

# **Rad Cube (working title)**

Control and design of reaction wheel balanced inverted pendulum

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# Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...

To be filled in:

Problem

Approach

Results

Conclusion



# Sammanfattning

## Stabilisering med svänghjul

Projektet gick ut på att bygga en cub som kan balansera på en kant med hjälp av ett reaktionshjul. Dessutom skulle den undersökas huruvida det gick att förbättra reglersystemet för cuben, så den klarade en större yttre störning. Ingengörsproblemet delades upp i mindre delproblem och kubens byggdes. Reglersystemet beräknades på formen State space och implementerades. Från resultatet drogs slutsatserna att...



# Preface

Here goes our thanks to sources of help, cooperation, inspiration  
To be filled in

Alexander Ramm  
Mikael Sjöstedt  
KTH, månad, 2015





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# Nomenclature

## Symbols - needs restructure

Symbol	Description
$E$	Elasticity module (Pa)
$r$	Radius (m)
$t$	Thickness (m)
$L$	Lagrange ( fixa)
$\theta$	Cube angle
$\phi$	Flywheel angle
Q and q	Lagrange operators
$E_k$	Kinetic energy
$E_p$	Potential Energy
$I_c$	Inertia of the cube
$I_f$	Inertia of the flywheel
$M_c$	Total mass of the cube
$i$	Current
$K_t$	Torque constant
$E_{emf}$	Induced voltage
$K_{emf}$	Induced voltage constant
$U$	Voltage across motor poles
$R_m$	Motor internal resitance
$\eta_m$	Motor efficiency
$z$	Measurement noise
$w$	Process noise

## Abbreviations

Abbreviation	Description
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
$I^2C$	Inter-Integrated circuit
USB	Universal Serial Bus

# Chapter 1

## Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

### 1.1 Background

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. The cost of MEMS sensors today are cheap due to high demand and production [ [Ciufu(2012)]]]. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadcopters.

### 1.2 Purpose

The purpose of this thesis was to examine what effected the stability of the mechanical system, especially the control system. Many parameters are to be accounted for in the state-space model and their effects on the mechanical system behaviour is not trivial. The aim was to determine if some of the parameters had an extra importance and if any conclusions were applicable on other mechanical systems. The research can be summarized with:

*‘What improvements can be made to the control system to better cope with an applied external force to the cube?’*

Sample frequency and clock frequency are other parameters which were to be examined as well.

The open-source community is growing but still lacks automated control systems for balancing robots. This project will hopefully contribute to some development within the open-source community. All results are available online, open source

(MIT license reference here), on GitHub (GitHub link here).

### **1.3 Scope**

The scope were to examine the parameters, of the state space controller and sensor sample frequency, affects on the overshoot behaviour of a balancing "1-DOF" inverted pendulum. The overshoot should be caused by an external force, disrupting the cubes balance.

## Chapter 2

# Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

### 2.1 Contruction

The main contruction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem where (uppställt) with Newtional mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

### 2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is allso imortant, but usually one can get suggestions on drivers from motor manufactores, which was the choosen path.

### 2.3 Sensor Reading

The IMU's parameters and filtering of the signals

## 2.4 System Control

The chosen control method where state space. The problem in to linearise and discretise with good enough precision.

## 2.5 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.



## Chapter 3

# Theory

*Den teoretiska fördjupningen är en sammanfattning av tillgänglig kunskap och resultat från forskning som tidigare har utförts inom examensarbetets område. Detta kapitel presenterar den teoretiska referensramen som utgör utgångspunkten för den utförda forskningen, produktutvecklingen eller konstruktionsuppgiften.*

Här måste det köttas på mycket inför tisdag 9/3

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due to the instability of pendulums. The latest development is on "2-DOF" pendulum structures using multiple orthogonal reaction wheels. This method is commonly used to rotate satellites and maintaining their attitude to increase performance and align solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research ie not only changing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be useful for land/sea based machines to overcome various obstacles without a separate system for balancing and movement.



## Chapter 4

# Demonstrator

*Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstratorn utvecklats enligt, dvs resultatet och vägen dit.*

### 4.1 Problem Formulation

Beskriv din problemställning för demonstratorn.

The engineering problem where to build a cube that, using a reaction wheel, could balance on its edge.

### 4.2 State space model

PICTURE TO BE ADDED

To create a state-space model the physical model has to be translated to a mathematical model. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_i = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \left( \frac{\partial L}{\partial q_i} \right) \quad (4.1)$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}} \right) - \left( \frac{\partial L}{\partial \theta} \right) \quad (4.2)$$

$$-M_a = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\phi}} \right) - \left( \frac{\partial L}{\partial \phi} \right) \quad (4.3)$$

Whereas  $\theta$  represents the angle of the cube and  $\phi$  is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \quad (4.4)$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \quad (4.5)$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos \theta}{\sqrt{2}} \quad (4.6)$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin \theta}{\sqrt{2}} = -M_a \quad (4.7)$$

$$I_s \cdot \ddot{\phi} = M_a \quad (4.8)$$

From these equations it is evident that  $M_a$  is the torque executed by the flywheel which is wielded by the motor torque  $\tau$ , it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \quad (4.9)$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \quad (4.10)$$

The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \quad (4.11)$$

$$\phi_r = \dot{\phi} - \dot{\theta} \quad (4.12)$$

$$\tau = \frac{K_t}{R_m} U - \frac{K_t K_{\text{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m} \dot{\theta} \quad (4.13)$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \quad (4.14)$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\theta} + \frac{M_t g l}{\sqrt{2} I_c} \sin \theta \quad (4.15)$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\theta} \quad (4.16)$$

With the equations (4.15) and (4.16) the system can be described with a state space model with a states  $x^T = [\theta, \dot{\theta}, \phi]$ . The system is hence described by

$$\dot{x} = Ax + Bu \quad (4.17)$$

where

### 4.3. KALMAN FILTER

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Mtg l}{\sqrt{2}I_c} & -\frac{K_t K_{emf} \eta_m}{R_m I_c} & \frac{K_t K_{emf} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{emf} \eta_m}{R_m I_f} & -\frac{K_t K_{emf} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

## 4.3 Kalman filter

**Nu har jag bara slängt ut nyckelord. Den ska styras upp :) Men 2 bra källor iaf tror jag**

It is an estimator, recursive, new measurements are processed on the go. optimal for gaussian error much like the one in piezoelectrothings, minimises the mean square error algorithm that uses observed measurement over time which in this case is the measurements from the gyro and accelerometer. The problem with the accelerometer is that it's very noisy? for small adjustments/accelerations. The gyro on the other hand, drifts over time [In med källa, finns hur många som helst..] and can be accounted for during short accelerations. [Simon(2001)] [Welch and Bishop(2006)] It minimizes the variance of the estimation error A kalman filter suits the needs of this project. Because to control the position of the cube, a good and reliable estimate of the position is needed.

### 4.3.1 The process

The discrete Kalman filter which is applied to the problem at hand is a state based estimator. By using the measurements from both the accelerometer and the gyro from the past, present (future??) it can derive a good estimate of the state in our linearised problem. The Kalman filter estimates the present state using the following equations

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (4.18)$$

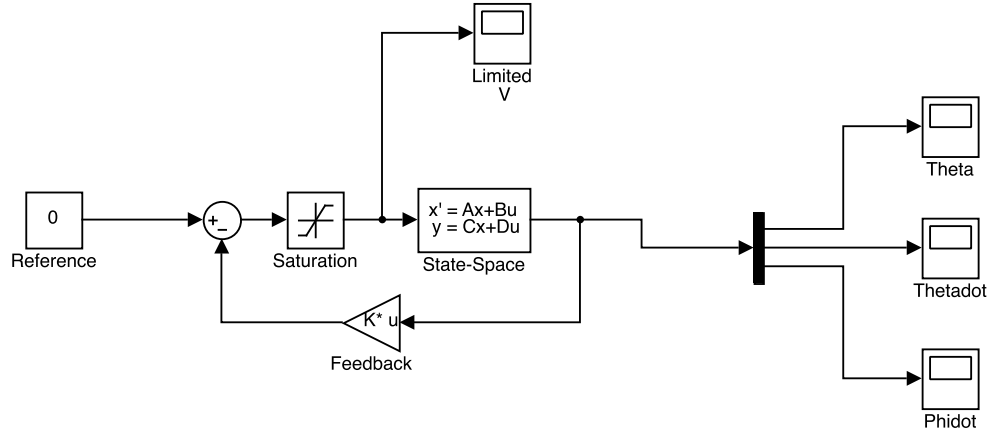
$$y_k = Hx_k + z_k \quad (4.19)$$

Where  $x$  is the states of the cube which cannot be measured directly i.e  $\theta$  and  $\dot{\theta}$ .  $y$  is the measured value which would be the serial data from the IMU , which is a function of  $x$  but is distorted by the measurement noise  $z$  The process noise  $w$  in equation (4.18) is a representation of variances? in the system behavior that cannot be mathematically predicted.  $z$ , the measurement noise is common in any measurement and represents fluctuations in the equipment or **develop this**.

## 4.4 Software

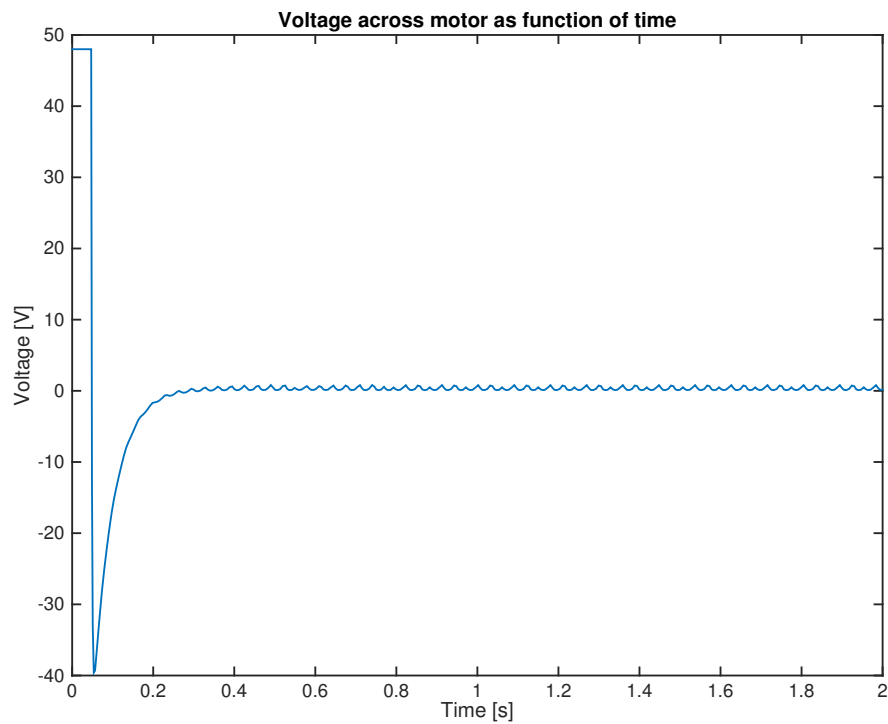
To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink<sup>®</sup> was used together with the math-

emational model. The Simulinkmodel seen in figure 4.1 describes the system



**Figure 4.1.** Simulink model.

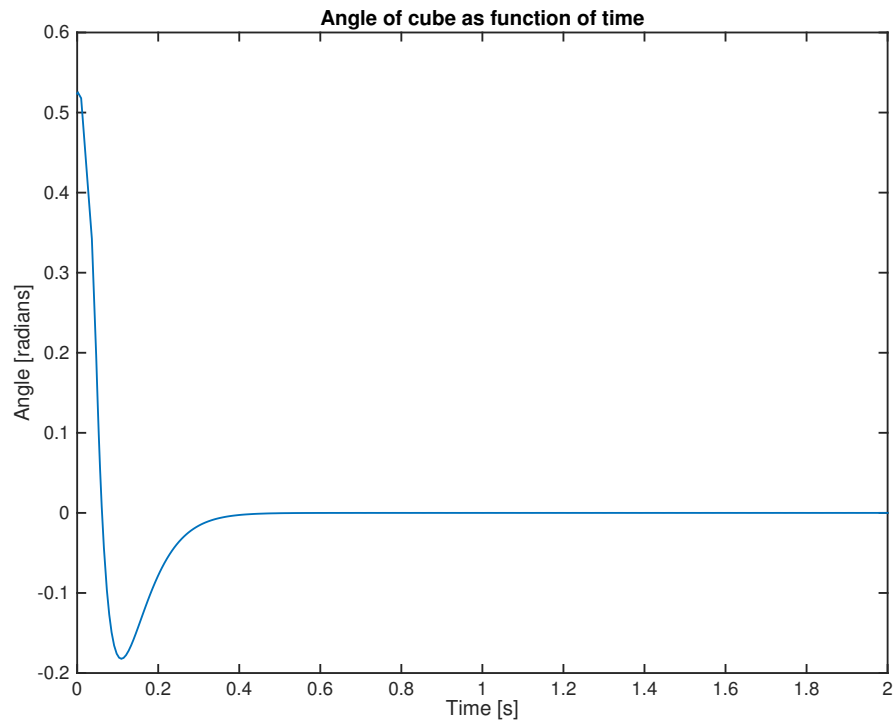
Something about the optimizing of the feedback control



**Figure 4.2.** Voltage across motor poles.

## 4.5. ELECTRONICS

The voltage supplied to the motor



**Figure 4.3.** Angle of the cube.

The angle of the cube. Very good such magic

## 4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema.

Motivera dina lösningar.

Sensors

Motor

Arduino

Motor control

## 4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is directly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.

Basic construction

## 4.7 Results

Beskriv resultatet.



## Chapter 5

# Discussion and conclusions

*I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.*

### 5.1 Discussion

Bla bla bla

### 5.2 Conclusions

Bla bla bla



## Chapter 6

# Recommendations and future work

### 6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

### 6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.



# Bibliography

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## **Appendix A**

### **Additional information**





## **Appendix B**

### **Proofs**



