

# **Balancing Cube (working title)**

Control and design of reaction wheel balanced inverted pendulum

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Approved: TBA 2015-month-day

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# Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...

To be filled in:

Problem

Approach

Results

Conclusion



# Sammanfattning

## Stabilisering med svänghjul

Projektet gick ut på att bygga en kub som kan balansera på en kant med hjälp av ett reaktionshjul. Dessutom skulle den undersökas huruvida det gick att förbättra reglersystemet för kuben, så den klarade en större yttre störning. Ingengörsproblemet delades upp i mindre delproblem och kuben byggdes. Reglersystemet beräknades på formen State space och implementerades. Från resultatet drogs slutsatserna att...



# Preface

Here goes our thanks to sources of help, cooperation, inspiration  
To be filled in

Alexander Ramm  
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KTH, månad, 2015





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# Nomenclature

## Symbols - needs restructure

Symbol	Description
$E$	Elasticity module (Pa)
$r$	Radius (m)
$t$	Thickness (m)
$L$	Lagrange (fixa)
$\theta$	Cube angle
$\phi$	Flywheel angle
Q and q	Lagrange operators
$E_k$	Kinetic energy
$E_p$	Potential Energy
$I_c$	Inertia of the cube
$I_f$	Inertia of the flywheel
$M_c$	Total mass of the cube
$i$	Current
$K_t$	Torque constant
$E_{emf}$	Induced voltage
$K_{emf}$	Induced voltage constant
$U$	Voltage across motor poles
$R_m$	Motor internal resistance
$\eta_m$	Motor efficiency
$z$	Measurement noise
$w$	Process noise

## Abbreviations

Abbreviation	Description
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
$I^2C$	Inter-Integrated circuit
USB	Universal Serial Bus

# Chapter 1

## Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

### 1.1 Background

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became more accessible with the introduction of cheap micro-controllers. The use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters.

One of the most basic systems that requires some control to become stable is the inverted pendulum. Altho it is simple to define controlling it is not a trivial task. A lot of work has been done on the topic but there's still not knowledge easily acquired by the public.

The method to achieve balance of the pendulum using reaction wheels is even more narrow. The use of reaction wheels to change the rotation is commonly used in satellites. The exact control is also required. In recent years prototypes of land based structures using reaction wheels have been a hot topic and the Cubli is truly remarkable.

It would be a great achievement to contribute knowledge about how such a mechanism could be built and evaluate the capabilities and restrictions of such a machine, on a level that doesn't require a PhD.

### 1.2 Purpose

The goal of the project was to build a structure that in one DOF can maintain balance using a reaction wheel and examine the behaviours of the system.

The behaviour is mostly effected by the control system, which is responsible for

accelerating the motor in the correct angular direction, to maintain balance. The parameters in the control system effects response time, overshoot and sinusoidal settling time. This project will hopefully contribute to some development within the open-source community. All results are available online, open source (MIT license reference here), on GitHub (GitHub link here).

As a mechatronical thesis the research task were to be along the line of how something physical is implemented in a, in some context, good or even correct way.

*If balance is maintained how does the maximum applied force correlate to the rise time and overshoot separately, can any conclusions be drawn from the results?*

or maybe

*How does sensor placement effect system performance -and can internal disturbances terminaTOR nej men något sånt???*

Where balance is defined as as the state where the cube is able to return to its reference angle/value?. The rise time and overshoot refers to the system angle. Can the results contribute to improve the overall performance?

### 1.3 Scope

The scope were to examine the parameters, of the state space controller and sensor sample frequency, affects on the overshoot behaviour of a balancing "1-DOF" inverted pendulum. The overshoot should be caused by an external force, disrupting the cubes balance. Moar "we will not do this"

## Chapter 2

# Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

### 2.1 Construction

The main construction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem were (uppställt) with Newtonian mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

### 2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is also important, but usually one can get suggestions on drivers from motor manufactures, which was the chosen path.

### 2.3 Sensor Reading

The IMU's parameters and filtering of the signals

## 2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

## 2.5 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.



## Chapter 3

# Theory

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due to the instability of pendulums. The latest development is on "2-DOF" pendulum structures using multiple orthogonal reaction wheels. This method is commonly used to rotate satellites and maintaining their attitude to increase performance and align solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research i.e. not only changing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be useful for land/sea based machines to overcome various obstacles without a separate system for balancing and movement. **Move some to background**

### 3.1 Moar theory ?

Where do we mention communication protocol, maybe some in-depth of accelerometer and gyro ?

### 3.2 Kalman filter

The problem with an IMU is that that the signal contains a lot of noise. For short periods the IMU supplies reliable data but over time the estimated position 'drifts'. This drift occurs because of integration of the measurements to acquire a position which in theory works, but the readings contain unwanted noise and often a bias as well. Integrating the angular motion to estimate a position would result in an angular drift for the gyro and an even worse drift for the accelerometer as it is integrated twice if it were to estimate a position [?]. By using a Kalman filter the drift can effectively be minimized. By using both readings from the IMU and with some help of probability theory the estimated state is not far from the true value. A Kalman *filter* is not what the name suggests, it's an estimator. Old and new measurements are processed real-time to calculate an estimation of the

current state. What could be asked of the filter? **is this too "friendly"** A good estimator produces states that are non biased, *values that have an average of the true value*. As well that the estimated state variance from the true state is as small as possible. [Simon(2001)]

### 3.2.1 Estimator (better subtitle...)

The Kalman filter is a state based estimator. By using the measurements from the past and present it can derive a good estimate of the current state. The Kalman filter estimates the present state using the following equations

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (3.1)$$

$$y_k = Hx_k + z_k \quad (3.2)$$

Where  $x$  is the states, in this case the states of the cube which can't be measured directly i.e a position. The measured value,  $y$  (see (3.2)) is information from another source, in this case the accelerometer. In an ideal world this would only be a function of  $x$ , but is distorted by the measurement noise  $z$ . The process noise  $w$  in equation (3.1) is a representation of variances in the gyroscope that cannot be mathematically predicted such as flaws in production. The measurement noise,  $z$  is common in any measurement and represents various fluctuations caused by the equipment. As the recursive filter uses old and new values a *priori* state  $\hat{x}_k^-$  is defined as the estimate of the current state at the time  $k$ . The *posteriori* state  $\hat{x}_k$  is the new estimated state. For the Kalman filter to work properly some criteria has to be fulfilled. The average value of the measurement noise  $z$  and process noise  $w$  has to be zero, i.e. a Gaussian error.  $z$  and  $w$  also has to be independent of each other. The noise and error in an IMU and many other devices have the characteristics of gaussian noise. By using the above definitions of process noise and measurement noise the noise covariance is defined as

$$Q = E(w_k w_k^T) \quad (3.3)$$

$$R = E(z_k z_k^T) \quad (3.4)$$

### 3.2.2 The process

The Kalman filter loops two stages. The *predict* and *update* stages.

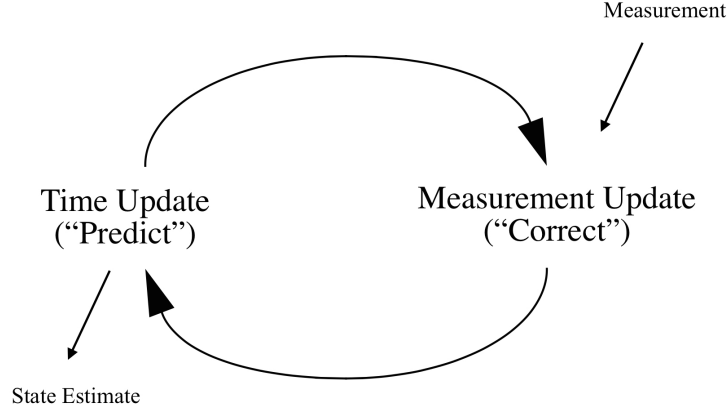
During the *predict* phase the filter estimates the states using the inputs from the process, i.e the gyroscope. 3.1

$$\hat{x}_k^- = A\hat{x}_{k-1}^- + Bu_{k-1} \quad (3.5)$$

As stated above **is this stated above?** .. the Kalman filter uses readings from both the gyro and accelerometer to estimate a position closer to the true value. A *priori* error covariance matrix is introduced to estimate how much the the readings from the process can be trusted.

$$P_k^- = AP_{k-1}^- A^T + Q_k \quad (3.6)$$

### 3.2. KALMAN FILTER



**Figure 3.1.** Kalman phases.

During the *update* the accelerometer values are used. **Do i mention earlier that this is the measurement?** The measurement *innovation* is calculated as

$$\tilde{y} = z_k - H\hat{x}_k^- \quad (3.7)$$

This residual reflects the relation between the predicted measurement and the actual measurement. A measurement *innovation* of zero indicates a perfect agreement. From there the measurement *innovation* covariance is calculated

$$S_k = HP_k^-H^T + R \quad (3.8)$$

This is very similar to the *priori* error covariance but represents how much the measurement can be trusted. From there the core of the Kalman filter can be calculated, the Kalman gain

$$K_k = P_k^-H^TS_k^{-1} \quad (3.9)$$

indicates how reliable the measurement is. Not that if the measurement covariance error (3.4) is large the Kalman gain will be small. By now the *posteriori* state can be estimated by

$$\hat{x}_k = \hat{x}_k^- + K_k\tilde{y}_k \quad (3.10)$$

The Kalman filter can be represented in many different ways, this is just one of them. It consists of an estimated step, a Kalman gain and estimation error covariance  $P$ . The new estimated step consists of two terms. One that considers the former estimated step and the input at that time and the second term is called the *correction term* considers how much the estimated value needs to be corrected due to the measurement. Inspection of the Kalman gain in equation (??) indicates that if the measurement contains a lot of error and noise the noise covariance (3.4)

will be large hence the Kalman gain will be small. That is if the credibility of the measurement is low, it wont effect the estimated state as much as if the credibility would be high. **Maybe needs restructuring ...** The Kalman implementation in discrete form can be seen in section 4.3.

For further reading, and mathematical proof see [Welch and Bishop(2006)].

## Chapter 4

# Demonstrator

*Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstratorn utvecklats enligt, dvs resultatet och vägen dit.*

### 4.1 Problem Formulation

The engineering problem were to build a cube that, using a reaction wheel, could balance on its edge. *To be continued*

### 4.2 State space model

PICTURE TO BE ADDED

To create a state-space model the physical model has to be translated to a mathematical model. The system can be estimated much like an inverted pendulum as a two-degree-of-freedom model ???. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_i = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \left( \frac{\partial L}{\partial q_i} \right) \quad (4.1)$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \left( \frac{\partial L}{\partial \theta} \right) \quad (4.2)$$

$$- M_a = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \left( \frac{\partial L}{\partial \phi} \right) \quad (4.3)$$

Whereas  $\theta$  represents the angle of the cube and  $\phi$  is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \quad (4.4)$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \quad (4.5)$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos \theta}{\sqrt{2}} \quad (4.6)$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin \theta}{\sqrt{2}} = -M_a \quad (4.7)$$

$$I_s \cdot \ddot{\phi} = M_a \quad (4.8)$$

From these equations it is evident that  $M_a$  is the torque executed by the flywheel which is wielded by the motor torque  $\tau$ , it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \quad (4.9)$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \quad (4.10)$$

**Mention that inductance is neglected due to something? see KTHpendulum reference** The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \quad (4.11)$$

$$\phi_r = \dot{\phi} - \dot{\theta} \quad (4.12)$$

$$\tau = \frac{K_t}{R_m} U - \frac{K_t K_{\text{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m} \dot{\theta} \quad (4.13)$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \quad (4.14)$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\theta} + \frac{M_t g l}{\sqrt{2} I_c} \sin \theta \quad (4.15)$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\theta} \quad (4.16)$$

With the equations (4.15) and (4.16) the system can be described with a state space model with a states  $x^T = [\theta, \dot{\theta}, \dot{\phi}]$ . The system is hence described by

$$\dot{x} = Ax + Bu \quad (4.17)$$

where

### 4.3. DISCRETE KALMAN FILTER

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{MtgI}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

### 4.3 Discrete Kalman filter

To implement the Kalman filter in algorithm it has to be discretized. This is done much like a feedback control. The filter firstly estimates the process state and then obtains feedback as noisy measurements. That means that the filter works in two steps, a *time update* and a *measurement update*. The names implicate that the *time update* projects the next state to obtain the *priori estimate* whilst the *measurement update* uses the feedback mentioned above to obtain an improved *posteriori* estimate. **We need a picture here, but not the same as david and viktor...**

Some of the implementation and discretization of the filter.

#### 4.3.1 Kalman implementation

An implementation of the Kalman filter on the IMU would look something like this for the gyroscope

$$\theta_k = \theta_{k-1} + (w_k \mp b_{k-1})\Delta t \quad (4.18)$$

$$b_k = b_{k-1} \quad (4.19)$$

$$z_k = a_k \quad (4.20)$$

With  $b$  as the gyro bias. Looking at equation (3.1) the system can be written

$$\mathbf{x}_k = \begin{bmatrix} \theta \\ \dot{\theta}_{b_k} \end{bmatrix} \quad (4.21)$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \quad (4.22)$$

$$\mathbf{B} = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \quad (4.23)$$

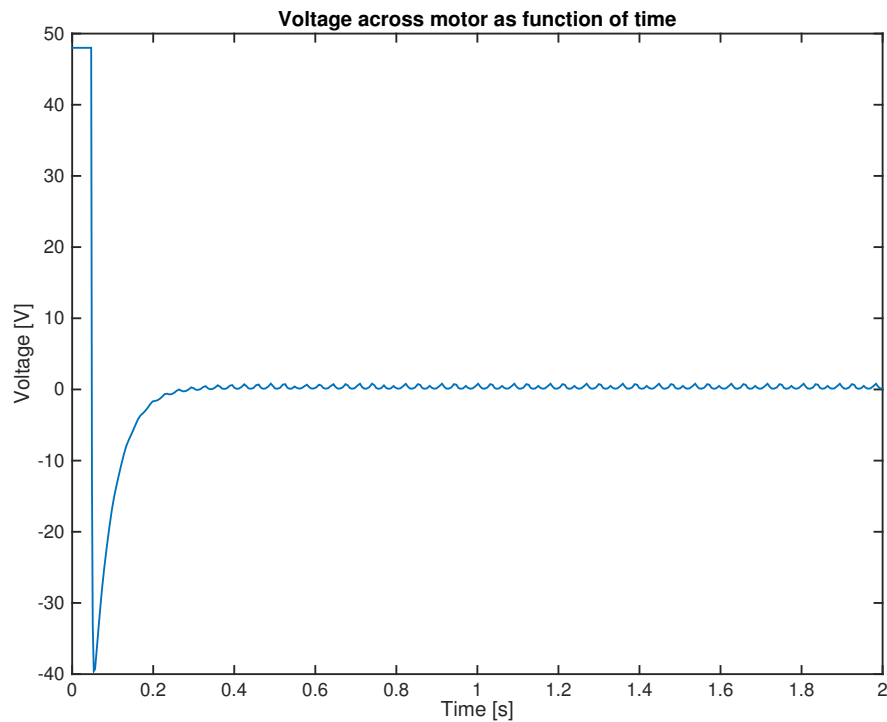
### 4.3.2 Error thingys

To improve the Kalman filter the characteristics of the model has to be (ta fram ??) . A way of determining the process noise and measurement noise of the IMU is the Allan variance method ?? . The gyro data is treated as an external input the system, so the error and bias from the gyro readings are characterised as process noise. The advantage of using the gyro data instead of accelerometer data is the reduction in states as the accelerometer would need to be integrated twice to retrieve an angle ( elaborate....)

## 4.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink<sup>®</sup> was used together with the mathematical model.

The Simulinkmodel seen in figure ?? describes the system  
Something about the optimizing of the feedback control

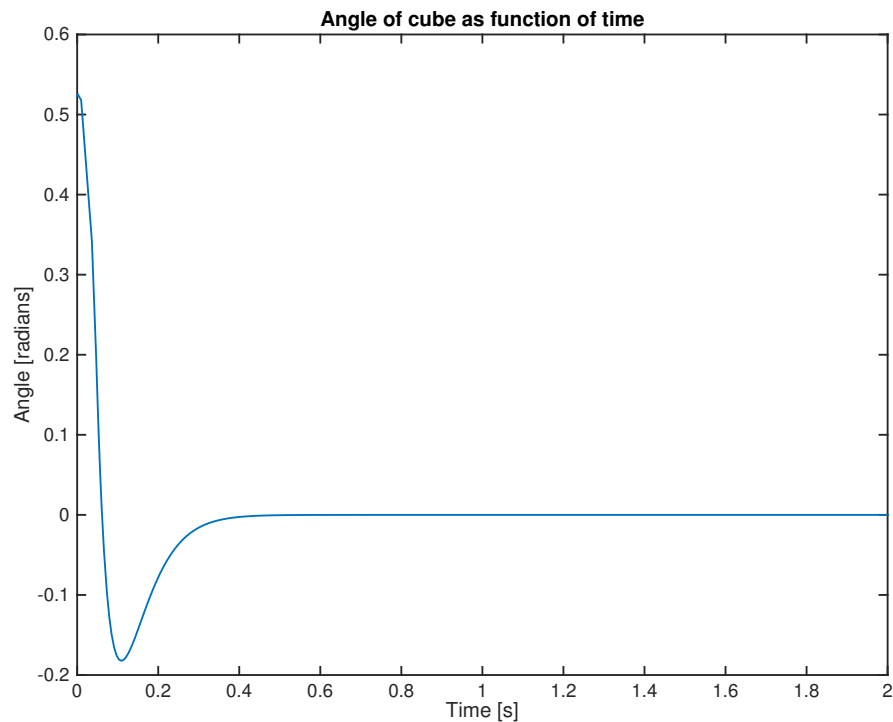


**Figure 4.1.** Voltage across motor poles.

The voltage supplied to the motor



## 4.5. ELECTRONICS



**Figure 4.2.** Angle of the cube.

The angle of the cube. Very good such magic

## 4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar. How do we send data?

Sensors

Motor

Arduino

Motor control

## 4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is directly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.

Basic construction

## 4.7 Results

Beskriv resultatet.

## Chapter 5

# Discussion and conclusions

*I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.*

### 5.1 Discussion

Motor choice osv

### 5.2 Conclusions

Successful victory



## Chapter 6

# Recommendations and future work

### 6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

### 6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.



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## **Appendix A**

### **Additional information**



## **Appendix B**

### **Proofs**



