1 Surface 2

Problem 1.1. *A surface is either a sphere or plane* $\Leftrightarrow H^2 = K$.

Solution. Note that $K = \kappa_1\kappa_2$ and $H = (\kappa_1 + \kappa_2)/2$. So $H^2 = K \Leftrightarrow \kappa_1 = \kappa_2$.

- $\Rightarrow$

  Let's calculate the curvatures for spheres and planes. $? = -\mathrm{d}\vec{r} \cdot \mathrm{d}\vec{n}$.

  - For planes we have $\mathrm{d}\vec{n} = 0$. So everything is identically 0.
  - For the unit sphere $r = (\cos u \cos v, \sin u \cos v, \sin v)$. And

    $$\vec{r_u} = (-\sin u \cos v, \cos u \cos v, 0), \vec{r_v} = (-\cos u \sin v, -\sin u \sin v, \cos v)$$

    $$\begin{aligned}&\vec{n} = \vec{r_u} \times \vec{r_v}/|\vec{r_u}||\vec{r_v}|\\ &= (\cos u \cos^2 v, \sin u \cos^2 v, \sin^2 u \cos v \sin v + \sin v \sin u \cos v \cos u)/\cos v\\ &= (\cos u \cos v, \sin u \cos v, \sin v) \quad (1)\end{aligned}$$

    We have that $\vec{r} = \vec{n}$. So

    $$\begin{aligned}&\mathrm{d}\vec{r}\\ &= (-\sin u \cos v, \cos u \cos v, 0)\mathrm{d}u + (-\cos u \sin v, -\sin u \sin v, \cos v)\mathrm{d}v\\ &= \mathrm{d}\vec{n} \quad (2)\end{aligned}$$

    And

    $$II = -\mathrm{d}\vec{r} \cdot \mathrm{d}\vec{n} = -\cos^2 v du^2 - \sin^2 v dv^2 \quad (3)$$

    $$I = \mathrm{d}\vec{r} \cdot \mathrm{d}\vec{r} = -II = \cos^2 v du^2 + \sin^2 v dv^2 \quad (4)$$

    So that the two principle curvatures are all 1.

- $\Leftarrow$

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Problem 1.2. *The helicoid* $\vec{r} = (u \cos v, u \sin v, bv)$ *is a minimal surface. And that besides planes, all ruled minimal surfaces must be helicoids.*

Solution. Let's recall the definition of minimal surfaces:
Let's calculate their principle curvature.

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Problem 1.3. *If the ?? surface* $\vec{r} = (u\cos v, u\sin v, \phi(v))$ *is a minimal surface, then it must be the helicoid.*

Solution. Let's calculate their principle curvatures.

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Problem 1.4. *A surface is a minimal surface* $\Leftrightarrow$ *there exist two families of orthogonal asymptotes.*

Solution. ◀