SWING UP AND STADILIZATION OF REACTION WHEEL PENDULUM

Pavol Seman [1], Martin Juhás [2]

Abstract: The article deals with swing up and stabilization of the inverted pendulum designed as pendulum with the reaction wheel. First, the mathematical model of the pendulum is created. On model basis, LQ controller for stabilization if pendulum in the upper position is synthesized. For swing up control of pendulum energy principle is used. Finally the pendulum control is implemented and verified using B&R automation PLC and physical laboratory pendulum. Implementation of code generated out of MATLAB Simulink model into industrial PLC application is also demonstrated.

Keywords: reaction wheel pendulum, model, LQ control, B&R automation

1. Introduction

Inverted pendulum is often used means for exploring nonlinear mechatronic systems with fast dynamics. Its use is especially suitable for the learning process, when it is relatively easy to develop a mathematical model of such a system and to design and implement various control techniques, whether to stabilize or swing up the pendulum. There are different possible layouts of the system with inverted pendulum. This paper deals with the reaction wheel pendulum. It is a planar system with two degrees of freedom and one actuator. The pendulum link is at one end freely attached at pivot pin and at the other end of the pendulum the reaction wheel is attached. Reaction wheel is driven by actuator - electric motor which is connected to pendulum link. When wheel speed changes, reaction torque that controls the pendulum is generated.

Physical pendulum system is created on the B&R automation platform. Pendulum is controlled by PLC. The control scheme has been developed within MATLAB Simulink and then implemented into the control application of PLC.

2. Mathematical model

A mathematical model was obtained by Lagrange equations of the second kind [1,3]. They were used for derivation of the equations of motion of the system. Table 1 lists the model parameters describing the pendulum it is a schematic representation in Figure 1.

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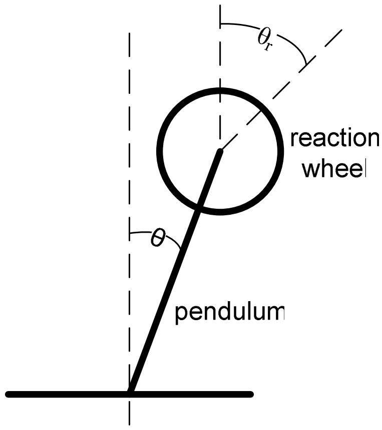

Fig. 1 Reaction wheel pendulum

Table 1. The basic parameters of the model

Symbol	Quantity
θ	pendulum position
θ_r	wheel position
m	mass of the complete pendulum
l	reduced pendulum length
g	gravitational acceleration
J	torque of inertia of the complete pendulum
J_r	torque of inertia of motor and rotor
i_p	reduction ratio of the transmission
η_m	motor efficiency
η_p	gear efficiency
R_m	terminal resistance of the motor
k_m	torque constant of the motor
k_{EMF}	motor back electromotive force (EMF)

The basic form of Lagrange equations is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = Q_i\,, \tag{1}$$

where L is the Lagrange function, q_i is the i-th generalized coordinate and Q_i is a generalized force in the direction of i-th coordinate. Lagrange equations for reaction wheel pendulum:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = -\tau_k \tag{2}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_r}\right) - \left(\frac{\partial L}{\partial \theta_r}\right) = \tau_k \tag{3}$$

Lagrange function is expressed as the difference between kinetic and potential energy of the system

$$E_k = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}J_r\dot{\theta}_r^2 \tag{4}$$

$$E_p = mgl(\cos\theta - 1) \tag{5}$$

$$L = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}J_r\dot{\theta}_r^2 - mgl(\cos\theta - 1) \tag{6}$$

Nonlinear equations of motion of the pendulum model have the following form:

$$J\ddot{\theta} - mgl\sin\theta = -\tau_k \tag{7}$$

$$J_r\ddot{\theta}_r = \tau_k \tag{8}$$

Torque τ_k, exerted by the reaction wheel is dependent on torque τ_m. This is created by the electric motor. It depends on the flowing current I_m and on voltage U respectively.

$$\tau_m = k_m I_m \tag{9}$$

Torque generated by the motor when electromotive force generated by a rotating motor is considered:

$$\tau_m = k_m \frac{U - E_{EMF}}{R_m} \tag{10}$$

$$E_{EMF} = k_{EMF}\dot{\theta}_{rt} \tag{11}$$

considering motor rotor position θ_{rt}:

$$\theta_{rt} = (\theta_r - \theta)i_p \tag{12}$$

$$\tau_m = k_m \frac{U}{R_m} - \frac{k_m k_{EMF}\dot{\theta}_r i_p}{R_m} + \frac{k_m k_{EMF}\dot{\theta}\, i_p}{R_m} \tag{13}$$

$$\tau_k = \tau_m \eta_m \eta_p i_p \tag{14}$$

auxiliary constant, used to simplify expressions:

$$k_p = k_m \eta_m \eta_p i_p \tag{15}$$

Based on previous entries and adjustments, system linear equations of motion of pendulum model can be expressed as:

$$\ddot{\theta} = \frac{mgl\theta}{J} - \frac{k_p U}{R_m J} + \frac{k_p k_{EMF}\dot{\theta}_r i_p}{R_m J} - \frac{k_p k_{EMF}\dot{\theta}\, i_p}{R_m J} \tag{16}$$

$$\ddot{\theta}_r = \frac{k_p U}{R_m J_r} - \frac{k_p k_{EMF}\dot{\theta}_r i_p}{R_m J_r} + \frac{k_p k_{EMF}\dot{\theta}\, i_p}{R_m J_r} \tag{17}$$

Based on the equations of motion (16) and (17) model of system can be created with state vector $x^T = [\theta \quad \dot{\theta} \quad \dot{\theta}_r]$ and input u:

$$\dot{x} = Ax + Bu \tag{18}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{mgl}{J} & \frac{-k_p k_{EMF} i_p}{R_m J} & \frac{k_p k_{EMF} i_p}{R_m J} \\ 0 & \frac{k_p k_{EMF} i_p}{R_m J_r} & \frac{-k_p k_{EMF} i_p}{R_m J_r} \end{bmatrix} \tag{19}$$

$$B = \begin{bmatrix} 0 \\ \frac{-k_p}{R_m J} \\ \frac{k_p}{R_m J_r} \end{bmatrix} \tag{20}$$

3. LQ controller

It is a quadratic optimal controller designed for linear systems [4]. Determination of controller parameters is based on minimizing the quadratic criterion (21). Standard procedure of minimizing criterion based on state space model description and leads to solving of Riccati equation (22).

$$J = \min\left\lfloor x^T Q \;\; x + u^T R \;\; u \right\rfloor \tag{21}$$

Matrix Q is a weight matrix of states and matrix R is a weight matrix of input of the system.

$$A^T P + PA + Q = PBR^{-1} B^T P \tag{22}$$

K is gain optimal controller and is obtained as:

$$K = R^{-1} B^T P \tag{23}$$

4. Swing up of pendulum

For swing up pendulum energy principle is used. It is based on adding energy to the system through vibrant pendulum by turning the reaction wheel. Wheel speed and thus the torque generated is dependent on the current energy of the system. It is defined as the sum of kinetic and potential energy of the system. And this value is compared with the value of the system energy when the pendulum is in the upper unstable equilibrium position. The value of energy at this point is:

$$E_0 = 2mgl \tag{24}$$

To determine the control output, following expression can be used:

$$u = sat_n\left(k(E - E_0)sign(\dot{\theta})\right), \tag{25}$$

where k is a constant determining the speed for swing up. Expression is a bit modified as to determine the direction of action of the resultant torque only direction of movement of pendulum is used. In original expression for Furuta pendulum [2] a direction depends on the quadrant in which the pendulum is situated and for determination of the direction the term $sign(\dot{\theta}\cos\theta)$ is used. In presented case, it is sufficient only to know direction of movement of pendulum. This change is due to a change in the position of actuator action on pendulum.

This algorithm does not work if the pendulum is located in the lower stable equilibrium position and therefore startup control pulse is generated to start movement of pendulum.

5. Physical pendulum model

As mentioned above model is realized on the B&R automation platform. This means that control runs on standard industrial products of this company. Model itself is made of freely rotating pendulum. Position of the pendulum to the base is measured by the relative position IRC sensor. On the free end of the pendulum DC motor with gearbox which drives the reaction wheel is mounted. Relative position of the motor rotor and pendulum link is also measured by the position IRC sensor. The motor is controlled by PWM signal.

The online control for model is cared by control unit X20CP1484 from company B&R automation. Model of pendulum is attached to the controller through the I/O card that tracks the position of the pendulum and the wheel and also generates a PWM signal with a width of period 20 microseconds for of motor control.

Control strategy to stabilize and swing up the pendulum is running online in PLC with period 0,001 s. It was created in MATLAB Simulink environment. From the created scheme it is possible to generate code in C language that is then integrated into control application of the project for PLC.

Figure 2 shows the above described way of connecting the individual elements of the model.

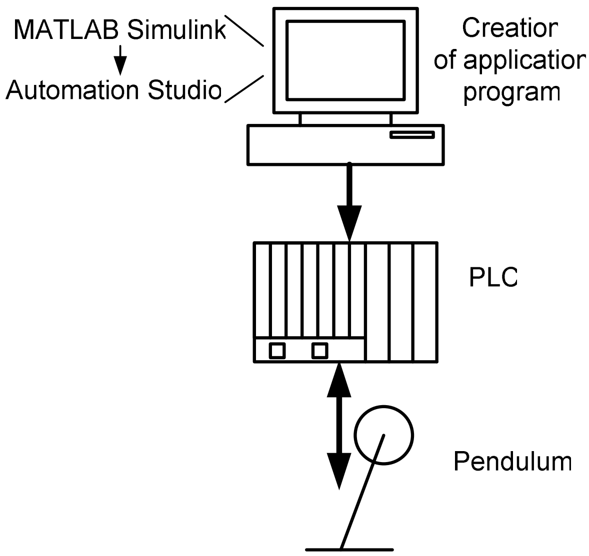

Fig. 2 Hardware configuration

6. Experimental verification of control algorithms

To verify designed algorithms for swing up and stabilization of the pendulum an experiment was conducted. The pendulum is initially situated in the lower equilibrium position. When the control is started, after few swing up oscillations, pendulum is placed around the upper unstable equilibrium position. At the beginning of the course of applied action intervention (Fig. 3) the impulse that moves the pendulum from the equilibrium position can be seen.

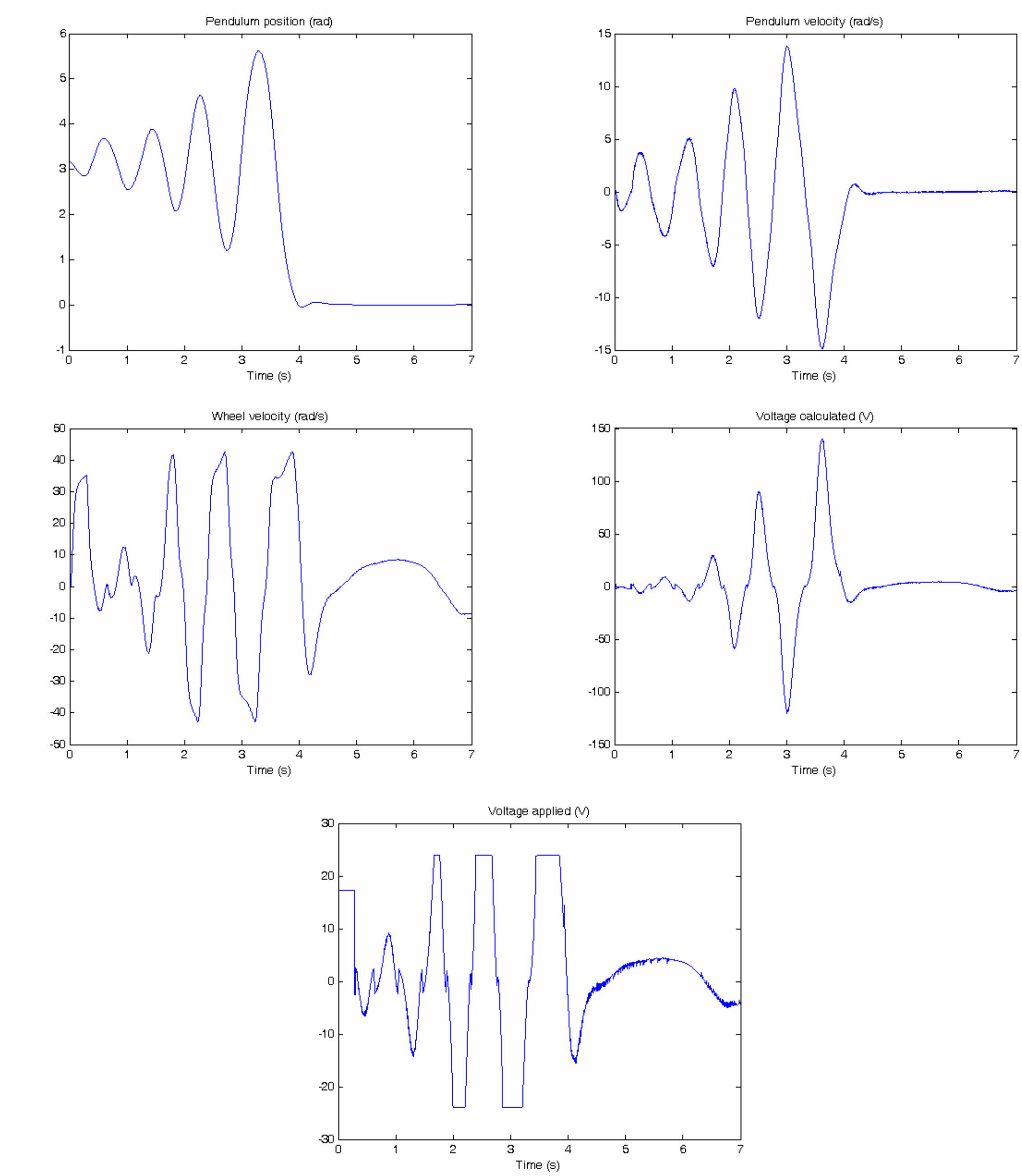

Fig. 3 Swing up and stabilization of pendulum

As can be seen from the waveforms, designed control strategy is able to swing up and stabilize the pendulum in the upper equilibrium position even when applied control action during swing up sequence is significantly cut off by limits of motor parameters. Its maximum voltage is 24 V.

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