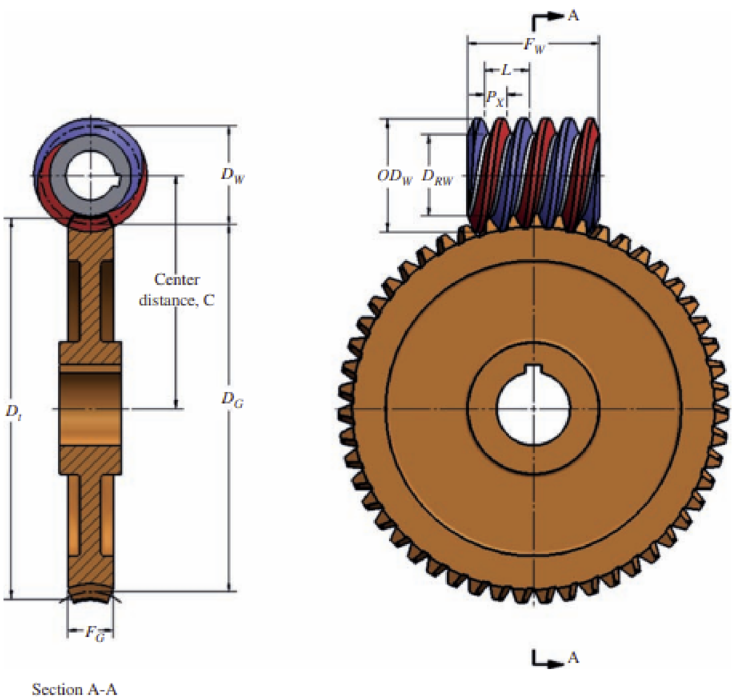
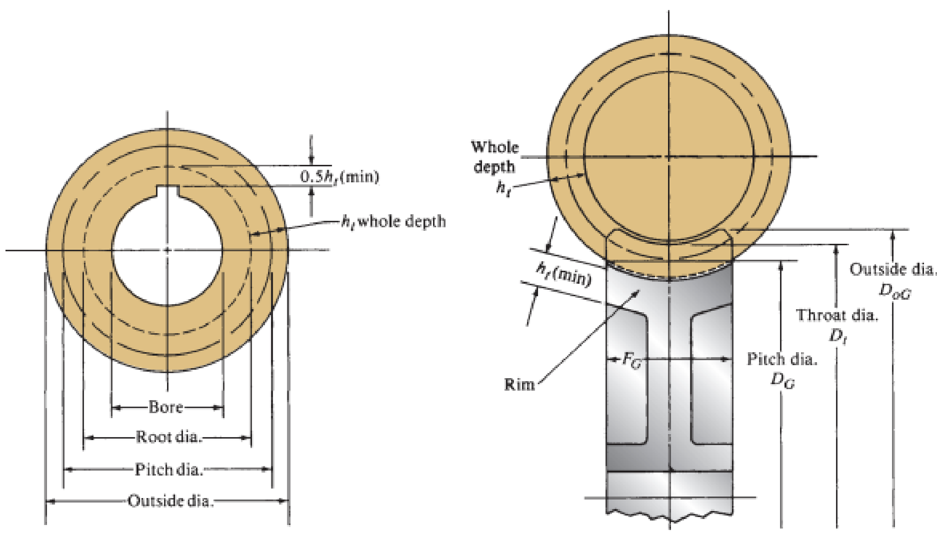
2.4 Worm Gears

2.4.1 Anatomy





2.4.2 Nomenclature

N_G = number of teeth on the gear
N_W = number of worm threads
D_G = pitch diameter of the gear (in)
D_W = pitch diameter of the worm (in)

p = circular pitch (in)
P_d = diametral pitch (teeth/in)
m = module
L = lead (in): axial distance if the worm completes one revolution
P_x = axial pitch
λ = lead angle
C = center distance (in)
ϕ_n = normal pressure angle
ϕ_t = transverse pressure angle
a = addendum (in)
h_t = whole depth (in)
h_k = working depth (in)
b = dedendum (in)
D_{rW} = root diameter of worm (in)
D_{oW} = outside diameter of worm (in)
D_{rG} = root diameter of gear (in)
D_t = throat diameter of gear (in)
F_G = face width of wormgear (in)
F_W = face length of worm (in)
n_W = speed of worm (rpm)
n_G = speed of gear (rpm)
v_{tW} = pitch line speed for worm (ft/min)
v_{tG} = pitch line speed for gear (ft/min)
VR = velocity ratio

2.4.3 Formulae

Geometry:

$$\begin{aligned}
\text{circular pitch: } & p = \frac{\pi D_G}{N_G} = \pi m \\
\text{diametral pitch: } & P_d = \frac{N_G}{D_G} \\
& P_d p = \pi \\
\text{module: } & m = \frac{D}{N} \\
\text{axial pitch: } & P_x = p \\
\text{lead: } & L = N_W P_x \\
\text{lead angle: } & \tan\lambda = \frac{L}{\pi D_W} \\
\text{center distance: } & C = \frac{D_W + D_G}{2} = \frac{N_W + N_G}{2P_d} \\
\text{angle relationship: } & \tan\phi_n = \tan\phi_t \cos\lambda \\
\text{addendum: } & a = 0.3183 P_x = \frac{1}{P_d}
\end{aligned}$$

$$\text{whole depth: } h_t = 0.6866P_x = \frac{2.157}{P_d}$$

$$\text{working depth: } h_k = 2a$$

$$\text{dedendum: } b = h_t - a$$

$$\text{root diameter of worm: } D_{rW} = D_W - 2b$$

$$\text{outside diameter of worm: } D_{oW} = D_W + 2a = D_W + h_k$$

$$\text{root diameter of gear: } D_{rG} = D_G - 2b$$

$$\text{throat diameter of gear: } D_t = D_G + 2a$$

$$\text{face width of wormgear: } F_G = \sqrt{D_{oW}^2 - D_{W^2}} = 2p = \frac{2\pi}{P_d} \approx \frac{6}{P_d}$$

$$\text{face length of worm: } F_W = 2\sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D_G}{2-a}\right)^2}$$

Speed:

$$\text{pitch line speed for worm: } v_{tW} = \frac{\pi D_W n_W}{12} \text{ or } v_{tW} = \frac{\pi D_W n_W}{60000}\ \text{m/s}$$

$$\text{pitch line speed for gear: } v_{tG} = \frac{\pi D_G n_G}{12} \text{ or } v_{tG} = \frac{\pi D_G n_G}{60000}\ \text{m/s}$$

$$\text{velocity ratio: } VR = \frac{n_W}{n_G} = \frac{N_G}{N_W}$$

$$\text{sliding speed: } v_s = \frac{v_{tG}}{\sin\lambda} = \frac{v_{tW}}{\cos\lambda}$$

Forces:

$$\text{force relationship: } W_{tG} = W_{xW},\ W_{xG} = W_{tW},\ W_{rG} = W_{rW}$$

$$\text{output torque: } T_o = \frac{63000P_o}{n_G} = \frac{W_{tG}D_G}{2}$$

$$\text{transmitted force: } W_{tG} = \frac{2T_o}{D_G}$$

$$\text{axial force: } W_{xG} = W_{tG}\frac{\cos\phi_n\sin\lambda + \mu\cos\lambda}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$$

$$\text{radial force: } W_{rG} = \frac{W_{tG}\sin\phi_n}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$$

$$\text{friction force: } W_f = \frac{\mu W_{tG}}{\cos\lambda\cos\phi_n - \mu\sin\lambda}$$

$$\text{power loss due to friction: } P_L = \frac{v_s W_f}{33000}$$

$$\text{input power: } P_i = P_o + P_L$$

$$\text{efficiency: } \eta = \frac{P_o}{P_i} = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \frac{\mu}{\tan\lambda}}$$

2.4.4 Design Selection

1. Compute the lead and lead angle

$$p = \frac{\pi}{P_d} = \frac{\pi D_G}{N_G}$$
$$P_x = p$$
$$L = N_W P_x$$
$$\lambda = \arctan\left(\frac{L}{\pi D_W}\right)$$

2. Compute the center distance

$$C = \frac{D_G + D_W}{2}$$

3. Compute the pitch line speed of the gear

$$v_{tG} = \frac{\pi D_G n_G}{12}$$

4. Compute the sliding speed

$$v_s = \frac{v_{tG}}{\sin\lambda}$$

5. Find the coefficient of friction

$$\mu = \begin{cases} 0.15 & v_s = 0 \\ 0.124e^{-0.074v_s^{0.645}} & 0 < v_s < 10 \\ 0.103e^{-0.11v_s^{0.45}} + 0.012 & v_s > 10 \end{cases}$$

6. Compute the forces on the gear

$$T_o = \frac{63000 P_o}{n_G} = \frac{W_{tG} D_G}{2}$$
$$W_{tG} = \frac{2T_o}{D_G}$$
$$W_{xG} = W_{tG}\frac{\cos\phi_n \sin\lambda + \mu\cos\lambda}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$$
$$W_{rG} = \frac{W_{tG}\sin\phi_n}{\cos\phi_n\cos\lambda - \mu\sin\lambda}$$

7. Compute the friction force

$$W_f = \frac{\mu W_{tG}}{\cos\lambda\cos\phi_n - \mu\sin\lambda}$$

8. Compute the power loss due to friction

$$P_L = \frac{v_s W_f}{33000}$$

9. Compute the input power $P_i = P_o + P_L$

10. Compute the efficiency

$$\eta = \frac{P_o}{P_i} = \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \frac{\mu}{\tan\lambda}}$$

11. Find the Lewis form factor y

TABLE 10-5 Approximate Lewis Form Factor for Wormgear Teeth

ϕ_n	y
$14\frac{1}{2}°$	0.100
20°	0.125
25°	0.150
30°	0.175

12. Find the normal circular pitch

$$p_n = p\cos\lambda = \frac{\pi\cos\lambda}{P_d}$$

13. Compute K_v

$$K_v = \frac{1200}{1200 + v_{tG}}$$

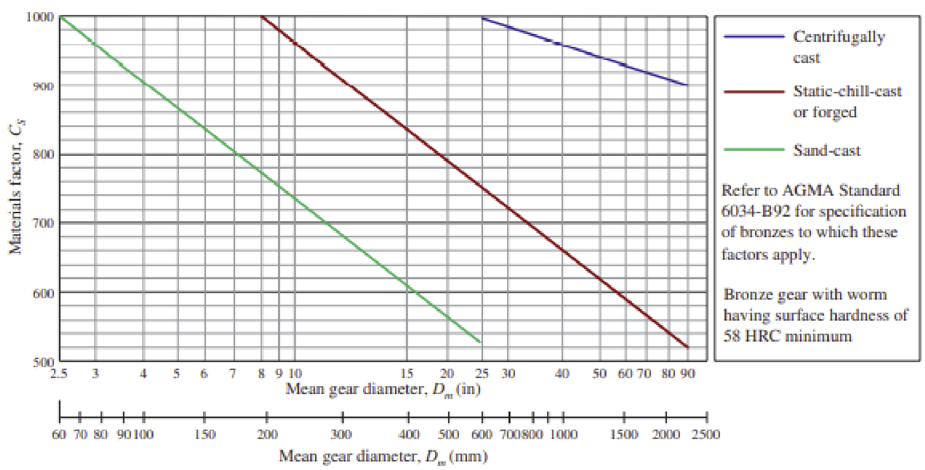
14. Compute the dynamic load

$$W_d = \frac{W_{tG}}{K_v}$$

15. Find the stress in the gear teeth

$$\sigma = \frac{W_d}{yFp_n}$$

16. Find C_s



For sand-cast bronze

$$C_s = \begin{cases} 1189.636 - 476.545 \log_{10}(D_G) & D_G > 2.5 \\ 1000 & D_G < 2.5 \end{cases}$$

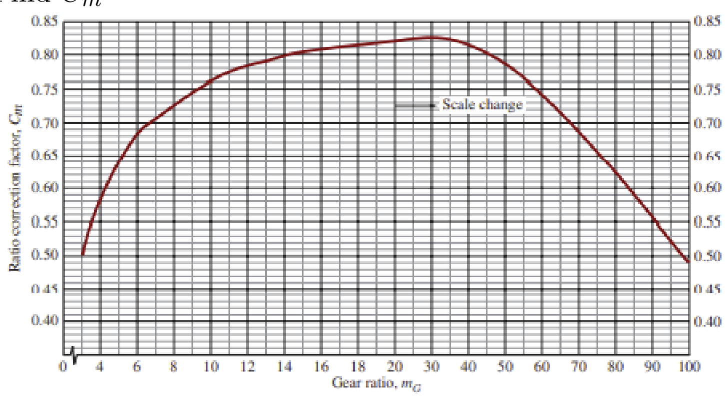
For static-chill-cast or forged bronze

$$C_s = \begin{cases} 1411.651 - 455.825 \log_{10}(D_G) & \\ 1000 & D_G < 8 \end{cases}$$

For centrifugally cast bronze

$$C_s = \begin{cases} 1251.291 - 179.75 \log_{10}(D_G) & \\ 1000 & D_G < 25 \end{cases}$$

17. Find C_m



$$C_m = \begin{cases} 0.02\sqrt{-m_G^2 + 40m_G - 76} + 0.46 & 6 < m_G < 20 \\ 0.0107\sqrt{-m_G^2 + 56m_G + 5146} & 20 < m_G < 76 \\ 1.1483 - 0.00658m_G & m_G > 76 \end{cases}$$

18. Find C_v

$$C_v = \begin{cases} 0.659e^{-0.0011v_s} & 0 < v_s < 700 \\ 13.31v_s^{-0.571} & 700 < v_s < 3000 \\ 65.52v_s^{-0.774} & v_s > 3000 \end{cases}$$

19. Find F_e

$$F_e = \begin{cases} F & F < \frac{D_W}{3} \\ \frac{D_W}{3} & F > \frac{D_W}{3} \end{cases}$$

20. Find the rated tangential load

$$W_{tR} = C_s D_G^{0.8} F_e C_m C_v$$

21. Check if the design is satisfactory to resist pitting:
If $W_{tR} > W_{tG}$ then the design is satisfactory