## Rad Cube (working title)

Control of reaction wheel balanced inverted pendulum

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Bacherlor's Thesis in Mechatronics

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### **Abstract**

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. ?This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites?

To be filled in:

Problem (

Approach (On the premiss of constructing a small box shaped robot components with desired properties were chosen....)

Results

Conclusion

# Sammanfattning

### Stabilisering med svänghjul

Skriv som abstract men på svenska

# **Preface**

Detta är rätt ställe att tacka för hjälp, råd, samarbete och inspiration för det presenterade projektet. Detta kapitel är valfritt. Förordet avslutas lämpligtvis med de båda raderna Namn och Plats, månad och år.

Alexander Ramm Mikael Sjöstedt KTH, månad, 2015

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# Nomenclature

## Symbols

Symbol	Description
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
$\theta$	Kubvinkel
$\phi$	Flywheel angle
Q och q	Lagrange operator
$E_k$	Kinetic energy
$E_p$	Potential Energy
$I_c$	Inertia of the cube
$I_f$	Inertia of the flywheel
$\dot{M}_c$	Total mass of the cube xi
i	Current
$K_t$	Torque constant
$\mathrm{E}_{\mathrm{emf}}$	Induced voltage
$K_{\mathrm{emf}}$	Induced voltage constant ????
U	Voltage over motor?
$R_m$	Motor resitance
	M-4

### **Abbreviations**

### Abbreviation Description

PLM	Product Lifecycle Management
CAE	Computer Aided Engineering
CAD	Computer Aided Design

PWM Pulse With Modulation DOF Degrees of freedom

MEMS Microelectromechanical Systems

...

### Introduction

Beskriv bakgrund, syfte, avgränsning för det utförda examensarbetet, samt även de metod(-er)som använts för att lösa uppgiften. Skriv exempelvis: Detta kapitel beskriver bakgrund, syfte, avgränsning och metod för det utförda examensarbetet.

This chapter decribes the backgroud, pupose, scope, and method of the theisis.

### 1.1 Background

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. The cost of MEMS sensors today are cheap due to high demand and production (http://www.reuters.com/article/2015/01/15/technavi research-idUSnBw155173a+100+BSW20150115, should refer to the source but cant gain access...). This growth has made automated control available in our every-day life in products lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters. The process

—-Balancing a inverted pendulum can be a challenging task. It requires knoledge of closed loop control systems and their stability, electronics and... — (JUST SOME BACKGROUND TO THE PROBLEM, AND SOME CONTEMPORARY EXAMPLES TO GET THE READERS ATTENTION)

### 1.2 Purpose

(instert RQ here) "What improvements can be made to the control system to better cope with an applied external force to the cube"

The purpose was to examine what affected the stability of the mechanical system, (specially) the control system. Many parameters goes in to the state space control system and their affect on the mechanical system behaviour is not trivial. The purpose was to detirmine if some of the parameters had an extra importance and if the conclutions were applicable on other mechanical systems. Sample frequency and clock frequency were other parameters which where to be examined aswell.

Also the results were to be contributed to the open source comunity. All results is available online, open source (referens till MIT licens här), on GitHub (ref till github länk)(summer 2015).

### 1.3 TÄNK PÅ DETTA

Figurer i avhandlingen skall centreras och figurtexten skrivs centrerat. Figurerna numereras Fig. 1.1, Fig. 1.2, och så vidare.



Figure 1.1. Logo of the ITM school.

Tabellrubriker skrivs centrerat. Tabeller numreras Table 1.1, Table 1.2, och så vidare. Tabelltext skrivs centrerat. Nedan visas ett exempel på en tabell med tabellrubrik.

Table 1.1. Table header

Rubrik	Rubrik
<i>'</i>	råd 1, kolumn 2 råd 2, kolumn 2
råd n, kolumn 1	råd n, kolumn 2

- Onumrerade listor skrivs lämpligtvis med i punktform ("bullets")
- 1. Numrerade listor numreras 1,2,3 etc.

Ekvationer centras. Varje ekvation numreras konsekutivt genom hela rapporten med arabiska siffror inom parantes: eq. (1.1), (1.2), etc. Ekvationsnumret högerjusteras på raden.

$$R = \frac{N_F}{N_F - N_R} \tag{1.1}$$

#### 1.4. SCOPE

Referenser bör göras med namn och år, som exempel, [Angell and Straub(1999)] hävdar att ..., eller ickelinjär finita element analys förutsätter såväl specialkunskaper som erfarenhet [de Borst et al.(2012)de Borst, Crisfield, Remmers, and Verhoosel]. Observera också lilla skillnaderna mellan referenser till tidskrifter [van Wezel and Jorna(2001)] och [Angell and Straub(1999)], en bok [de Borst et al.(2012)de Borst, Crisfield, Remmers, and Verhoosel] och en konferens [Eppinger and Salminen(2001)]. Ett exempel på en Internetreferens är [Standish Group(1994)]. Huvudrubriken för referenskapitlet [Angell and Straub(1999)]behandlas som en helt vanlig huvudrubrik.

#### Angående forskningsfrågor och vetenskaplig frågeställning

Beskriv, med utgångspunkt i den bakgrund som presenterats i föregående avsnitt, den frågeställning som rapporten undersöker, diskuterar och drar slutsatser runt. Själva frågeställningen ska beskrivas i 1-2 meningar som tydligt pekar ut vad som ska mätas inom de begränsningar och med de metoder som presenteras i efterföljande avsnitt. Med andra ord, undvik vaga formuleringar ("Går det att..."), kriterier som egentligen är funktioner ("Ska kunna...") eller subjektiva formuleringar ("...ska vara bra/acceptabel/tillräckligt snabb/etc.")

### 1.4 Scope

Det är god sed att definiera och beskriva projektets/uppgiftens avgränsning i det introducerande kapitlet. // The scope were to examine the parameters, of a state space controller and other electronical desitions, affect on the behaviour of a balancing "1-DOF" inverted pendulum.

### Method

Den metod eller de metoder, som huvudsakligen används för att angripa den uppgift eller det problem som definieras ovan, kan antingen definieras i introduktionskapitlet eller förklaras mera ingående i ett följande metodkapitel.

The engineering tast was devided into smaller problems (In med punktlista?). (Behandlede) seperatly to later asseble the entire system. The choosen subproblems where:

- Construction
- Motor Control
- Sensor Reading
- System Control

The main goal of this project build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts

- Constructing the main frame
- Choosing motor and motor control
- Filter sensors and "get good readings"
- "Make them all play in the same kindergarten"
- Final assembly

#### 2.1 Contruction

The main contruction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes abillity to balance. The problem where (uppställt) with Newtionial mechanics.

### 2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha

### 2.3 Sensor Reading

The IMU's parameters and filtering of the signals

### 2.4 System Control

The choosen conrol methot where state space. The problem in to linareise and discretise with good enough precition. ">>> origin/master

======  $\rangle\rangle\rangle$  parent of 6c0af1e... Ska vi börja trycka till dev ?

## Theory

Den teoretiska fördjupningen är en sammanfattning av tillgänglig kunskap och resultat från forskning som tidigare har utförts inom examensarbetets område. Detta kapitel presenterar den teoretiska referensramen som utgör utgångspunkten för den utförda forskningen, produktutvecklingen eller konstruktionsuppgiften.
Här måste det köttas på mycket inför tisdag 9/3

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due tu the instability of pendulums. The latest development is on "2-DOF" pendulum structures unsing multiple orthogonal reaction wheels. This method is commonly used to rotate sattelites and maintaining their attitude to increase performance and allign solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research ie not only changeing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be usefull for land/sea based machines to overcome various obstacles without a seperate system for balancing and movement.

## **Demonstrator**

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstartorn utvecklats enligt, dvs resultatet och vägen dit.

#### 4.1 Problem Formulation

Beskriv din problemställning för demonstratorn.

The engineering problem where to build a cube that, using a reaction wheel, could balance on its edge.

### 4.2 State space model

To create a state-space model the physical model has to be translated to a mathematical model. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_{i} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_{i}} \right) - \left( \frac{\partial L}{\partial q_{i}} \right) \tag{4.1}$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \left( \frac{\partial L}{\partial \theta} \right) \tag{4.2}$$

$$-M_{a} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \left( \frac{\partial L}{\partial \phi} \right) \tag{4.3}$$

Whereas  $\theta$  represents the angle of the cube and  $\phi$  is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \tag{4.4}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{4.5}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos \theta}{\sqrt{2}} \tag{4.6}$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin \theta}{\sqrt{2}} = -M_a \tag{4.7}$$

$$I_s \cdot \ddot{\phi} = M_a \tag{4.8}$$

From these equations it is evident that  $M_a$  is the torque executed by the flywheel which is wielded by the motor torque  $\tau$ , it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \tag{4.9}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \tag{4.10}$$

The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\rm emf} = K_{\rm emf} \cdot \dot{\phi_r} \tag{4.11}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{4.12}$$

$$\tau = \frac{K_t}{R_m} U - \frac{K_t K_{\text{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m} \dot{\theta}$$
(4.13)

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \tag{4.14}$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\theta} + \frac{M t g l}{\sqrt{2} I_c} \sin \theta \tag{4.15}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\theta}$$
(4.16)

based on equations (4.15) and (4.16) the system can be described with a state space model with a states  $x^T = [\theta, \dot{\theta}, \dot{\phi}]$ . The system is hence described by

$$\dot{x} = Ax + Bu \tag{4.17}$$

where

#### 4.3. SOFTWARE

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Mtgl}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}}\eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}}\eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}}\eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}}\eta_m}{R_m I_f} \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

#### 4.3 Software

Beskriv hur din mjukvara fungerar. Använd bl.a. flödesscheman för att åskådliggöra programmets struktur.

#### 4.4 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar.

Sensors

Motor

Arduino

Motor control

#### 4.5 Hardware

Beskriv din mekaniska konstruktion (om du har någon) The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other componets are mounted on the motor-body side of the cube.

Basic construction

#### 4.6 Results

Beskriv resultatet.

## Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

### 5.1 Discussion

Bla bla bla

### 5.2 Conclusions

Bla bla bla

## Recommendations and future work

I detta kapitel ges rekommendationer for mera detaljerade lösningar och/eller framtida arbete.

### 6.1 Recommendations

Lorem ipsum dolor sit amet, consetetur sadipscing elitr, sed diam nonumy eirmod tempor invidunt ut labore et dolore magna aliquyam erat, sed diam voluptua. At vero eos et accusam et justo duo dolores et ea rebum. Stet clita kasd gubergren, no sea takimata sanctus est Lorem ipsum dolor sit amet.

### 6.2 Future work

An extention of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

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# Appendix A

# **Additional information**

# Appendix B

# **Proofs**