Connection and Curvature

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Problem 4.1. *Suppose* $\alpha \in A^1(M)$*, then*

$$\int_M (\delta\alpha)\eta = 0$$

.

Proof. □

Problem 4.2. *Suppose M is compact smooth Riemannian manifold without boundary, $h, f \in C^2(M)$. Prove the follow Green formula:*

$$\int_M (h\Delta f - f\Delta h)\eta = 0$$

.

Proof. $\square$

Problem 4.3. *Take the geodesic polar coordinates on (M, g):*

$$g = (\mathrm{d}r)^2 + g_{ij}(r,\theta)\mathrm{d}\theta^i\mathrm{d}\theta^j, 1 \leq i, j, k \leq m-1$$

. Suppose $f = f(r)$ is independent from $\{\theta^i\}$, prove that

$$\Delta f = f'' + \frac{m-1}{r}f' + f'\frac{\partial}{\partial r}\log\sqrt{G}$$

, where $G = \det(g_{ij})$ and $'$ denotes ordinary differentiation with respect to r.

Proof. $\square$