Introduction to Computer Vision

Lecture 2. Convolution & filters, template matching, edge detection

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Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

1	2	0	-1

Output:

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

1	2	0	-1

Output:

9						

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

1	2	0	-1

Output:

9	0					

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

1	2	0	-1

Output:

9	0	1				

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

				1	2	0	-1		

Output:

9	0	1	3			

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

1	2	0	-1

Output:

9	0	1	3	-5		

Source: François Fleuret's deep learning course

Convolution Example

Input:

1	4	-1	0	2	-2	1	3	3	1

Filter:

						1	2	0	-1

Output:

9	0	1	3	-5	-3	

Source: François Fleuret's deep learning course

Convolution Example

How to calculate convolution output size?

- 1D: Input size: n, filter size: k, output size:

Convolution Example

How to calculate convolution output size?

- 1D: Input size: n, filter size: k, output size: $n - k + 1$

Convolution Example

How to calculate convolution output size?

- 1D: Input size: n, filter size: k, output size: $n - k + 1$
- 2D: Input size: $m \times n$, filter size: $k \times k$, output size:

Convolution Example

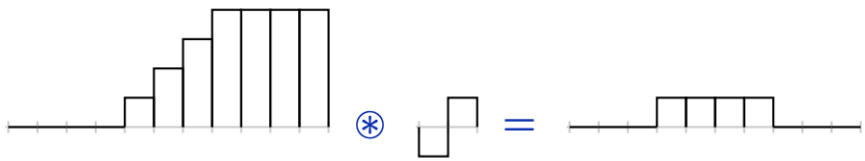
How to calculate convolution output size?

- 1D: Input size: n, filter size: k, output size: $n - k + 1$
- 2D: Input size: $m \times n$, filter size: $k \times k$, output size: $m - k + 1 \times n - k + 1$

Convolution Example

Convolution can implement in particular differential operators, e.g.

$$(0, 0, 0, 0, 1, 2, 3, 4, 4, 4, 4) \circledast (-1, 1) = (0, 0, 0, 1, 1, 1, 1, 0, 0, 0)$$

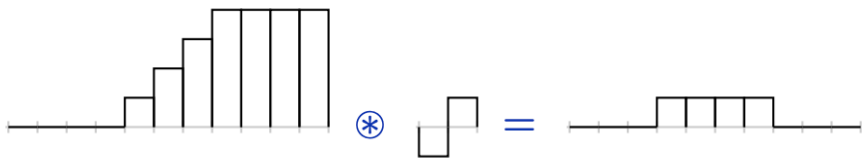


Source: François Fleuret's deep learning course

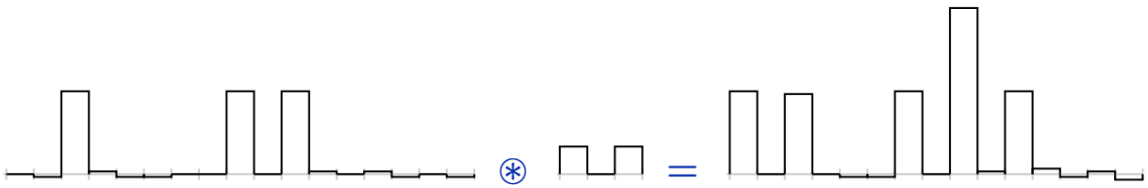
Convolution Example

Convolution can implement in particular differential operators, e.g.

$$(0,0,0,0,1,2,3,4,4,4,4) \circledast (-1,1) = (0,0,0,1,1,1,1,0,0,0)$$

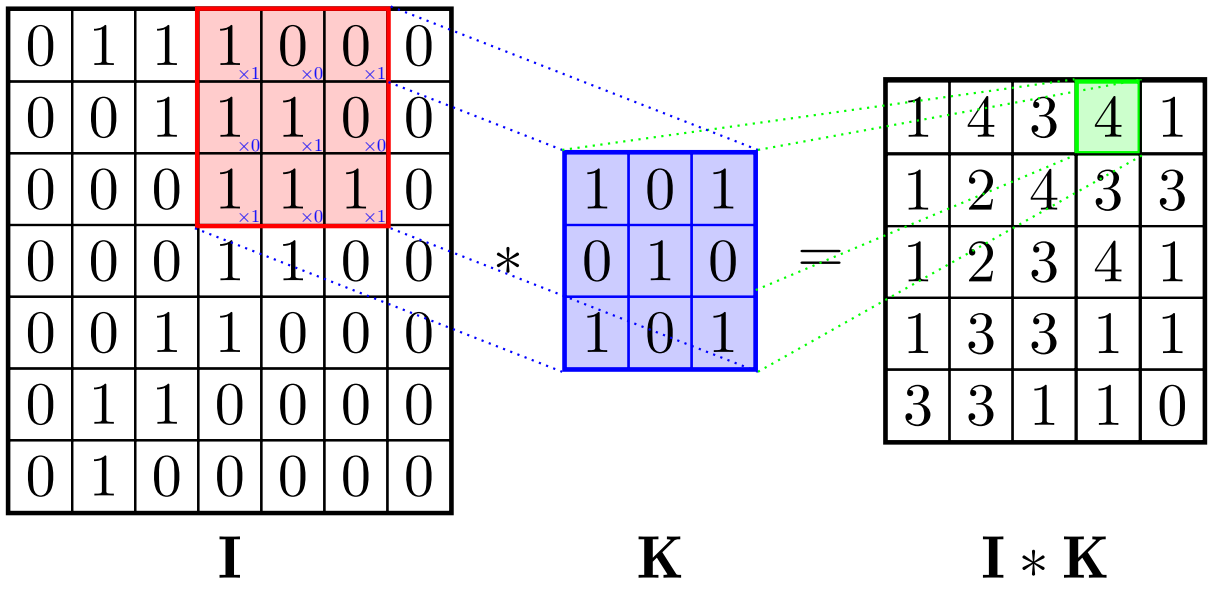


or crude “template matcher”, e.g.



Source: François Fleuret’s deep learning course

Convolutions in 2D



Source: https://tex.stackexchange.com/questions/522118/visualizing-matrix-convolution

Convolutions: some math

Question: what is g for moving average?

$$(f * g)(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x - x', y - y') dx'\, dy' \qquad (1)$$

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This operation & the corresponding matrix are also called filters, kernels, convolutional matrices.

Convolutions: moving average

How does it modify images?

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Source: J. Niebles

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Convolution Vs Correlation

Cross correlation is another important operation we can apply to images:

$$(f \circ g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y') \tag{2}$$

Is it equivalent to convolution?

Convolution Vs Correlation

Cross correlation is another important operation we can apply to images:

$$(f \circ g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y') \tag{2}$$

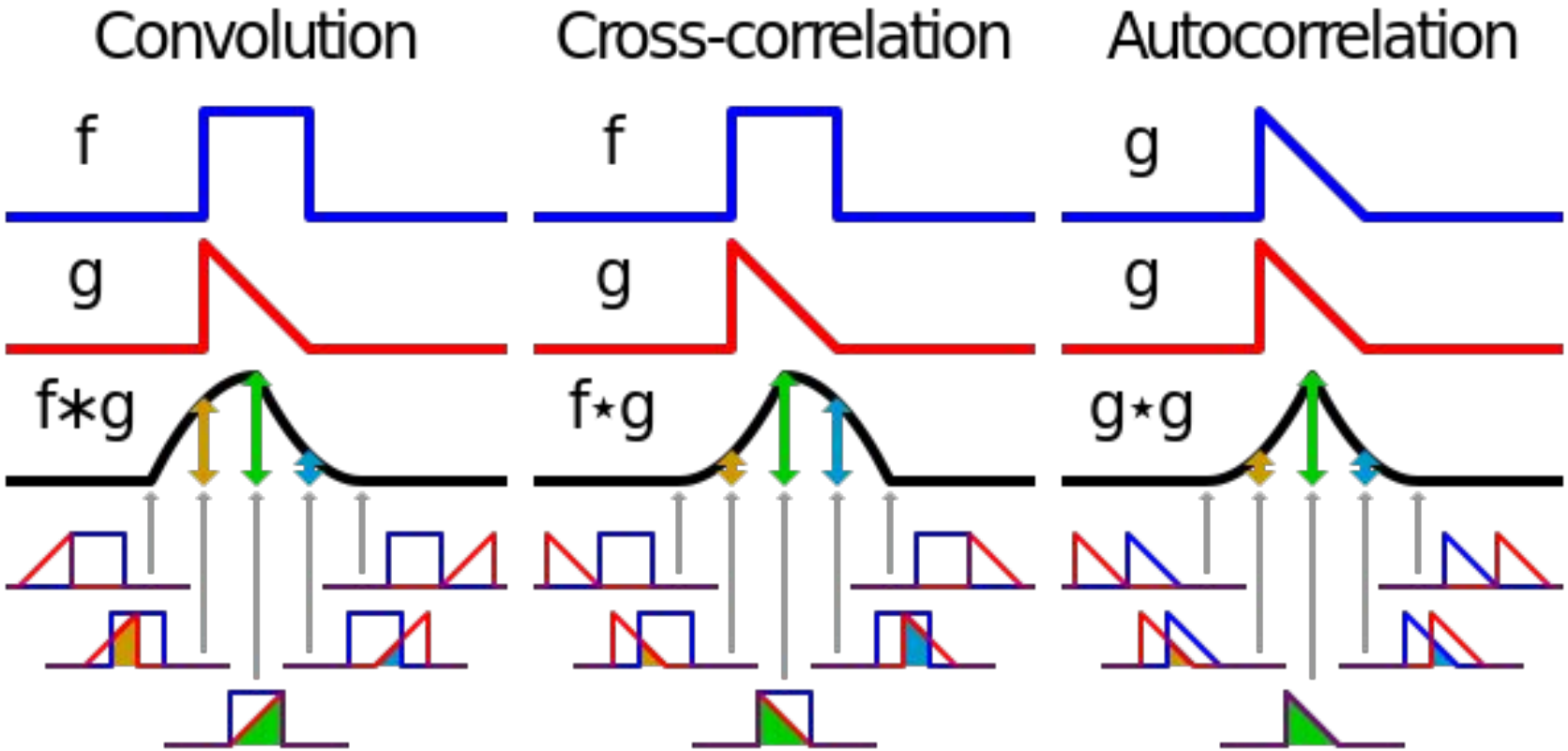
Is it equivalent to convolution? Not exactly!

$$(f * g)(x, y) = \sum_{x'=0}^{X} \sum_{y'=0}^{Y} f(x', y') g(x - x', y - y') \tag{3}$$

See also this blog post:

`https://glassboxmedicine.com/2019/07/26/convolution-vs-cross-correlation/`

Convolution vs Correlation



Convolution = Cross-correlation with the flipped version of the kernel (along both axes for 2D)

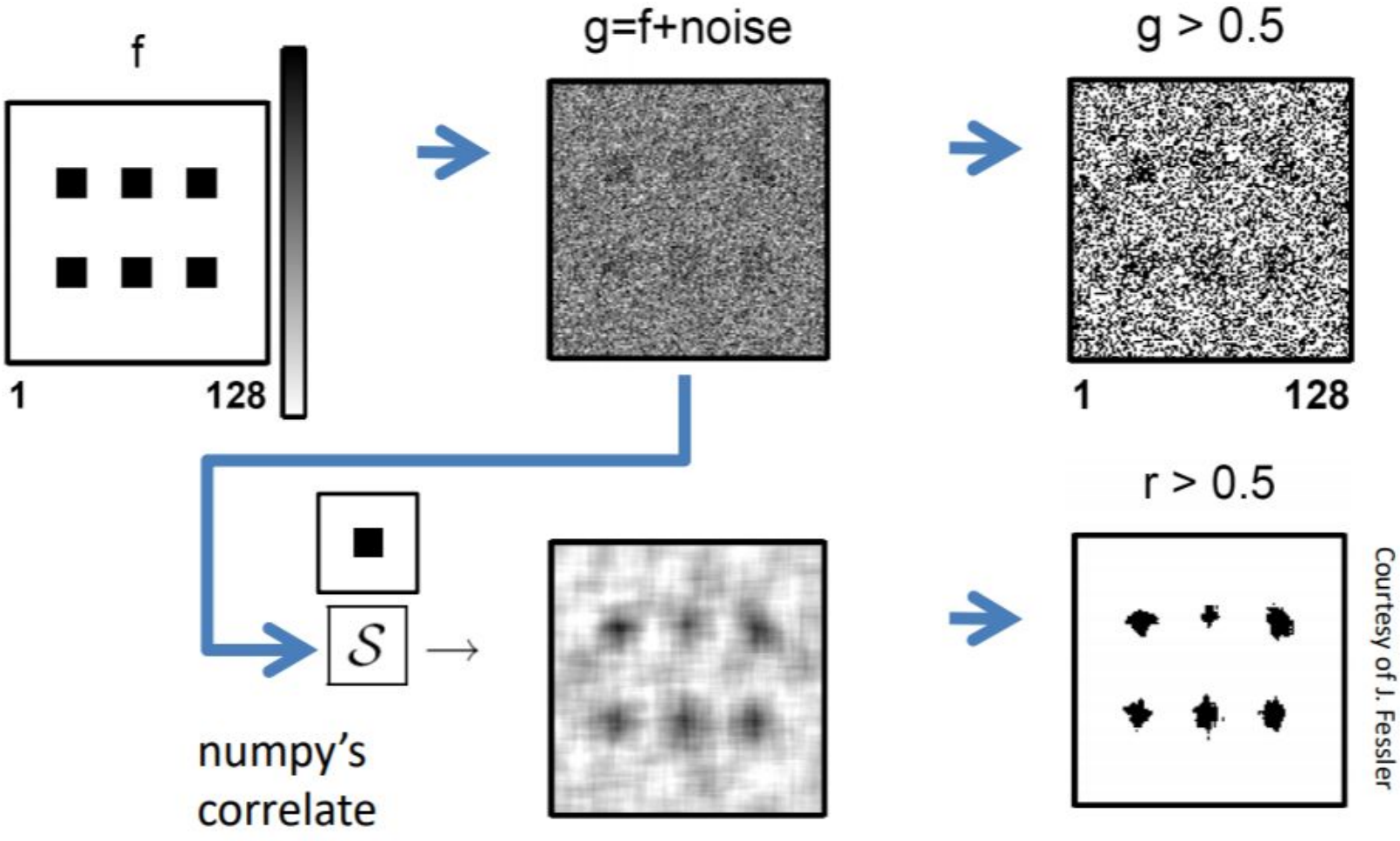
Convolution vs Correlation

```
def convolve(a, v, mode='full'):
    """
    Returns the discrete, linear convolution of two one-dimensional sequences.

    The convolution operator is often seen in signal processing, where it
    models the effect of a linear time-invariant system on a signal [1]_.  In
    probability theory, the sum of two independent random variables is
    distributed according to the convolution of their individual
    distributions.
    a, v = array(a, copy=False, ndmin=1), array(v, copy=False, ndmin=1)
    if (len(v) > len(a)):
        a, v = v, a
    if len(a) == 0:
        raise ValueError('a cannot be empty')
    if len(v) == 0:
        raise ValueError('v cannot be empty')
    return multiarray.correlate(a, v[::-1], mode)
```

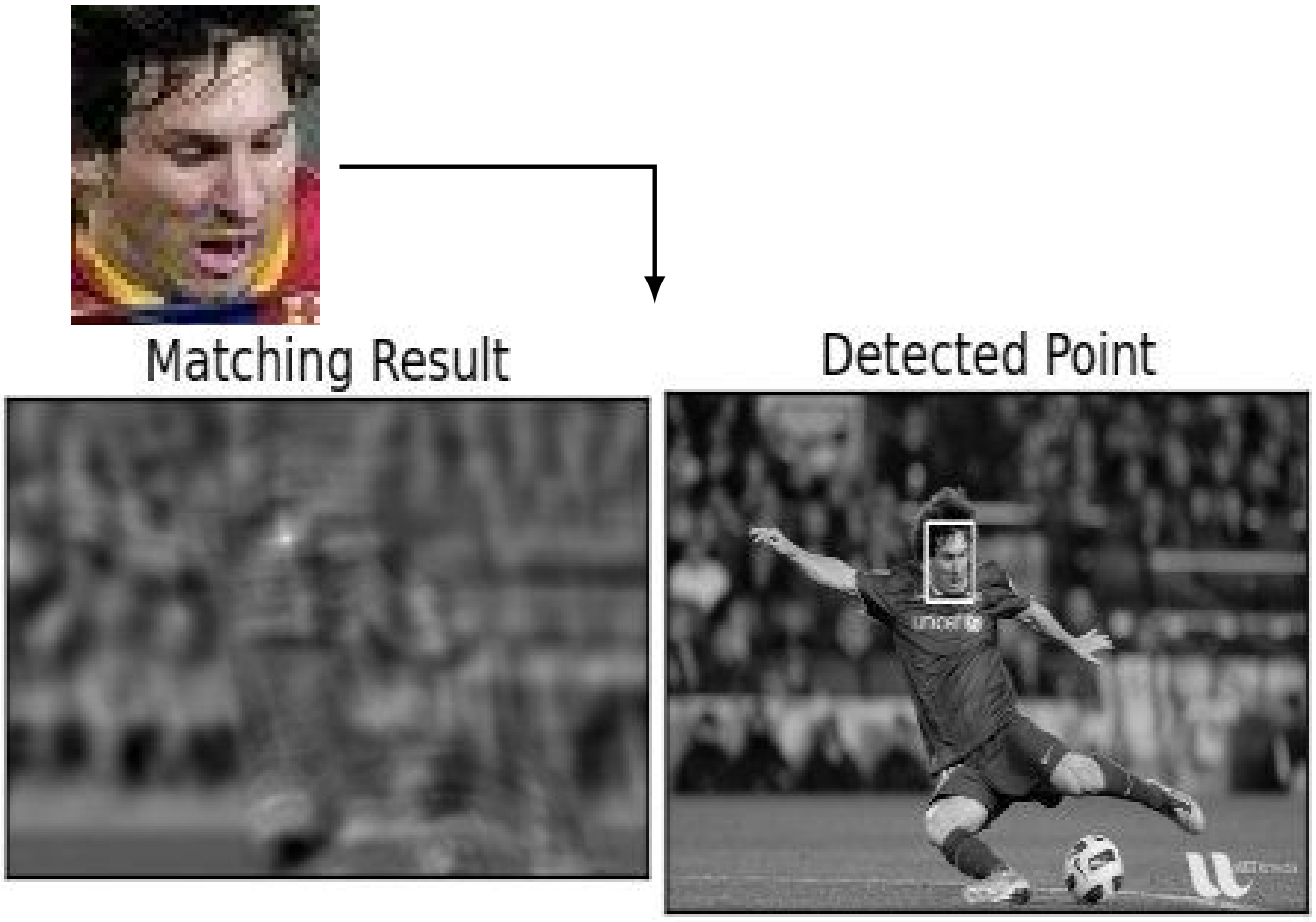
https://github.com/numpy/numpy/blob/main/numpy/_core/numeric.py#L736

Convolution vs Correlation



Source: J. Niebles

Template matching



https://docs.opencv.org/4.x/d4/dc6/tutorial_py_template_matching.html

Template matching

How can we estimate that a part of the image is similar to the template?

Template matching

How can we estimate that a part of the image is similar to the template?

$$R(x, y) = \sum_{x', y'} (T(x', y') - I(x + x', y + y'))^2 \tag{4}$$

Template matching

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$$R(x,y) = \sum_{x',y'} (T(x',y') \cdot I(x+x',y+y')) \tag{5}$$

Template matching

How can we estimate that a part of the image is similar to the template?

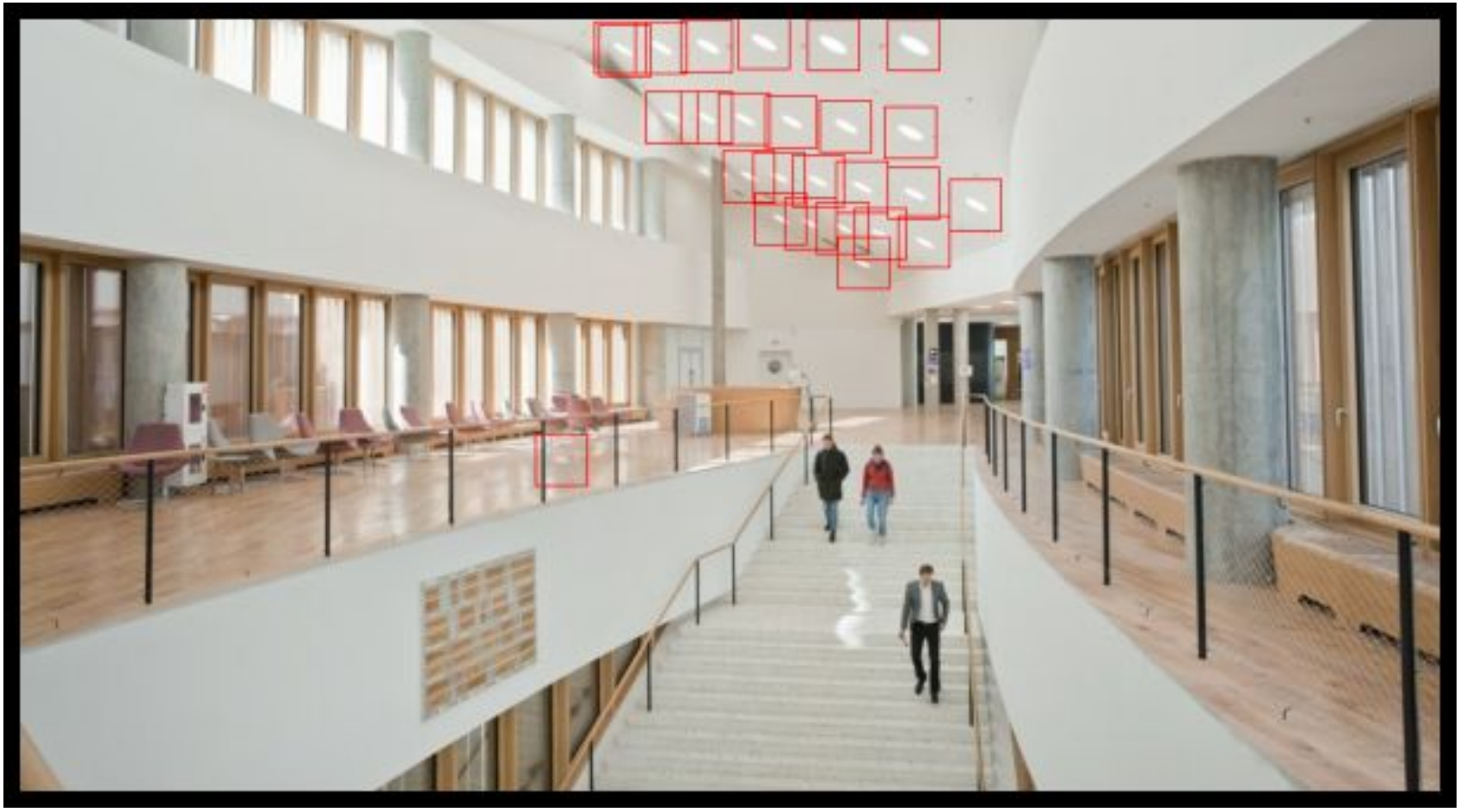
$$R(x,y) = \sum_{x',y'} (T(x',y') - I(x+x',y+y'))^2 \tag{4}$$

$$R(x,y) = \sum_{x',y'} (T(x',y') \cdot I(x+x',y+y')) \tag{5}$$

$$R(x,y) = \frac{\sum_{x',y'} (T(x',y') \cdot I(x+x',y+y'))}{\sqrt{\sum_{x',y'} T(x',y')^2 \cdot \sum_{x',y'} I(x+x',y+y')^2}} \tag{6}$$

Template matching

Exercise: count lamps using template matching!



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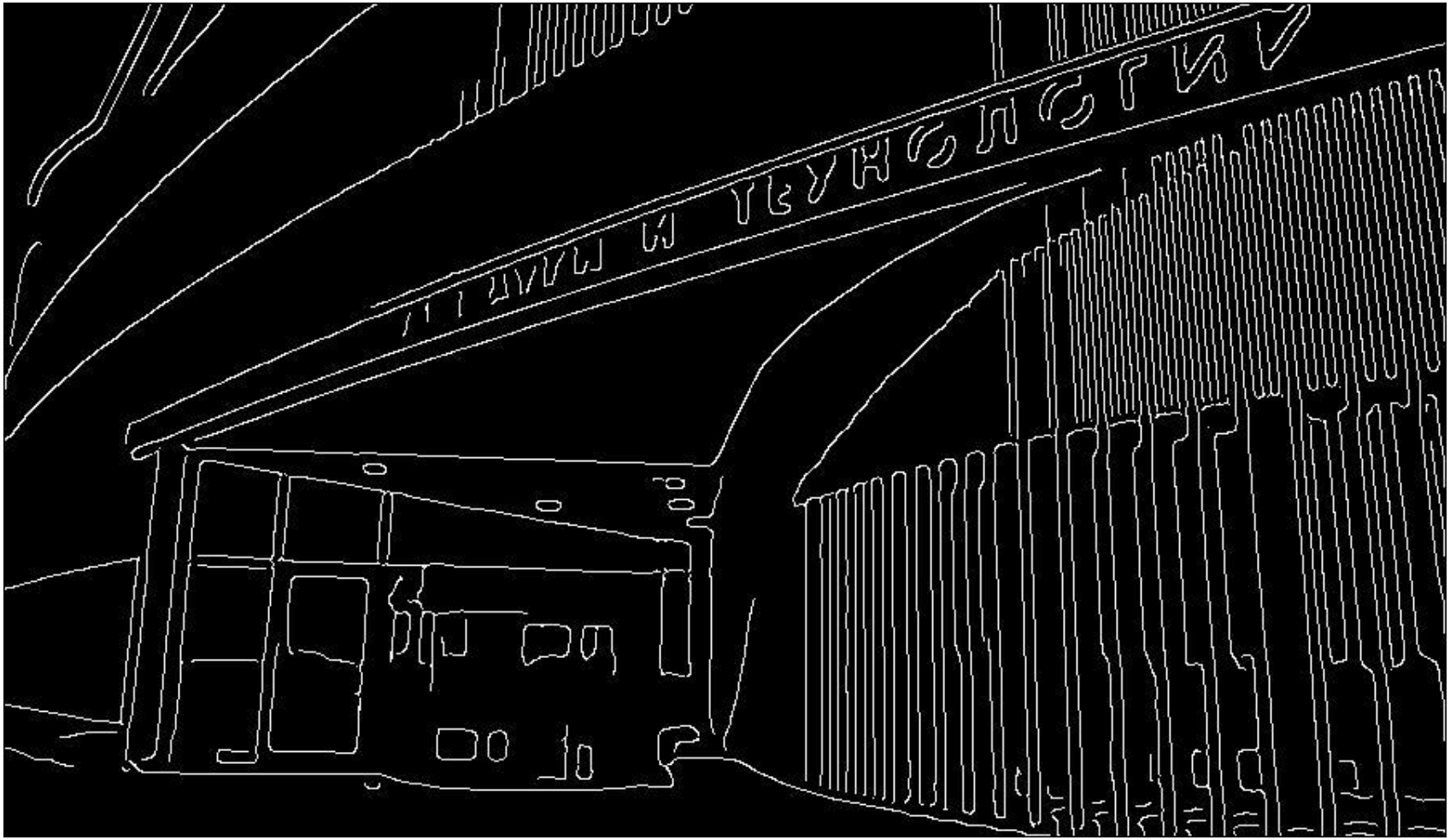
Why do we need to detect edges?



Why do we need to detect edges?

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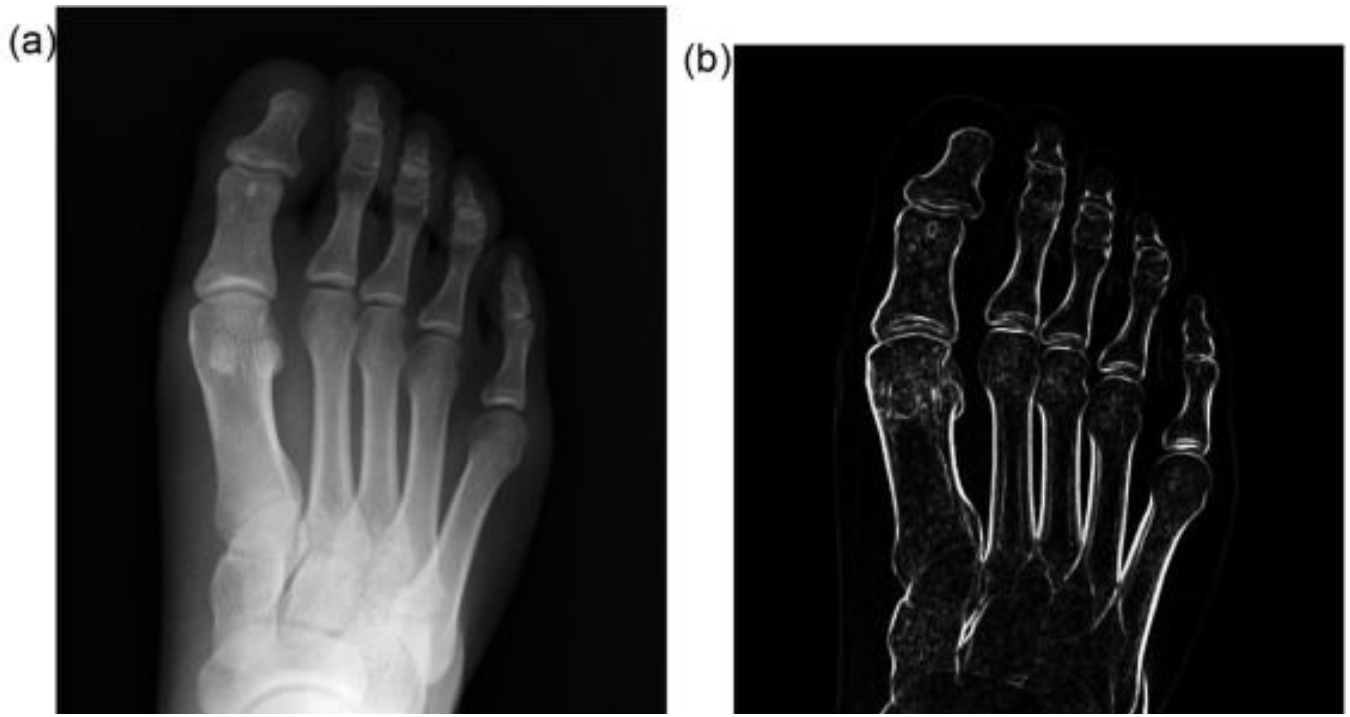
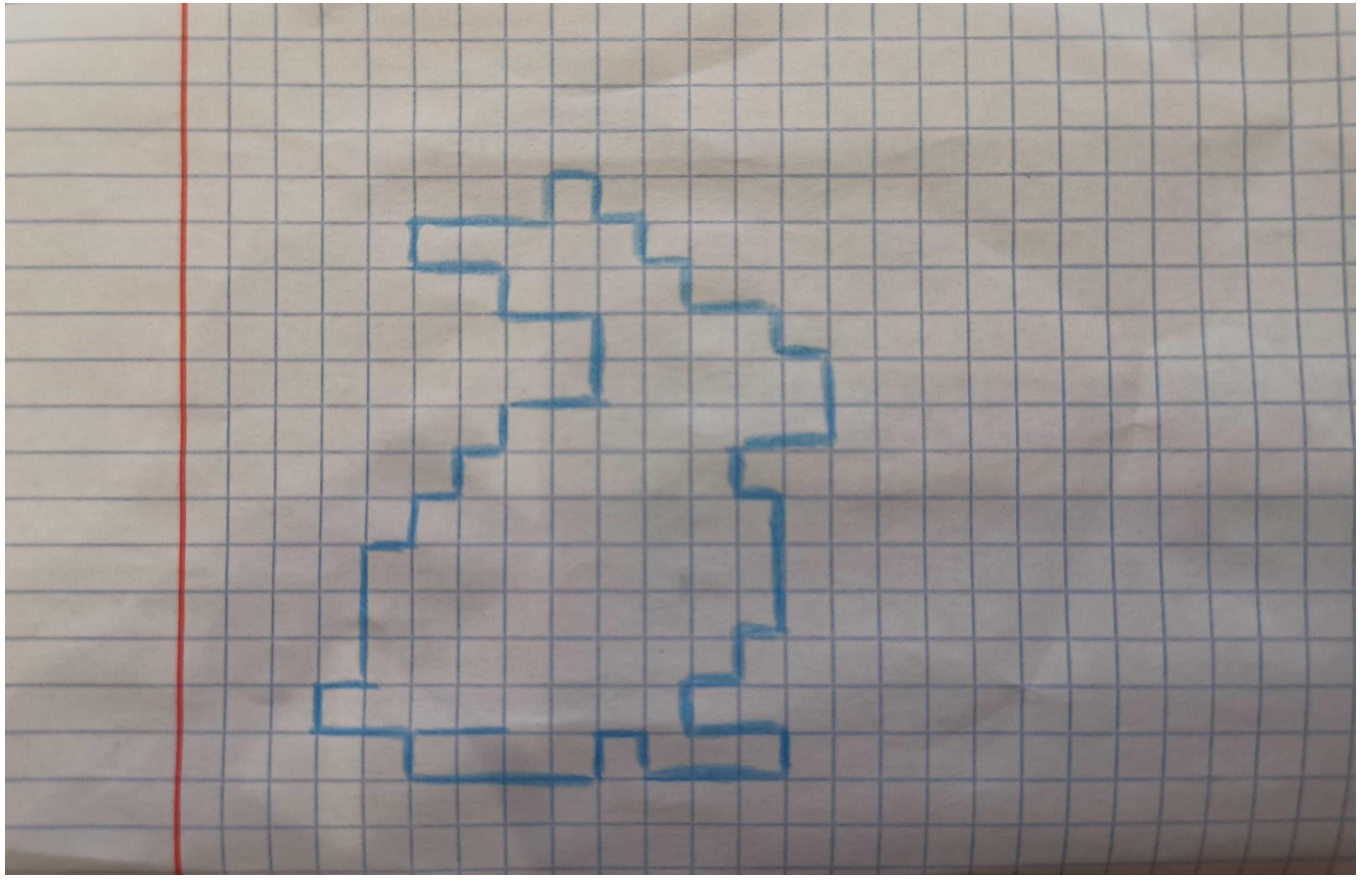


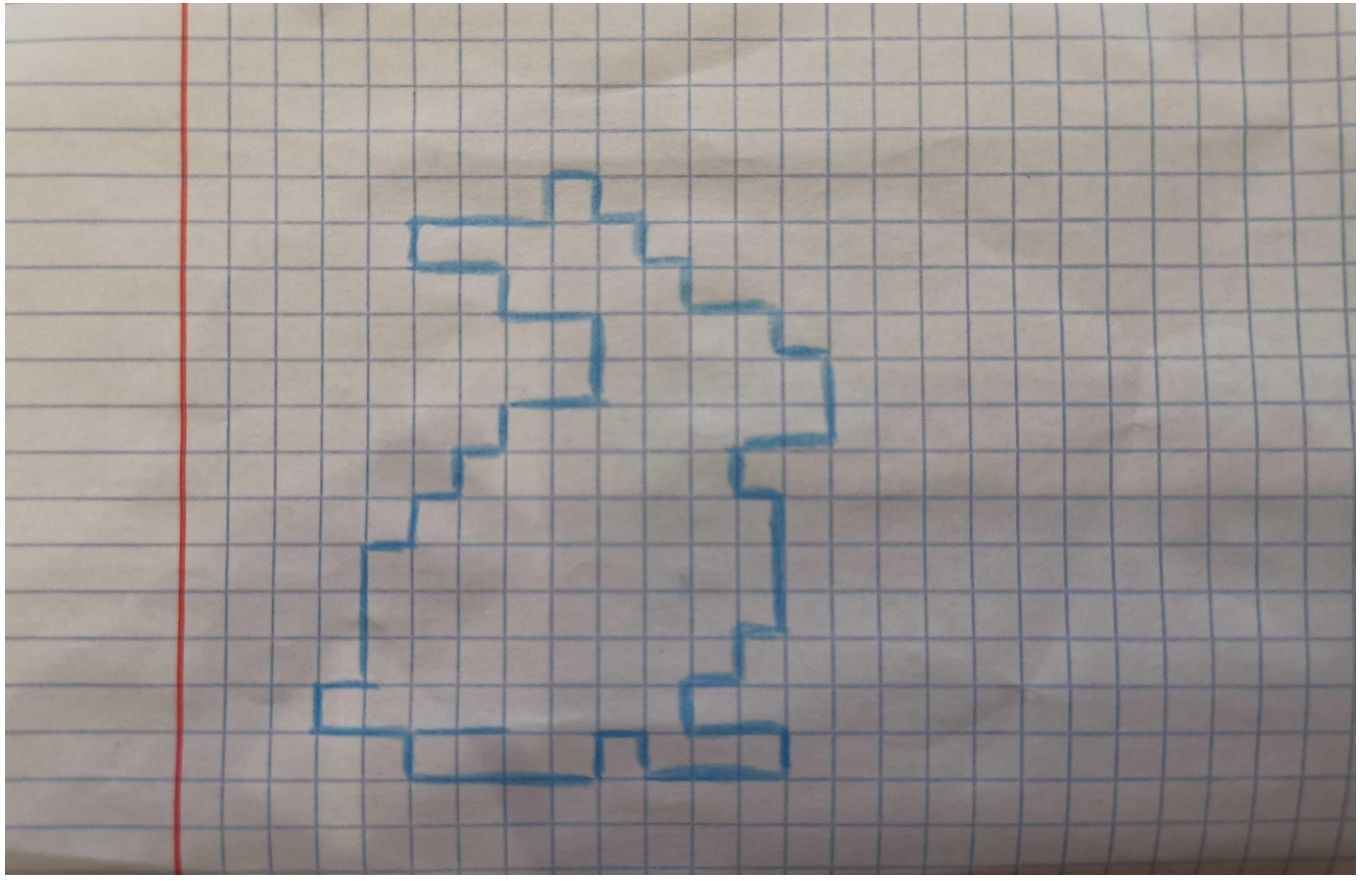
Source: Edge detection in medical images with quasi high-pass filter based on local statistics

Why do we need to detect edges?

Why do we need to detect edges?

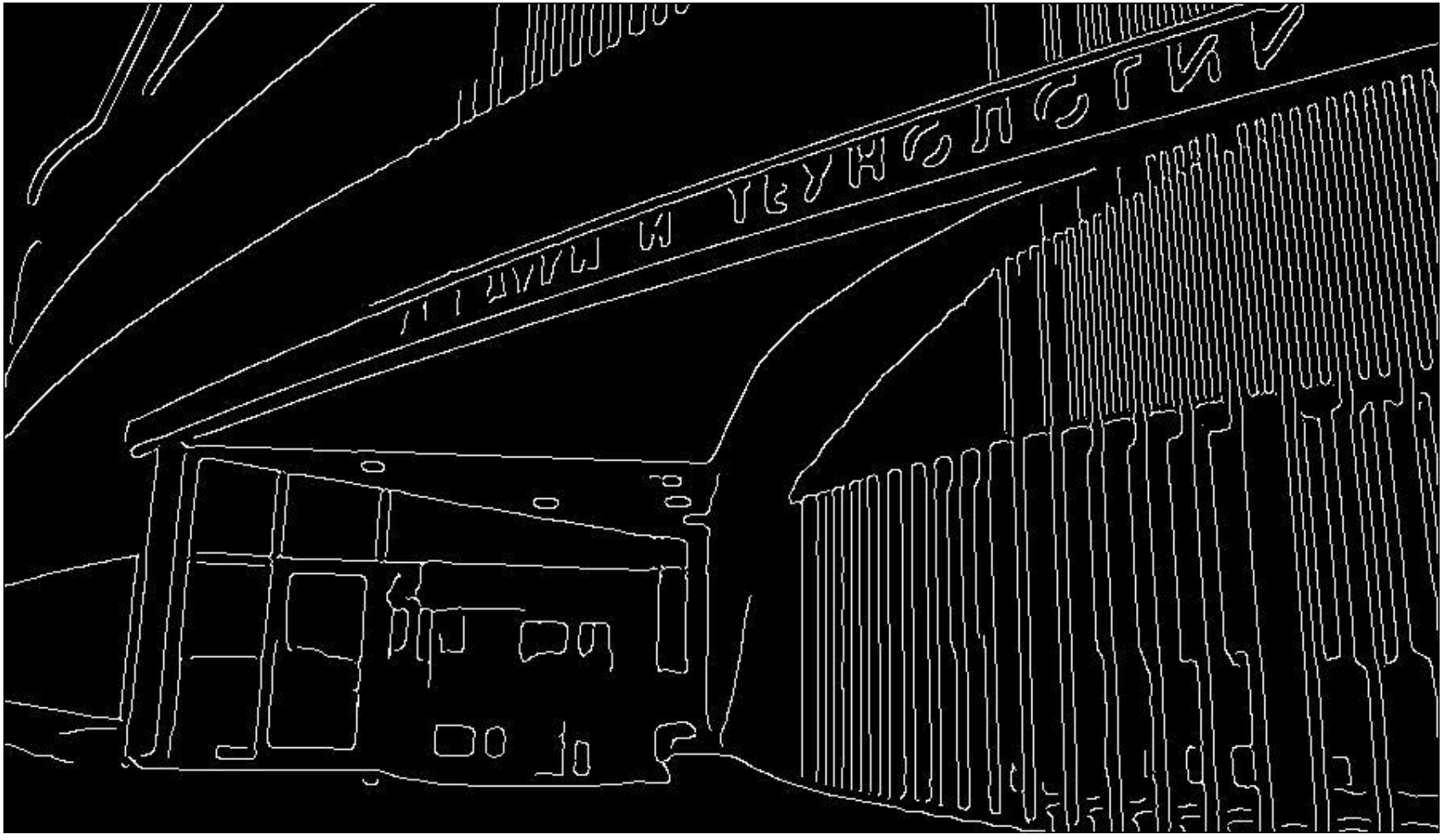


Why do we need to detect edges?



- Edges contain a lot of information about the image
- Generated by discontinuities, edges give a lot of information about separate objects, or their separate parts

Where do edges come from?



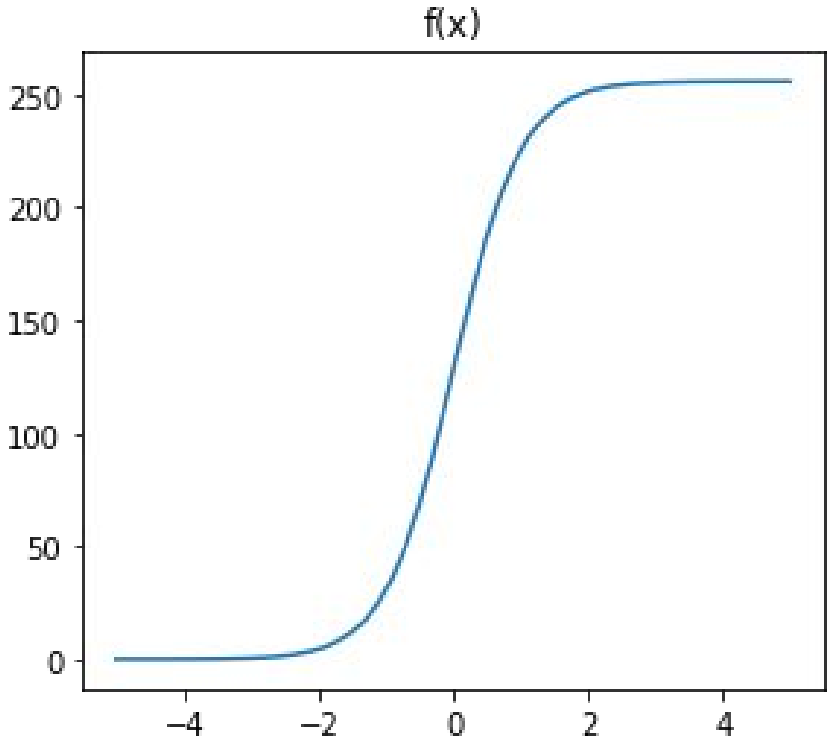
Please name 3 “sources” of edges on this photo

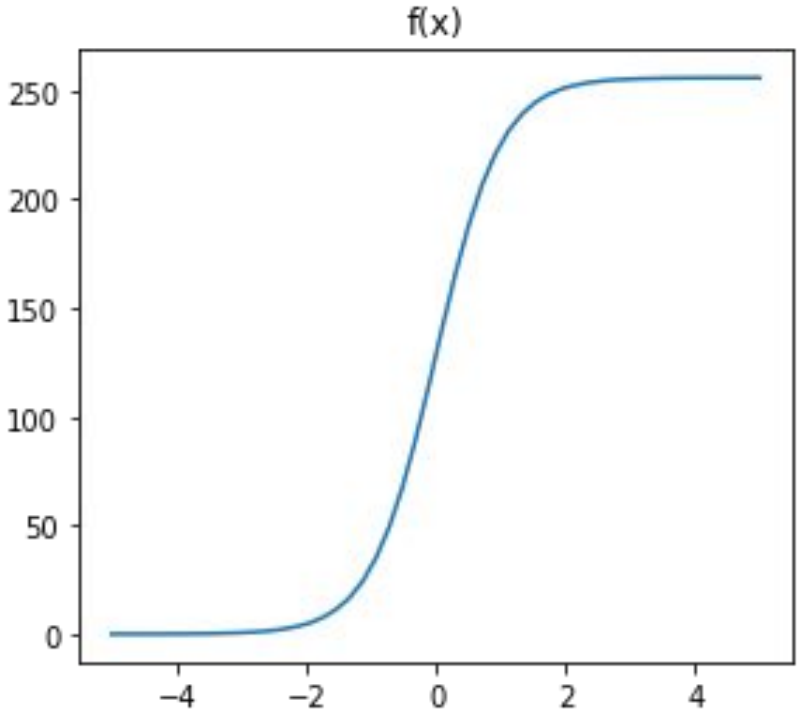
Where do edges come from?

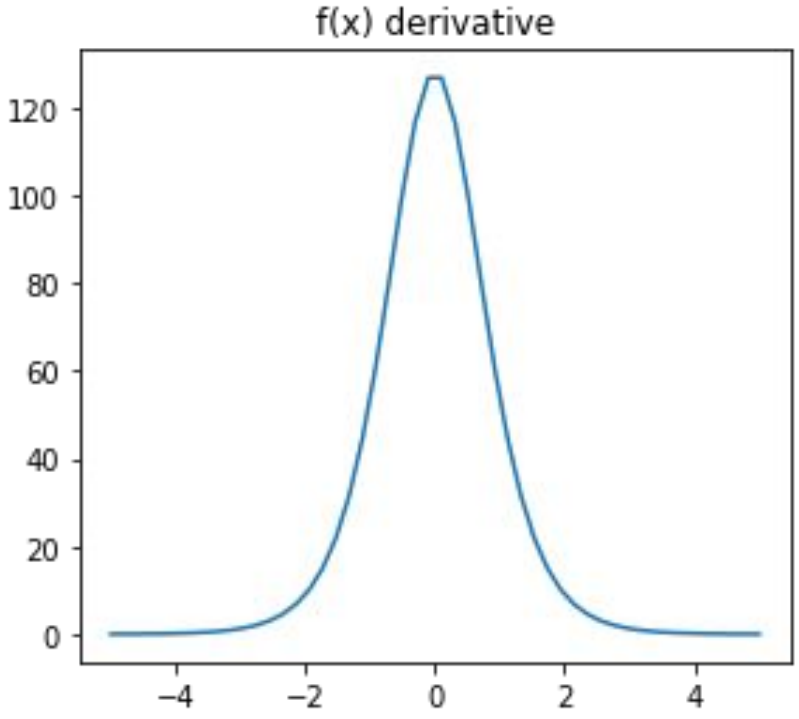
Please name 3 “sources” of edges on this photo

Where do edges come from?

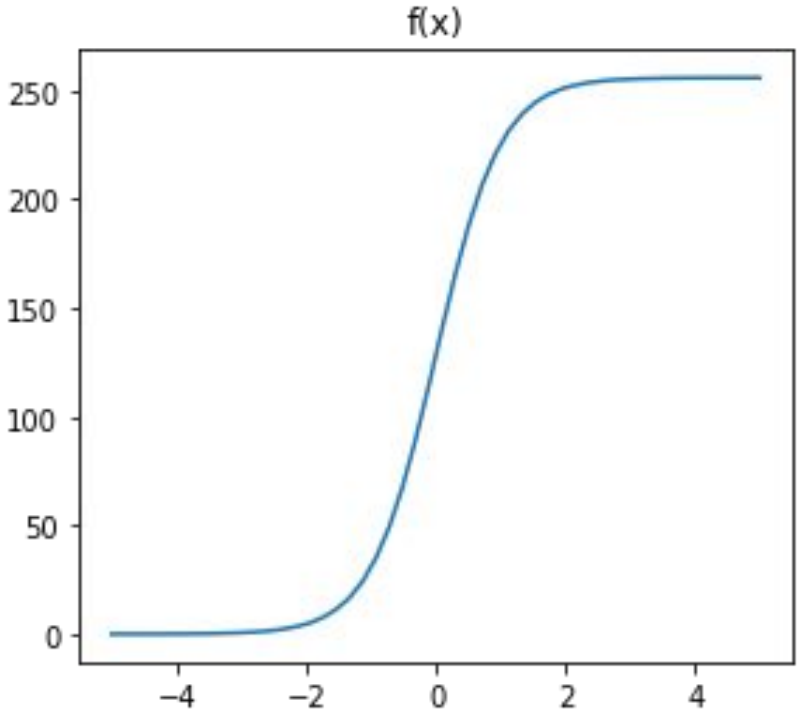
- Surface / Depth
- Colors
- Ilumination

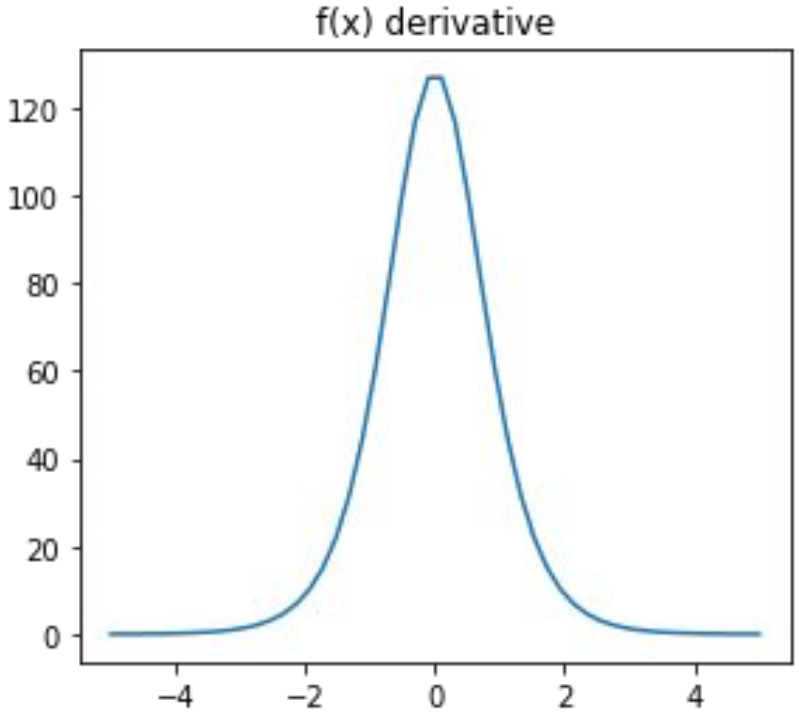






- Derivative is a good way to find edges

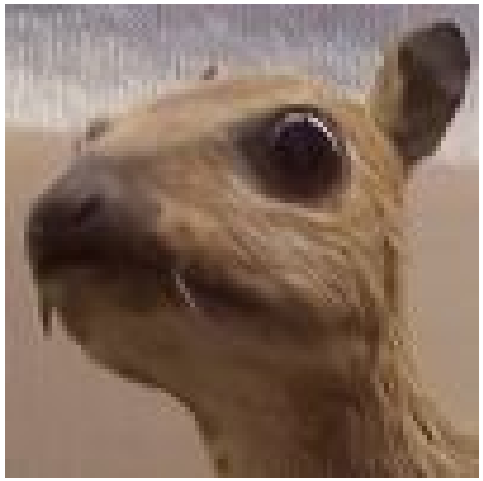




- Derivative is a good way to find edges
- But what can we do with 2D discrete function?

Convolution: examples of filters

How does it work?

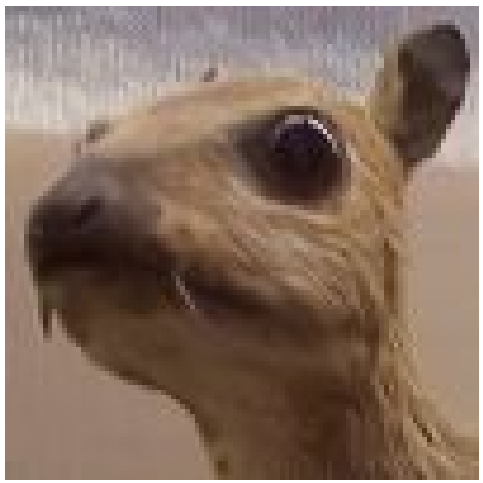


$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Source: Wikipedia

Convolution: examples of filters

How does it work?

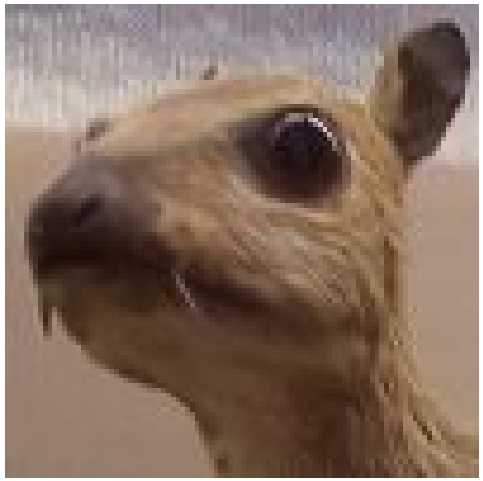


$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Source: Wikipedia

Convolution: examples of filters

How does it work?



$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

Source: Wikipedia

Separable convolutions

What is the time complexity of a naive convolution algorithm for an image of shape (n, m) and a kernel of shape (k, l)?

Separable convolutions

Sometimes kernels can be decomposed into outer product of two vectors:

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \tag{7}$$

In this case we can exploit the associativity of convolution:

$$f \star (v \star w) = f \star (vw) = (f \star v) \star w$$

What is the time complexity of a naive separable convolution algorithm for an image of shape (n, m) and a kernel of shape (k, l)?

Sobel filter

The x-derivative of smoothed image

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \tag{8}$$

Gaussian smoothing Differentiation

Sobel filter

Approximation of differentiation for both directions on the image:

$$G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

Horizontal changes

$$G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Vertical changes

Sobel filter

Approximation of differentiation for both directions on the image:

$$\mathrm{G}_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$$

Horizontal changes

$$\mathrm{G}_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

Vertical changes

Gradient magnitude

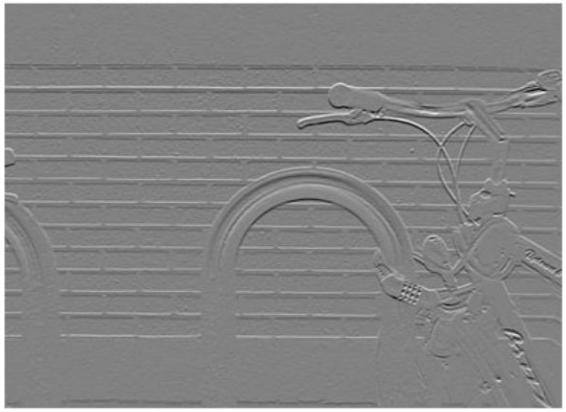
$$\mathrm{G} = \sqrt{\mathrm{G}_x^2 + \mathrm{G}_y^2}$$

Gradient direction

$$\Theta = \arctan\left(\frac{\mathrm{G}_y}{\mathrm{G}_x}\right)$$

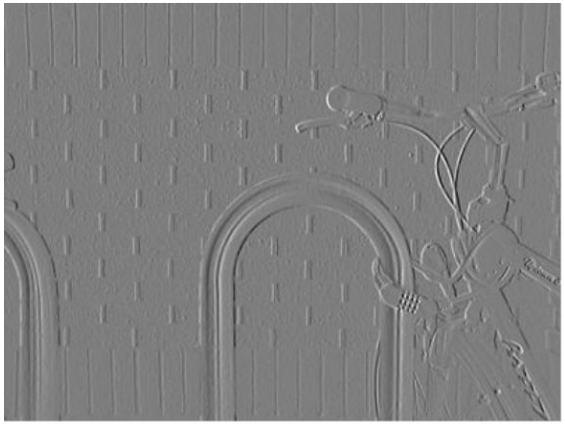
Sobel filter (example)

Grayscale image



y-derivative G_y

x-derivative G_x



Gradient magnitude $|\mathrm{G}|$

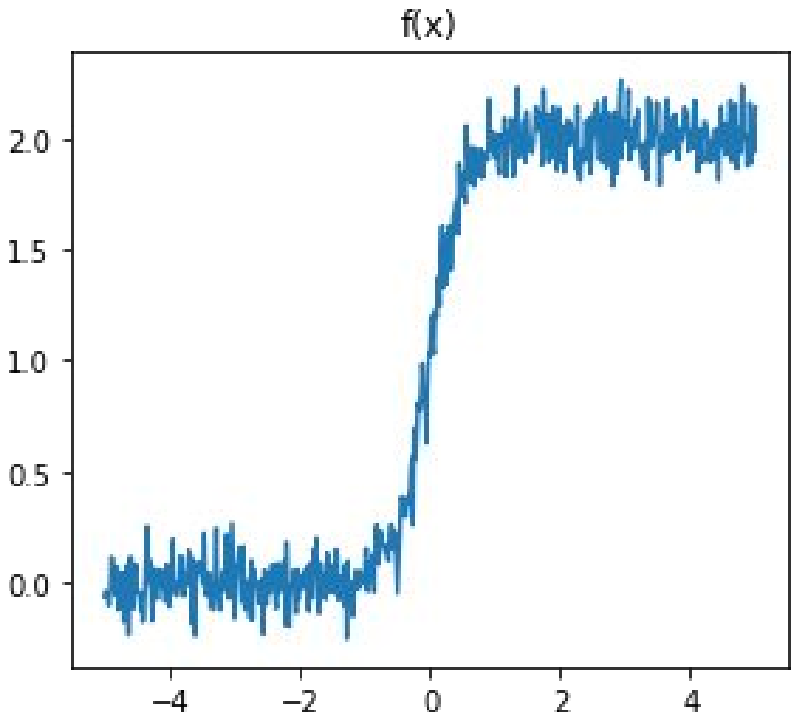
Source: wikipedia

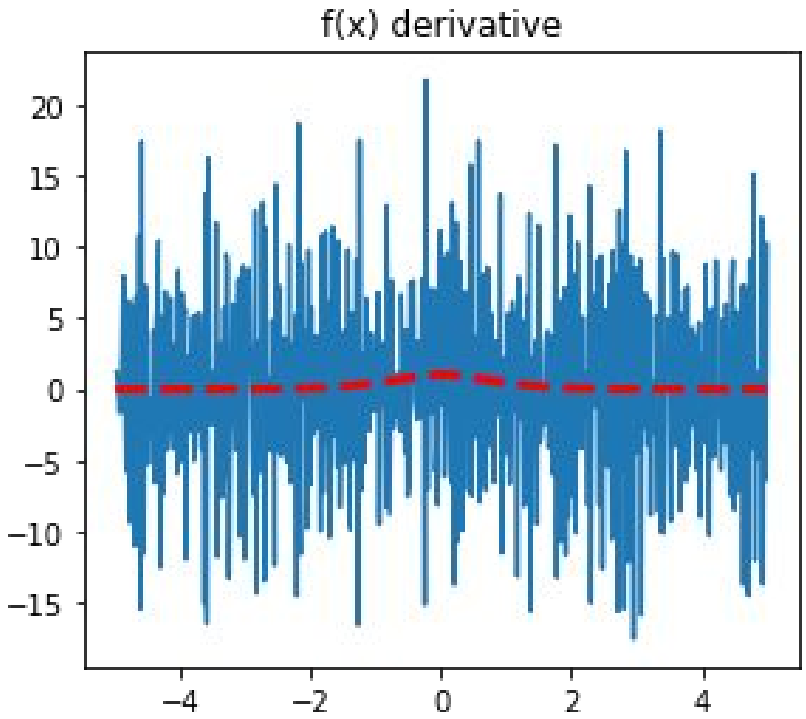
Edge detection by Sobel

1. Convolve the image with filters G_x and G_y to estimate image derivatives.
2. Calculate gradient magnitude $\mathrm{G} = \sqrt{\mathrm{G}_x^2 + \mathrm{G}_y^2}$

Note: these edges aren't binary.

Sobel filter (issues)

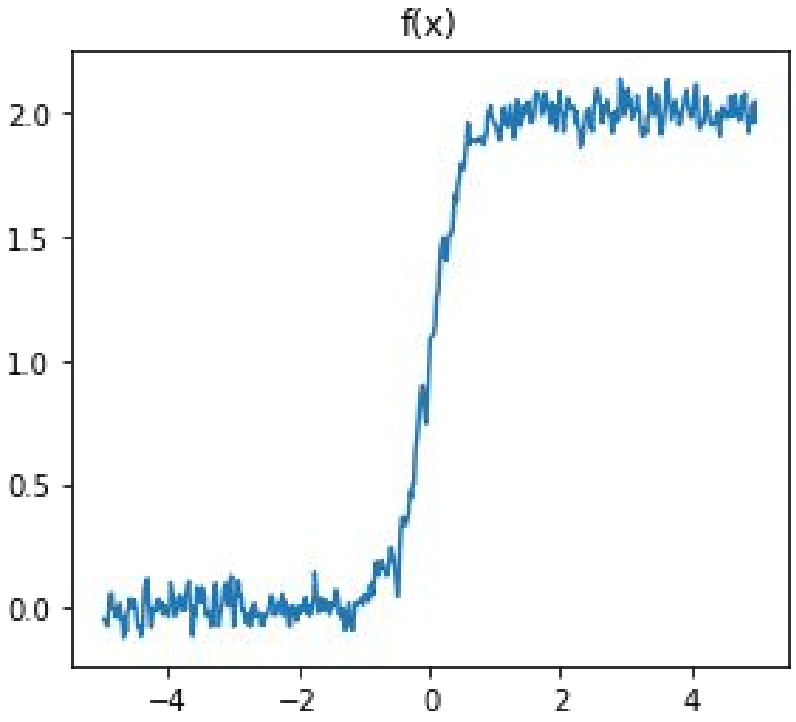


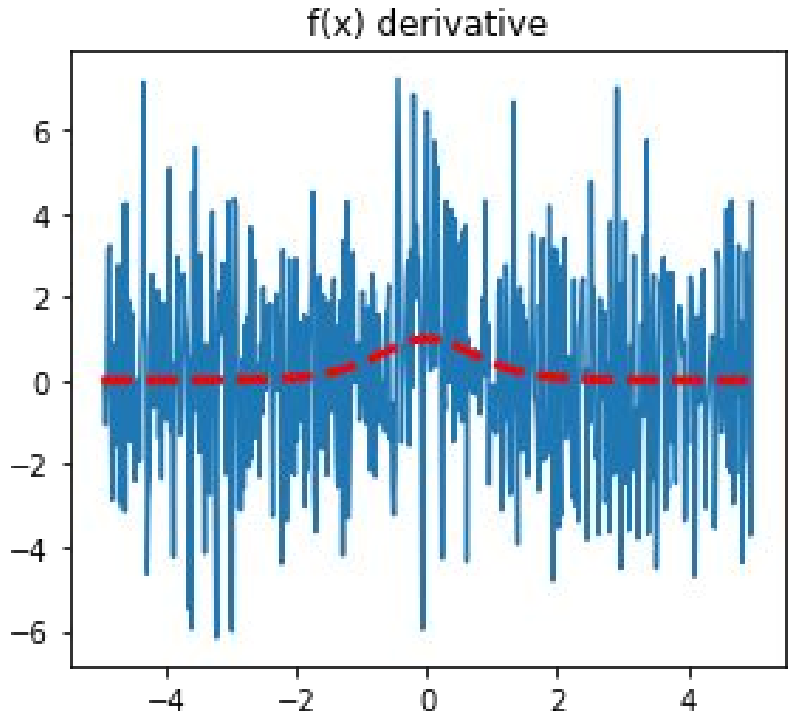


Numerical derivatives are sensitive to noise (dashed red line represents the analytical derivative).

What can we do with our image?

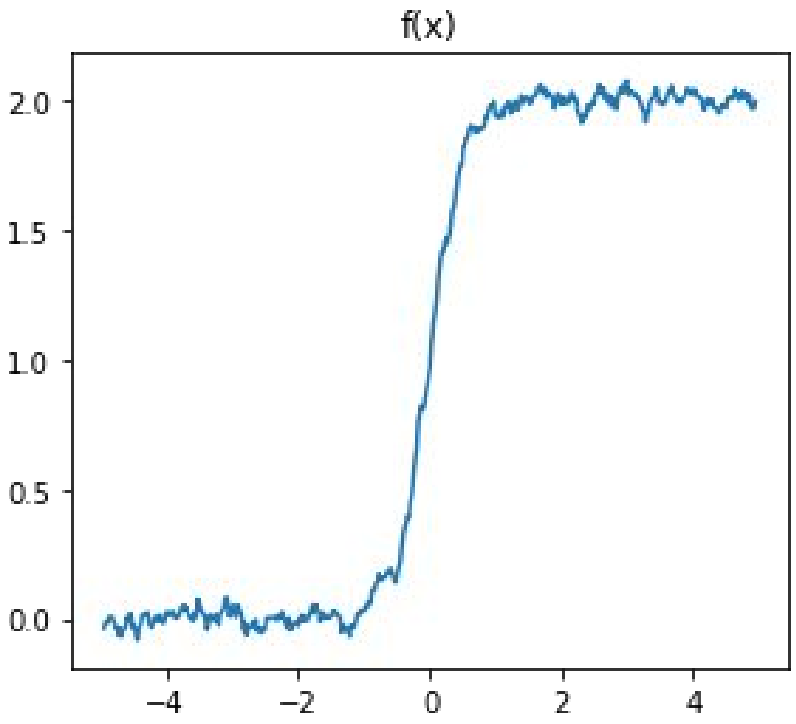
Sobel filter (issues)

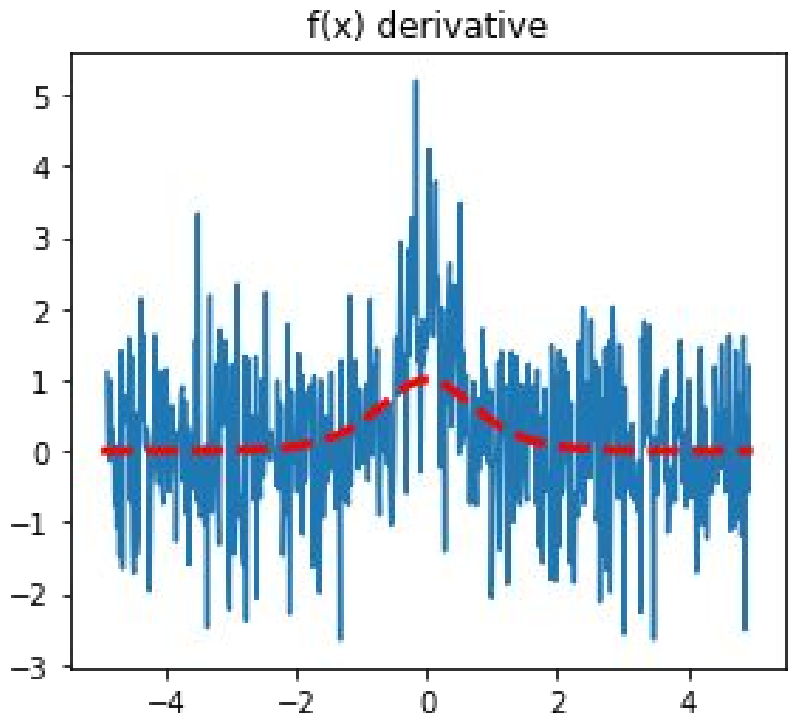




Smoothing! For example, moving average with window=3

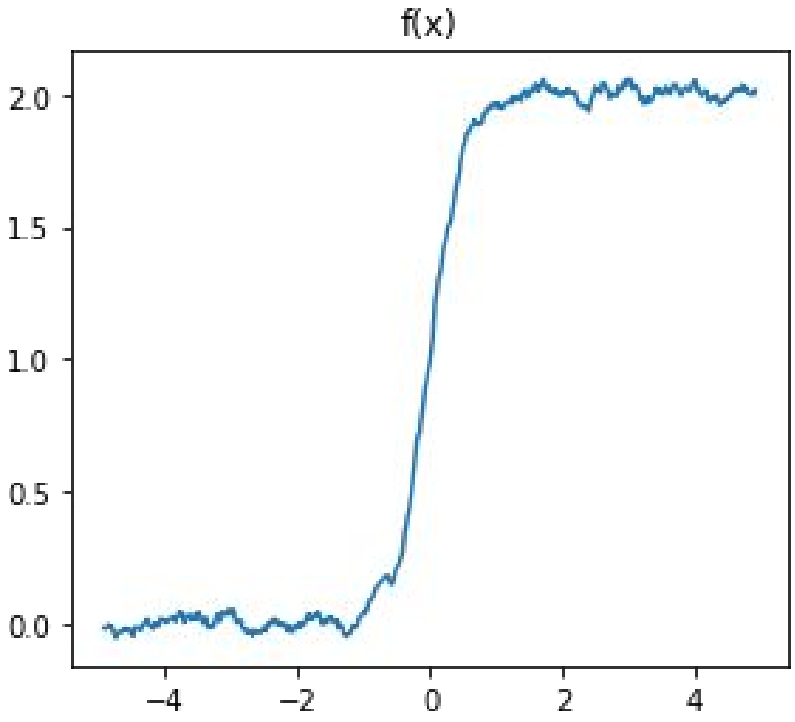
Sobel filter (issues)

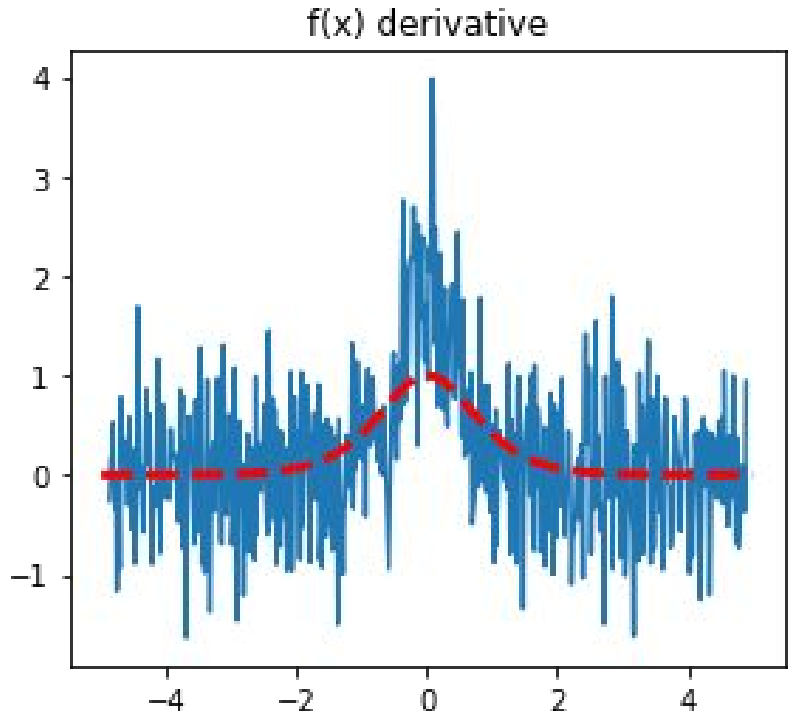




Smoothing! For example, moving average with window=7

Sobel filter (issues)





Smoothing! For example, moving average with window=11

Edge detection - desired features

- Detection of edge with low false negative error rate, which means that the detection should accurately catch as many edges shown in the image as possible.
- The edge point should accurately localize on the center of the edge.
- A given edge in the image should only be marked once, and where possible, image noise should not create false edges (low false positive errors rate).

Canny edge detector - steps

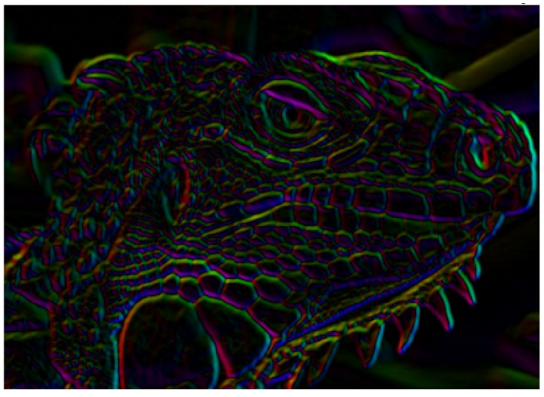
1. Noise suppression (e.g. Gaussian filter, median filter)
2. Gradient magnitude and direction (e.g. via Sobel filter)
3. Non-Maximum Suppression
4. Hysteresis thresholding and connectivity analysis

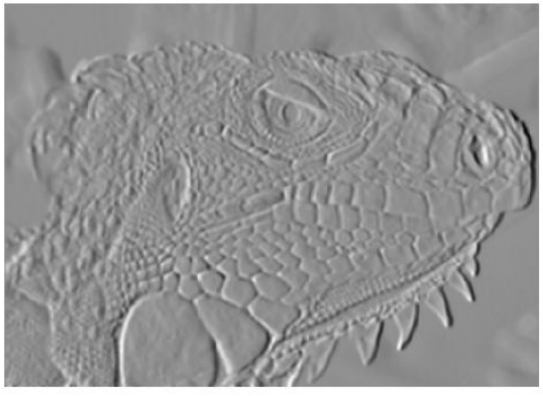
Canny, J. (1986). A computational approach to edge detection. In IEEE Transactions on Pattern Analysis and Machine Intelligence (Vol. 8), 679-714.

Canny edge detector (step-by-step)

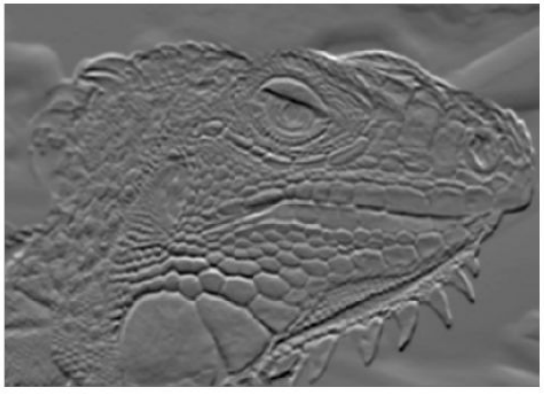
orig. image

gradient orientation





X-Derivative of Gaussian



Y-Derivative of Gaussian

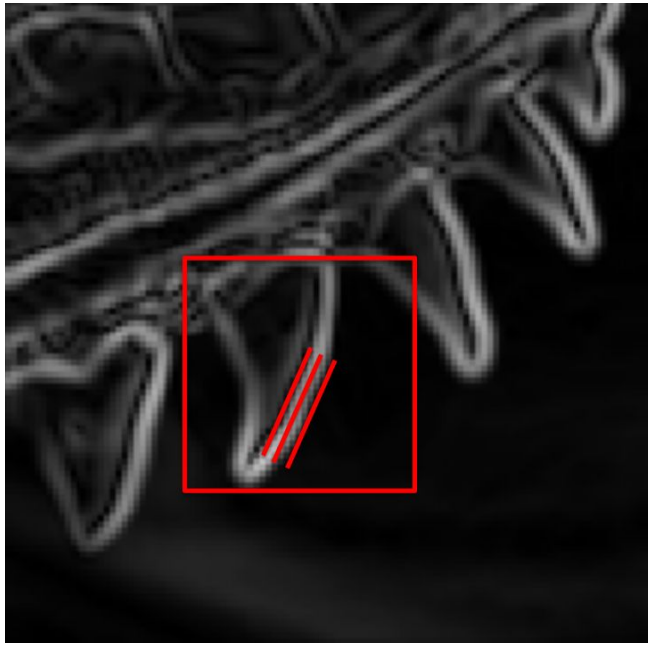
Gradient Magnitude

Source: J. Hayes

Canny edge detector (step-by-step)

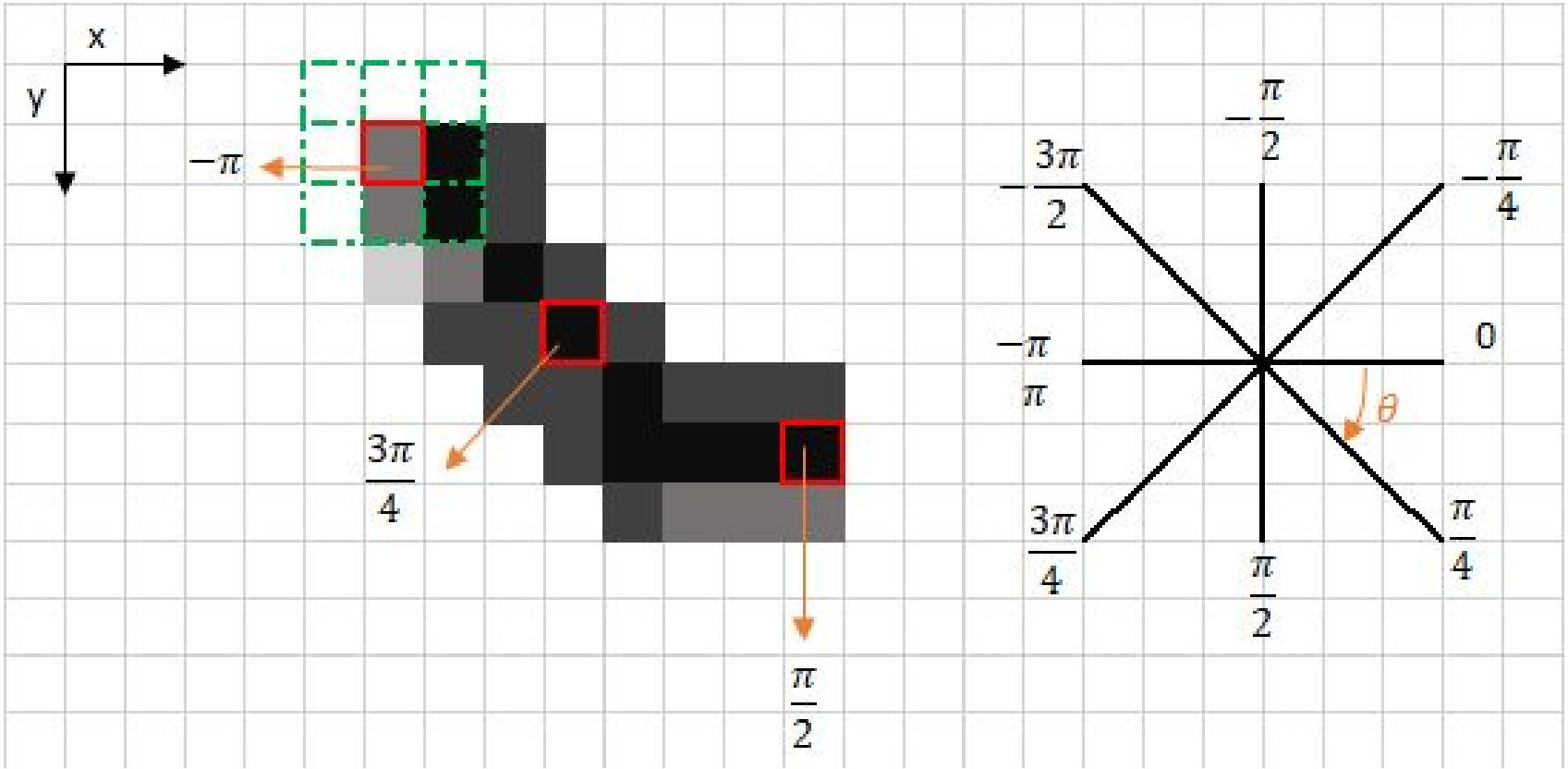
Non-Maximum Suppression

- Edge occurs where gradient reaches a (local) maxima
- Consider only 8 angle directions (e.g. 45°, 90°, 135°, ...)
- Suppress all pixels in each direction which are not maxima



Canny edge detector (step-by-step)

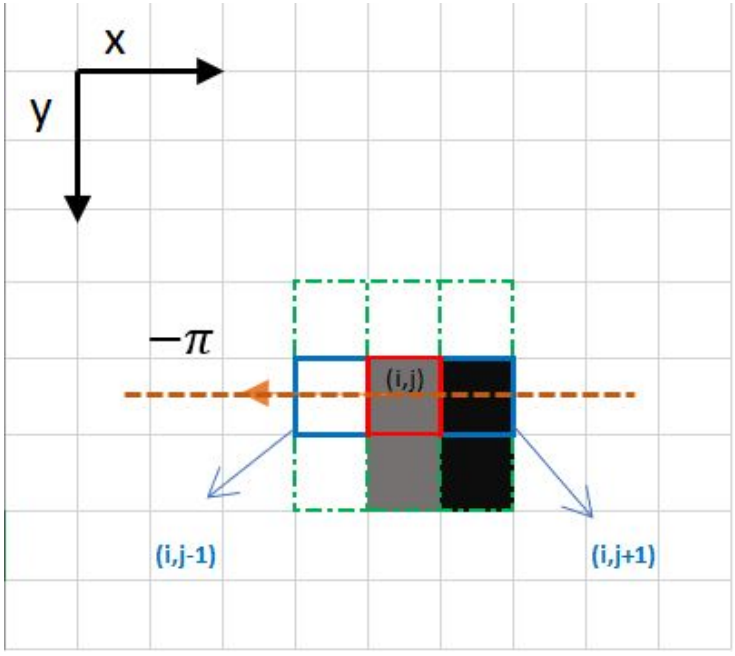
Non-Maximum Suppression



Canny edge detector (step-by-step)

Non-Maximum Suppression

- the pixel (i, j) is being processed
- Gradient orientation is approximately - (orange line).
- We consider pixels on the same gradient direction: (i, j-1) and (i, j+1)
- if (i, j) is more intense than these two neighbors, then it is kept
- overwise, it is suppressed (set to 0)



Canny edge detector (step-by-step)

Non-Maximum Suppression

- the pixel (i, j) is being processed
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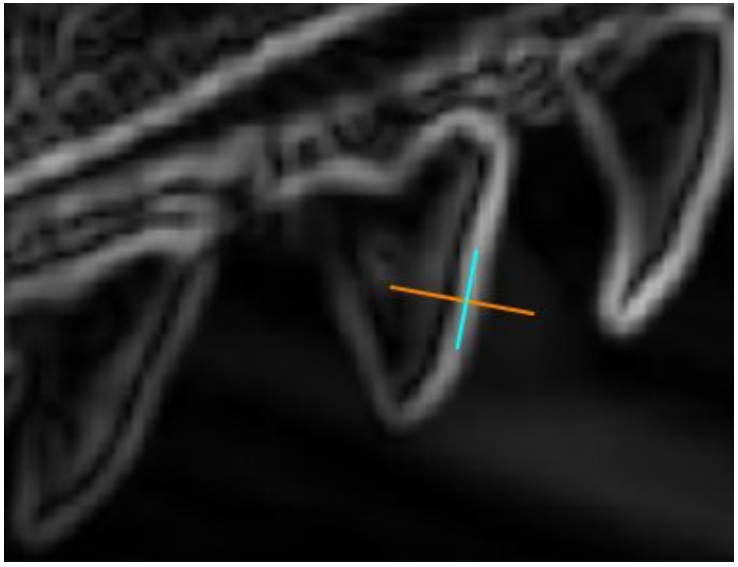
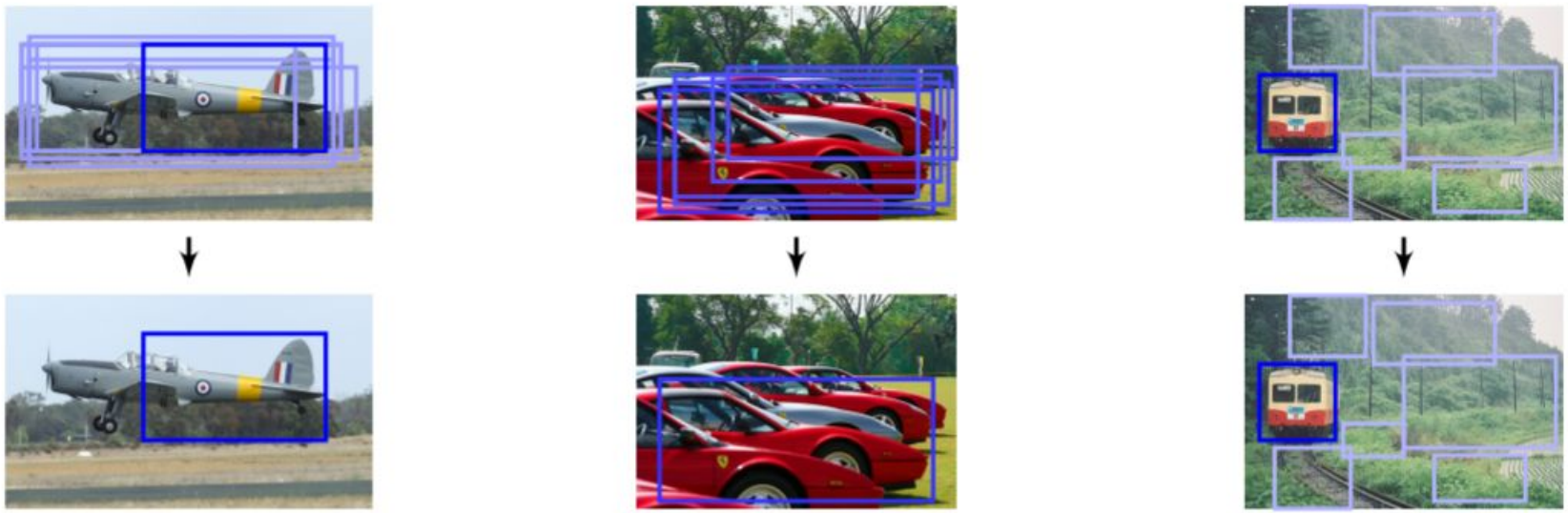


The blue line is aligned with the edge, so we don't want to apply NMS in this direction

Canny edge detector (step-by-step)

Non-Maximum Suppression - a side note

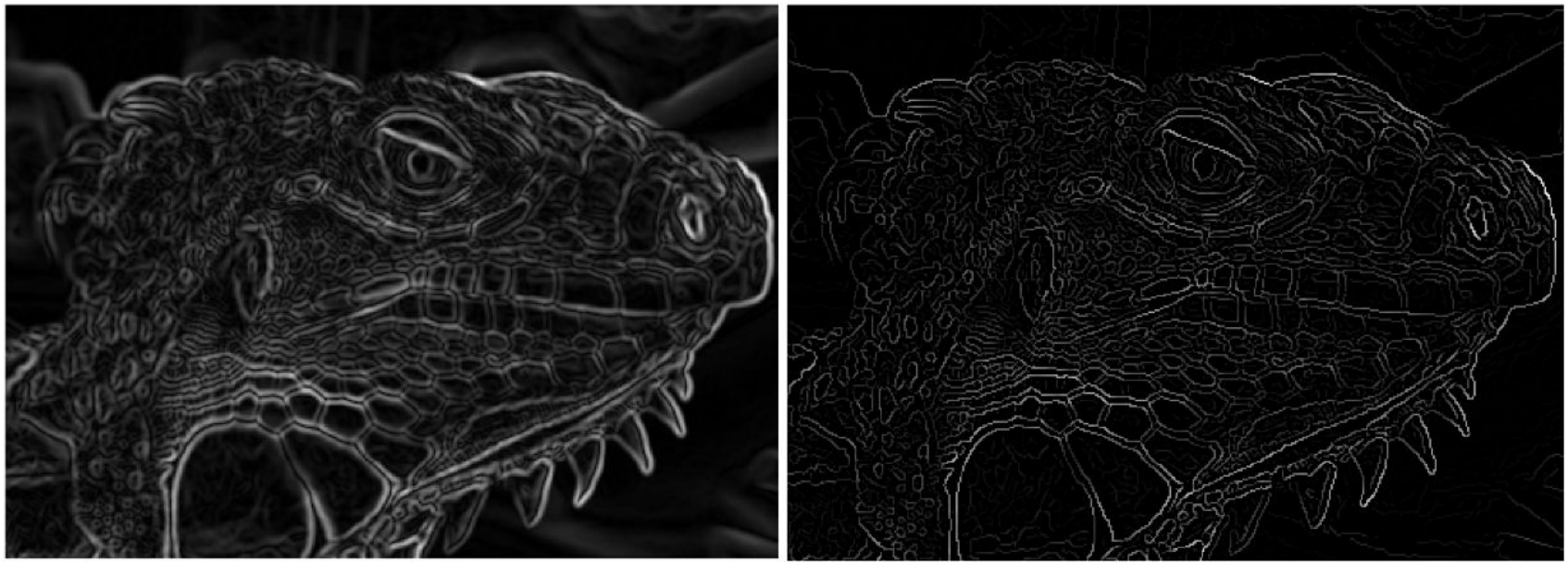
- NMS (the same idea, but different algorithms) is very useful for object detection



Source: Non-Maximum Suppression for Object Detection by Passing Messages between Windows

Canny edge detector (step-by-step)

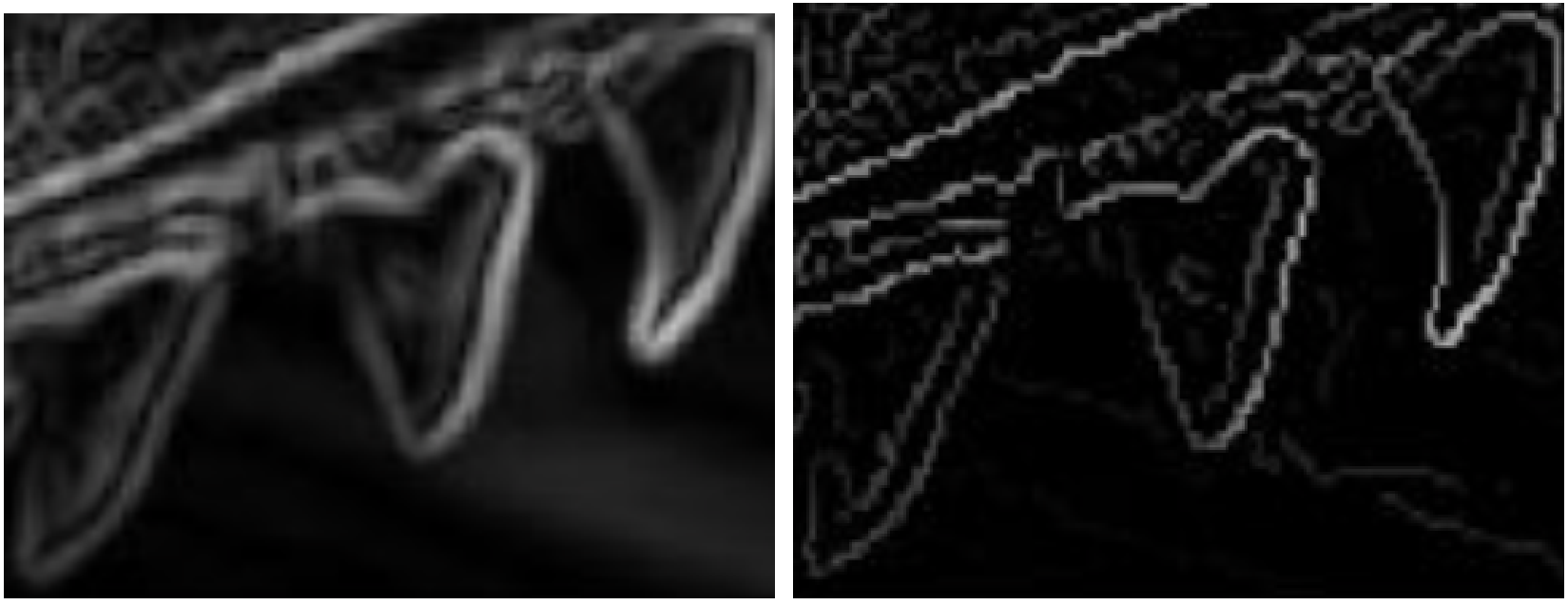
Non-Maximum Suppression



Gradient Magnitude $\rightarrow$ Non-Maximum Suppression

Canny edge detector (step-by-step)

Non-Maximum Suppression



Gradient Magnitude $\rightarrow$ Non-Maximum Suppression

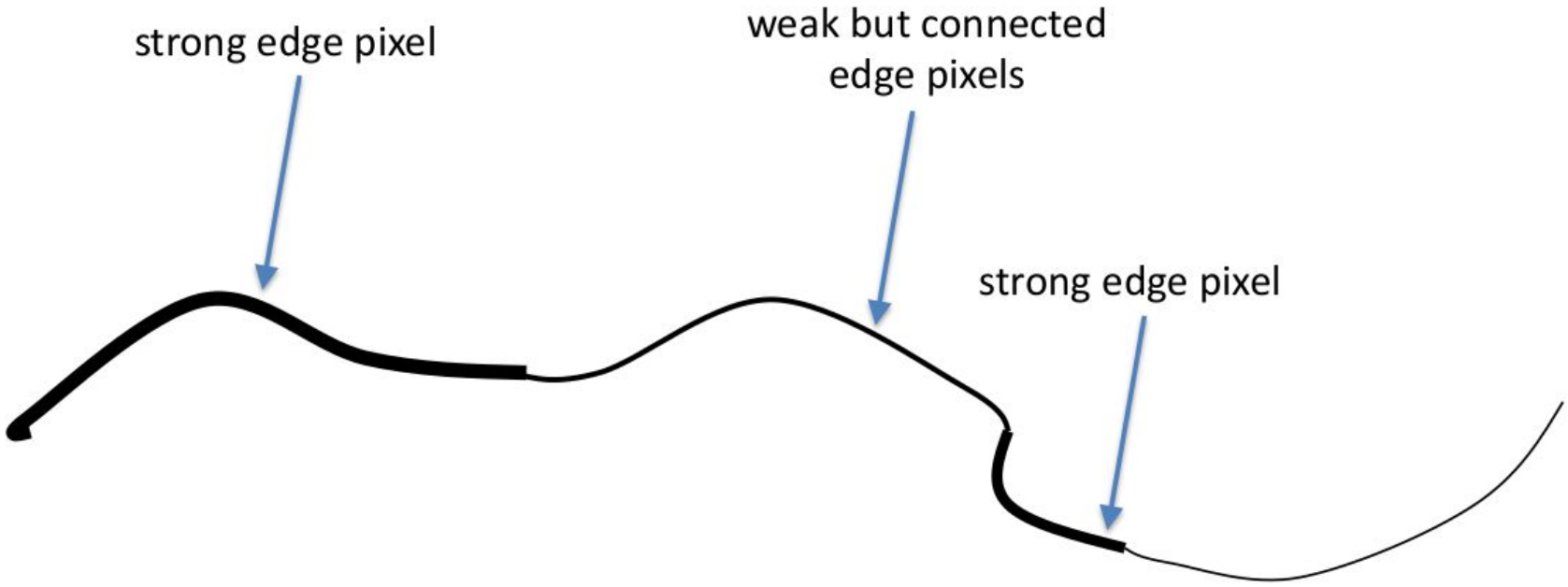
Canny edge detector (step-by-step)

Hysteresis thresholding and connectivity analysis

- Define two thresholds: Low and High
  - pixel $<$ Low: **not an edge**
  - pixel $>$ High: **strong edge**
  - Low $<$ pixel $<$ High: **weak edge**

Canny edge detector (step-by-step)

Hysteresis thresholding and connectivity analysis



- Re-declare weak edge as strong edge if it is in the same connected component with a strong edge
- Re-declare weak edge as not an edge if it has no strong edges in the connected component

Canny edge detector (step-by-step)

Non-Maximum Suppression

Non-Maximum Suppression

Hysteresis thresholding

Canny edge detector

Hysteresis thresholding

Edge detection for a noisy image

Practical exercise: Sobel filter (in jupyter)