Rad Cube (working title)

Control and design of reaction wheel balanced inverted pendulum

MIKAEL SJÖSTEDT
ALEXANDER RAMM

Bachelor's Thesis in Mechatronics

Supervisor: Daniel Frede
Examiner:Martin Edin Grimheden
Approved: TBA 2015-month-day

TRITA xxx yyyy-nn

Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...
To be filled in:
Problem
Approach
Results
Conclusion

Sammanfattning

Stabilisering med svänghjul

Projektet gick ut på att bygga en kub som kan balansera på en kant med hjälp av ett reaktionshjul. Dessutom sulle den undersökas huruvida det gick att förbättra reglersystemet för cuben, så den klarade en större yttre störning. Ingengörsproblemet delades upp i mindre delproblem och kuben byggdes. Reglestystemet beräknades på formen State spaceöch implementerades. Från resultatet drogs slutsatserna att...

Preface

Here goes our thanks to sources of help, cooperation, inspiration
To be filled in

Alexander Ramm
Mikael Sjöstedt
KTH, månad, 2015

Contents

Appendices

Nomenclature

Symbols - needs restructure

Symbol	**Description**
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
θ	Cube angle
ϕ	Flywheel angle
Q and q	Lagrange operators
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_c	Total mass of the cube
i	Current
K_t	Torque constant
$\mathrm{E_{emf}}$	Induced voltage
$\mathrm{K_{emf}}$	Induced voltage constant
U	Voltage across motor poles
R_m	Motor internal resitance
η_m	Motor efficiency
z	Measurement noise
w	Process noise

Abbreviations

Abbreviation	**Description**
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

1.1 Background

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. The cost of MEMS sensors today are cheap due to high demand and production [[Ciufo(2012)]]. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters. The open-source community is growing but still lacks automated control systems for balancing robots. (out of context atm...)

1.2 Purpose

The goal of this project was to build a balancing mechanical system and to examine the behaviour of the particular system.
The behaviour is mostly effected by the control system, which is responsible for accelerating the motor in the correct angular direction, to maintain balance. The parameters in the control system effects response time, overshoot and sinusoidal settling time. This project will hopefully contribute to some development within the open-source community. All results are available online, open source (MIT license reference here), on GitHub (GitHub link here).

> *If balance is maintained how does the maximum applied force correlate to the rise time and overshoot separately, can any conclusions be drawn from the results?*

Where balance is defined as as the state where the cube is able to return to its reference angle/value?. The rise time and overshoot refers to the system angle. Can the results contribute to improve the overall performance?

1.3 Scope

The scope were to examine the parameters, of the state space controller and sensor sample frequency, affects on the overshoot behaviour of a balancing "1-DOF" inverted pendulum. The overshoot should be caused by an external force, disrupting the cubes balance. Moar "we will not do this"

Chapter 2

Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

2.1 Contruction

The main construction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem were (uppställt) with Newtonian mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is also important, but usually one can get suggestions on drivers from motor manufactures, which was the chosen path.

2.3 Sensor Reading

The IMU's parameters and filtering of the signals

2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

2.5 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.

Chapter 3

Theory

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due tu the instability of pendulums. The latest development is on "2-DOF" pendulum structures using multiple orthogonal reaction wheels. This method is commonly used to rotate satellites and maintaining their attitude to increase performance and align solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research i.e. not only changing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be useful for land/sea based machines to overcome various obstacles without a separate system for balancing and movement.

Chapter 4

Demonstrator

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstartorn utvecklats enligt, dvs resultatet och vägen dit.

4.1 Problem Formulation

The engineering problem where to build a cube that, using a reaction wheel, could balance on its edge. *To be continued*

4.2 State space model

PICTURE TO BE ADDED
To create a state-space model the physical model has to be translated to a mathematical model. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) \tag{4.1}$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) \tag{4.2}$$

$$-M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \left(\frac{\partial L}{\partial \phi}\right) \tag{4.3}$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \tag{4.4}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{4.5}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{4.6}$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin\theta}{\sqrt{2}} = -M_a \tag{4.7}$$

$$I_s \cdot \ddot{\phi} = M_a \tag{4.8}$$

From these equations it is evident that M_a is the torque executed by the flywheel which is wielded by the motor torque τ, it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \tag{4.9}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \tag{4.10}$$

The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \tag{4.11}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{4.12}$$

$$\tau = \frac{K_t}{R_m} U - \frac{K_t K_{\text{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m} \dot{\theta} \tag{4.13}$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \tag{4.14}$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\theta} + \frac{M t g l}{\sqrt{2} I_c} \sin\theta \tag{4.15}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\theta} \tag{4.16}$$

With the equations (4.15) and (4.16) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \tag{4.17}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Mtgl}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

4.3 Kalman filter

Still a bit messy
The problem with the accelerometer is that that the signal contains a lot of noise. It can be accounted for measurements over a longer period but not short term. The gyroscope on the other hand, drifts over time and is only reliable during short accelerations. A Kalman filter suits the needs of this project. It is a recursive estimator, where old and new measurements are processed realtime. A good estimator produces states that are non biased, *values that have an average of the true value.* And that the estimated state variance from the true state is as small as possible. Hence the Kalman filter is optimal with the use of an accelerometer as the error is gaussian much like many other measurement devices. [Simon(2001)] [Welch and Bishop(2006)]

4.3.1 The process

The discrete Kalman filter which is used is a state based estimator. By using the measurements from both the accelerometer and the gyroscope from the past, present (future??) it can derive a good estimate of the state in our linearised problem. The Kalman filter estimates the present state using the following equations

$$x_k = Ax_{\text{k-1}} + Bu_{\text{k-1}} + w_{\text{k-1}} \tag{4.18}$$

$$y_k = Hx_k + z_k \tag{4.19}$$

Where x is the states of the cube which cannot be measured directly i.e θ and $\dot{\theta}$. y is the measured value which would be the serial data from the IMU , which is a function of x but is distorted by the measurement noise z The process noise w in equation (4.18) is a representation of variances? in the system behavior that cannot be mathematically predicted. z, the measurement noise is common in any measurement and represents fluctuations in the equipment or **develop this thought**. For the Kalman filter to work properly some criteria has to be fulfilled. The average value of the measurement noise z and process noise w has to be zero, a gaussian error. z and w also has to be independent of each other as the noise covariance can then be expressed by

$$S_z = E(z_k {z_k}^{\mathrm{T}}) \tag{4.20}$$

$$S_w = E(w_k {w_k}^{\mathrm{T}}) \tag{4.21}$$

To be continued...

4.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink® was used togheter with the mathematical model. The Simulinkmodel seen in figure 4.1 describes the system

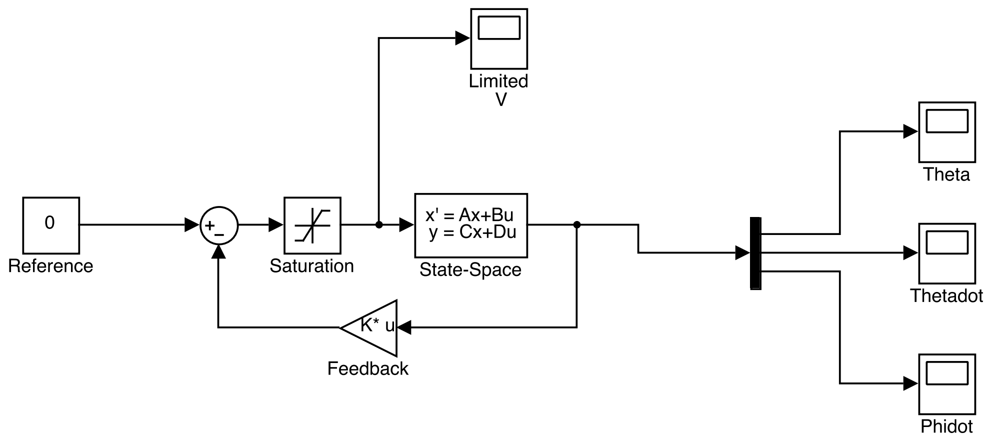

Figure 4.1. Simulink model.

Something about the optimizing of the feedback control

The voltage supplied to the motor

The angle of the cube. Very good such magic

4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar. How do we send data?
Sensors
Motor
Arduino
Motor control

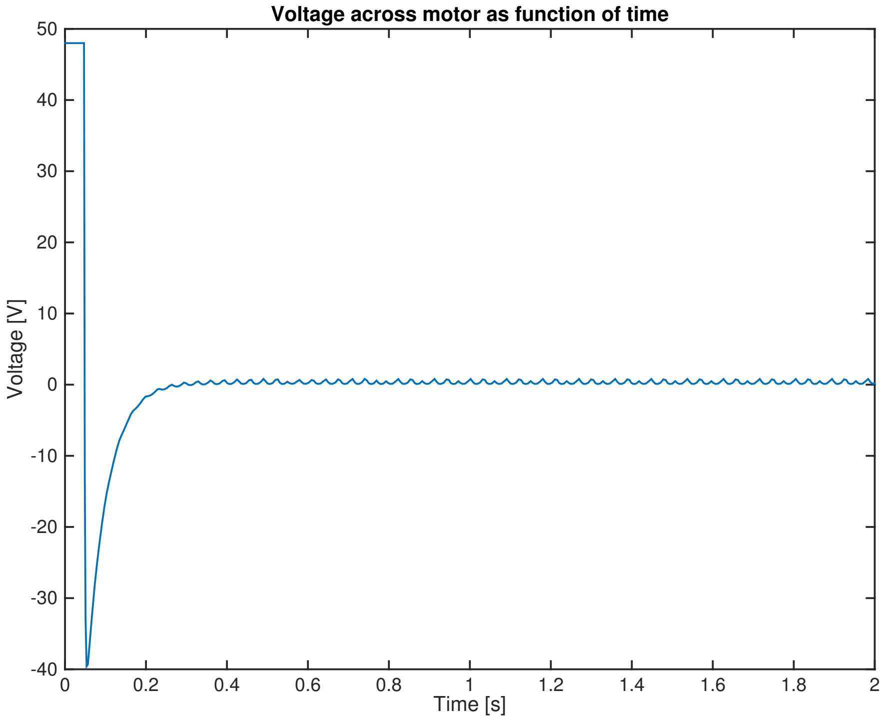

Figure 4.2. Voltage across motor poles.

4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.
Basic construction

4.7 Results

Beskriv resultatet.

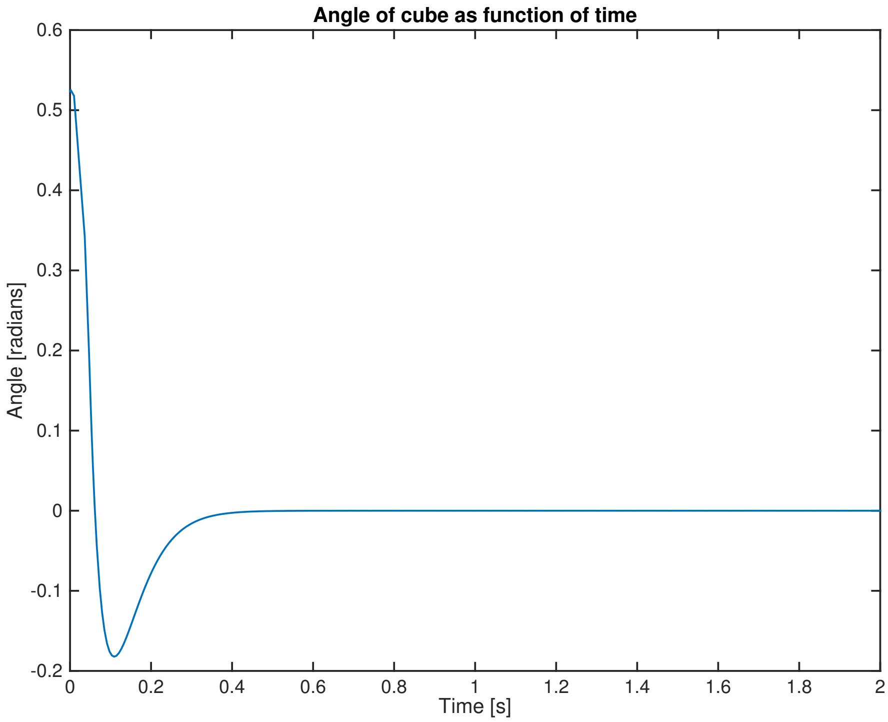

Figure 4.3. Angle of the cube.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Motor choice osv

5.2 Conclusions

Successful victory

Chapter 6

Recommendations and future work

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

Bibliography

[Ciufo(2012)] C. A. Ciufo, MEMS Trends: Smaller, Cheaper, Everywhere, URL http://eecatalog.com/sensors/2012/05/24/mems-trends-smaller-cheaper-everywhere/, 2012.

[Simon(2001)] D. Simon, Kalman Filtering, Embedded System Programming 1 (6) (2001) 72–79.

[Welch and Bishop(2006)] G. Welch, G. Bishop, Introduction to the Kalman Filter, Tech. Rep., Department of Computer Science, University of North Carolina at Chapel Hill, 2006.

[Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea] M. Gajamohan, M. Muehlebach, T. Widmer, R. D'Andrea, The Cubli: A Reaction Wheel Based 3D Inverted Pendulum, in: proc. European Control Conference, 268–274, 2013.

Appendix A

Additional information

Appendix B

Proofs