

Balancing Cube (working title)

Control and design of reaction wheel balanced inverted pendulum

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Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...

To be filled in:

Problem

Approach

Results

Conclusion

Sammanfattning

Stabilisering med svänghjul

Projektet gick ut på att bygga en kub som kan balansera på en kant med hjälp av ett reaktionshjul. Dessutom skulle den undersökas huruvida det gick att förbättra reglersystemet för kuben, så den klarade en större yttre störning. Ingengörsproblemet delades upp i mindre delproblem och kuben byggdes. Reglersystemet beräknades på formen State space och implementerades. Från resultatet drogs slutsatserna att...

Preface

Here goes our thanks to sources of help, cooperation, inspiration
To be filled in

Alexander Ramm
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Nomenclature

Symbols - needs restructure

Symbol	Description
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
θ	Cube angle
ϕ	Flywheel angle
Q and q	Lagrange operators
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_c	Total mass of the cube
i	Current
K_t	Torque constant
E_{emf}	Induced voltage
K_{emf}	Induced voltage constant
U	Voltage across motor poles
R_m	Motor internal resistance
η_m	Motor efficiency
z	Measurement noise
w	Process noise

Abbreviations

Abbreviation	Description
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

1.1 Background

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became more accessible with the introduction of cheap micro-controllers. The use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters.

One of the most basic systems that requires some control to become stable is the inverted pendulum. Altho it is simple to define controlling it is not a trivial task. A lot of work has been done on the topic but there is still not knowledge easily acquired by the public.

The method to achieve balance of the pendulum using reaction wheels is even more narrow. The use of reaction wheels to change the rotation is commonly used in satellites. The exact control is also required. In recent years prototypes of land based structures using reaction wheels have been a hot topic and the cubli is truly remarkable.

It would be a great achievement to contribute knowledge about how such a mechanism could be built and evaluate the capabilities and restrictions of such a machine, on a level that does not require a PhD.

1.2 Purpose

The goal of the project was to build a structure that in one DOF can maintain balance using a reaction wheel and examine the behaviours of the system.

The behaviour is mostly effected by the control system, which is responsible for

accelerating the motor in the correct angular direction, to maintain balance. The parameters in the control system effects response time, overshoot and sinusoidal settling time. This project will hopefully contribute to some development within the open-source community. All results are available online, open source (MIT license reference here), on GitHub (GitHub link here).

As a mechatronical thesis the research task were to be along the line of how something physical is implemented in a, in some context, good or even correct way.

If balance is maintained how does the maximum applied force correlate to the rise time and overshoot separately, can any conclusions be drawn from the results?

or maybe

How does sensor placement effect system performance -and can internal disturbances terminaTOR nej men något sånt???

Where balance is defined as as the state where the cube is able to return to its reference angle/value?. The rise time and overshoot refers to the system angle. Can the results contribute to improve the overall performance?

1.3 Scope

The scope were to examine the parameters, of the state space controller and sensor sample frequency, affects on the overshoot behaviour of a balancing "1-DOF" inverted pendulum. The overshoot should be caused by an external force, disrupting the cubes balance. Moar "we will not do this"

Chapter 2

Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

2.1 Construction

The main construction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem were (uppställt) with Newtonian mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is also important, but usually one can get suggestions on drivers from motor manufactures, which was the chosen path.

2.3 Sensor Reading

The IMU's parameters and filtering of the signals

2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

2.5 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.

Chapter 3

Theory

This chapter cover some theory that is required if one want to build a similar machine. It is assumed that the reader has some understanding of Newtonian mechanics, signal analysis and control theory. Basic understanding of DC motor is also an advantage. The first part covers the Kalman filter theory, required for getting high quality data from the IMU. The filter task noisy data from sensors and digitaly filters it to a more reliable quality. The second part will cover the theory of the mechanical system behaviour that is used to device the state space control system. Those equations are responsible for the actual balance part and thus important. There is also a part on sensor characteristics and why they are important to the system as a whole and the research question in particular. When using a sensor such as an IMU one can choose between different resolution settings, depending of the specific usage. Altho the resolutions were not changed between measurements these settings could be important when comparing to another machine.

3.1 Moar theory ?

Where do we mention communication protocol, maybe some in-depth of accelerometer and gyro ?

3.2 Kalman filter

The signal from an IMU contains data of angular velocities and transversal acceleration, but also a lot of noise. An estimated position of an untreated signal from an IMU would work for short periods but over time the estimated position 'drifts'. This drift occurs because of integration of the measurements to acquire a position, but as the readings contain unwanted noise and often a bias as well this small error grows. Integrating the angular motion to estimate a position would result in an angular drift for the gyro and an even worse drift for the accelerometer as it is integrated twice if it were to estimate a position [?]. By using a Kalman filter the drift can effectively be minimized. By using both readings from the IMU and

with some help of probability theory the estimated state is not far from the true value. A Kalman *filter* is not what the name suggests, it's an estimator. Old and new measurements are processed real-time to calculate an estimation of the current state. What could be asked of the filter? **is this too "friendly"** A good estimator produces states that are non biased, *values that have an average of the true value*. As well that the estimated state variance from the true state is as small as possible. [Simon(2001)]

3.2.1 State Estimator

The Kalman filter is a state based estimator. By using measurements from the past and present it can derive a good estimate of the current state. The true state and the measured value at a time k would be

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \quad (3.1)$$

$$y_k = Hx_k + z_k \quad (3.2)$$

The true state x is expressed with the the old state, an input u , that is data from the gyroscope. But the signal also contains a process noise w . The process noise w in equation (3.1) is a representation of variances in the gyroscope that cannot be mathematically predicted such as flaws in production. The measured value, y (see (3.2)) is another observed measurement, in this case the accelerometer. Ideally this would only be a function of x , but is distorted by the measurement noise z . The measurement noise, z , much like the process noise is common in any measurement and represents various fluctuations caused by the equipment.

As this recursive filter uses old and new values a *priori* and *posteriori* state is defined

$$\hat{x}_k^- \quad (3.3)$$

$$\hat{x}_k \quad (3.4)$$

The *priori* (3.3) state is defined as the estimate of the current state at the time k . The *posteriori* state (3.4) is the new estimated state. For the Kalman filter to work properly some criteria has to be fulfilled. The average value of the measurement noise z and process noise w has to be zero, i.e. a Gaussian error. z and w also has to be independent of each other. The noise and error in an IMU and many other devices have the characteristics of gaussian noise.

3.2.2 The process

The Kalman filter loops two stages. The *predict* and *update* stages.

During the *predict* phase the filter estimates the states using the inputs from the process, i.e the gyroscope. 3.1

$$\hat{x}_k^- = A\hat{x}_{k-1}^- + Bu_{k-1} \quad (3.5)$$

3.2. KALMAN FILTER

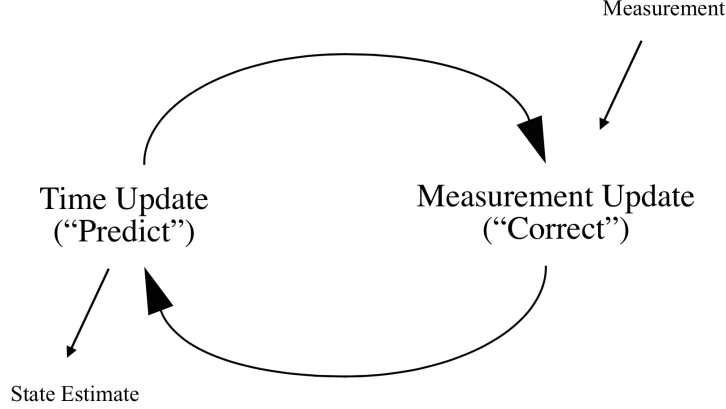


Figure 3.1. Kalman phases.

As stated above **För slentriant?** the Kalman filter uses readings from both the gyro and accelerometer to estimate a position closer to the true value. To determine how reliable the process and measurement readings are a noise covariance is defined as

$$Q = E(w_k w_k^T) \quad (3.6)$$

$$R = E(z_k z_k^T) \quad (3.7)$$

From here a *priori* error covariance matrix is introduced to symbolize the noise in the process measurement.

$$P_k^- = A P_{k-1}^- A^T + Q_k \quad (3.8)$$

During the *update* the accelerometer values are used. The measurement *innovation* is calculated as

$$\tilde{y} = z_k - H \hat{x}_k^- \quad (3.9)$$

The *innovation* is a residual that reflects the relation between the predicted measurement and the actual measurement. A measurement *innovation* of zero indicates a perfect agreement. The measurement *innovation* covariance is calculated as

$$S_k = H P_k^- H^T + R \quad (3.10)$$

This is very similiar to the *priori* error covariance but represents the measurement instead. From here the core of the Kalman filter can be calculated, the Kalman gain

$$K_k = P_k^- H^T S_k^{-1} \quad (3.11)$$

indicates how reliable the measurement is. Note that if the measurement covariance error (3.7) is large the Kalman gain will be small and the opposite if the *priori* error

covariance is large **superlång mening** ?. By now the *posteriori* state can be estimated by

$$\hat{x}_k = \hat{x}_k^- + K_k \tilde{y}_k \quad (3.12)$$

A current state has been estimated and the Kalman filter returns to the measurement phase seen in figure 3.1. For further reading, and mathematical proof see [Welch and Bishop(2006)].

Chapter 4

Demonstrator

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstratorn utvecklats enligt, dvs resultatet och vägen dit.

4.1 Problem Formulation

The engineering problem were to build a cube that, using a reaction wheel, could balance on its edge. *To be continued*

4.2 State space model

PICTURE TO BE ADDED

To create a state-space model the physical model has to be translated to a mathematical model. The system can be estimated much like an inverted pendulum as a two-degree-of-freedom model ???. *Lagrangian* Dynamics can be used to derive the systems behaviour. Firstly by expressing the generalized forces, the energy functions and lagrangian. And then acquire the equations of motion with the Lagrange equation ??.

$$Q_i = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \left(\frac{\partial L}{\partial q_i} \right) \quad (4.1)$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \left(\frac{\partial L}{\partial \theta} \right) \quad (4.2)$$

$$- M_a = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\phi}} \right) - \left(\frac{\partial L}{\partial \phi} \right) \quad (4.3)$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \quad (4.4)$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \quad (4.5)$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos \theta}{\sqrt{2}} \quad (4.6)$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin \theta}{\sqrt{2}} = -M_a \quad (4.7)$$

$$I_s \cdot \ddot{\phi} = M_a \quad (4.8)$$

From these equations it is evident that M_a is the torque executed by the flywheel which is wielded by the motor torque τ , it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \quad (4.9)$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \quad (4.10)$$

Mention that inductance is neglected due to something? see KTHpendulum reference The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \quad (4.11)$$

$$\phi_r = \dot{\phi} - \dot{\theta} \quad (4.12)$$

$$\tau = \frac{K_t}{R_m} U - \frac{K_t K_{\text{emf}}}{R_m} \dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m} \dot{\theta} \quad (4.13)$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \quad (4.14)$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \dot{\theta} + \frac{M_t g l}{\sqrt{2} I_c} \sin \theta \quad (4.15)$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f} U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \dot{\theta} \quad (4.16)$$

With the equations (4.15) and (4.16) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \quad (4.17)$$

where

4.3. DISCRETE KALMAN FILTER

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{MtgI}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

4.3 Discrete Kalman filter

To implement the Kalman filter in algorithm it has to be discretized. This is done much like a feedback control. The filter firstly estimates the process state and then obtains feedback as noisy measurements. That means that the filter works in two steps, a *time update* and a *measurement update*. The names implicate that the *time update* projects the next state to obtain the *priori estimate* whilst the *measurement update* uses the feedback mentioned above to obtain an improved *posteriori* estimate. **We need a picture here, but not the same as david and viktor...**

Some of the implementation and discretization of the filter.

4.3.1 Kalman implementation

An implementation of the Kalman filter on the IMU would look something like this for the gyroscope

$$\theta_k = \theta_{k-1} + (w_k \mp b_{k-1})\Delta t \quad (4.18)$$

$$b_k = b_{k-1} \quad (4.19)$$

$$z_k = a_k \quad (4.20)$$

With b as the gyro bias. Looking at equation (3.1) the system can be written

$$\mathbf{x}_k = \begin{bmatrix} \theta \\ \dot{\theta}_{b_k} \end{bmatrix} \quad (4.21)$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \quad (4.22)$$

$$\mathbf{B} = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \quad (4.23)$$

Figure 4.1. Comparison of Kalman filtered signal and original signal

4.3.2 Error thingys

For the Kalman filter to properly work it is essential to know how reliable the process and measurement input is. A way of determining the process noise and measurement noise of the IMU is the Allan variance method ???. The gyro data is treated as an external input the system, so the error and bias from the gyro readings are characterised as process noise. The advantage of using the gyro data instead of accelerometer data is the reduction in states as the accelerometer would need to be integrated twice to retrieve and angle (elaborate....)

4.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink® was used together with the mathematical model.

The Simulinkmodel seen in figure ?? describes the system
Something about the optimizing of the feedback control

Figure 4.2. Voltage across motor poles.

The voltage supplied to the motor

Figure 4.3. Angle of the cube.

The angle of the cube. Very good such magic

4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar. How do we send data?

Sensors

Motor

Arduino

Motor control

4.6. HARDWARE

4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is directly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.

Basic construction

4.7 Results

Beskriv resultatet.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Motor choice osv

5.2 Conclusions

Successful victory

Chapter 6

Recommendations and future work

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

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Appendix A

Additional information

Appendix B

Proofs

