Balancing Cube (working title)

Stabilization and design of reaction wheel based inverted pendulum. –?– Control and design of reaction wheel balanced inverted pendulum

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Bachelor's Thesis in Mechatronics

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Abstract

Robots that moves requires a high degree of precision from position tracking sensors. This paper studies how the placement of these sensors affect the robots ability to determine its position. A robot with a cubical frame were built, which were able to balance on a edge with help of a reaction wheel. The robot could determine its rotation using a sensor type called –inertial measurement unit–. Different sensor positions were evaluated empirically and...

Sammanfattning

Stabilisering med svänghjul Utevkcla...

Robotar som förflyttar sig kraver mycket precis nogrannhet från sina positionerings sensorer. Det här rapporten tar upp hur placeringen av dessa sensorer påverkar robotens förmåga att bestämma sin position. Från en kubformad ram byggdes en robot, som med hjälp av ett motordrivet svänghjul kan applicera ett internt moment för att balansera på en kant. Roboten använde en sensor av typen –inertial measurment unit– för att bestämma sin position. Olika placeringar av sensorn utvärderases empiriskt och ...

Preface

Here goes our thanks to sources of help, cooperation, inspiration
To be filled in

Alexander Ramm
Mikael Sjöstedt
KTH, månad, 2015

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Nomenclature

Symbols - needs restructure

Symbol	**Description**
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
$\mathcal{L}$	Lagrange
θ	Cube angle
ϕ	Flywheel angle
q	Lagrange operator
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_{tot}	Total mass of the cube
M_f	Mass of the flywheel
i	Current
K_t	Motor torque constant
$\mathrm{E_{emf}}$	Induced voltage
$\mathrm{K_{emf}}$	Motor voltage constant
U	Voltage across motor poles
R_m	Motor internal resistance
η_m	Motor efficiency
η_g	Gear efficiency
u	Gear ratio
z	Measurement noise
w	Process noise

Abbreviations

Abbreviation	**Description**
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
RMS	Root Mean Square
MCU	Microcontroller
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus
UAV	Unmanned Aerial Vehicle

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH, Sweden. The work was carried out during the spring 2015.

1.1 Background

A reaction wheel is a wheel that is accelerated to apply torque to something. The most wide spread use of reaction wheels is in human made satellites. The reaction wheels, usually three of them in the case of satellites, are used to change the attitude of the satellite by applying torque in a favourable manner. This is imperative to direct solar panels towards the sun or pointing antennas to assure maximum performance and connectivity to the satellite. Compared to most machines satellites are quite uncommon (there are 1100 satellites currently in orbit [source]), and their technology can at times seem alien. But reaction wheels should not be alienated, they can be used in many contexts and this paper will cover one of them.

Balancing 1 *degree of freedom* (DOF) inverted pendulum type structures using reaction wheels is no new concept, and became more accessible with the introduction of cheap microcontrollers. The use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. This growth has made automated control together with sensors available more now than ever. It can be seen in the every-day life in product lines such as mobile phones, gaming controllers, cars and UAV's such as quadrocopters.

One of the most basic systems that requires some control to become stable is the inverted pendulum. Although it is simple to define controlling it is not a trivial task. A lot of work has been done on the topic but there are still no knowledge easily acquired by the public available.

The method to achieve balance of the pendulum using reaction wheels is even more narrow. In recent years prototypes of land based structures using reaction wheels have been a hot topic and some of these robots are truly remarkable.

It would be a great achievement to contribute knowledge about how such a mechanism could be built and evaluate the capabilities and restrictions of such a machine, on a level that does not require a PhD.

1.2 Purpose

The goal of the project was to build a structure that in one degree of freedom that can maintain balance using a reaction wheel and examine the behaviours of the system. The behaviour (samma ord igen) is mostly effected by the control system, which is responsible for accelerating the motor in the correct angular direction, to maintain balance. The parameters in the control system effects response time, overshoot and sinusoidal settling time. This project will hopefully contribute to some development within the open-source community. All results are available online, open source (MIT license reference here), on GitHub [Sjöstedt and Ramm(2015)]. As a mechatronical thesis, this paper can be divided into two parts. One engineering part which focus is to implement knowledge in mechanics, electronics and control theory to result in a functioning robot. And then a research part which topic could be concentrated to a question

How does the sensor placement effect the quality of the sensor data.

The only sensor that can be placed arbitrarily in the system is the *Inertial Measurement Unit* (IMU). Certain positions might have an advantage in terms of how usable the raw data is. The IMU is a sensitive devise and disturbances such as high current and fast oscillations in its vicinity might ruin the data entirely.[citation PLZ]. With quality defined as the usability of the data given by the sensor.

1.3 Scope

The only sensor to be examined was the IMU. The encoder for the motor is fixed to the motor shaft and was not examined. Only a few key positions of the sensor were examined.

The effects that were looked at were the ones linked to the control system. Mainly the overshoot behaviour and the settling time of the system. Only data used for the specific control system were examined, other DOF measurements were not taken into consideration. I.e. the results may only be applicable in similar machines and not in general.

For every position the same parameters and constants were used in all systems and software. The comparisons were made in between measurements while balance was maintained and no external disturbance is applied. All measurements where taken during a limited time frame (We dont know this yet....).

1.4 Method

The sensor was placed in the upper corner, one of the side corners, between the mentioned corners and in the center of the cube (insert figure reference). Both raw data and filtered data were collected and sent over serial to a computer. Using Matlab [MATLAB(2014)] the readings where analysed for stablization behaviour. Other obvious observations were noted. To have equal conditions, all measurements were made on the same horizontal space with the same external voltage supply.

Chapter 2

Theory

This chapter cover some theory that is required if one want to build a similar robot. It is assumed that the reader has some understanding of Newtonian mechanics, signal analysis and control theory. Basic understanding of DC motor operation is also an advantage.

The first part is about the inertial reference unit that covers issues of sensor characteristics and why they are important to the system as a whole and the research question in particular.

There is a part that discuss Kalman filter theory, a filter required for getting high quality data from the IMU. The filter interprets noisy data from the sensor and digitally filters the signal to a more trustworthy output.

The last part will cover the theory of the mechanical system behaviour that is used to develop the state space control system. The equations are responsible for the actual balance part and thus important.

2.1 Inertial Measurement Unit

The data collected for calculating the angle of the cube is gathered from an IMU. This is an unit that uses both an accelerometer and a gyroscope to track the orientation and position. An IMU is often rated for several degrees of freedom, a unit specified as 6-DOF uses three orthogonal accelerometers and gyroscopes. These measures linear acceleration and angular velocity in each direction seperately. There are also units that are rated for additional degrees of freedom that usually includes features such as magnetometer or barometer sensors. To understand the fundamentals of an inertial system a cartesian coordinate system is defined

The inertial navigation system used in this project is a small *microelectromechanical system* (MEMS). A micromechanical sensor is more or less a very small unit that take use of its mechanical properties to sense alteration in the environment SOURCE . The advantages of these small units are low production costs, small size and low power consumption. As the research of these fairly modern units continues the reliability increases but they still hold a disadvantage versus the optical units

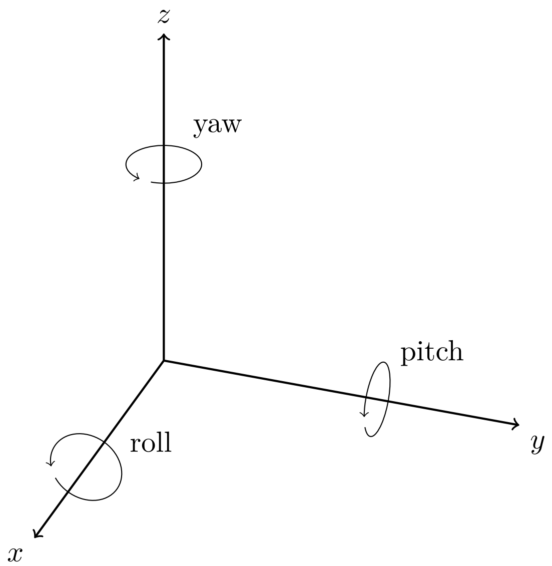

Figure 2.1. Cartesian coordinate system

that is accuracy. [Woodman(2007)]

2.1.1 Accelerometer

Accelerometers are used to measure linear acceleration. Or rather, the device measure forces due to acceleration. These forces can be divided into two groups

- Static forces, such as gravity
- Dynamic forces, due to movement

The force is then converted to an acceleration, this is done by measuring the change in capacitance when a spring mass system is moving. The typical accelerometer consists of a movable mass that is attached via a mechanical spring or suspension system to a frame that is used as a reference.

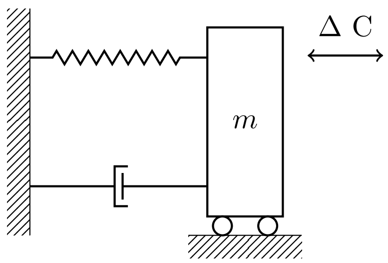

Figure 2.2. Accelerometer M

The change of capacitance is converted to a voltage that is sent to the microcontroller for further use. The typical noise sources in an accelerometer is mechanical vibration of the springs, the circuitry and the measurement as well. These noise terms can be characterized by a white noise. Relevant for this project is how this noise effects the integrated value which is represented by the *velocity random walk* (VRW). The accelerometer also outputs a constant bias, it is essential to determine the bias when estimating a position with the help of an accelerometer. [Andrejasic(2008)]

2.1.2 Gyroscope

Gyroscopes unlike accelerometers, does not measure linear acceleration. Gyroscopes, or gyros as they are referred as in everyday speech, measure the angular rate of velocity. This is done by making use of the Coriolis effect to measure the angular rate.

Gyroscope pic here

A mass is vibrating along an axis, with the momentary velocity v, and when the mass is rotated, a secondary perpendicular vibration is induced which is explained by the coriolis force

$$\mathbf{F}_c = -2m(\boldsymbol{w} \times \boldsymbol{v}) \tag{2.1}$$

The result is a physical displacement due to the Coriolis force and a capacitance is measured just like the accelerometer. So for example if a rotation occurs along the x-axis the gyroscope would output a *roll* rate.

A micromechanical gyroscope is, like the accelerometer, effected by a constant bias. This is often due to friction caused by moving parts or production variations that induces stress on the construction resulting in an offset of the output. If a constant error is integrated the angular error grows linearly with time. This is easily corrected by subtracting the bias from the output. The constant bias introduced above is not entirely constant either. The small size and sensitivity of this device is making the bias wander due to flickering noise in the electronics. Hence a *bias stability* is introduced as a measurement of how the bias may change during a period of time. More troublesome errors that occur in MEMS gyroscopes thermomechanical white noise similiar to the accelerometer, a more or less uncorrelated error. This can be translated to a phenomena known as *Angle Random Walk* or ARW that indicates how the integrated value is effected. The concepts of ARW, ARW and bias stability that has been introduced are more or less an indication of how precise the are. [Woodman(2007)]

2.2 Kalman filter

The signal from an IMU contains data of angular velocities and acceleration, but also a lot of noise. An estimated position of an untreated signal from an IMU could work for short periods, but over time the estimated position *drifts* [Shiau et al.(2012)Shiau, Huang, and Chang]. This drift occurs when measurements containing noise is integrated to acquire a position, the readings contain both white noise and often a bias which is making the error to grow for every calulation. Integrating the angular motion from the gyro to estimate a position would result in an angular drift and an even worse drift for the accelerometer as it is integrated twice to estimate a position. By using a Kalman filter the drift can effectively be minimized. If the readings from both the gyroscope and accelerometer is considered, and with some help of probablity theory the estimated state is not far from the true value. A Kalman *filter* is not what the name suggests, it is an estimator. Old and new measurements are processed real-time to calculate an estimation of the current state. Keep in mind that there are some regards that should be taken into consideration when choosing an estimator. A good estimator produces states that are non biased, *values that have an average of the true value.* As well that the estimated state variance from the true state is as small as possible. [Simon(2001)]

2.2.1 State Estimator

The Kalman filter is, as stated above, a state based estimator. By using the last measurement and the one before that it can derive a better estimate of the current state. The true state and the measured value at a time k would be

$$x_k = Ax_{\text{k-1}} + Bu_{\text{k-1}} + w_{\text{k-1}} \tag{2.2}$$

$$z_k = Hx_k + v_k \tag{2.3}$$

The true state x is expressed with the the old state, an input u, in this case data from the gyroscope. But the signal also contains a process noise w. The process noise w in equation (2.2) is a representation of variances in the process that cannot be mathematically predicted. When using a gyro this reflects the error characteristics mention in section2.1.2. The measured value, z (see (2.3)) is an observed measurement, in this case the accelerometer. Ideally this would only be a function of x, but is distorted by the measurement noise v. The measurement noise, v, much like the process noise is common in any measurement and represents various fluctuations caused by the equipment.

As this recursive filter uses old and new values a *priori* and *posteriori* state is defined

$$\hat{x}_k^- \tag{2.4}$$

$$\hat{x}_k \tag{2.5}$$

The *priori* (2.4) state is defined as the estimate of the current state at the time k. The *posteriori* state (2.5) is the new estimated state. For the Kalman filter to work properly some criteria has to be fulfilled. The average value of the measurement noise z and process noise w has to be zero, i.e. a Gaussian error. z and w also has to be independent of each other. The noise and error in an IMU and many other devices have the charecteristics of gaussian noise.

2.2.2 The process

The Kalman filter loops two stages. The *predict* and *update* stages.

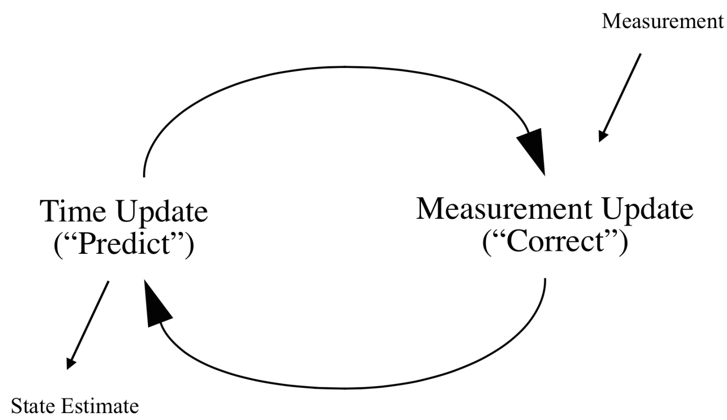

Figure 2.3. Kalman phases.

During the *predict* phase the filter estimates the states using the inputs from the process, i.e the gyroscope. It then moves on to the *update* phase where it compares the state to the measurement, the accelerometer. See figure 2.3

$$\hat{x}_k^- = A\hat{x}_{\text{k-1}} + Bu_{\text{k-1}} \tag{2.6}$$

As stated above the Kalman filter uses readings from both the gyroscope and accelerometer to estimate a position closer to the true value. To determine how reliable the process and measurement readings are a noise covariance is defined as

$$Q = E(w_k {w_k}^{\mathrm{T}}) \tag{2.7}$$

$$R = E(v_k {v_k}^{\mathrm{T}}) \tag{2.8}$$

How to determine these covariances are further investigated in section 3.3.1 From here a *priori* error covariance matrix is introduced to symbolize the noise in the process measurement

$$P_k^- = AP_{\text{k-1}}A^T + Q_k \tag{2.9}$$

During the *update* the accelerometer values are used. The measurement *innovation* is calculated as

$$\tilde{y} = z_k - H\hat{x}_k^- \tag{2.10}$$

The *innovation* is a residual that reflects the relation between the predicted measurement and the actual measurement. A measurement *innovation* of zero indicates a perfect agreement. The measurement *innovation* covariance is calculated as

$$S_k = HP_k^- H^T + R \tag{2.11}$$

The *innovation* covariance is very similiar to the *priori* error covariance but represents the measurement instead. From here the core of the Kalman filter can be calculated, the Kalman gain

$$K_k = P_k^- H^T S^{\text{-}1}{}_k \tag{2.12}$$

indicates how reliable the measurement is. Note that if the measurement covariance error (2.8) is large the Kalman gain will be small and vice versa if the *priori* error covariance is large. By now the *posteriori* state can be estimated by

$$\hat{x}_k = \hat{x}_k^- + K_k\tilde{y}_k \tag{2.13}$$

A current state has been estimated and the Kalman filter and finally the *priori* error covariance is updated

$$P_k = (I - K_kH)P_k^- \tag{2.14}$$

The filter now returns to the measurement phase seen in figure 2.3. For further reading, and mathematical proof see [Welch and Bishop(2006)].

2.3 Model dynamics

To create a state-space model the physical model has to be translated to a mathematical model. The system can be estimated much like an inverted pendulum two-degree-of-freedom model [Chauveau et al.(2005)Chauveau, Chazal, Nakayama, Olsen, and Palm].

Lagrangian Dynamics have been used to derive the systems behaviour. Firstly by expressing the generlized forces, the energy functions and lagrangian. And then acquire the equations of motion from the Lagrange equation [Block et al.(2007)Block, Åström, and Spong]. Consider the Lagrangian equation

$$\tau_i = \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}_i}\right) - \left(\frac{\partial\mathcal{L}}{\partial q_i}\right) \tag{2.15}$$

Where τ is generilzed force, in this case a torque. The cube's angular momentum is counteracted by the flywheel and the system can be divided into two parts, One considering the movement of the cube, the other the flywheel.

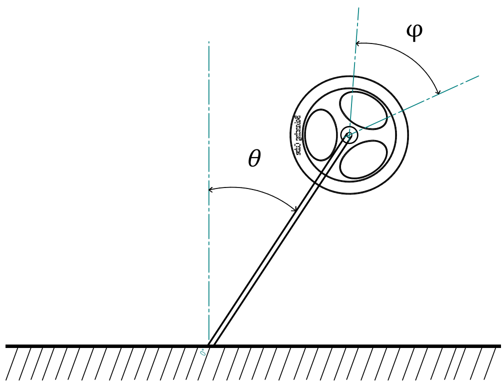

Figure 2.4. Cube modelled as a reaction wheel pendulum

$$\tau_k = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) - \left(\frac{\partial \mathcal{L}}{\partial \theta}\right) \tag{2.16}$$

$$-\tau_k = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) - \left(\frac{\partial \mathcal{L}}{\partial \phi}\right) \tag{2.17}$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$\mathcal{L} = E_k - E_p \tag{2.18}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{2.19}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{2.20}$$

The lagrangian (2.18) is then

$$\mathcal{L} = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} - \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{2.21}$$

The kinetic energy depends on the angular velocities of the cube construction as well as the flywheel fixed to the motor. Note that the total moment of inertia I_c is defined around the pivot point of the cube. The potential energy has been defined as being at its maximum when the cube is balancing in an upright position.

The construction is considered to be symmetric and hence the gravitational force is applied on the center of the cube. Equation (2.16) and (2.17) with (2.18)

$$I_c \cdot \ddot{\theta} + \frac{M_c \cdot g \cdot l \cdot \sin\theta}{\sqrt{2}} = -\tau_k \tag{2.22}$$

$$I_s \cdot \ddot{\phi} = \tau_k \tag{2.23}$$

From these equations it is evident that τ_k is the torque executed on the flywheel which is wielded by the motor torque τ_m, it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau_m = K_t \cdot i_m \tag{2.24}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau_m = K_t \cdot \frac{U - E_{\mathrm{emf}}}{R_m} \tag{2.25}$$

Note that the motor inductance in neglected in equation (2.25), that is due to the time constant which is fast considering the rest of the system and is not vital for the control system [Chauveau et al.(2005)Chauveau, Chazal, Nakayama, Olsen, and Palm]. The induced voltage can be described as a function of motor speed.

$$E_{\mathrm{emf}} = K_{\mathrm{emf}} \cdot \dot{\phi}_r \tag{2.26}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{2.27}$$

$$\tau_m = \frac{K_t}{R_m} U - \frac{K_t K_{\mathrm{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\mathrm{emf}}}{R_m} \dot{\theta} \tag{2.28}$$

The torque executed on the flywheel can then be described with the torque on the motor shaft, efficiency and gearing.
Not completely finished with the part below

$$\tau_k = \tau_m \cdot \eta_m \cdot \eta_g \cdot u \tag{2.29}$$

Based on equation (2.17), (2.16) and (2.29) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\mathrm{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\mathrm{emf}} \eta_m}{R_m I_c} \dot{\theta} - \frac{M t g l}{\sqrt{2} I_c} \sin\theta \tag{2.30}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\mathrm{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\mathrm{emf}} \eta_m}{R_m I_f} \dot{\theta} \tag{2.31}$$

To use linear control methods the model has to be linearised. This is done at the instable equilibrium where the cube is balancing. Consider the sinus term at the equilibrium point where θ equals 0. The term can then be expressed with taylor/macloaruin expansion

$$sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}... \approx \theta \tag{2.32}$$

With the equations (2.30) and (2.31) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \tag{2.33}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mtgl}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

2.4 Control theory

To create a state space feedback loop... Use Ackermann instead of place? Why ?

2.5 Pulse Width Modulation

Pulse Width Modulation: According to en.wikipedia.org: "Pulse-width modulation (PWM) [...] is a technique used to encode a message into a pulsing signal." This is useful in many power applications, from dimming LED's to motor control. The idea of PWM is to alter the voltage over a device while still only having a set level voltage to supply. Simplified the voltage is cut into a square wave. This is done by using two transistors, one that can short the motor poles and one that distributes the supply voltage to be applied to the motor [Johansson(2003)]. If these transistors are switched on and of fast compared to the time constant of the motor, the root mean square (RMS) voltage will be the acting voltage. This allows for a DC voltage that is perceived as lower than the acual supply voltage. They duty cycle can be changed during operation unlike liner type voltage regulators.

Chapter 3

Demonstrator

"NUTS"

3.1 Problem Formulation

The construction of the cube can be seen as the engineering problem. The cube should be a robot that, using a reaction wheel can balance on its edge. All components were to be mounted in the robot, then only requiring a power source. The main componets was a frame, reaction wheel. motor, motor controller, arduino, IMU, and encoder.

The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

All these individual system had to be implemented and joined in the final assembly.

3.2 Model validation

To synthesize a mathematical model from a real world problem it's often beneficial to simplify the reality. Examples of assumption made for this application would be that center of mass is located at the center of the cube, the friction in the motor is ignored and the frame is considered stiff etcetera. To validate the model from chapter 2.3, events with known results can be tested. To do so, Simulink [MATLAB(2014)] is

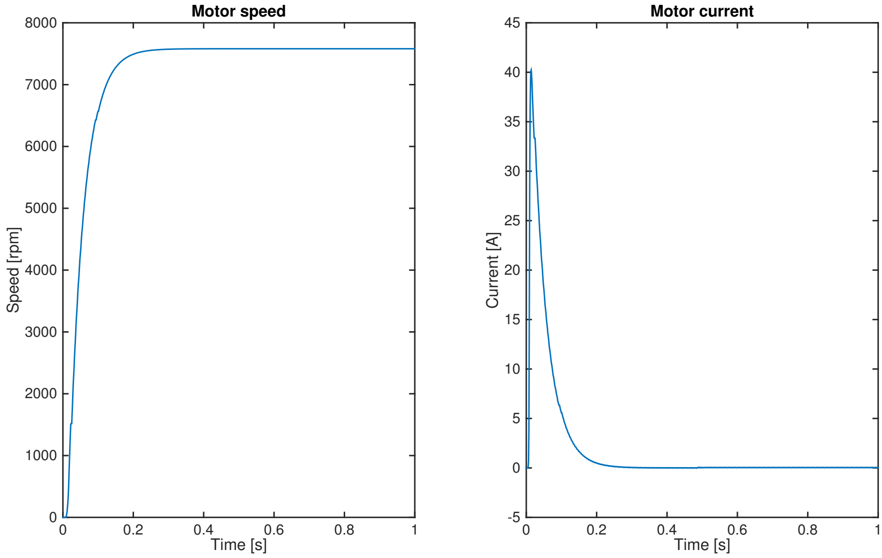

Figure 3.1. Validation of motor model

used. First of all the DC-motor model is validated to known characteristics, such as no load speed and current.

The graphs in figure 3.1 displays the speed and current of the unloaded motor. Showing that the motor model correlates with the specified speed and current of an unloaded motor.

The dynamics of the cube is simplified as an inverted pendulum. That means if there is no control input to the system it should behave as pendulum in free movement. That is, it should oscillate at a constant amplitude. As there is no torque applied to the flywheel rotor should be static at all times.

3.3 Kalman implementation

For the implementation of Kalman filter a library made by Lauszus which is available a the Github repository was used [Lauszus(2012)]. For an expanded explanation of the Kalman implementation, see appendix A.1. The results of the Kalman filtering can be seen in figure 3.2

The figure shows a comparison of the estimated angle when using the accelerometer and gyroscope separately and then using the Kalman filter. Plain to see from the plot is that the data sent from the accelerometer contains a lot of noise while the gyroscope drifts.

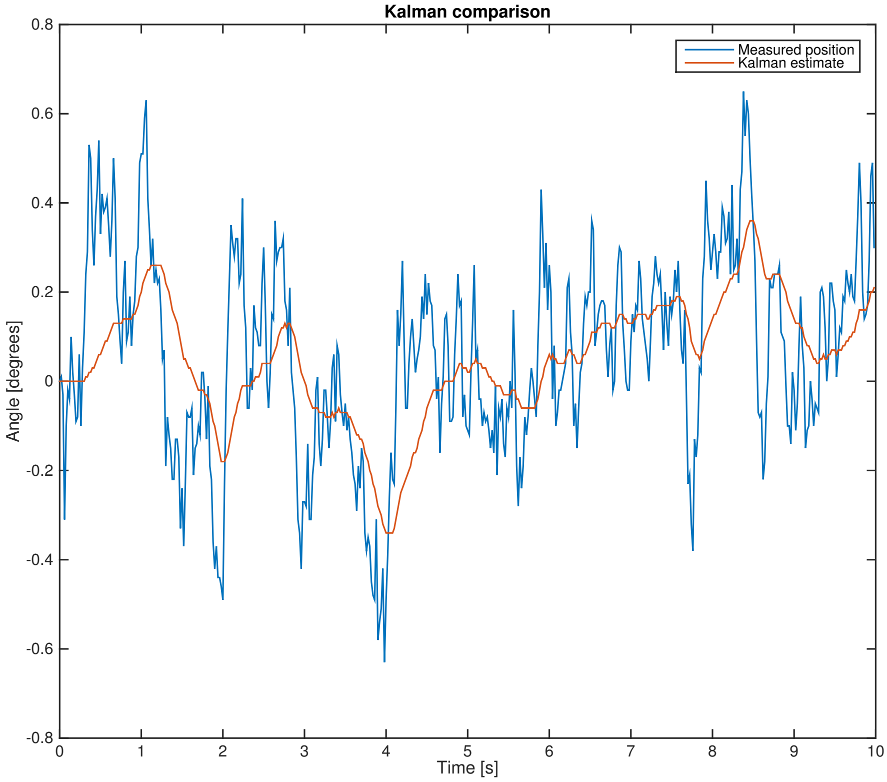

Figure 3.2. Comparison of Kalman filtered signal and original signal (TO BE UPDATED)

3.3.1 Measurement and process noise

For the Kalman filter to properly work it is essential to know how reliable the process and measurement inputs are. A way of determining the process noise and measurement noise of the IMU is the Allan variance method REF. The gyro data is treated as an external input to the system, so the error and bias from the gyro readings are characterised as process noise. This is then compared to the measurement, the accelerometer which contains a measurement noise. By gathering samples from the gyroscope and accelerometer at a stationary state the Allan variance can be calculated. The Allan variance is then used to determine the noise ans stability of the system. The interesting components of the variance for the IMU are the ARW, VRV and bias stability mentioned in section 2.1. The theory of Allan variance is out of the scope of this thesis but for reader reference is a time domain analysis technique commonly used to determine the characteristics of errors for inertial sensors [Han et al.(2009)Han, Wang, and Knight]. Data from the gyro were collected during twelve hours to achieve/för att få a reliable estimate of the gyro bias. The root Allan variance was calculated and can is described by figure 3.3 where the variance is a function of averaging time *tau*

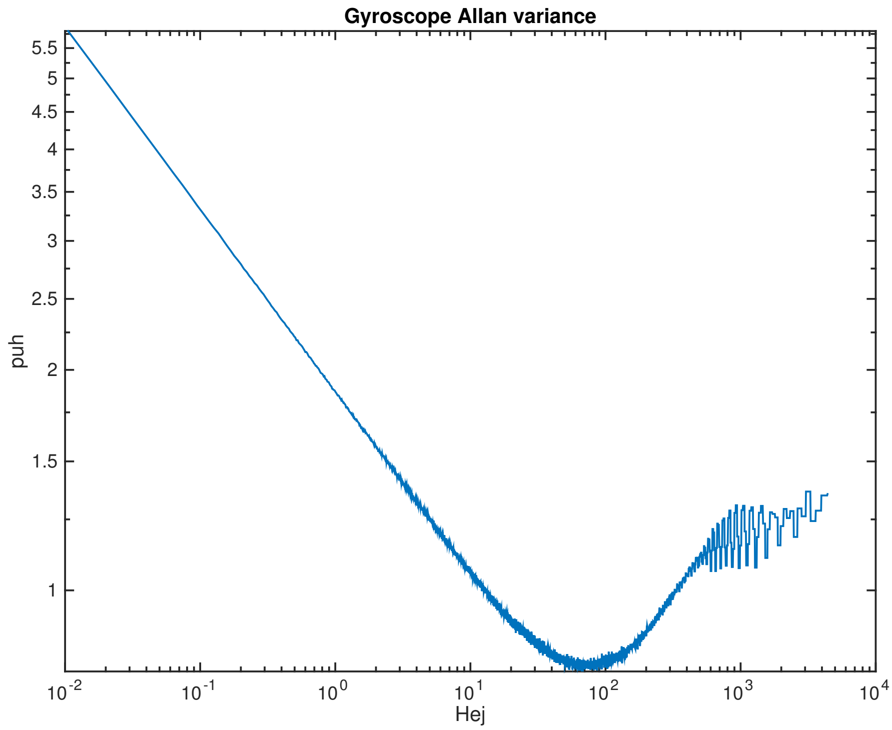

Figure 3.3. Gyroscope root Allan variance FIX PIC

The angle (velocity) random walk can be depicted from the plot at an angle of -0.5 while the bias instability is determined by looking at the lowest point of the plot.

3.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime, the model were simulated with Simulink®.

The Simulinkmodel seen in figure **??** describes the system
Something about the optimizing of the feedback control

The voltage supplied to the motor

The angle of the cube. Very good such magic

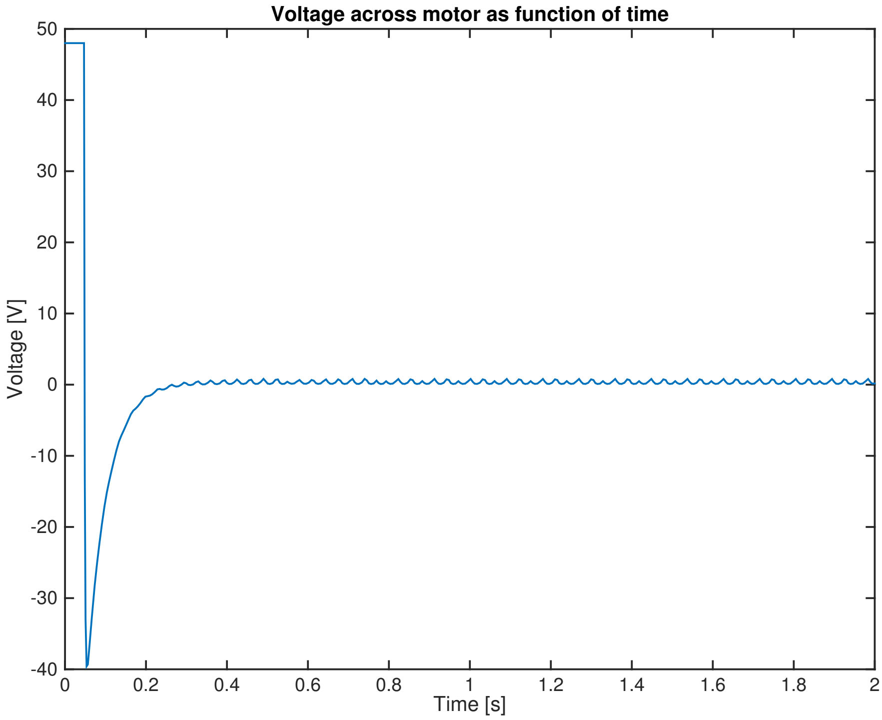

Figure 3.4. Voltage across motor poles.

3.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar. How do we send data?
Sensors
Motor
Arduino
Motor control

3.5.1 PWM

skriv lite om PWM hax

3.5.2 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

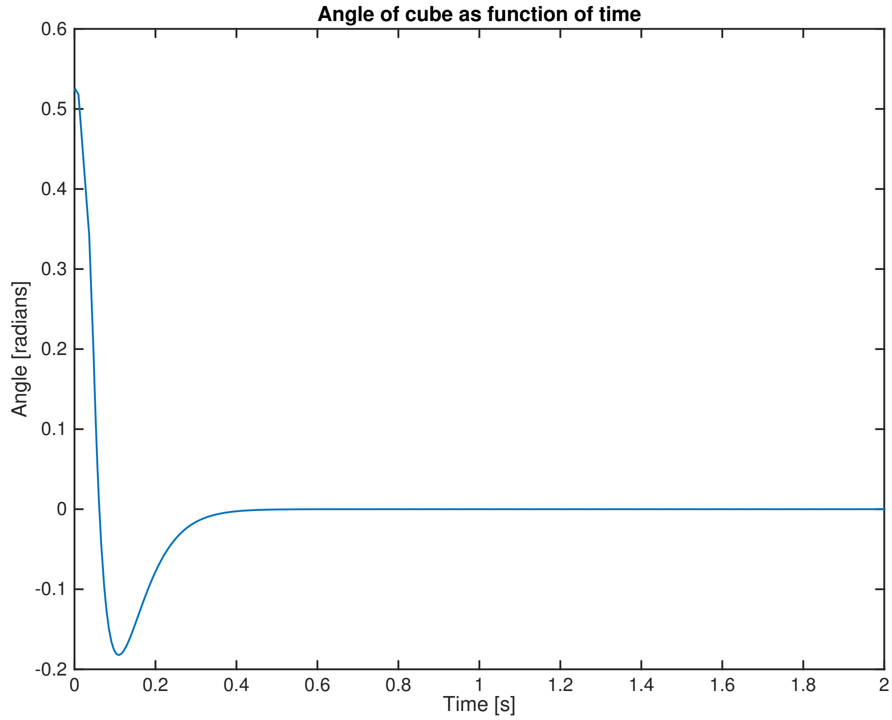

Figure 3.5. Angle of the cube.

3.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.
Basic construction

3.6.1 Construction

The main construction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem were (uppställt) with Newtonian mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

3.6.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is also important, but usually one can get suggestions on drivers from motor manufactures, which was the chosen path.

3.6.3 Sensor Reading

The IMU's parameters and filtering of the signals

3.6.4 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.

Chapter 4

Results

Beskriv resultatet.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Motor choice osv

5.2 Conclusions

Successful victory

Chapter 6

Recommendations and future work

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

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Appendix A

Additional information

A.1 Kalman implementation, more elaborative

KALMAN IMPLEMENTATION GOES HERE The Kalman filter cycles two states, the *predict* and *update* phases. Kalman is using discretized steps making the filter implemented on a microprocessor fairly simple. An implementation of the Kalman filter on the IMU would look something like this

Consider the equation (2.6) from theory chapter. At first the filter predicts the state

$$\hat{x}_k^- = A\hat{x}_{\text{k-1}} + Bu_{\text{k-1}} \tag{A.1}$$

$$\mathbf{x}_k = \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_k u_{\text{k-1}} = \dot{\theta} \tag{A.2}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \tag{A.3}$$

The states used here are the angle as well as the bias. Next up is the *priori* error covariance

$$P_k^- = AP_{\text{k-1}}A^T + Q_k \tag{A.4}$$

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^-_{\text{k}} = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}_{\text{k-1}} \begin{bmatrix} 1 & 0 \\ -\Delta t & 1 \end{bmatrix} + \begin{bmatrix} Q_{\theta} & 0 \\ 0 & Q_{\dot{\theta}_b} \end{bmatrix} \Delta t \tag{A.5}$$

The innovation covariance mentioned in 2.11

$$S_k = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^-_k \begin{bmatrix} 1 \\ 0 \end{bmatrix} + R \tag{A.6}$$

Kalman gain

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_k = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^-_k \begin{bmatrix} 1 \\ 0 \end{bmatrix} S_k^{-1} \tag{A.7}$$

The predicted states are then updated with the WEIGHED measures from the accelerometer

$$\begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_k = \begin{bmatrix} \theta \\ \dot{\theta}_b \end{bmatrix}_k^- + \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_k \tilde{y} \tag{A.8}$$

And then at last the error covariance

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}_{\text{k}} = (\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}_k \begin{bmatrix} 1 & 0 \end{bmatrix}) \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}^-_{\text{k}} \tag{A.9}$$

Appendix B

Proofs