Balancing Cube (working title)

Control and design of reaction wheel balanced inverted pendulum

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Bachelor's Thesis in Mechatronics

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Abstract

This thesis is about implementing automated control and balance a simple construction using a reaction wheel commonly used in satellites...
To be filled in:
Problem
Approach
Results
Conclusion

Sammanfattning

Stabilisering med svänghjul

Projektet gick ut på att bygga en kub som kan balansera på en kant med hjälp av ett reaktionshjul. Dessutom sulle den undersökas huruvida det gick att förbättra reglersystemet för cuben, så den klarade en större yttre störning. Ingengörsproblemet delades upp i mindre delproblem och kuben byggdes. Reglestystemet beräknades på formen State spaceöch implementerades. Från resultatet drogs slutsatserna att...

Preface

Here goes our thanks to sources of help, cooperation, inspiration
To be filled in

Alexander Ramm
Mikael Sjöstedt
KTH, månad, 2015

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Appendices

Nomenclature

Symbols - needs restructure

Symbol	Description
E	Elasticity module (Pa)
r	Radius (m)
t	Thickness (m)
L	Lagrange (fixa)
θ	Cube angle
ϕ	Flywheel angle
Q and q	Lagrange operators
E_k	Kinetic energy
E_p	Potential Energy
I_c	Inertia of the cube
I_f	Inertia of the flywheel
M_c	Total mass of the cube
i	Current
K_t	Torque constant
$\mathrm{E_{emf}}$	Induced voltage
$\mathrm{K_{emf}}$	Induced voltage constant
U	Voltage across motor poles
R_m	Motor internal resitance
η_m	Motor efficiency
η_g	Gear efficiency
Γ	Gear ratio
z	Measurement noise
w	Process noise

Abbreviations

Abbreviation	**Description**
CAD	Computer Aided Design
CAE	Computer Aided Engineering
PLM	Product Lifecycle Management
PWM	Pulse With Modulation
DOF	Degrees of freedom
MEMS	Microelectromechanical Systems
MATLAB	Matrix Laboratory, computational program
IC	Integrated circuit
I^2C	Inter-Integrated circuit
USB	Universal Serial Bus

Chapter 1

Introduction

This chapter describes the background, purpose and scope of this project conducted at the mechatronics department at the Royal Institute of Technology, KTH. The work was carried out during the spring 2015.

1.1 Background

Balancing 1-DOF inverted pendulum type structures using reaction wheels is no new concept, and became more acsessible with the introduction of cheap micro-controllers. The use of automated control is growing in a rapid pace and is being implemented more and more in consumer related products. This growth has made automated control available more now than ever, in our every-day life in product lines as mobile phones, gaming controllers, cars and UAV's such as quadrocopters.

One of the most basic systems that requires some control to become stable is the inverted pendulum. Altho it is simple to define controlling it is not a trivial task. A lot of work has been done on the topic but their is still not knowledge easely acquired by the public.

The method the achieve balance of the pendulum using reaction wheels is even more narrow. The use of reaction wheels to change the rotation is commonly used in satellites. The exact control is also required. In recent years prototypes of land based structures using reaction wheels have been a hot topic and the cubli is truly remarkable.

It would be a great achievement to contribute knowledge about how such a mechanism could be built and evaluate the capabilities and restrictions of such a machine, on a level that does not require a PhD.

1.2 Purpose

The goal of the project was to build a stucture that in one DOF can maintain balace using a reaction wheel and examine the bahaviours of the system.
The behaviour is mostly effected by the control system, which is responsible for

accelerating the motor in the correct angular direction, to maintain balance. The parameters in the control system effects response time, overshoot and sinusoidal settling time. This project will hopefully contribute to some development within the open-source community. All results are available online, open source (MIT license reference here), on GitHub (GitHub link here).
As a mechatronical thesis the research task were to be along the line of how something physical is implemented in a, in some context, good or eaven correct way.

> *If balance is maintained how does the maximum applied force correlate to the rise time and overshoot separately, can any conclusions be drawn from the results?*

or maybe

> *How does sensor placement effect system performance -and can internal disturbances terminaTOR nej men något sånt???*

Where balance is defined as as the state where the cube is able to return to its reference angle/value?. The rise time and overshoot refers to the system angle. Can the results contribute to improve the overall performance?

1.3 Scope

The scope were to examine the parameters, of the state space controller and sensor sample frequency, affects on the overshoot behaviour of a balancing "1-DOF" inverted pendulum. The overshoot should be caused by an external force, disrupting the cubes balance. Moar "we will not do this"

Chapter 2

Method

The engineering task The main goal of this project was to build a structure which remain stable in an unstable condition. A process of this sort can be divided into several parts.

- Construction
- Motor Control
- Sensor Reading
- System Control
- Final Assembly

2.1 Construction

The main construction problem where deciding the size of the cube and reaction wheel. A too big reaction wheel for the motor has a large affect on the cubes ability to balance. The problem were (uppställt) with Newtonian mechanics. Also idealy the cube should be nice looking, easy to produce and simple to assemble.

2.2 Motor and Motor Control

The motors nominal and stall torque are very important for the system blaha. The motor driver is also important, but usually one can get suggestions on drivers from motor manufactures, which was the chosen path.

2.3 Sensor Reading

The IMU's parameters and filtering of the signals

2.4 System Control

The choosen control method where state space. The problem in to linareise and discretise with good enough precition.

2.5 Final Assembly

When the subproblems above are solved and constructed, the final machine can be built. Here cabling and disturbances from other subsystems must be taken into consideration. The IMU placement would provisoricly be tried to se a placement were bad due to more disturbances form other compunents i.e. netsupply and motor lining.

Chapter 3

Theory

This chapter cover some theory that is required if one want to build a similar machine. It is assumed that the reader has some understanding of Newtonian mechanics, signal analysis and control theory. Basic understanding of DC motor is also an advantage. The first part covers the Kalman filter theory, required for getting high quality data from the IMU. The filter take noisy data from sensors and digitaly filters it to a more reliable quality. The second part will cover the theory of the mechanical system behaviour that is used to device the state space control system. Those equations are responsible for the actual balance part and thus important. There is also a part on sensor characteristics and why they are important to the system as a whole and the research question in particular. When using a sensor such as an IMU one can choose between different resolution settings, depending of the specific usage. Although the resolutions were not changed between measurements these settings could be important when comparing to another machine.

3.1 Moar theory ?

Where do we mention communication protocol, maybe some in-depth of accelerometer and gyro ?

3.2 Kalman filter

The signal from an IMU contains data of angular velocities and transversal acceleration, but also a lot of noise. An estimated position of an untreated signal from an IMU would work for short periods but over time the estimated position 'drifts´. This drift occurs because of integration of the measurements to acquire a position, but as the readings contain unwanted noise and often a bias as well this small error grows. Integrating the angular motion to estimate a position would result in an angular drift for the gyro and an even worse drift for the accelerometer as it is integrated twice if it were to estimate a position [Jaw-Kuen Shiau and Chang(2012)]. By using a Kalman filter the drift can effectively be minimized. By using both readings from the IMU and with some help of probablity theory the estimated state is not far from the true value. A Kalman *filter* is not what the name suggests, it's an estimator. Old and new measurements are processed real-time to calculate an estimation of the current state. What could be asked of the filter? **is this too "friendly"** A good estimator produces states that are non biased, *values that have an average of the true value.* As well that the estimated state variance from the true state is as small as possible. [Simon(2001)]

3.2.1 State Estimator

The Kalman filter is a state based estimator. By using measurements from the past and present it can derive a good estimate of the current state. The true state and the measured value at a time k would be

$$x_k = Ax_{\text{k-1}} + Bu_{\text{k-1}} + w_{\text{k-1}} \tag{3.1}$$

$$y_k = Hx_k + z_k \tag{3.2}$$

The true state x is expressed with the the old state, an input u, that is data from the gyroscope. But the signal also contains a process noise w. The process noise w in equation (3.1) is a representation of variances in the gyroscope that cannot be mathematically predicted such as flaws in production. The measured value, y (see (3.2)) is another observed measurement, in this case the accelerometer. Ideally this would only be a function of x, but is distorted by the measurement noise z. The measurement noise, z, much like the process noise is common in any measurement and represents various fluctuations caused by the equipment.

As this recursive filter uses old and new values a *priori* and *posteriori* state is defined

$$\hat{x}_k^- \tag{3.3}$$

$$\hat{x}_k \tag{3.4}$$

The *priori* (3.3) state is defined as the estimate of the current state at the time k. The *posteriori* state (3.4) is the new estimated state. For the Kalman filter to work properly some criteria has to be fulfilled. The average value of the measurement noise z and process noise w has to be zero, i.e. a Gaussian error. z and w also has to be independent of each other. The noise and error in an IMU and many other devices have the charecteristics of gaussian noise.

3.2.2 The process

The Kalman filter loops two stages. The *predict* and *update* stages.

During the *predict* phase the filter estimates the states using the inputs from the process, i.e the gyroscope. 3.1

$$\hat{x_k^-} = A\hat{x}^-{}_{\text{k-1}} + Bu_{\text{k-1}} \tag{3.5}$$

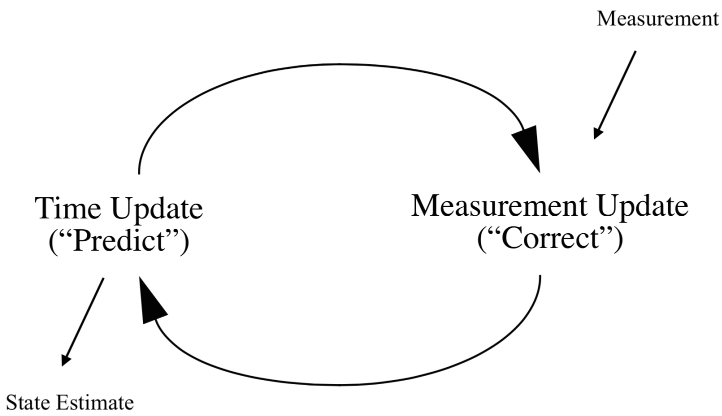

Figure 3.1. Kalman phases.

As stated above **För slentriant?** the Kalman filter uses readings from both the gyro and accelerometer to estimate a position closer to the true value. To determine how reliable the process and measurement readings are a noise covariance is defined as

$$Q = E(w_k {w_k}^{\mathrm{T}}) \tag{3.6}$$

$$R = E(z_k {z_k}^{\mathrm{T}}) \tag{3.7}$$

From here a *priori* error covariance matrix is introduced to symbolize the noise in the process measurement.

$$P_k^- = AP^-1_{\text{k-1}}A^T + Q_k \tag{3.8}$$

During the *update* the accelerometer values are used. The measurement *innovation* is calculated as

$$\tilde{y} = z_k - H\hat{x}_k^- \tag{3.9}$$

The *innovation* is a residual that reflects the relation between the predicted measurement and the actual measurement. A measurement *innovation* of zero indicates a perfect agreement. The measurement *innovation* covariance is calculated as

$$S_k = HP_k^- H^T + R \tag{3.10}$$

This is very similiar to the *priori* error covariance but represents the measurement instead. From here the core of the Kalman filter can be calculated, the Kalman gain

$$K_k = P_k^- H^T {S^{\text{-1}}}_k \tag{3.11}$$

indicates how reliable the measurement is. Note that if the measurement covariance error (3.7) is large the Kalman gain will be small and the opposite if the*priori* error covariance is large **superlång mening ?**. By now the *posteriori* state can be estimated by

$$\hat{x}_k = \hat{x}_k^- + K_k \tilde{y}_k \tag{3.12}$$

A current state has been estimated and the Kalman filter returns to the measurement phase seen in figure 3.1. For further reading, and mathematical proof see [Welch and Bishop(2006)].

3.3 Model dynamics

To create a state-space model the physical model has to be translated to a mathematical model. The system can be estimated much like an inverted pendulum two-degree-of-freedom model [**?**].

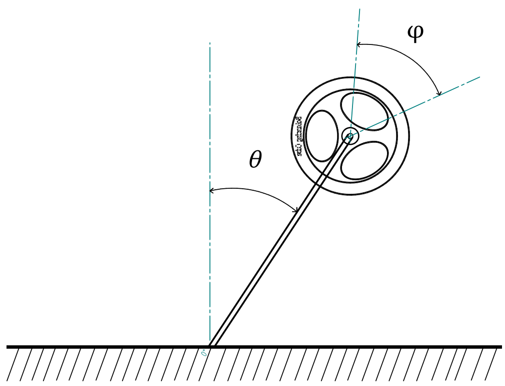

Figure 3.2. Cube modelled as a reaction wheel pendulum

Lagrangian Dynamics have been used to derive the systems behaviour. Firstly by expressing the generlized forces, the energy functions and lagrangian. And then acquire the equations of motion from the Lagrange equation [**?**]. Consider the Lagrangian

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) \tag{3.13}$$

Where τ is generilzed force, in this case a torque. The cube's angular momentum is counteracted by the flywheel and the system can be divided into two parts, One considering the movement of the cube, the other the flywheel.

$$\tau_k = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) \tag{3.14}$$

$$-\tau_k = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \left(\frac{\partial L}{\partial \phi}\right) \tag{3.15}$$

Whereas θ represents the angle of the cube and ϕ is the position of the flywheel. The Lagrange equation is derived from the difference in kinetic energy and potential energy of the cube

$$\mathcal{L} = E_k - E_p \tag{3.16}$$

$$E_k = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} \tag{3.17}$$

$$E_p = \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{3.18}$$

The lagrangian (3.16) is then

$$\mathcal{L} = \frac{I_c \cdot \dot{\theta}^2}{2} + \frac{I_f \cdot \dot{\phi}^2}{2} - \frac{M_c \cdot g \cdot l \cdot \cos\theta}{\sqrt{2}} \tag{3.19}$$

The kinetic energy depends on the angular velocities of the cube construction as well as the flywheel fixed to the motor. Note that the total moment of inertia I_c is defined around the pivot point of the cube. The potential energy has been defined as being at its maximum when the cube is balancing in an upright position. The construction is considered to be symmetric and hence the gravitational force is applied on the center of the cube. Equation (3.14) and (3.15) with (3.16)

$$I_c \cdot \ddot{\theta} + \frac{M_c \cdot g \cdot l \cdot \sin\theta}{\sqrt{2}} = -\tau_k \tag{3.20}$$

$$I_s \cdot \ddot{\phi} = \tau_k \tag{3.21}$$

From these equations it is evident that τ_k is the torque executed on the flywheel which is wielded by the motor torque τ_m, it can be described by a relation between the torque constant and the current flowing through the motor.

$$\tau_m = K_t \cdot i_m \tag{3.22}$$

The current can be described by the voltage across the two poles of the motor.

$$\tau_m = K_t \cdot \frac{U - E_{\text{emf}}}{R_m} \tag{3.23}$$

Note that the motor inductance in neglected in equation (3.23), that is due to the time constant which is fast considering the rest of the system.**Do we need source ?**

$$E_{\text{emf}} = K_{\text{emf}} \cdot \dot{\phi}_r \tag{3.24}$$

$$\phi_r = \dot{\phi} - \dot{\theta} \tag{3.25}$$

$$\tau_m = \frac{K_t}{R_m}U - \frac{K_t K_{\text{emf}}}{R_m}\dot{\phi} + \frac{K_t K_{\text{emf}}}{R_m}\dot{\theta} \tag{3.26}$$

The torque executed **byta ord och symbol för gear ?** on the flywheel can then be described with the torque on the motor shaft, efficiency and gearing.

$$\tau_k = \tau_m \cdot \eta_m \cdot \eta_g \cdot \Gamma \tag{3.27}$$

Based on equation (3.15), (3.14) and (3.27) the system can be described by

$$\ddot{\theta} = -\frac{K_t \eta_m}{R_m I_c}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c}\dot{\theta} - \frac{Mtgl}{\sqrt{2} I_c}\sin\theta \tag{3.28}$$

$$\ddot{\phi} = \frac{K_t \eta_m}{R_m I_f}U + \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\phi} - \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f}\dot{\theta} \tag{3.29}$$

To use linear control methods the model has to be linearised. This is done at the instable equilibrium where the cube is balancing. Consider the sinus term at the equilibrium point where θ equals 0. The term can then be expressed with taylor/macloaruin expansion

$$sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} ... \approx \theta \tag{3.30}$$

With the equations (3.28) and (3.29) the system can be described with a state space model with a states $x^T = [\theta, \dot{\theta}, \dot{\phi}]$. The system is hence described by

$$\dot{x} = Ax + Bu \tag{3.31}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{Mtgl}{\sqrt{2} I_c} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_c} \\ 0 & \frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} & -\frac{K_t K_{\text{emf}} \eta_m}{R_m I_f} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -\frac{K_t \eta_m}{R_m I_c} \\ \frac{K_t \eta_m}{R_m I_f} \end{bmatrix}$$

3.3.1 Control theory

To create a state space feedback loop... Use Ackermann instead of place? Why ?

Chapter 4

Demonstrator

Detta kapitel beskriver både den utvecklade demonstratorn och den aktuella arbetsprocessen som demonstartorn utvecklats enligt, dvs resultatet och vägen dit. VAD FAN ÄR PROBLEMET MED MIN STATE SPACE DET KNASAR JU GRANDE WTF MODE

4.1 Problem Formulation

The engineering problem were to build a cube that, using a reaction wheel, could balance on its edge. *To be continued*

4.2 Model validation

To synthesize a mathematical model from a real world problem it's often beneficial to simplify the reality. Examples of assumption made for this application would be that center of mass is located at the center of the cube, the friction in the motor is ignored and the frame is considered stiff etcetera. To validate the model from chapter 3.3, events with known results can be tested. To do so, Simulink [MATLAB(2014)] is used. First of all the DC-motor model is validated to known characteristics, such as no load speed and current.

Graph here

The graphs in figure (ref) displays the speed and current of the unloaded motor. Showing that it

The dynamics of the cube is simplified as an inverted pendulum. That means if there is no control input to the system it should behave as pendulum in free movement. That is, it should oscillate at a constant amplitude. As there is no torque applied to the flywheel the rotor should remain zero at all times.

4.3 Discrete Kalman filter

To implement the Kalman filter in algorithm it has to be discretizied This is done much like a feedback control. The filter firstly? estimates the process state and then obtains feedback as noisy measurements. That means that the filter works in two steps, a *time update* and a *measurement update*. The names implicate that the *time update* projects the next state to obtain the *priori estimate* whilst the *measurement update* uses the feedback mentioned above to obtain an improved *posteriori* estimate.

Some of the implementation and discretization of the filter.

4.3.1 Kalman implementation

An implementation of the Kalman filter on the IMU would look something like this for the gyroscope

$$\theta_k = \theta_{\text{k-1}} + (w_k \mp b_{\text{k-1}})\Delta t \tag{4.1}$$

$$b_k = b_{\text{k-1}} \tag{4.2}$$

$$z_k = a_k \tag{4.3}$$

With b as the gyro bias Looking at equation (3.1) the system can be written

$$\mathbf{x}_k = \begin{bmatrix} \theta \\ \dot{\theta}_{bk} \end{bmatrix} \tag{4.4}$$

$$\mathbf{A} = \begin{bmatrix} 1 & -\Delta t \\ 0 & 1 \end{bmatrix} \tag{4.5}$$

$$\mathbf{B} = \begin{bmatrix} \Delta t \\ 0 \end{bmatrix} \tag{4.6}$$

4.3.2 Measurement and process noise

For the Kalman filter to properly work it is essential to know how reliable the process and measurement inputs are. A way of determining the process noise and measurement noise of the IMU is the Allan variance method ref. The gyro data is treated as an external input the system, so the error and bias from the gyro readings are characterised as process noise. This is then compared to the measurement, the accelerometer which contains a measurement noise. More to come

4.4 Software

To develop and improve a system such as this is an iterative process. To verify changes and improvements in realtime Simulink® was used togheter with the mathematical model.

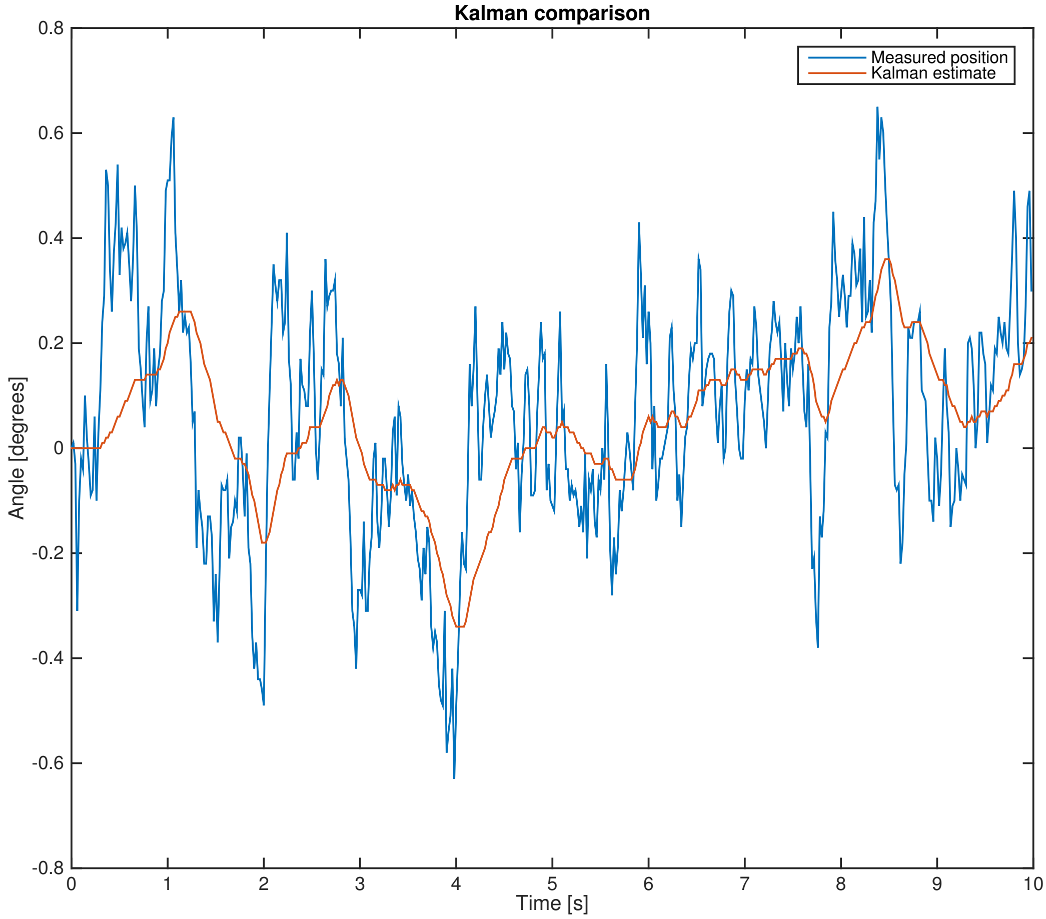

Figure 4.1. Comparison of Kalman filtered signal and original signal

The Simulinkmodel seen in figure **??** describes the system
Something about the optimizing of the feedback control

The voltage supplied to the motor

The angle of the cube. Very good such magic

4.5 Electronics

Beskriv din elektroniska konstruktion. Använd figurer och förenklade blockschema. Motivera dina lösningar. How do we send data?
Sensors
Motor

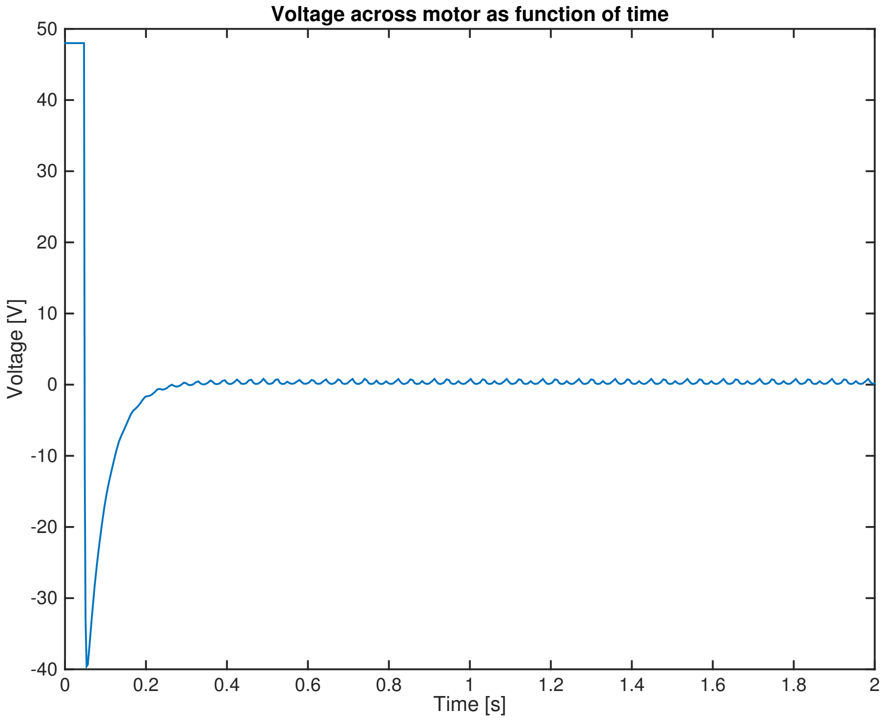

Figure 4.2. Voltage across motor poles.

Arduino
Motor control

4.5.1 PWM

skriv lite om PWM hax

4.6 Hardware

The motor is fixed through the middle wall in the cube, the shaft on one side and the body on the other. The flywheel is dicrectly mounted to the motor shaft. All other components are mounted on the motor-body side of the cube.
Basic construction

4.7 Results

Beskriv resultatet.

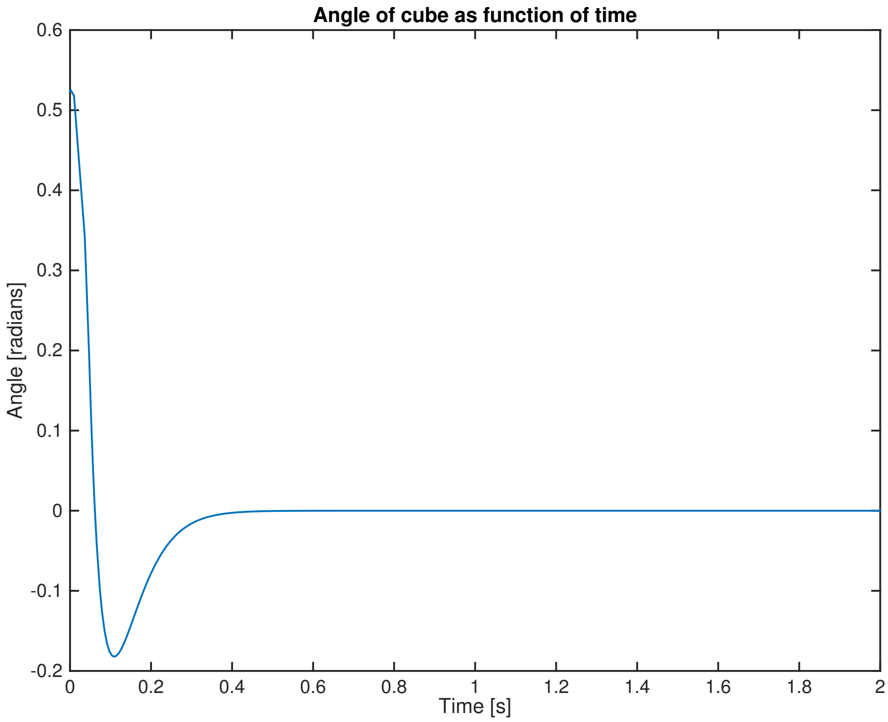

Figure 4.3. Angle of the cube.

Chapter 5

Discussion and conclusions

I detta kapitel diskuteras och sammanfattas de resultat som presenterats i föregående kapitel. Sammanfattningen baseras på en resultatanalys och syftar till att svara på den fråga eller de frågor som formuleras i kapitel i.

5.1 Discussion

Motor choice osv

5.2 Conclusions

Successful victory

Chapter 6

Recommendations and future work

6.1 Recommendations

A more extensive research with non-linear control systems has been done at ETH, with the name Cubli, [Gajamohan et al.(2013)Gajamohan, Muehlebach, Widmer, and D'Andrea]

6.2 Future work

An extension of the project would be balancing the cube not only on it's edge but it's corner. To achieve this multiple reaction wheels must be used and a more complicated control system due to changes in moment of inertia caused by angular velocities in the other reaction wheels.

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Appendix A

Additional information

Appendix B

Proofs