

Allan Variance: Noise Analysis for Gyroscopes

1 Introduction

Allan variance is a method of analyzing a sequence of data in the time domain, to measure frequency stability in oscillators. This method can also be used to determine the intrinsic noise in a system as a function of the averaging time. The method is simple to compute and understand, it is one of the most popular methods today for identifying and quantifying the different noise terms that exist in inertial sensor data. The results from this method are related to five basic noise terms appropriate for inertial sensor data. These are quantization noise, angle random walk, bias instability, rate random walk, and rate ramp.

The Allan variance analysis of a time domain signal $\Omega(t)$ consists of computing its root Allan variance or Allan deviation as a function of different averaging times τ and then analyzing the characteristic regions and log-log scale slopes of the Allan deviation curves to identify the different noise modes.

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2 Creating an Allan Deviation Plot for Noise Identification in a Gyroscope

The following describes the steps to be followed in order to create an Allan deviation plot. The overlapping Allan variance method is used here for computing the Allan variance and creating the Allan deviation plot for noise analysis. The steps are:

1. Acquire a time history $\Omega(t)$ of the gyroscope's output (keeping the gyro static) using an experimental setup. Let the number of samples be N and the sample period be τ_0 .
2. Set the averaging time to be $\tau = m\tau_0$, where m is the averaging factor. The value of m can be chosen arbitrarily and $m < (N-1)/2$.
3. Divide the time history of the signal which is the sequence of Gyro output data over time, into clusters of finite time duration of $\tau = m\tau_0$.

In this method of overlapped Allan variance, the time stride between two consecutive data clusters is always equal to the sample period τ_0 . This overlapping method forms all possible overlapping sample clusters with the averaging time, τ , making maximum use of the dataset. This can be seen in Figure 1.

The figure illustrates the clusters of samples in the overlapping method with Averaging factor $m = 3$. Also shown in this figure, the clusters have overlapping samples. Each cluster is associated with a duration equal to $\tau = 3\tau_0$. Each cluster is separated from the other by the sample period τ_0 (Stride).

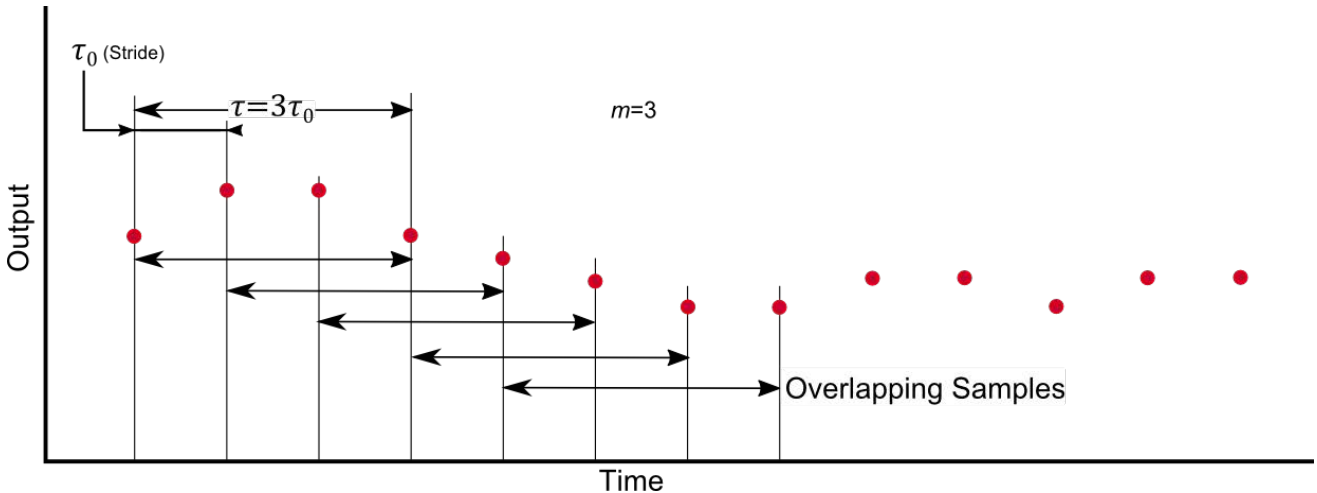


Figure 1. Overlapping Samples

4. Once the clusters are formed, compute the Allan variance (by the overlapping method) in two ways.
 - Averages of the output rate samples (over each cluster), section 2.1
 - Output angles θ corresponding to each gyro rate sample, section 2.2
5. Finally, calculate the Allan deviation value for that particular value of τ , and then obtain the Allan deviation plot by repeating the steps for multiple values of τ (Section 2.3).

2.1 Calculate Allan Variance using Output Angles

The Allan variance can be computed in terms of the output angles θ , corresponding to each gyro rate sample.

1. Calculate θ corresponding to each gyro output sample. This can be accomplished as in:

$$\theta(t) = \int^t \Omega(t') dt' \quad (1)$$

These angle measurements are made at times $t = k\tau_0$ ($\tau_0, 2\tau_0, 3\tau_0 \dots$) where k varies from 1 to N . For a discrete set of samples, a cumulative sum can also be used to give N values of θ . Here the cumulative sum of the gyro output samples at each $k\tau_0$ is taken and each sum obtained is then multiplied by sample period τ_0 to give N values of θ .

Example: Calculate three values of θ , assuming k to vary from one to three samples.

Suppose the observed Ω_k values are 10, 12 and 15.

By using a cumulative sum, the following values are obtained; 10, $10 + 12 = 22$ and $10 + 12 + 15 = 37$.

On further multiplying by τ_0 , we now get the corresponding values of θ_k : $\theta_1 = 10\tau_0$, $\theta_2 = 22\tau_0$, $\theta_3 = 37\tau_0$.

2. Once N values of θ have been computed, calculate the Allan variance using equation (2).

$$\sigma^2(\tau) = \frac{1}{2\tau^2} \langle (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 \rangle \quad (2)$$

$\sigma^2(\tau)$ represents Allan variance as a function of τ and $\langle \rangle$ is the ensemble average. On expanding the ensemble average in equation (2) we get:

$$\sigma^2(\tau) = \frac{1}{2\tau^2(N-2m)} \sum_{k=1}^{N-2m} (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 \quad (3)$$

where N is the total number of samples, m is the averaging factor, $\tau = m\tau_0$ is the averaging time and K is a set of discrete values varying from 1 to N .

Equation (3) computes the final rate Allan variance (by overlapping method) value using output angle θ for a gyroscope for one particular value of τ . Once we know the output angle values of θ , the sample period and the value of m , we can use this equation (3) to compute Allan variance.

Creating an Allan Deviation Plot for Noise Identification in a Gyroscope

2.2 Calculate Allan Variance Using Averages of Output Rate Samples

The Allan variance can be computed in terms of averages of output rate samples over each cluster.

1. Average the output rate samples across each cluster between times $K\tau_0$ and $K\tau_0 + \tau$.

$$\bar{\Omega}_K(\tau) = \frac{1}{\tau} \int_{K\tau_0}^{K\tau_0 + \tau} \Omega(t) dt \quad (4)$$

Also, as N values of θ have been calculated In Section 2.1, “Calculate Allan Variance using Output Angles”, calculate the average output rate $\bar{\Omega}_K(\tau)$ using the output angle. The average output rate in terms of output angle θ_K between times $K\tau_0$ and $K\tau_0 + \tau$ is given as:

$$\bar{\Omega}_K(\tau) = \frac{\theta_{K+m} - \theta_K}{\tau} \quad (5)$$

Example: $\bar{\Omega}_2(\tau)$ with $k = 2$ and supposing $m = 3$ denotes the average output of the gyro over the cluster between times $2\tau_0$ and $2\tau_0 + 3\tau_0 = 5\tau_0$. This would be equal to:

$$\bar{\Omega}_2(\tau) = \frac{\theta_5 - \theta_2}{3\tau_0} \quad (6)$$

By either equation (4) or (5), all possible values of $\bar{\Omega}(\tau)$ (over each cluster) are calculated for a particular value of τ . As there are $N - m$ possible clusters formed in this method $N - m$ values of $\bar{\Omega}(\tau)$ are computed.

All possible values of $\bar{\Omega}(\tau)$ (over each cluster) are now known, the Allan variance can be computed as follows:

$$\sigma^2(\tau) = \frac{1}{2} < (\bar{\Omega}_{K+m}(\tau) - \bar{\Omega}_K(\tau))^2 > \quad (7)$$

This is the definition of Allan variance. $\sigma^2(\tau)$ represents Allan variance as a function of τ and $< >$ is the ensemble average.

On expanding the ensemble average in equation (7), we get the following equation:

$$\sigma^2(\tau) = \frac{1}{2m^2(N - 2m)} \sum_{j=1}^{N-2m} \left\{ \sum_{i=K}^{j+m-1} (\bar{\Omega}_{K+m}(\tau) - \bar{\Omega}_K(\tau))^2 \right\} \quad (8)$$

This equation represents the final rate Allan variance (overlapping method) for a gyroscope for a particular value τ by using the averages of output rate value, $\bar{\Omega}(\tau)$ across each cluster.

Note: The Allan variance formula in equation (2) (in term of output angle θ) is derived by substituting the formula of $\bar{\Omega}_K(\tau)$ (in equation (5)) in the Allan variance definition in equation (7).

Using the two ways in the overlapping method of computing Allan variance, $N - 2m$ Allan variances are computed.

2.3 Calculate Allan Deviation and Create an Allan Deviation Plot

Finally, take the square root of the result obtained from either equation (3) or (8) to obtain the value of the root Allan variance or the Allan deviation for a particular value of τ . This result will now be used to characterize the noise in a gyroscope.

$$AD(\tau) = \sqrt{AVAR(\tau)} \quad (9)$$

Note: The overlapping method of Allan variance is most commonly used for time domain frequency measurements, the term *AVAR* has come to be used mainly for the overlapping method of Allan variance and *ADEV* for its square root.

The described sequence of steps yields the estimated value of the root Allan variance or Allan deviation for the chosen averaging time τ . In order to obtain the whole curve, the computation is repeated multiple times for a sequence of τ values.

As previously mentioned, the range of τ values to obtain a complete Allan variance curve can be chosen arbitrarily, for example, powers of two or log spaced numbers could be used. The Allan deviation plot is normally plotted as values of Allan deviation over τ on a log-log plot.

3 Noise Identification

Different types of random process cause slopes with different gradients to appear on the Allan deviation plot. The different processes usually appear in different regions of τ , allowing their presence to be easily identified. Having identified a process it is then possible to read its numerical parameters directly from the plot. For a MEMS device such as a gyroscope, the important processes to be measured are random walk and bias instability (sometimes also called bias stability), which can be identified and read as follows:

- **White Noise/Random Walk** appears on the Allan variance plot as a slope with gradient -0.5 . The random walk measurement for this noise (ARW for a rate-gyroscope, VRW for an accelerometer) is obtained by fitting a straight line through the slope and reading its value at $\tau = 1$. ARW in gyroscope is a measure for gyro noise and is given in units of dps/rt(Hz).
- **Bias Instability** appears on the plot as a flat region around the minimum. The numerical value is the minimum value on the Allan deviation plot. For a gyroscope, the bias stability measures how the bias of the gyroscope changes over a specified period of time at constant temperature. This is typically presented in units of dps/sec or dps/hr.

Noise Identification

Figure 2 shows various different noise processes the can be observed from the Allan deviation plot. From the figure below, the noise processes to be observed, calculated and analyzed for gyroscopes are Angle Random Walk and Bias Instability.

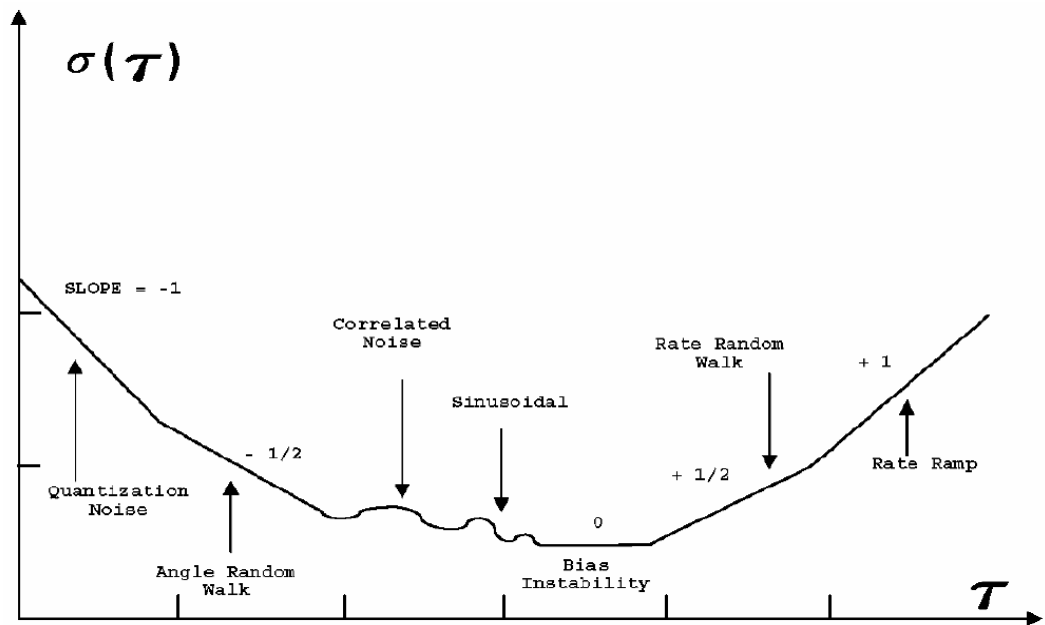


Figure 2. Different Noise processes on an Allan Variance Plot

Figure 3 shows the Allan variance curve corresponding to Freescale's Gyroscope FXAS21002C. The Bias Stability and ARW values are distinctly marked on the figure below.

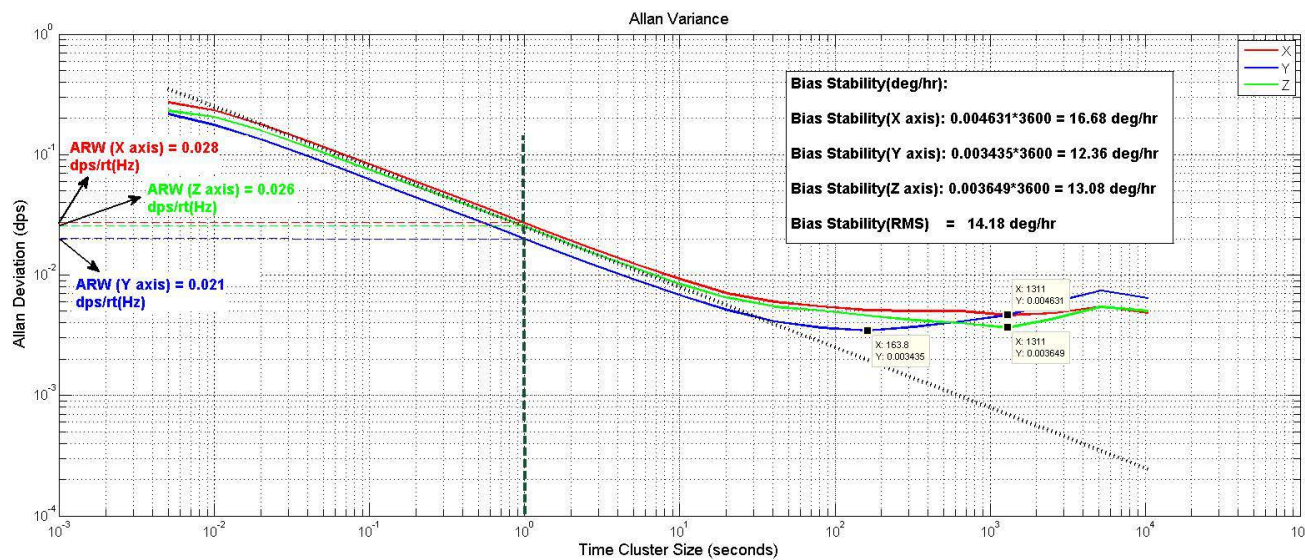


Figure 3. Allan Variance Curve for Gyroscope FXAS21002C

The following is a code snippet to compute Allan variance and the Allan deviation:

```
function [T,sigma] = allan(omega,fs,pts)
[N,M] = size(omega); % figure out how big the output data set is
n = 2.^(0:floor(log2(N/2)))'; % determine largest bin size
maxN = n(end);
endLogInc = log10(maxN);
m = unique(ceil(logspace(0,endLogInc,pts)))'; % create log spaced vector average factor
t0 = 1/fs; % t0 = sample interval
T = m*t0; % T = length of time for each cluster
theta = cumsum(omega)/fs; % integration of samples over time to obtain output angle  $\theta$ 
sigma2 = zeros(length(T),M); % array of dimensions (cluster periods) X (#variables)
for i=1:length(m) % loop over the various cluster sizes
    for k=1:N-2*m(i) % implements the summation in the AV equation
        sigma2(i,:) = sigma2(i,:) + (theta(k+2*m(i),:) - 2*theta(k+m(i),:) + theta(k,:)).^2;
    end
end
sigma2 = sigma2./repmat((2*T.^2.*(N-2*m)),1,M);
sigma = sqrt(sigma2)
```

Code Explanation

Section References are provided below with each point to explain the above code better in terms of the detailed steps provided in Section 2 of the document.

- Omega is the gyro rate output. (Section 2, Step 1)
- m is chosen arbitrarily and the τ value is defined as m *sample period. (Section 2, Step 2)
- In the code above, the methodology described in Section 2.1 is used to compute Allan variance.
- Theta is obtained by the cumulative sum of the output rate data divided by the sampling frequency (exactly as explained in equation (1)). This Theta is the output angle obtained corresponding to each sample. (Section 2.1, Step 1).
- As m is available and so is the output angle corresponding to each output rate sample, equation (3) is directly implemented in the above code to compute Allan variance (sigma2 in the code) (Section 2.1, Step 2).
- Allan deviation (sigma in the code) is then computed by taking the square root of AVAR, equation (9) (Section 2.3).

4 References

1. IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros, IEEE Std 647-1995 , vol., no., pp.i., 1996.
doi: 10.1109/IEEESTD.1996.80221
URL: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=494457&isnumber=10718>
2. W. J. Riley. Handbook of Frequency Stability Analysis. National Institute of Standards and Technology (NIST), U.S. Department of Commerce, NIST Special Publication 1065, July 2008.

Revision History

5 Revision History

Table 1. Revision History

Rev. No.	Date	Description
0	2/2015	Initial Public Release

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