Fundamental Theorems of the Theories of Curves and Surfaces

April 21, 2015

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Problem 1.1. *Prove with the Liouville formula that*

(1) The geodesics on the plane are exactly the straight lines.

(2) The geodesics on the cylinder are the straight generatrices and helicoids.

Solution. We remember that the Liouville formula is that

$$k_g = \frac{\mathrm{d}\theta}{\mathrm{d}s} - \frac{1}{2\sqrt{g_{22}}}\frac{\partial \ln g_{11}}{\partial u^2}\cos\theta + \frac{1}{2\sqrt{g_{11}}}\frac{\partial \ln g_{22}}{\partial u^1}\sin\theta$$

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