Rad Cube (working title)

Control of reaction wheel balanced inverted pendulum

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Bacherlor's Thesis in Mechatronics

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Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...
To be filled in:
Problem
Approach
Results
Conclusion

Sammanfattning

Stabilisering med svänghjul

Skriv som abstract men på svenska

Preface

Here goes our thanks to sources of help, cooperation, inspiration
To be filled in

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KTH, månad, 2015

Contents

Nomenclature

Symbols - needs restructure

Symbol	**Description**
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
θ	Cube angle
ϕ	Flywheel angle
Q and q	Lagrange operators
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_c	Total mass of the cube
i	Current
K_t	Torque constant
$\mathrm{E_{emf}}$	Induced voltage
$\mathrm{K_{emf}}$	Induced voltage constant
U	Voltage across motor poles
R_m	Motor internal resitance
η_m	Motor efficiency

Abbreviations

Abbreviation	Description
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

1.1 Background

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. The cost of MEMS sensors today are cheap due to high demand and production [[Ciufo(2012)]]. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters.

1.2 Purpose

The purpose of this thesis was to examine what effected the stability of the mechanical system, especially the control system. Many parameters are to be accounted for in the state-space model and their effects on the mechanical system behaviour is not trivial. The aim was to determine if some of the parameters had an extra importance and if any conclusions were applicable on other mechanical systems. The research can be summarized with:

> *'What improvements can be made to the control system to better cope with an applied external force to the cube?'*

Sample frequency and clock frequency are other parameters which were to be examined as well.
The open-source community is growing but still lacks automated control systems for balancing robots. This project will hopefully contribute to some development within the open-source community. All results are available online, open source

(MIT license reference here), on GitHub (GitHub link here).

1.3 Scope

The scope were to examine the parameters, of a state space controller and other electronical desitions, affect on the behaviour of a balancing "1-DOF" inverted pendulum.

Chapter 2

Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

2.1 Contruction

The main contruction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes abillity to balance. The problem where (uppställt) with Newtionial mechanics.

2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha

2.3 Sensor Reading

The IMU's parameters and filtering of the signals

2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

Chapter 3

Theory

Den teoretiska fördjupningen är en sammanfattning av tillgänglig kunskap och resultat från forskning som tidigare har utförts inom examensarbetets område. Detta kapitel presenterar den teoretiska referensramen som utgör utgångspunkten för den utförda forskningen, produktutvecklingen eller konstruktionsuppgiften.
Här måste det köttas på mycket inför tisdag 9/3

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due tu the instabillity of pendulums. The latest development is on "2-DOF" pendelum structures unsing multiple orthogonal reaction wheels. This method is commonly used to rotate sattelites and maintaining their attitude to increase performance and allign solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research ie not only changeing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be usefull for land/sea based machines to overcome various obstacles without a seperate system for balancing and movement.

Chapter 4

Demonstrator

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstartorn utvecklats enligt, dvs resultatet och vägen dit.

4.1 Problem Formulation

Beskriv din problemställning för demonstratorn.

The engineering problem where to build a cube that, using a reaction wheel, could balance on its edge.

4.2 State space model

PICTURE TO BE ADDED

To create a state-space model the physical model has to be translated to a mathematical model. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) - \left(\frac{\partial L}{\partial q_i}\right) \tag{4.1}$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) \tag{4.2}$$

$$-M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \left(\frac{\partial L}{\partial \phi}\right) \tag{4.3}$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \tag{4.4}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{4.5}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{4.6}$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin\theta}{\sqrt{2}} = -M_a \tag{4.7}$$

$$I_s \cdot \ddot{\phi} = M_a \tag{4.8}$$

From these equations it is evident that M_a is the torque executed by the flywheel which is wielded by the motor torque τ, it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \tag{4.9}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \tag{4.10}$$

The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \tag{4.11}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{4.12}$$

$$\tau = \frac{K_t}{R_m}U - \frac{K_t K_{\text{emf}}}{R_m}\dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m}\dot{\theta} \tag{4.13}$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \tag{4.14}$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\theta} + \frac{Mtgl}{\sqrt{2} I_c}\sin\theta \tag{4.15}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\theta} \tag{4.16}$$

With the equations (4.15) and (4.16) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \tag{4.17}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Mtgl}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

4.3 Kalman filter

Nu har jag bara slängt ut nyckelord. Den ska styras upp :) Men 2 bra källor iaf tror jag
It is an estimator, recursive, new measurements are processed on the go. optimal for gaussian error much like the one in piezoelectrothings, minimises the mean square error algorithm that uses observed measurement over time which in this case is the measurements from the gyro and accelerometer. The problem with the accelerometer is that it's very noisy even tough the cube only is making fine adjustments. The gyro on the other hand, drifts over time [Simon(2001)] [Welch and Bishop(2006)]

4.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink® was used togheter with the mathematical model. The Simulinkmodel seen in figure 4.1 describes the system

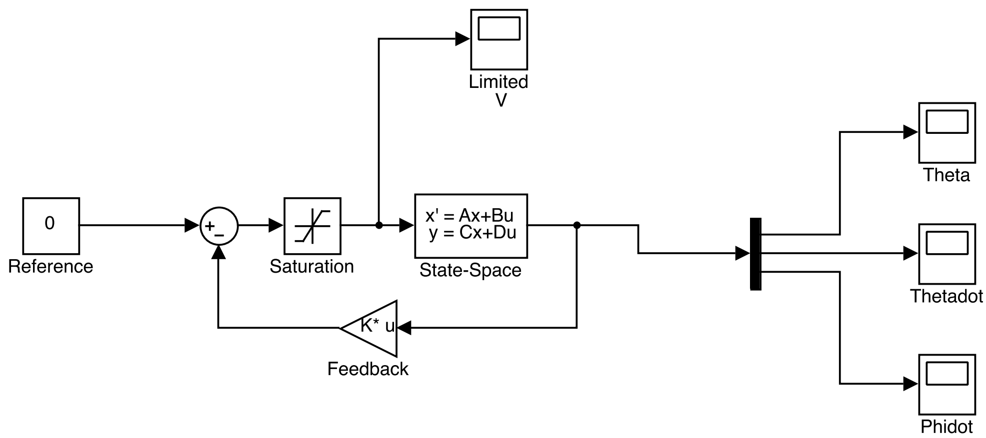

Figure 4.1. Simulink model.

Something about the optimizing of the feedback control

The voltage supplied to the motor

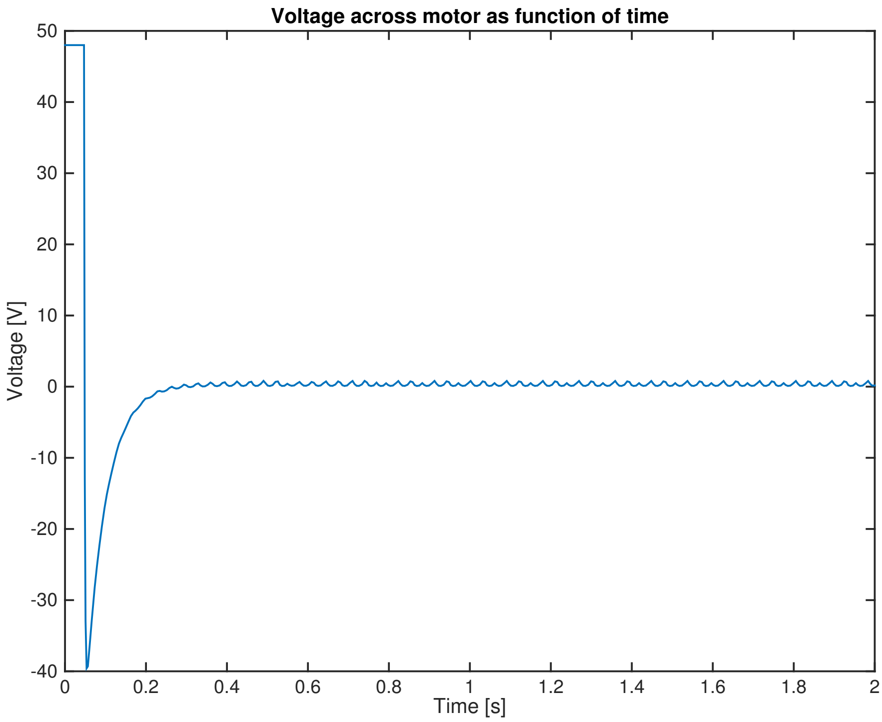

Figure 4.2. Voltage across motor poles.

The angle of the cube. Very good such magic

4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar.
Sensors
Motor
Arduino
Motor control

4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.
Basic construction

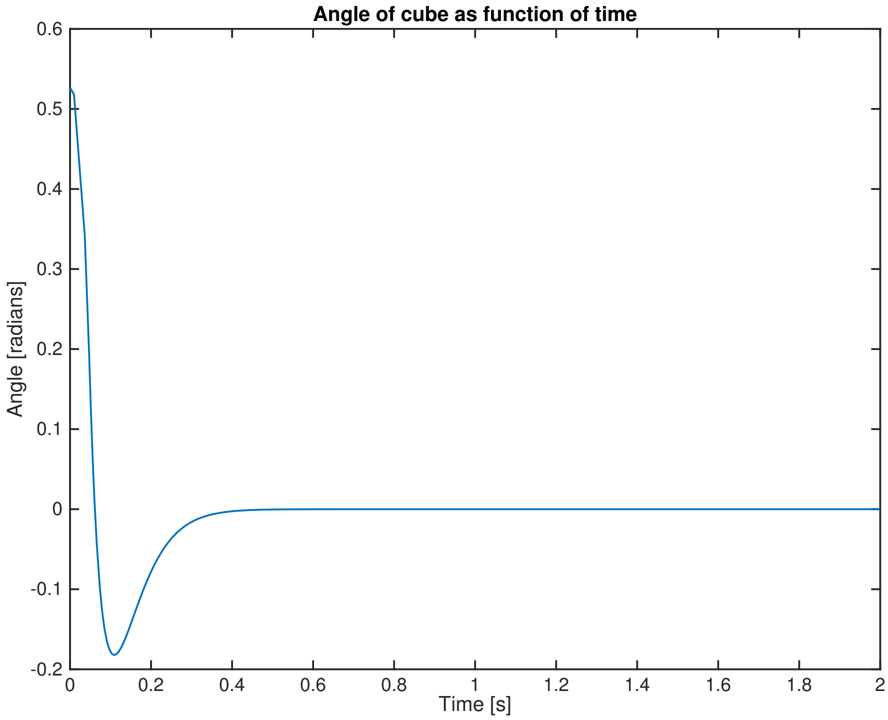

Figure 4.3. Angle of the cube.

4.7 Results

Beskriv resultatet.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Bla bla bla

5.2 Conclusions

Bla bla bla

Chapter 6

Recommendations and future work

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

Bibliography

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Appendix A

Additional information

Appendix B

Proofs