Rad Cube (working title)

Control of reaction wheel balanced inverted pendulum

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Bacherlor's Thesis in Mechatronics

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Abstract

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. ?This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites?
To be filled in:
Problem
Approach
Results
Conclusion

Sammanfattning

Stabilisering med svänghjul

Skriv som abstract men på svenska

Preface

Detta är rätt ställe att tacka för hjälp, råd, samarbete och inspiration för det presenterade projektet. Detta kapitel är valfritt. Förordet avslutas lämpligtvis med de båda raderna Namn och Plats, månad och år.

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KTH, månad, 2015

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Nomenclature

Symbols

Symbol	Description
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
θ	Kubvinkel
ϕ	Flywheel angle
Q och q	Lagrange operator
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_c	Total mass of the cube
i	Current
K_t	Torque constant
$\mathrm{E}_{\mathrm{emf}}$	Induced voltage
$\mathrm{K}_{\mathrm{emf}}$	Induced voltage constant
U	Voltage over motor ?
R_m	Motor resitance
η	Motor efficiency

Abbreviations

Abbreviation	Description
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus
...	...

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

1.1 Background

Today the use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. The cost of MEMS sensors today are cheap due to high demand and production(source?). This growth has made automated control available in our every-day life in products lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters. The process

—-Balancing a inverted pendulum can be a challenging task. It requires knoledge of closed loop control systems and their stability, electronics and... —
(JUST SOME BACKGROUND TO THE PROBLEM, AND SOME CONTEMPORARY EXAMPLES TO GET THE READERS ATTENTION)

1.2 Purpose

(instert RQ here)"What improvements can be made to the control system to better cope with an applied external force to the cube"

The purpose was to examine what affected the stability of the mechanical system, (speciellt) the control system. Many parameters goes in to the state space control system and their affect on the mechanical system behaviour is not trivial. The purpose was to detirmine if some of the parameters had an extra importance and if the conclutions were applicable on other mechanical systems. Sample frequency and clock frequency were other parametesr which where to be examined aswell. Also the results were to be contributed to the open source comunity. All results are availible online, open source (referens till MIT licens här), on GitHub (ref till github länk)(summer 2015).

1.3 Scope

The scope were to examine the parameters, of a state space controller and other electronical desitions, affect on the behaviour of a balancing "1-DOF" inverted pendulum.

Chapter 2

Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

2.1 Contruction

The main contruction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes abillity to balance. The problem where (uppställt) with Newtionial mechanics.

2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha

2.3 Sensor Reading

The IMU's parameters and filtering of the signals

2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

Chapter 3

Theory

Den teoretiska fördjupningen är en sammanfattning av tillgänglig kunskap och resultat från forskning som tidigare har utförts inom examensarbetets område. Detta kapitel presenterar den teoretiska referensramen som utgör utgångspunkten för den utförda forskningen, produktutvecklingen eller konstruktionsuppgiften.
Här måste det köttas på mycket inför tisdag 9/3

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became common with the introduction of cheap microcontrollers. A lot work has been done on the topic but it's still no easy task due tu the instabillity of pendulums. The latest development is on "2-DOF" pendelum structures unsing multiple orthogonal reaction wheels. This method is commonly used to rotate sattelites and maintaining their attitude to increase performance and allign solar panels. Also creating transversal movement using only the reaction wheels is a recent topic for research ie not only changeing direction of something but actually moving it. This is of course impossible in orbit, but could possibly be usefull for land/sea based machines to overcome various obstacles without a seperate system for balancing and movement.

Chapter 4

Demonstrator

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstartorn utvecklats enligt, dvs resultatet och vägen dit.

4.1 Problem Formulation

Beskriv din problemställning för demonstratorn.

The engineering problem where to build a cube that, using a reaction wheel, could balance on its edge.

4.2 State space model

PICTURE TO BE ADDED
To create a state-space model the physical model has to be translated to a mathematical model. To do this, *Euler-Lagrange* equations is used where a system in motion can be described by:

$$Q_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) - \left(\frac{\partial L}{\partial q_i}\right) \tag{4.1}$$

In this case the cube's angular momentum is counteracted by the flywheel and the system can be written as follows

$$M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) \tag{4.2}$$

$$-M_a = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \left(\frac{\partial L}{\partial \phi}\right) \tag{4.3}$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$L = E_k - E_p \tag{4.4}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{4.5}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{4.6}$$

Equation (4.2) and (4.3) with (4.4)

$$I_k \cdot \ddot{\theta} - \frac{M_c \cdot g \cdot l \cdot \sin\theta}{\sqrt{2}} = -M_a \tag{4.7}$$

$$I_s \cdot \ddot{\phi} = M_a \tag{4.8}$$

From these equations it is evident that M_a is the torque executed by the flywheel which is wielded by the motor torque τ, it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau = K_t \cdot i_m \tag{4.9}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \tag{4.10}$$

The induced voltage is related to the induced voltage constant and the rotor rotation

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \tag{4.11}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{4.12}$$

$$\tau = \frac{K_t}{R_m}U - \frac{K_t K_{\text{emf}}}{R_m}\dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m}\dot{\theta} \tag{4.13}$$

The torque on the executed by the flywheel can then be described with the efficiency of the motor

$$M_a = \tau \cdot \eta_m \tag{4.14}$$

Based on equation (4.3), (4.2) and (4.14) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\theta} + \frac{Mtgl}{\sqrt{2} I_c}\sin\theta \tag{4.15}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\theta} \tag{4.16}$$

based on equations (4.15) and (4.16) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \tag{4.17}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Mtgl}{\sqrt{2}I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

4.3 Simulation

To confirm the mathematical model Simulink® was used.

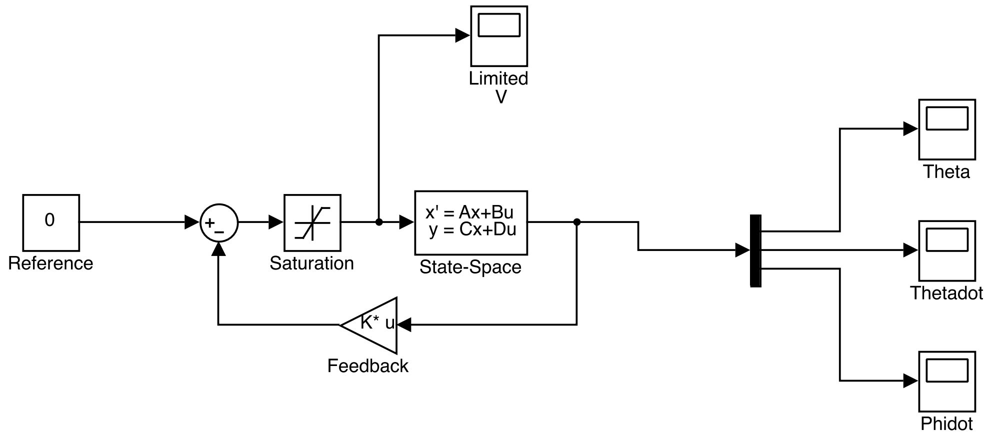

Figure 4.1. Simulink model.

The voltage supplied to the motor
The angle of the cube. Very good

4.4 Software

Beskriv hur din mjukvara fungerar. Använd bl.a. flödesscheman för att åskådliggöra programmets struktur.

4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar.
Sensors
Motor
Arduino
Motor control

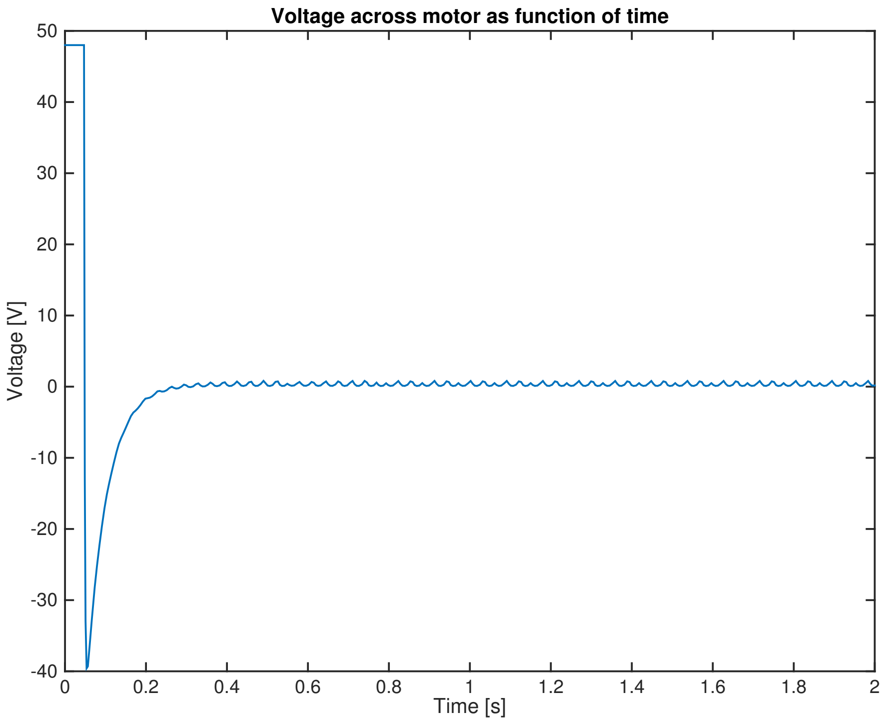

Figure 4.2. Voltage across motor poles.

4.6 Hardware

Beskriv din mekaniska konstruktion (om du har någon) The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other componets are mounted on the motor-body side of the cube.
Basic construction

4.7 Results

Beskriv resultatet.

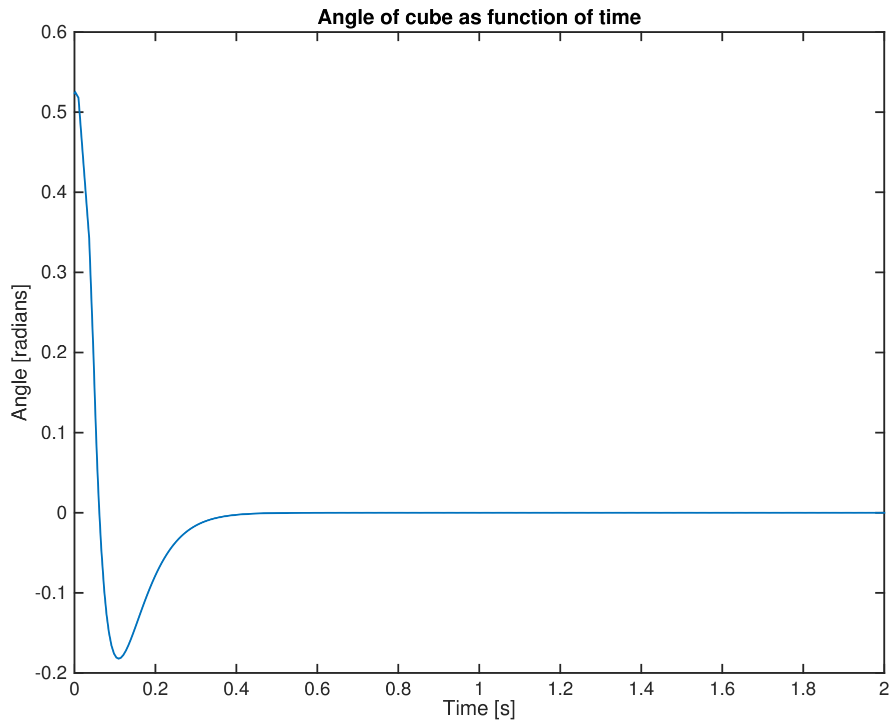

Figure 4.3. Angle of the cube.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Bla bla bla

5.2 Conclusions

Bla bla bla

Chapter 6

Recommendations and future work

I detta kapitel ges rekommendationer for mera detaljerade lösningar och/eller framtida arbete.

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

Bibliography

[Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea] M. Gajamohan, M. Muehlebach, T. Widmer, R. D'Andrea, The Cubli: A Reaction Wheel Based 3D Inverted Pendulum, in: proc. European Control Conference, 268–274, 2013.

Appendix A

Additional information

Appendix B

Proofs