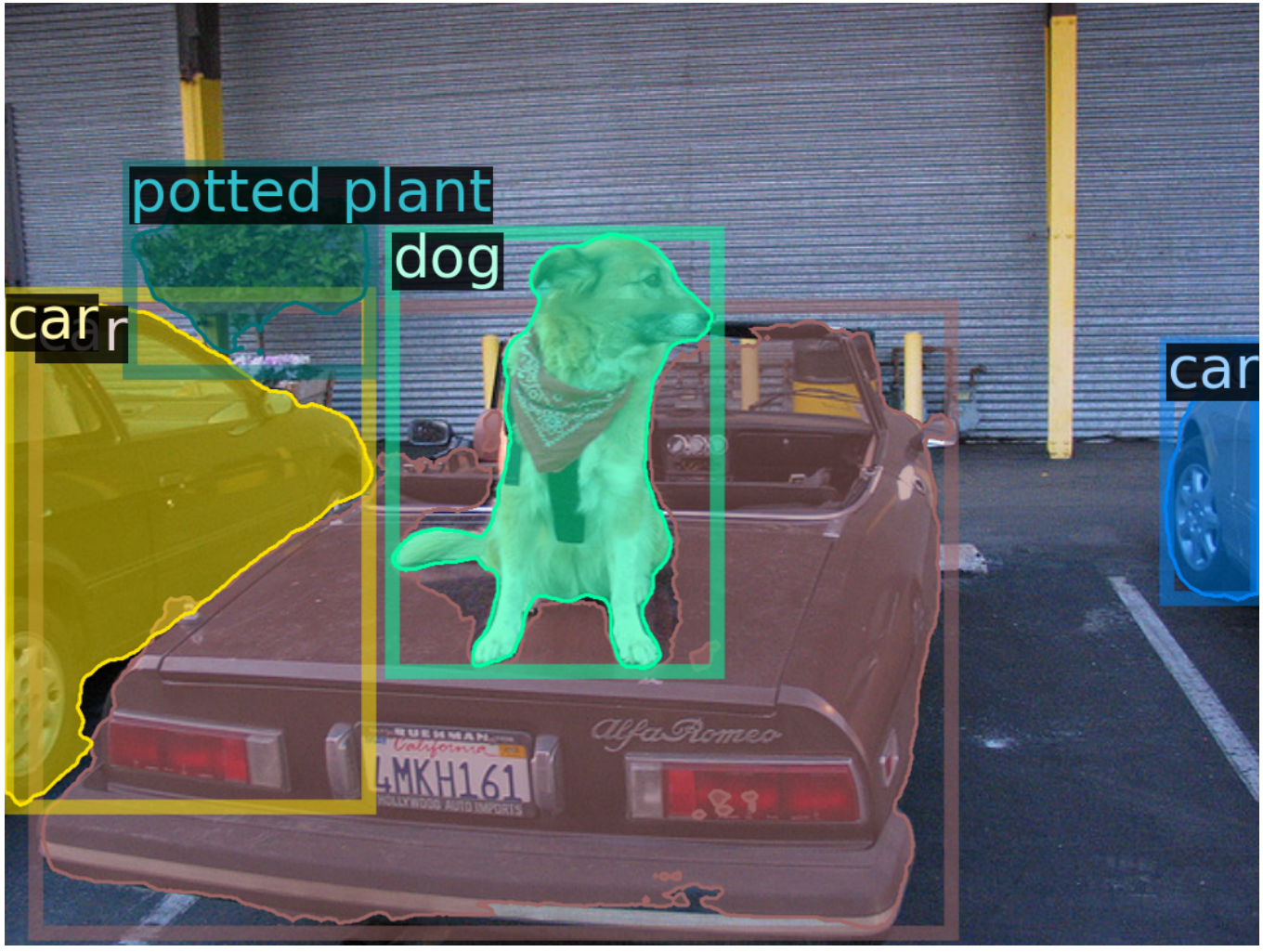
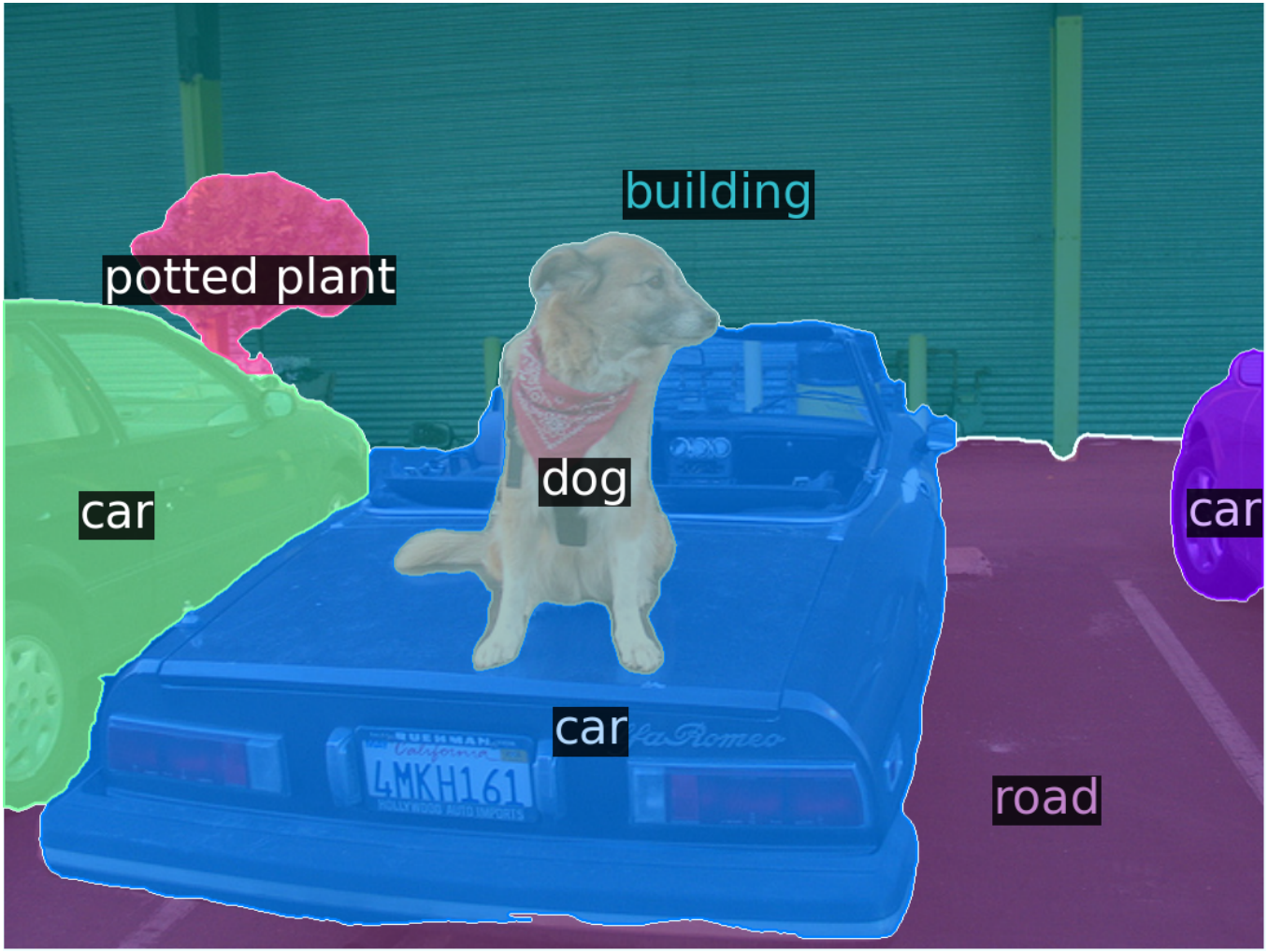


Image source: Vinyals et al. Show and Tell: A Neural Image Caption Generator

CV problems: Panoptic Segmentation



CV problems: Image Captioning

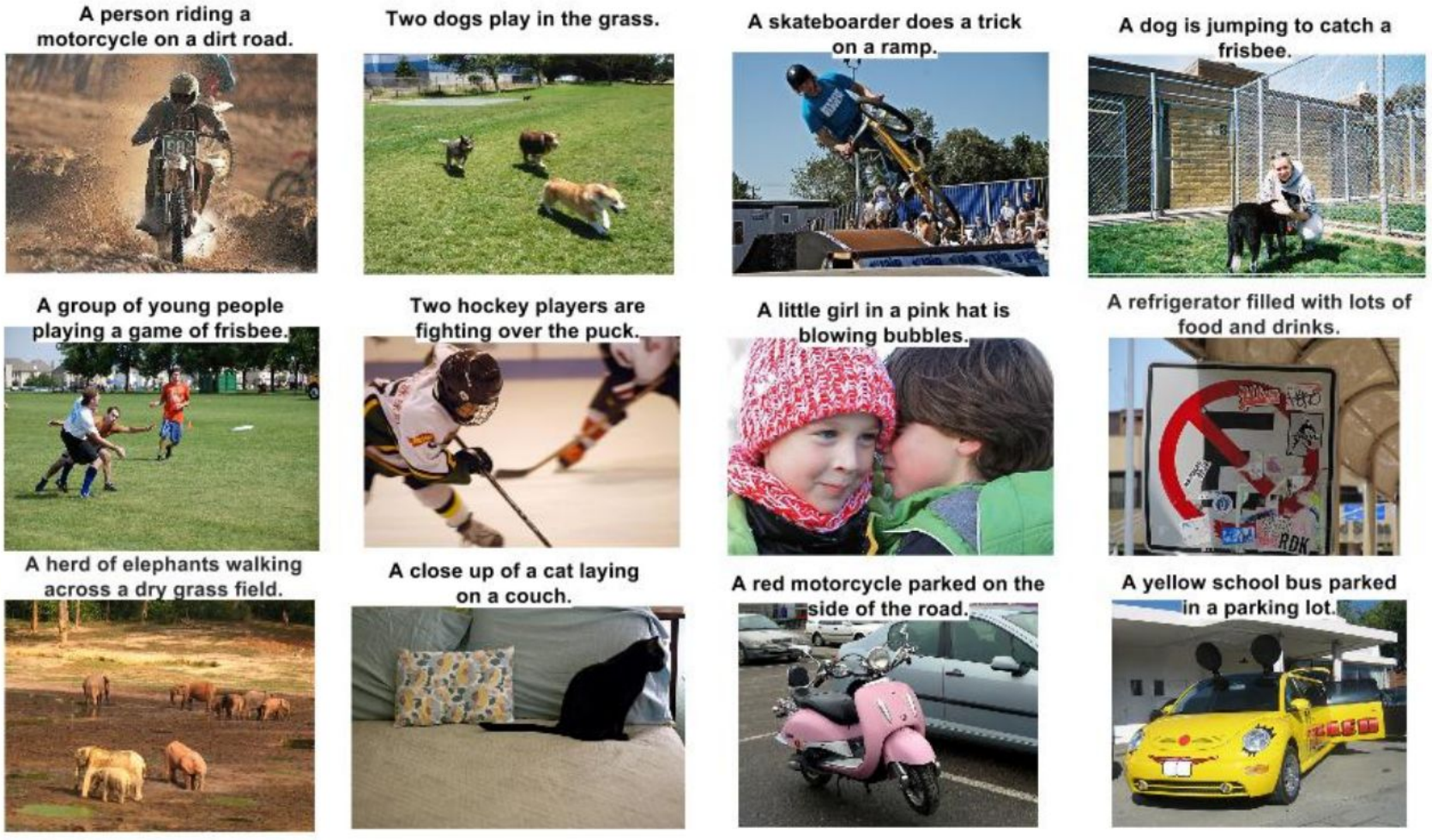


Image source: Vinyals et al. Show and Tell: A Neural Image Caption Generator

CV problems: Depth Estimation

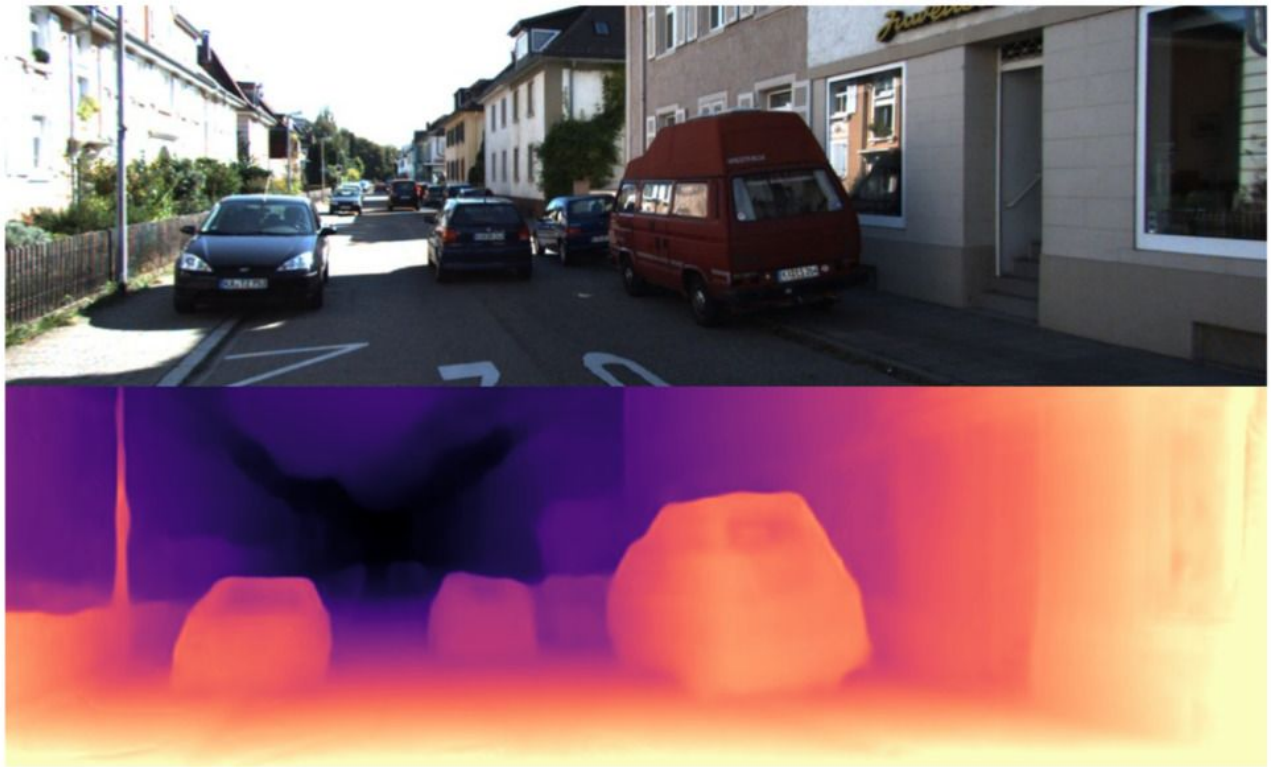
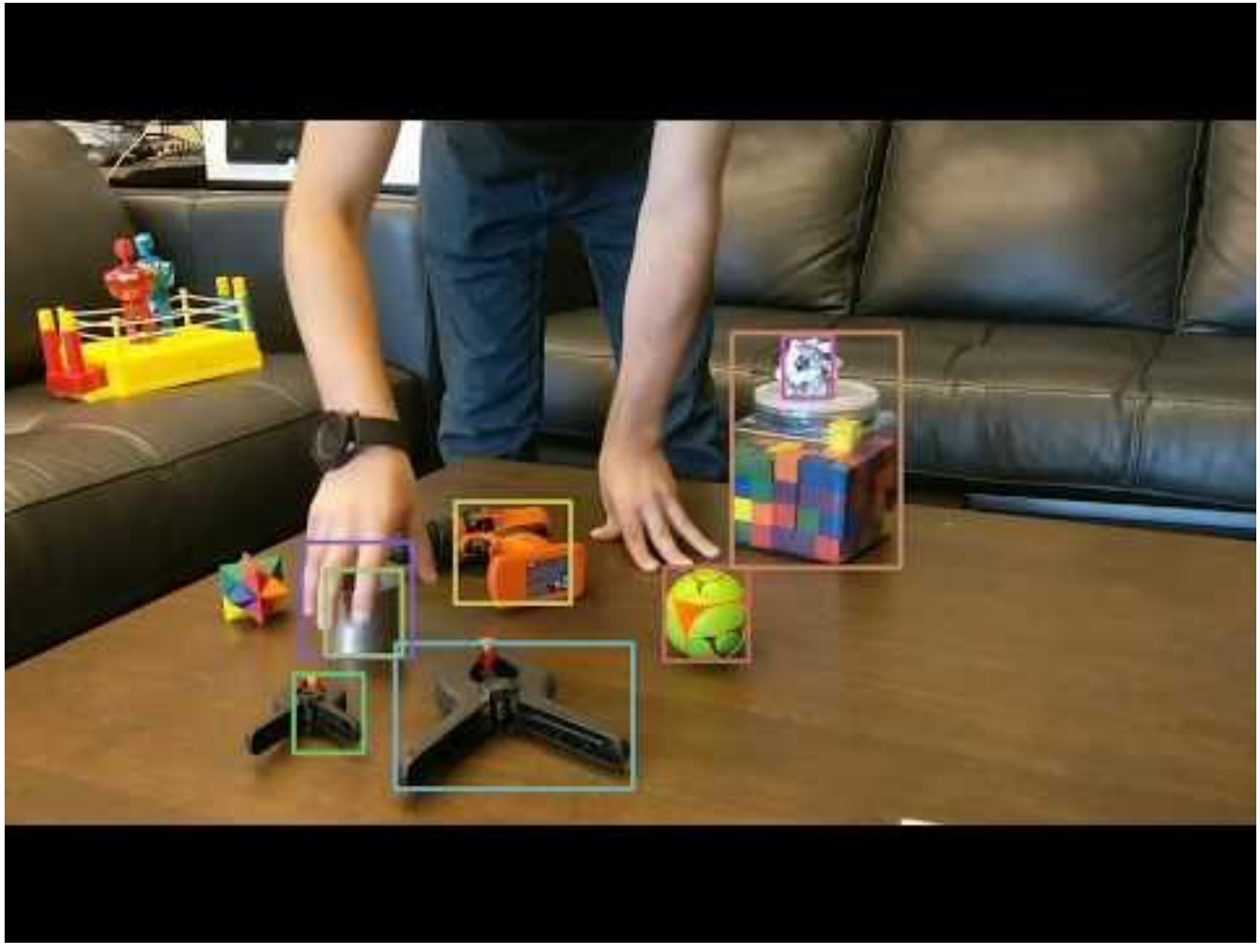


Image source: Vinyals et al. Show and Tell: A Neural Image Caption Generator

CV problems: Object Tracking



Video source: https://arxiv.org/abs/1705.06368

Image Processing problems: Super-resolution

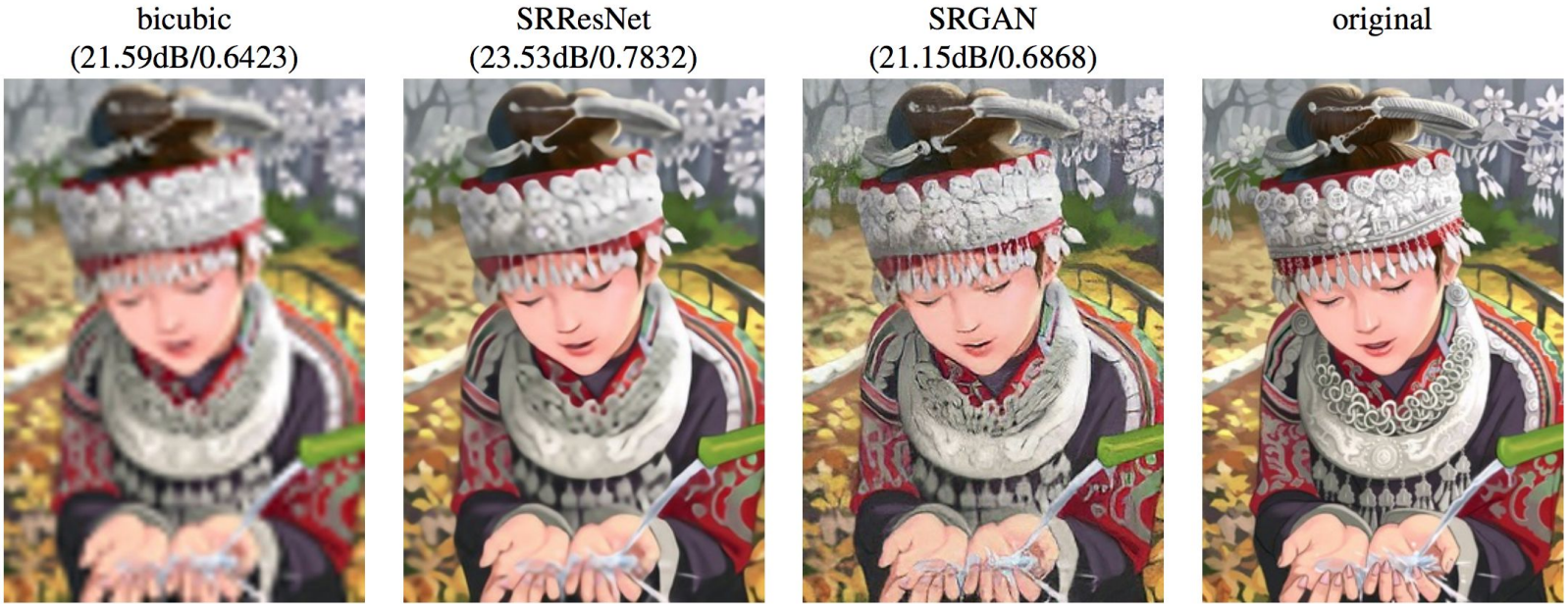


Figure 2: From left to right: bicubic interpolation, deep residual network optimized for MSE, deep residual generative adversarial network optimized for a loss more sensitive to human perception, original HR image. Corresponding PSNR and SSIM are shown in brackets. [4× upscaling]

Image source: Ledig et al. Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network

Image Processing problems: Inpainting

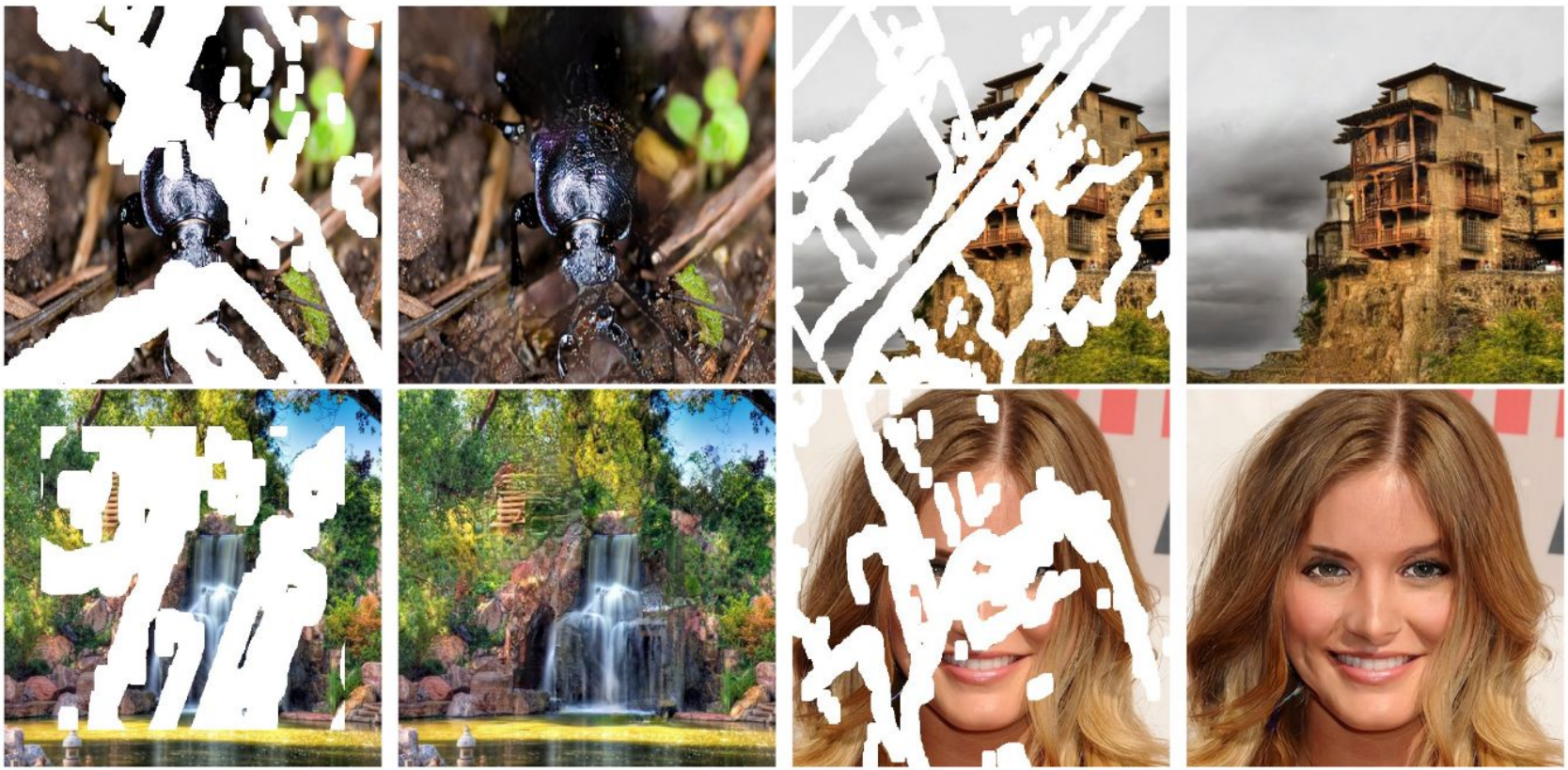


Image source: Liu et al. Image Inpainting for Irregular Holes Using Partial Convolutions

Image Processing problems: Image generation

Image sources: Gravity, Jurassic Park, Avatar

Image Processing problems: Style Transfer

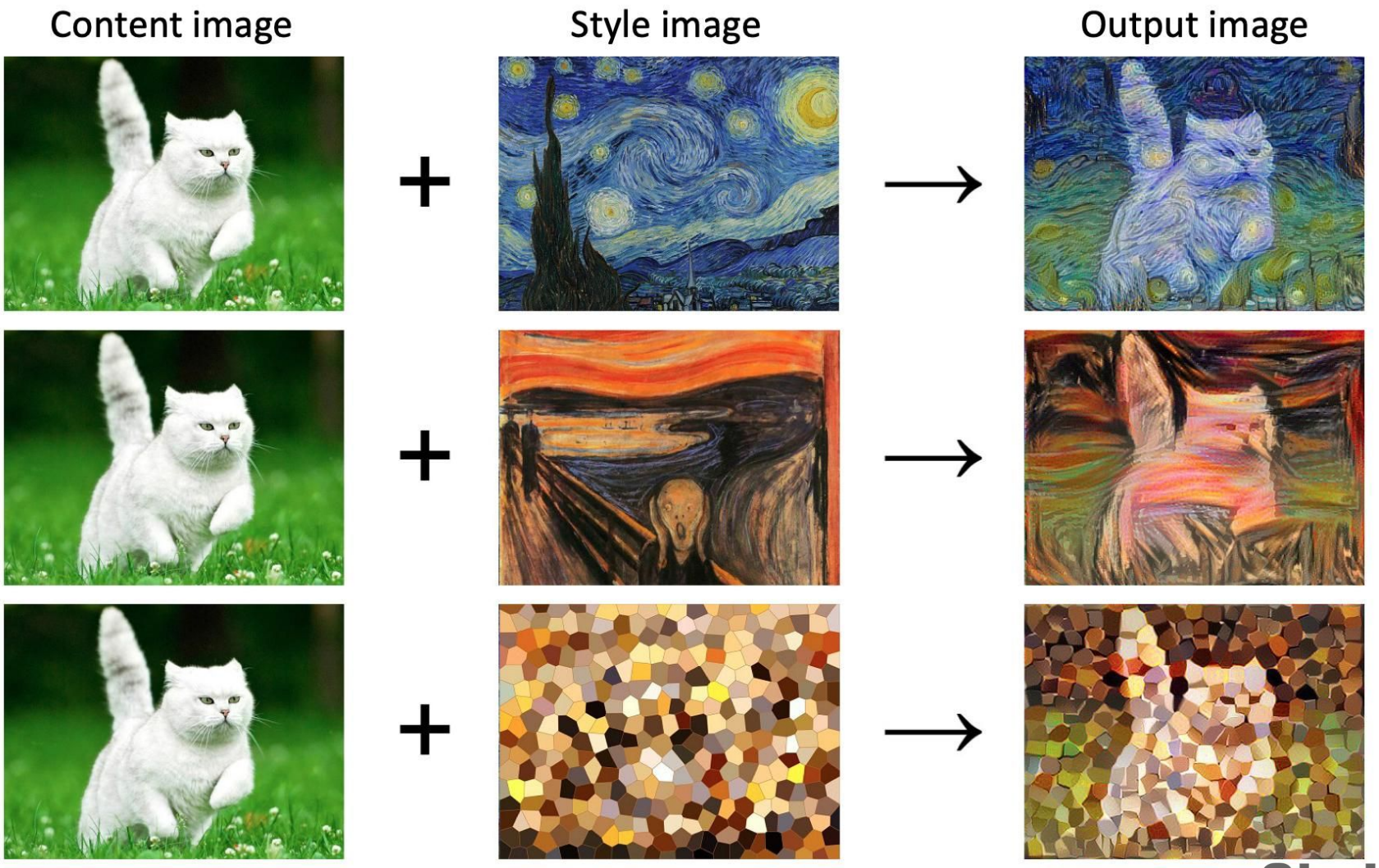
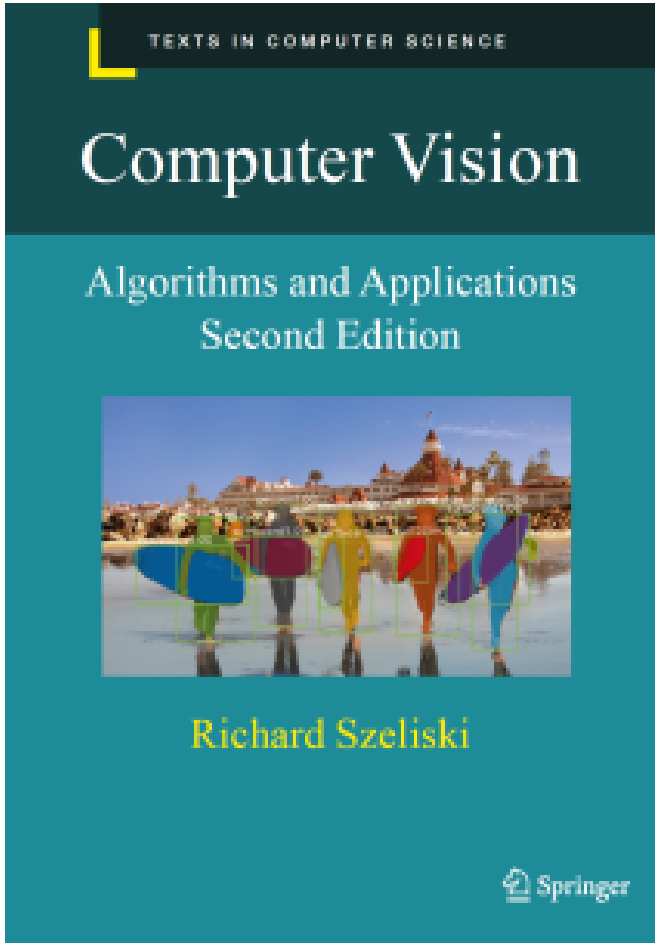


Image sources: https://blog.godatadriven.com/images/how-to-style-transfer

Image Processing problems: Face generation

Image sources: `https://arxiv.org/abs/1812.04948`

Reading



The book is available online

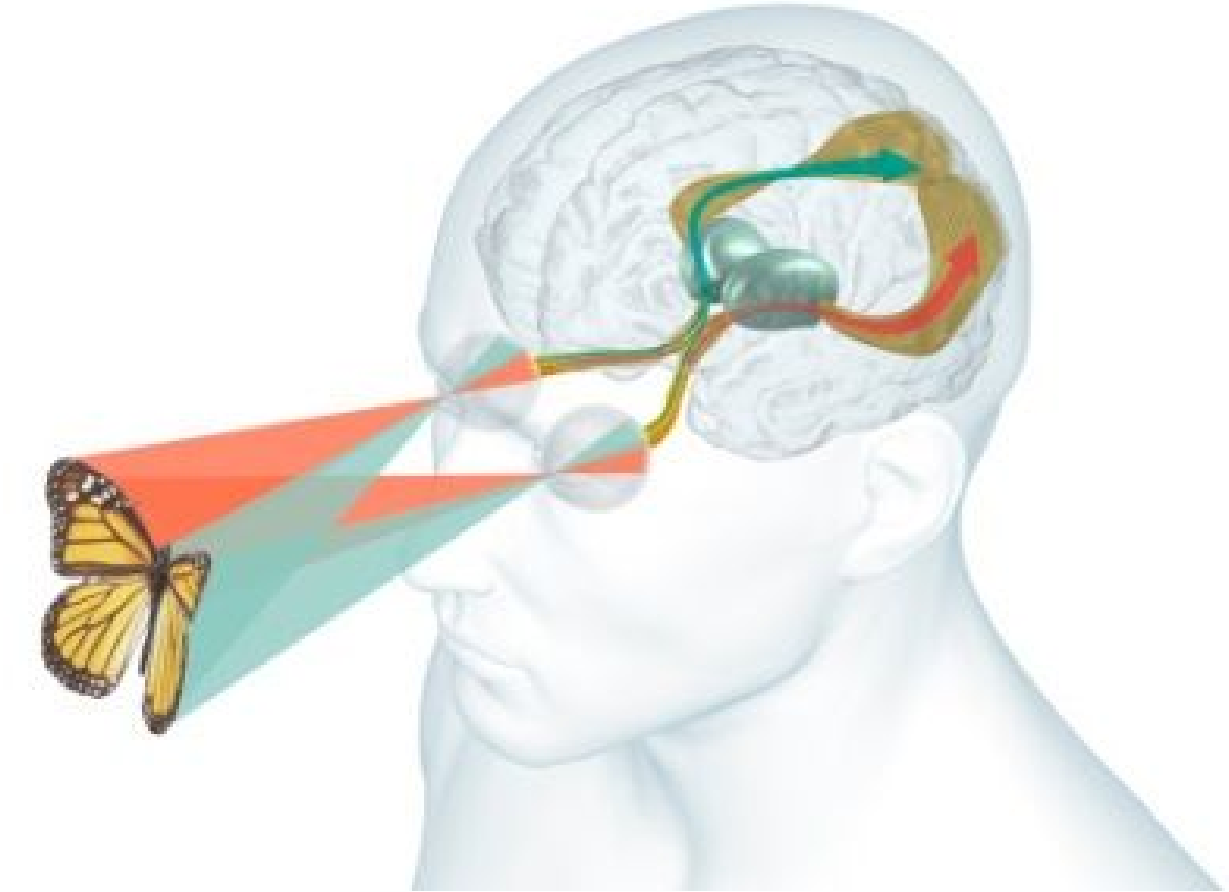
Good reading for this lecture.

`https://szeliski.org/Book`

2nd edition

What is an image?

How do humans perceive images?



How do humans perceive images?

What are colors of this dress?

How do humans perceive images?

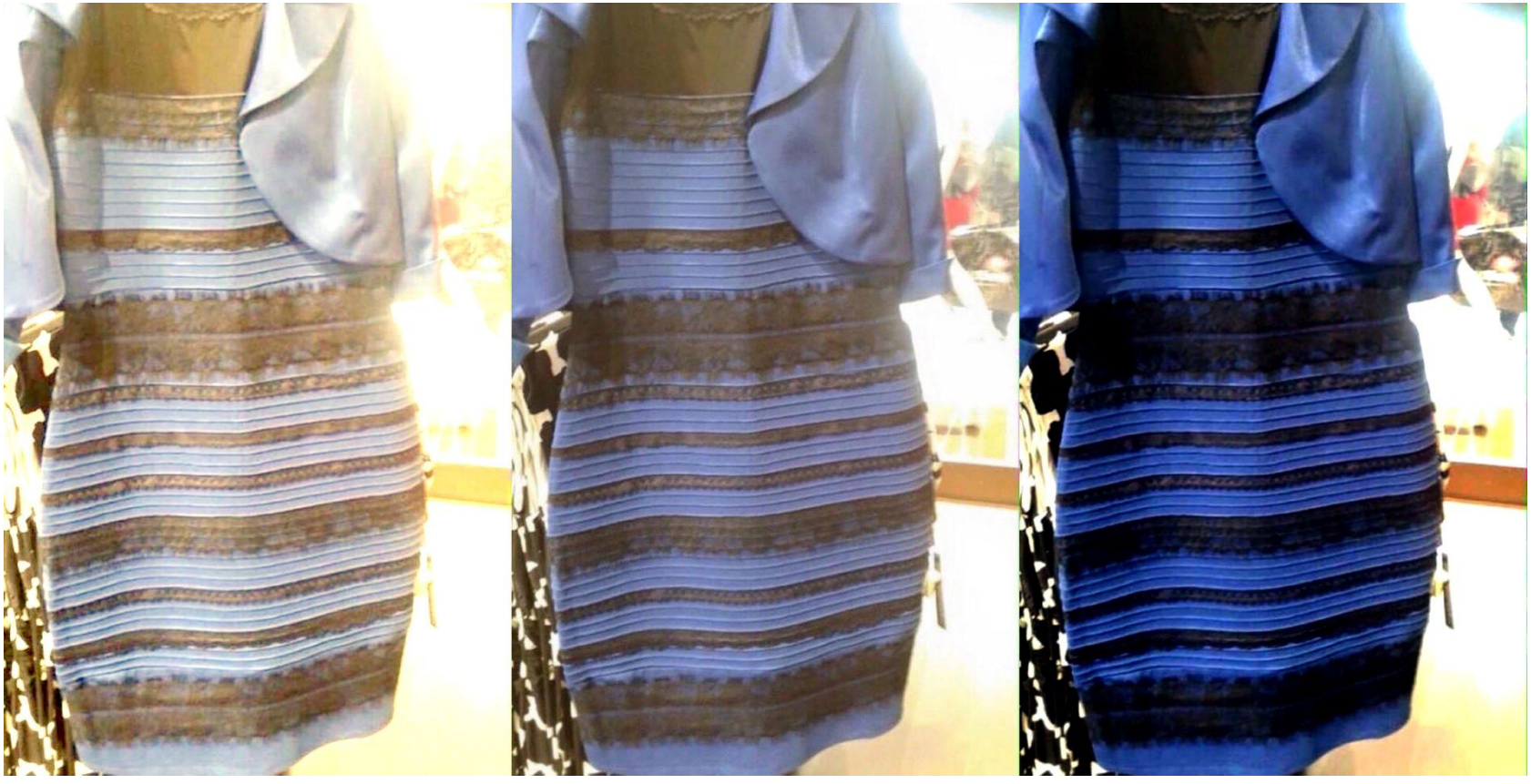
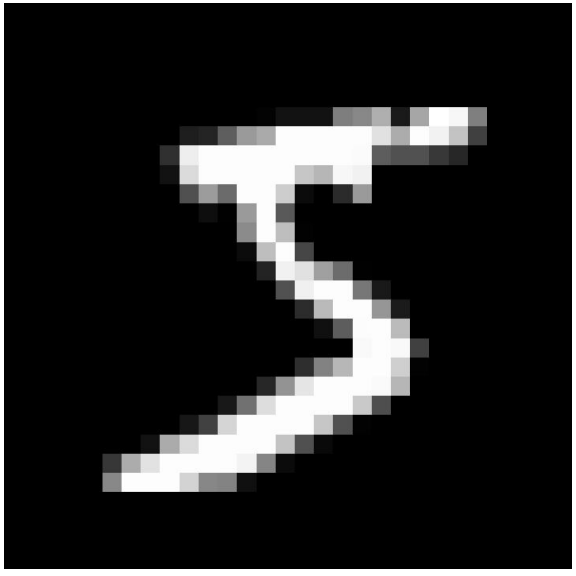


Image perception is subjective: different people will see this dress (left) either as blue and black or as white and brown.

How do humans perceive images?

Image perception is subjective: do you see a young woman or an old one?

How do machines store images?



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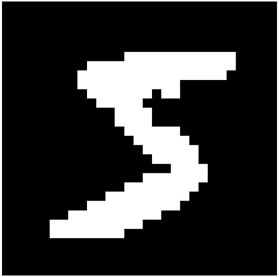
How do machines store images: Binary



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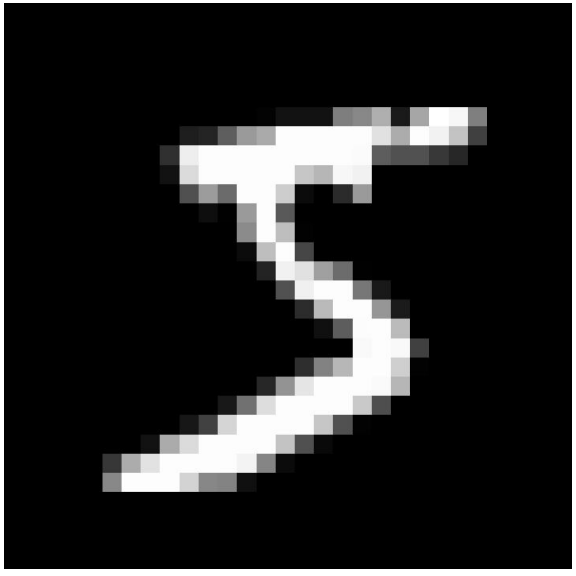
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

How do machines store images: Grayscale



```
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   3  18  18  18 126 136 175  26 166 255 247 127   0   0   0   0
  0   0   0   0   0   0   0   0  30  36  94 154 170 253 253 253 253 253 225 172 253 242 195  64   0   0   0   0
  0   0   0   0   0   0   0  49 238 253 253 253 253 253 253 253 253 251  93  82  82  56  39   0   0   0   0   0
  0   0   0   0   0   0   0  18 219 253 253 253 253 253 198 182 247 241   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0  80 156 107 253 253 205  11   0  43 154   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0  14   1 154 253  90   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0 139 253 190   2   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0  11 190 253  70   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0  35 241 225 160 108   1   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0  81 240 253 253 119  25   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0  45 186 253 253 150  27   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  16  93 252 253 187   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0 249 253 249  64   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  46 130 183 253 253 207   2   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0  39 148 229 253 253 253 250 182   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0  24 114 221 253 253 253 253 201  78   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0  23  66 213 253 253 253 253 198  81   2   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0  18 171 219 253 253 253 253 195  80   9   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0  55 172 226 253 253 253 253 244 133  11   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0 136 253 253 253 212 135 132  16   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

How do machines store images: RGB

Red

Green

Blue

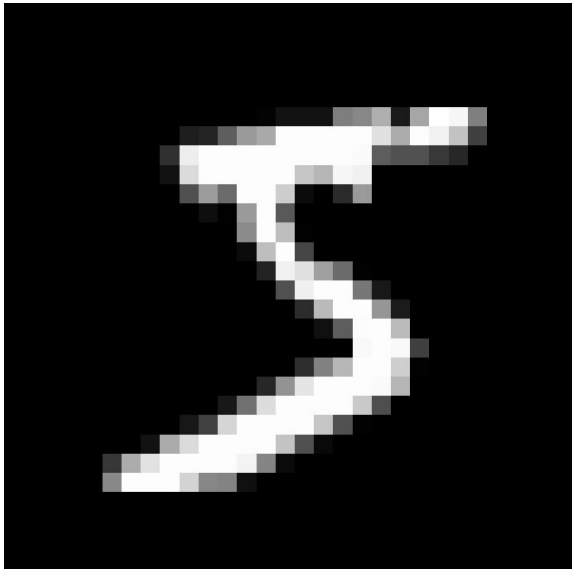
How do machines store images: RGB

Red

Green

Blue

How do machines store images?

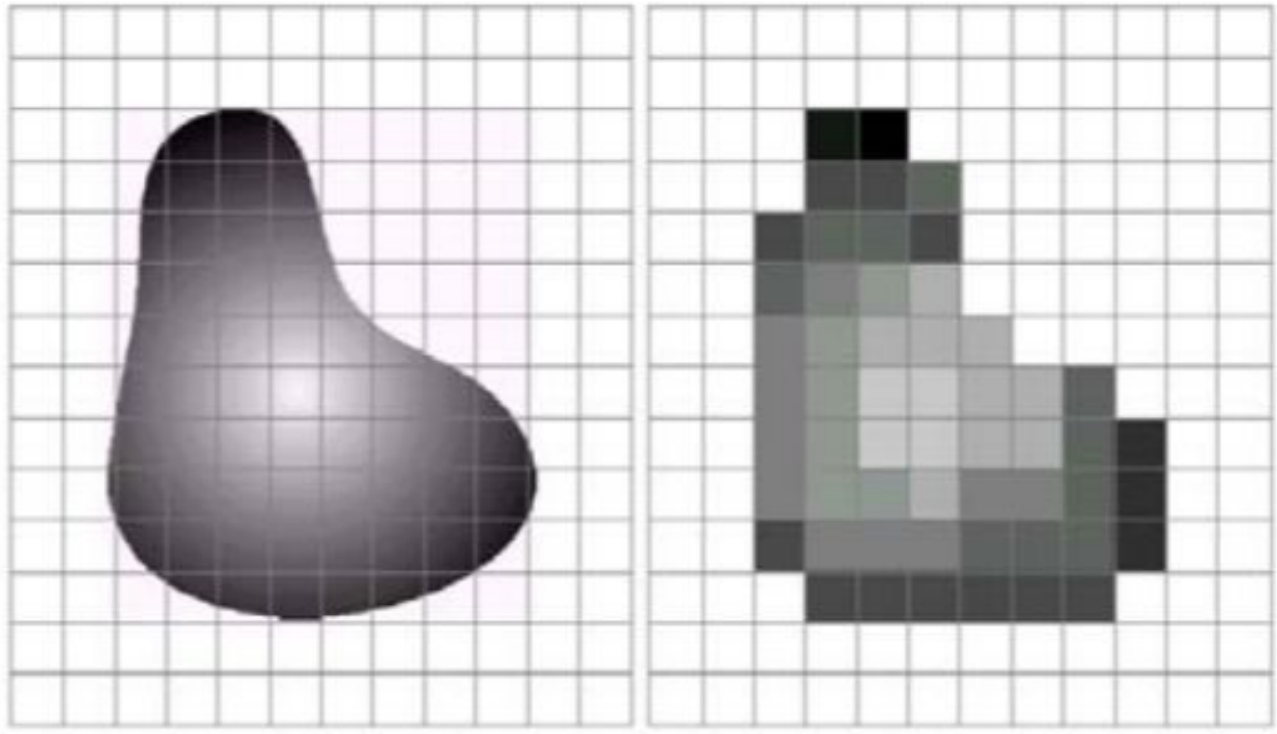


```
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   3  18  18  18 126 136 175  26 166 255 247 127   0   0   0   0
  0   0   0   0   0   0   0   0  30  36  94 154 170 253 253 253 253 253 225 172 253 242 195  64   0   0   0   0
  0   0   0   0   0   0   0  49 238 253 253 253 253 253 253 253 253 251  93  82  82  56  39   0   0   0   0   0
  0   0   0   0   0   0   0  18 219 253 253 253 253 253 198 182 247 241   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0  80 156 107 253 253 205  11   0  43 154   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0  14   1 154 253  90   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0 139 253 190   2   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0  11 190 253  70   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0  35 241 225 160 108   1   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0  81 240 253 253 119  25   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0  45 186 253 253 150  27   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  16  93 252 253 187   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0 249 253 249  64   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0  46 130 183 253 253 207   2   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0  39 148 229 253 253 253 250 182   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0  24 114 221 253 253 253 253 201  78   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0  23  66 213 253 253 253 253 198  81   2   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0  18 171 219 253 253 253 253 195  80   9   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0  55 172 226 253 253 253 253 244 133  11   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0 136 253 253 253 212 135 132  16   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
  0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
```

This a matrix. But from mathematical points of view we have another option.

How do machines store images?

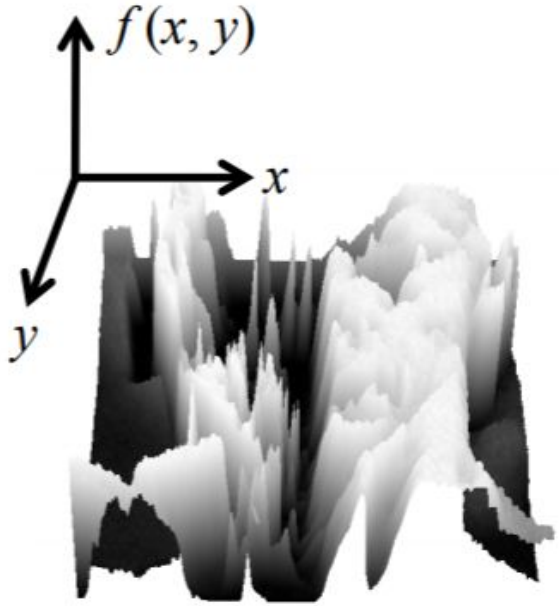
We can consider image as a discrete representation of a 2D function.



Source: D. Hoiem

How do machines store images?

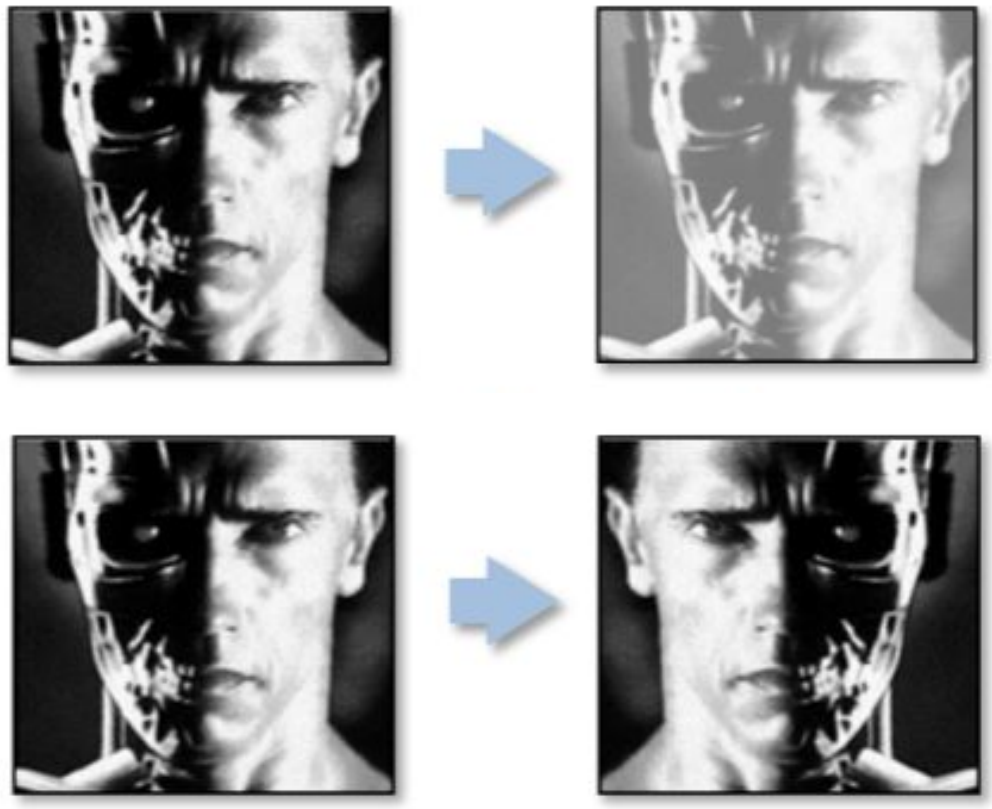
We can consider image as a discrete representation of a 2D function.



Source: N. Snavely

How do machines store images?

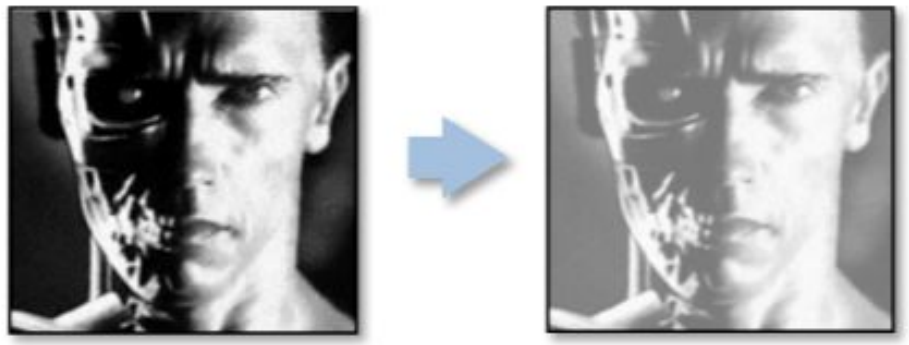
So some standard operations can be applied



Source: N. Snavely

How do machines store images?

So some standard operations can be applied



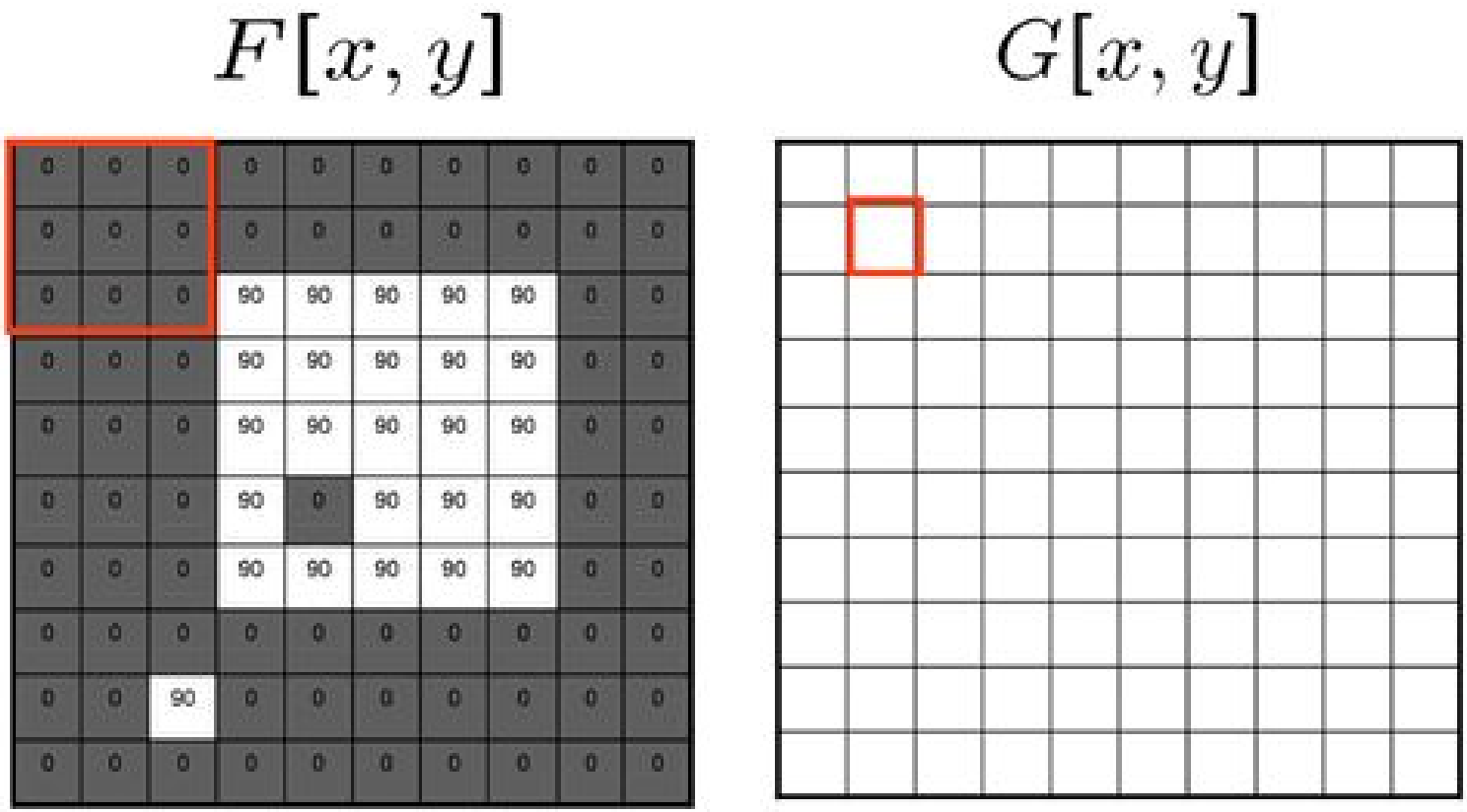
$$g(x,y) = f(x,y) + 20$$

$$g(x,y) = f(-x,y)$$

Source: N. Snavely

Table of Contents

Convolutions: moving average example



Source: S. Seitz

Convolutions: moving average example

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Source: S. Seitz

Convolutions: some math

Let us start with general 2 dimensional functions

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (1)$$

Convolutions: some math

Let us start with general 2 dimensional functions

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (1)$$

But images are discrete representations of 2D functions

$$(f * g)(x, y) = \sum_{x'=-\infty}^{\infty} \sum_{y'=-\infty}^{\infty} f(x', y') g(x - x', y - y') \qquad (2)$$

Convolutions: some math

Let us start with general 2 dimensional functions

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (1)$$

But images are discrete representations of 2D functions

$$(f * g)(x, y) = \sum_{x'=-\infty}^{\infty} \sum_{y'=-\infty}^{\infty} f(x', y') g(x - x', y - y') \qquad (2)$$

And, finally, images have finite support

$$(f * g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y') \qquad (3)$$

Convolutions: some math

Question: what is g for moving average?

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (4)$$

Convolutions: some math

Question: what is g for moving average?

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (4)$$

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This operation & the corresponding matrix are also called filters, kernels, convolutional matrices.

Convolutions: moving average

How does it modify images?

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

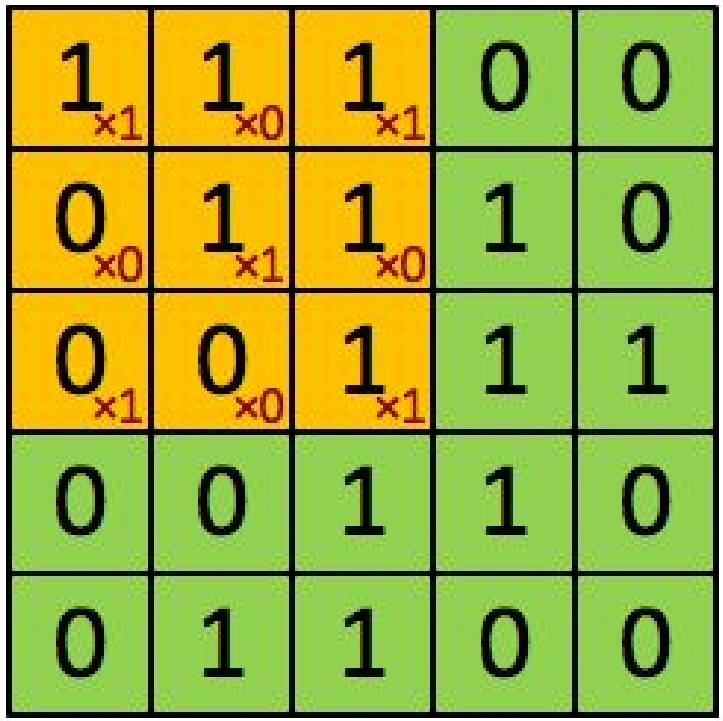
Convolutions: moving average

How does it modify images?

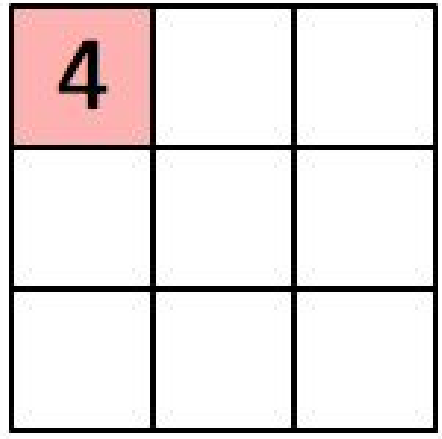
$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Source: J. Niebles

Convolutions in 2D



Image



Convolved Feature

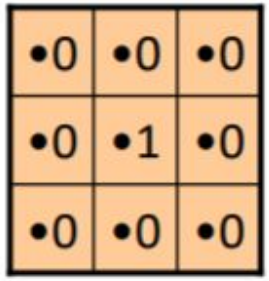
Convolutions in 2D: Practical exercise 1

Let's implement a Python function that convolves an image with a kernel.

$$(f * g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y') \tag{5}$$

Convolutions: examples of filters

How does it modify images?



Convolutions: examples of filters

How does it modify images?

•0	•0	•0
•0	•1	•0
•0	•0	•0

Original

*

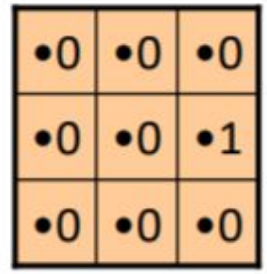
•0	•0	•0
•0	•1	•0
•0	•0	•0

=

Filtered
(no change)

Convolutions: examples of filters

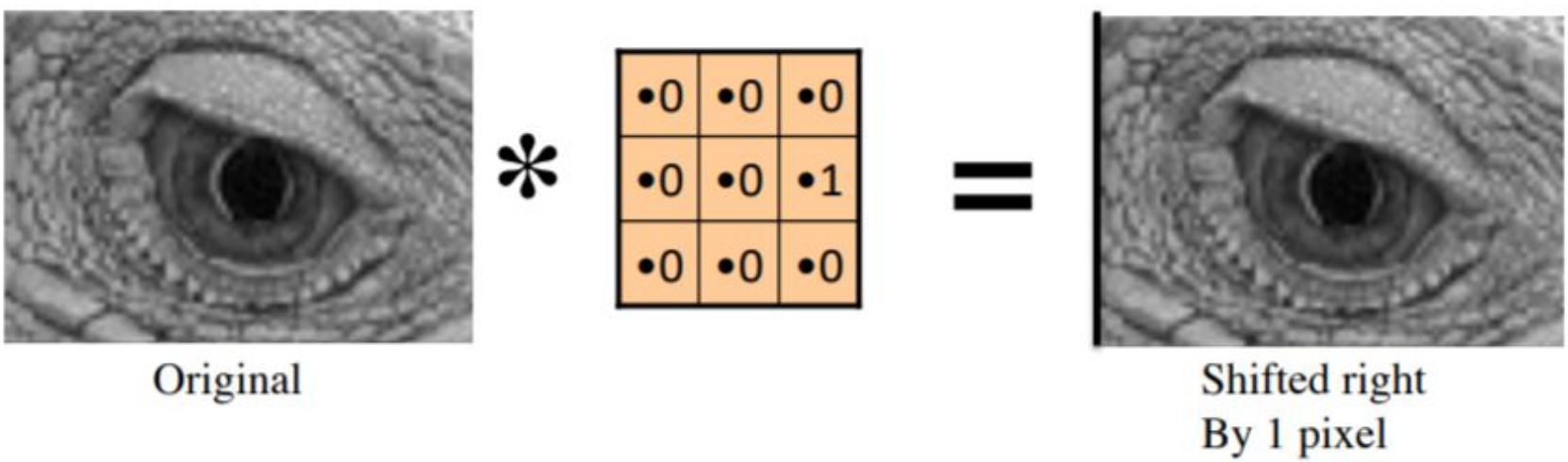
How does it modify images?



Convolutions: examples of filters

How does it modify images?

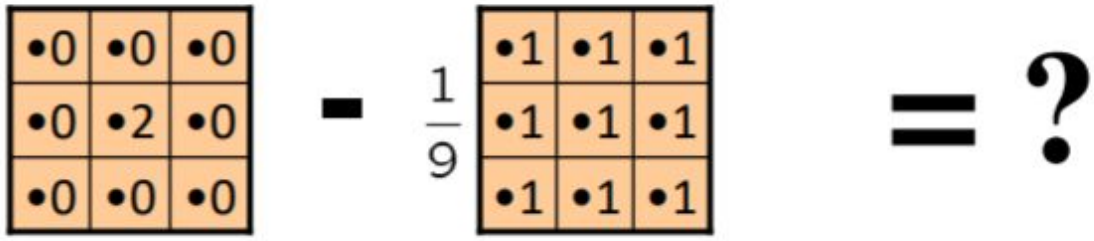
•0	•0	•0
•0	•0	•1
•0	•0	•0



Source: J. Niebles

Convolutions: examples of filters

How does it modify images?



Convolutions: examples of filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

(Note that filter sums to 1)

"details of the image"

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Source: J. Niebles

Convolutions: examples of filters

- Let's add it back:

Convolutions: examples of filters

Original

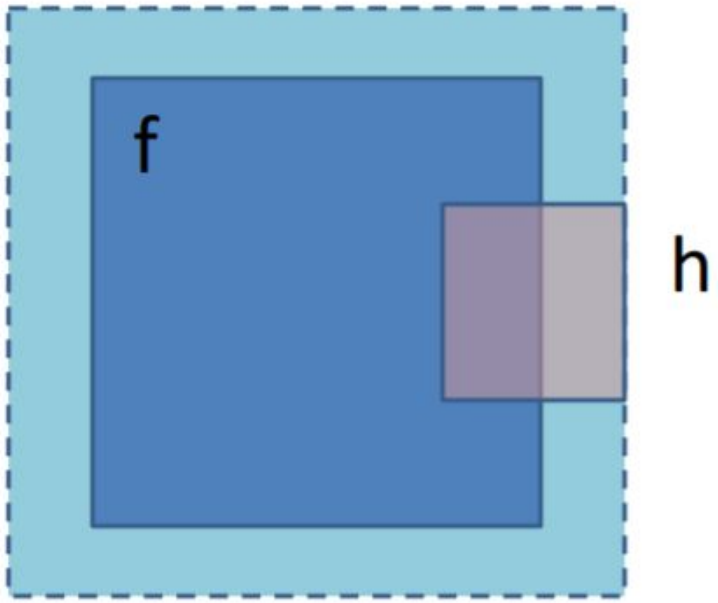
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

(Note that filter sums to 1)

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$

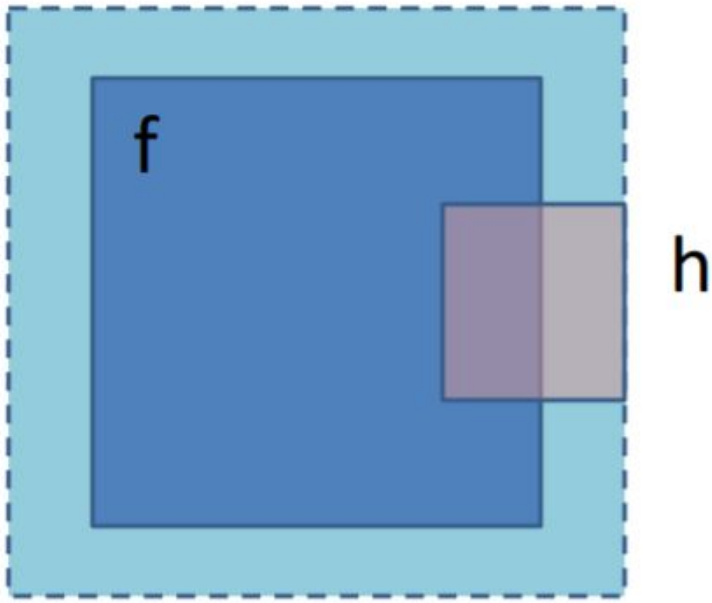
Convolutions: boundary conditions



How to process image boundaries?

Source: J. Niebles

Convolutions: boundary conditions



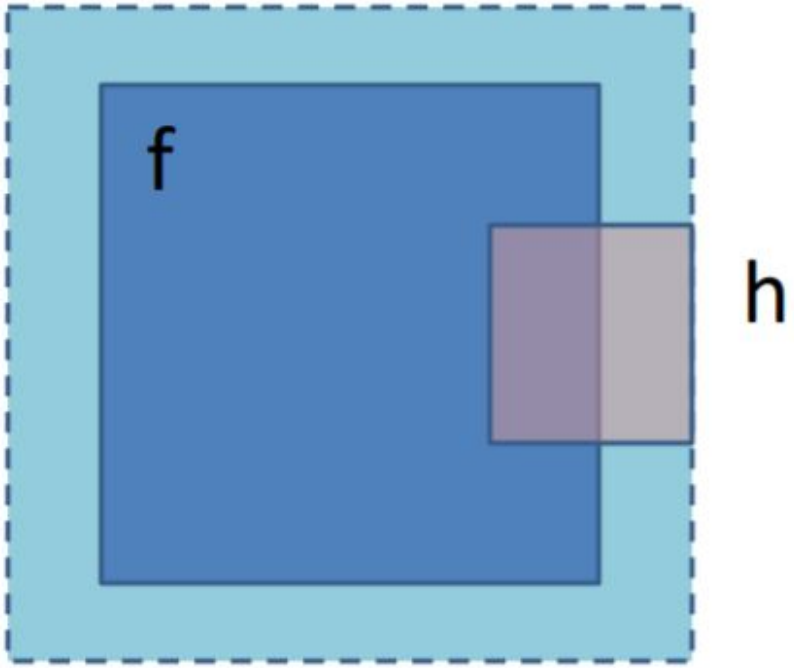
Source: J. Niebles

How to process image boundaries?

- **Reduce size** of the processed image
- **Add zeros** or constants
- Mirror the image

Convolutions: boundary conditions - Task 2

Let's extend our convolution function with simple zero padding.



Source: J. Niebles

Welcome!

Course telegram channel