9.3.3 Stress Calculations

Stress Analysis (This guide Follows Mott)

- Typically design using pinion values as the gear will be the same material
- Always use 4 decimal places for your values

9.3.3.1 Spur Gears **using Mott*

Here is the bending stress formulas for a pinion and a gear.

$$s_{tP} = \frac{W_t P_d}{F J_p} K_o K_s K_m K_B K_v$$

$$s_{tG} = s_{tP} \frac{J_P}{J_G}$$

The contact stress formula is:

$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_B K_v}{F D_p I}}$$

The allowable bending stress formulas are:

$$s_{at,P} > s_{tP} \frac{K_R(SF)}{Z_{NP}}$$

$$s_{at,G} > s_{tG} \frac{K_R(SF)}{Z_{NP}}$$

Follow steps 1 - 24 to calculate the bending stress values and to specify a material and SF for it.

Follow steps 25 - 30 to calculate the contact stress value and to specify a material and SF for it.

Let the fun begin!

1. **Find your shock for the input and output using the following:**

> ***Uniform:*** Electric motor or constant-speed gas turbine
>
> ***Light shock:*** Water turbine, variable-speed drive
>
> ***Moderate shock:*** Multicylinder engine
>
> Examples of the roughness of driven machines include the following:
>
> ***Uniform:*** Continuous-duty generator, paper, and film winders.
>
> ***Light shock:*** Fans and low-speed centrifugal pumps, liquid agitators, variable-duty generators, uniformly loaded conveyors, rotary positive displacement pumps, and metal strip processing.
>
> ***Moderate shock:*** High-speed centrifugal pumps, reciprocating pumps and compressors, heavy-duty conveyors, machine tool drives, concrete mixers, textile machinery, meat grinders, saws, bucket elevators, freight elevators, escalators, concrete mixers, plastics molding and processing, sewage disposal equipment, winches, and cable reels.
>
> ***Heavy shock:*** Rock crushers, punch press drives, pulverizers, processing mills, tumbling barrels, wood chippers, vibrating screens, railroad car dumpers, log conveyors, lumber handling equipment, metal shears, hammer mills, commercial washers, heavy-duty hoists and cranes, reciprocating feeders, dredges, rubber processing, compactors, and plastics extruders.

2. **Find your overload factor (K_o) based on the shocks for the driven machine and the power source using Table 9-1 below**

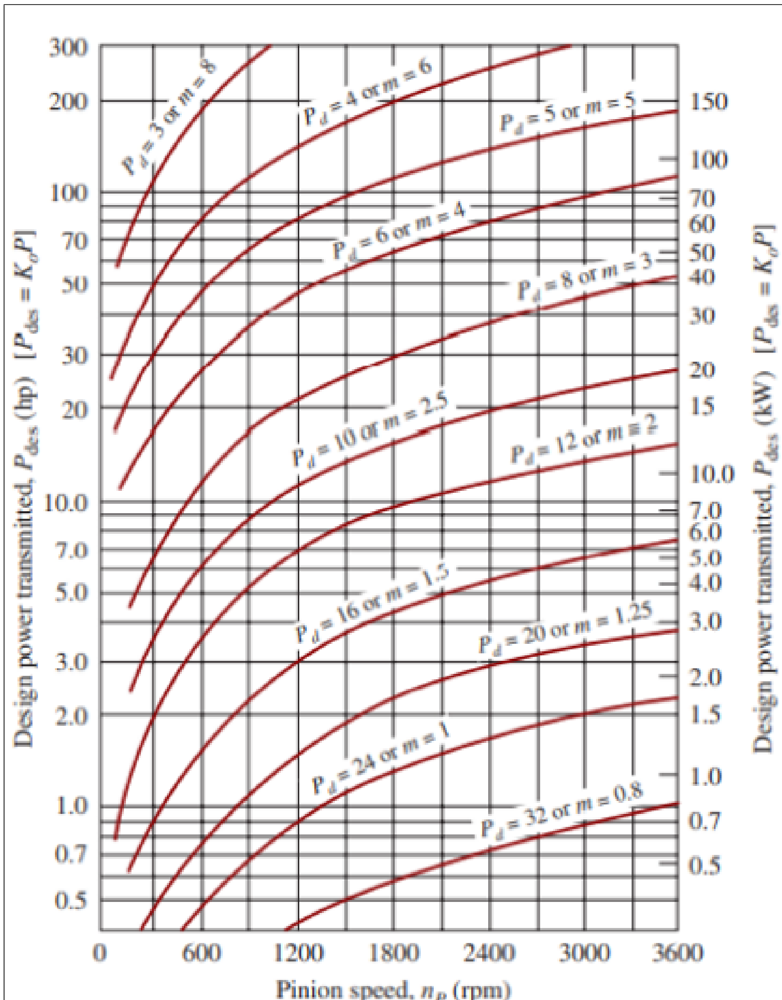
TABLE 9–1 Suggested Overload Factors, K_o

	Driven Machine			
Power source	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

3. **Using your K_o and your input power, calculate your design power (HP) using the following formula:**

$$P_{design} = PK_o$$

4. **Using your design power and the pinion speed, find your diametral pitch (P_d) using the following figure 9-11)**[53]



For all curves: 20° full depth teeth;
$N_P = 24$; $N_G = 96$; $m_G = 4.00$; $F = 12/P_d$; $A_v = 11$
Steel gears, HB 300; $s_{at} = 36000$ psi (250 MPa); $s_{ac} = 126000$ psi (869 MPa)

[53] Round to the smallest number!

5. **Now, choose the number of teeth for the pinion (N_P), using the following range:**

$$17 < N_P < 20$$

6. **Choose the speed of the gear based on the range given in the problem. Typically want to assume a value that is in the middle of the range.**

7. **Calculate the velocity ratio using the following formula:**

$$VR = \frac{n_P}{n_G}$$

Where "n_P" is the rotational speed of the pinion given in the question and "n_G" is the rotational speed of the gear

8. **Calculate the number of teeth in the output gear (N_G) using the following formula:**

$$N_G = N_P(VR)$$

Round to the nearest integer

9. **Now recalculate your velocity ratio using the number of teeth in the pinion and gear:**

$$VR = \frac{N_G}{N_P}$$

10. **Calculate the actual output speed (n_G) using this formula:**

$$n_G = n_P\left(\frac{N_P}{N_G}\right)$$

Make sure this value is within the specified range given in the question, if not, test a new value for N_P.

11. **Find the pitch diameters in inches for the two gears:**

$$D_P = \frac{N_P}{P_d}$$

$$D_G = \frac{N_G}{P_d}$$

12. **Calculate the Centre Distance (CD in inches), pitch line speed, v_t $[ft/min]$, transmitted load W_t $[lbs]$, and radial load W_r (lbs) using the following formulas:**

$$C = \frac{N_P + N_G}{2P_d}$$

$$v_t = \frac{\pi}{12} D_P n_P$$

$$W_t = \frac{33,000P}{v_t}$$

$$W_r = W_t \tan\phi$$

13. **Calculate your face width value, F (inches), given the following ranges:**

    (a) Lower Limit $F = \frac{8}{P_d}$

    (b) Nominal $F = \frac{12}{P_d}$

    (c) Upper Limit $F = \frac{16}{P_d}$

    Just go for the nominal value if the question doesn't mention anything specific

14. **If the quality number A_v is not given you can calculate it using this table:**

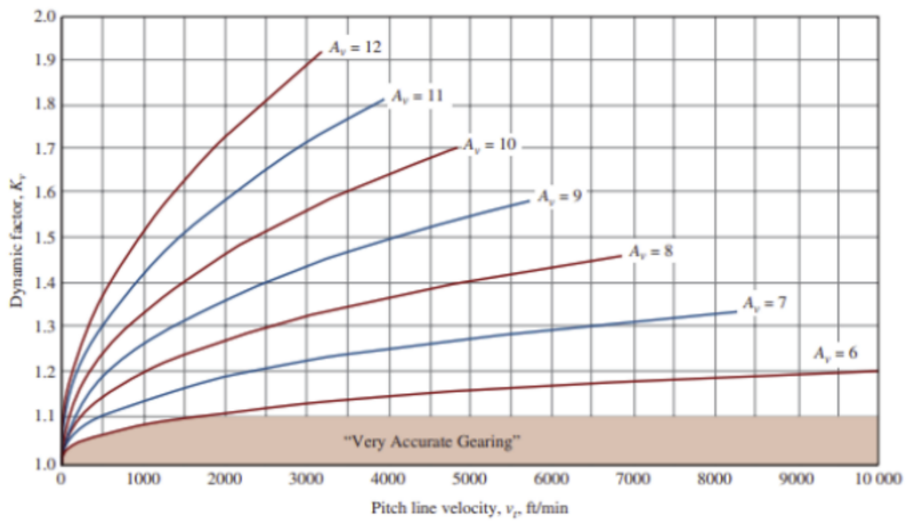
TABLE 9–5 Recommended AGMA Quality Numbers

Application	Quality number	Application	Quality number
Cement mixer drum drive	A11	Small power drill	A9
Cement kiln	A11	Clothes washing machine	A8
Steel mill drives	A11	Printing press	A7
Grain harvester	A10	Computing mechanism	A6
Cranes	A10	Automotive transmission	A6
Punch press	A10	Radar antenna drive	A5
Mining conveyor	A10	Marine propulsion drive	A5
Paper-box-making machine	A9	Aircraft engine drive	A4
Gas meter mechanism	A9	Gyroscope	A2

Machine tool drives and drives for other high-quality mechanical systems

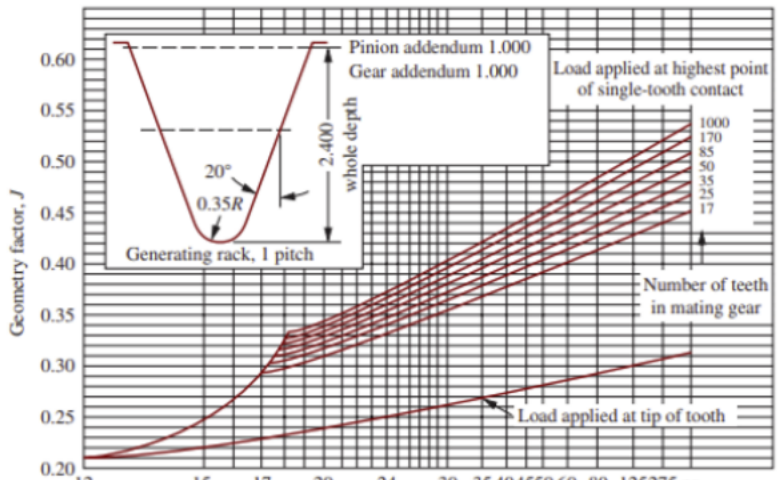
Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0–800	A10	0–4
800–2000	A8	4–11
2000–4000	A6	11–22
Over 4000	A4	Over 22

15. **Find the the dynamic factor K_v in the following figure using A_v and v_t:**

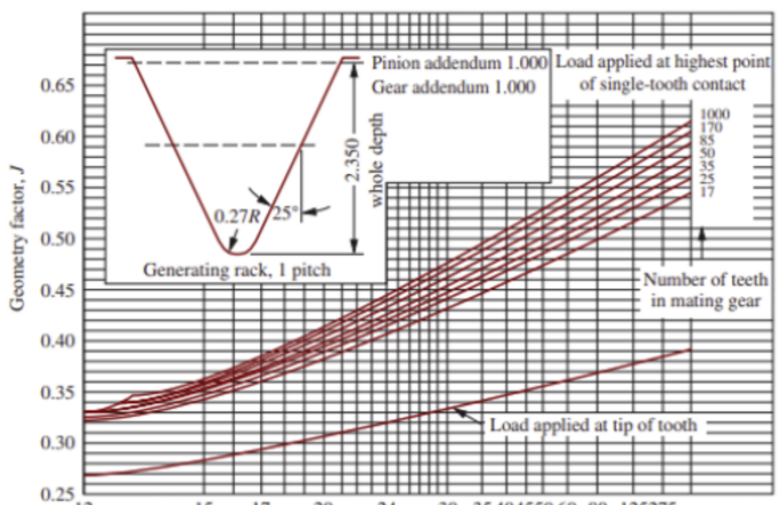


(*a*) U.S. Customary units

16. **Using N_P and N_G, Find the geometry factor "J" using the following graphs. Typically we assume a 20 degree pressure angle spur gear so use that graph and <u>don't dog it too much</u>.**



(a) 20° spur gear: standard addendum



(b) 25° spur gear: standard addendum

17. **Now to find the size factor, Ks, using the following table using your diametral pitch value**[54]

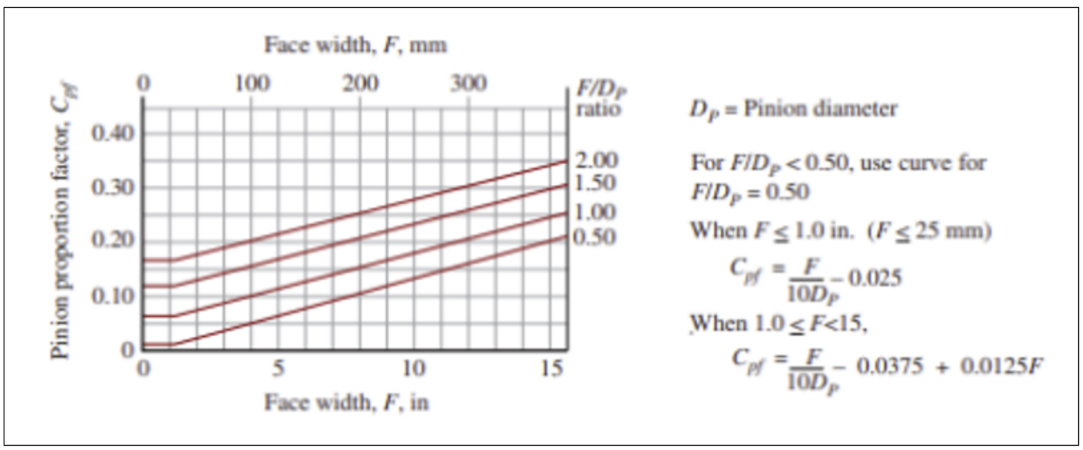
TABLE 9–2 Suggested Size Factors, K_s

Diametral pitch, P_d	Metric module, m	Size factor, K_s
≥ 5	≤ 5	1.00
4	6	1.05
3	8	1.15
2	12	1.25
1.25	20	1.40

18. **In order to find your load-distribution factor, Km, perform the following steps given this formula for K_m.**

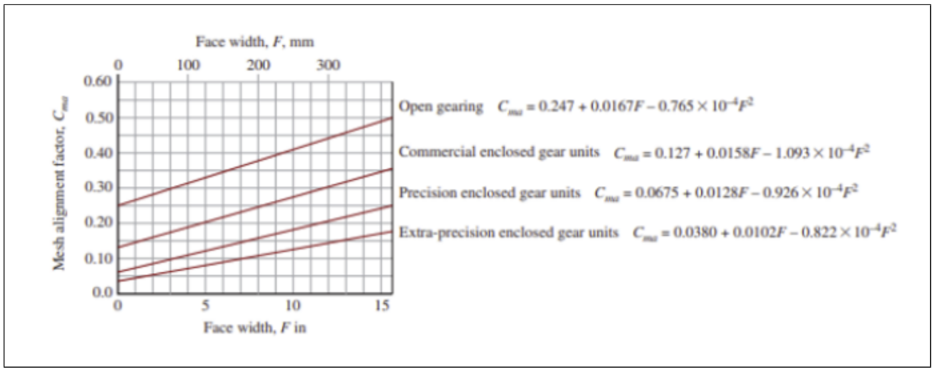
$$K_m = 1 + C_{pf} + C_{ma}$$

Using your face width value and $\frac{F}{D_p}$ ratio, find the pinion proportion factor (C_{pf}) using figure 9-12 below:



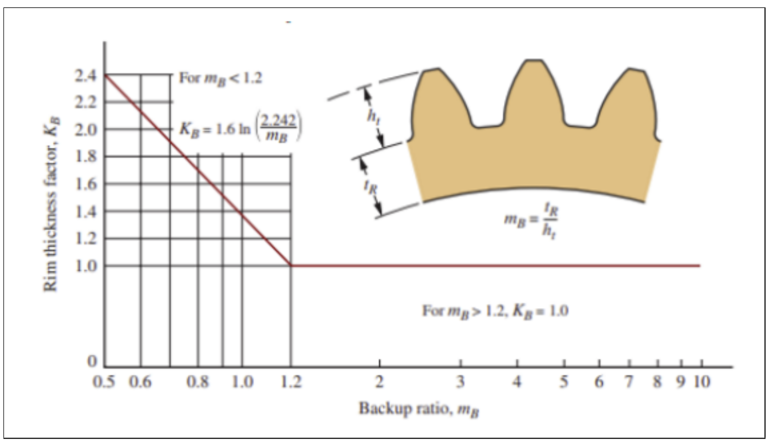
Now we need to find the mesh alignment factor, C_{ma}, using figure 9-13 below. Typically assume that the gears are "Commercial Enclosed Gear Units"

[54] or if you're enough of a dog, you can check the module



Finally find K_m using the formula given in this step.

19. **Find the rim thickness factor, K_b.** Typically this value is 1 for commercially made gears. If this is not the case then use the following figure 9-14 below:



20. **We now have all the factors needed to calculate the bending stress**. Plug everything back into the formula we initially introduced:

$$s_{t,P} = \frac{W_t P_d}{F J_p} K_o K_s K_m K_B K_v$$

$$s_{t,G} = s_{t,P} \frac{J_P}{J_G}$$

21. **You can also calculate the allowable bending stress using the following formula:**

$$s_{at\ P} > s_{tP}\frac{K_R(SF)}{Y_{NP}}$$
$$s_{at\ G} > s_{tG}\frac{K_R(SF)}{Y_{NG}}$$

(a) In order to find the Reliability factor, K_R, use this table:

TABLE 9–11 Reliability Factor, K_R

Reliability	K_R
0.90, one failure in 10	0.85
0.99, one failure in 100	1.00
0.999, one failure in 1000	1.25
0.9999, one failure in 10 000	1.50

Typically, Jon tell us to assume 99% Reliability, so use $K_R = 1$

(b) For the safety factor (SF), assume 1 for now. General range is between 1 and 1.5

(c) To find the stress cycle factor Y_N, we need to first calculate the number of loading cycles, N_c using this formula:

$$N_{cP} = 60 \times \text{lifetime} \times n_P$$
$$N_{cP} = 60 \times \text{lifetime} \times n_G$$

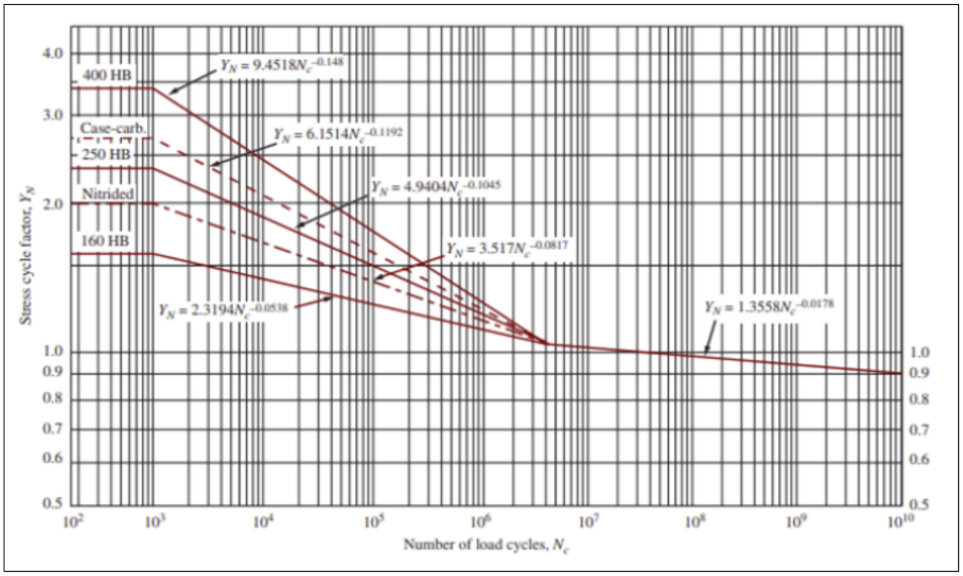
(d) The lifetime could be specified in the question in hours, or can be found using the following table:

TABLE 9–12 Recommended Design Life

Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000

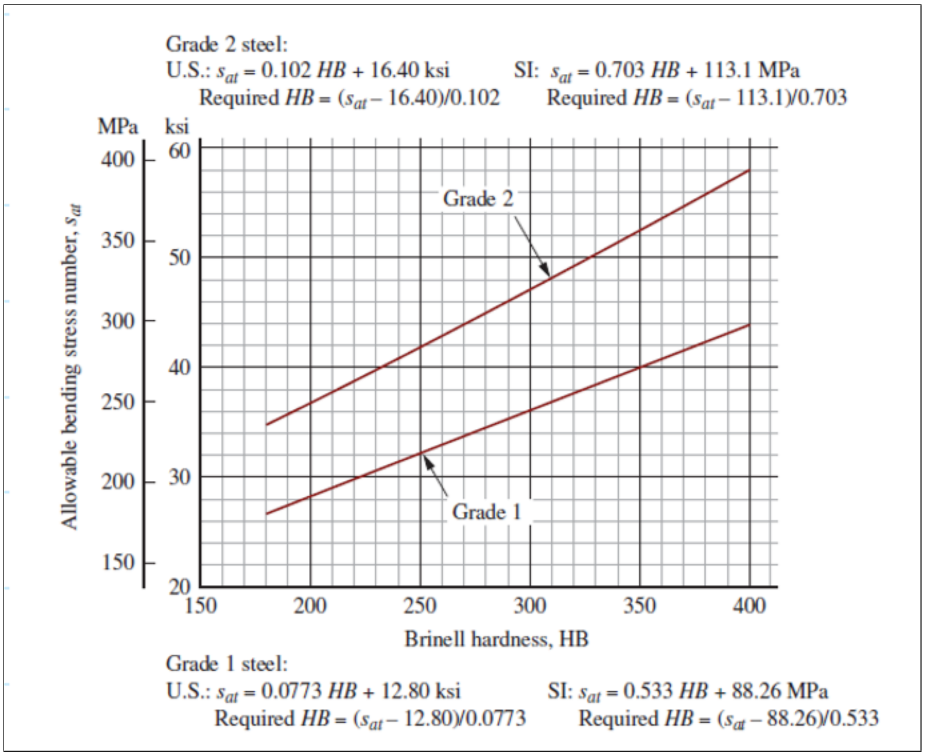
Source: Eugene A. Avallone and Theodore Baumeister III, eds. *Marks' Standard Handbook for Mechanical Engineers.* 9th ed. New York: McGraw-Hill, 1986.

(e) Now use N_c to calculate Y_N using figure 9-21:



22. **Now you can plug everything in to get your allowable bending stress, S_{at}.**

23. **We now have to choose a material. Use figure 9-18 below to find the required Brinell Hardness (HB) for the material.**



We typically use grade 1 steel values. Use ksi for every calculation here.

Now that we have the required hardness, look at appendix 3 or 5 in the textbook and choose any material that has a hardness above the required one. We normally use Appendix 3.

APPENDIX 3 Design Properties of Carbon and Alloy Steels

Material designation (SAE number)	Condition	Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 in)	Brinell hardness (HB)
1020	Hot-rolled	55	379	30	207	25	111
1020	Cold-drawn	61	420	51	352	15	122
1020	Annealed	60	414	43	296	38	121
1040[1]	Hot-rolled	72	496	42	290	18	144
1040	Cold-drawn	80	552	71	490	12	160
1040	OQT 1300	88	607	61	421	33	183
1040	OQT 400	113	779	87	600	19	262
1050	Hot-rolled	90	620	49	338	15	180
1050	Cold-drawn	100	690	84	579	10	200
1050	OQT 1300	96	662	61	421	30	192
1050	OQT 400	143	986	110	758	10	321
1117	Hot-rolled	65	448	40	276	33	124
1117	Cold-drawn	80	552	65	448	20	138
1117	WQT 350	89	614	50	345	22	178
1137	Hot-rolled	88	607	48	331	15	176
1137	Cold-drawn	98	676	82	565	10	196
1137	OQT 1300	87	600	60	414	28	174
1137	OQT 400	157	1083	136	938	5	352
1144[1]	Hot-rolled	94	648	51	352	15	188
1144	Cold-drawn	100	690	90	621	10	200
1144	OQT 1300	96	662	68	469	25	200
1144	OQT 400	127	876	91	627	16	277
1213	Hot-rolled	55	379	33	228	25	110
1213	Cold-drawn	75	517	58	340	10	150
12L13	Hot-rolled	57	393	34	234	22	114
12L13	Cold-drawn	70	483	60	414	10	140
1340[1]	Annealed	102	703	63	434	26	207
1340	OQT 1300	100	690	75	517	25	235
1340	OQT 1000	144	993	132	910	17	363
1340	OQT 700	221	1520	197	1360	10	444
1340	OQT 400	285	1960	234	1610	8	578
3140	Annealed	95	655	67	462	25	187
3140	OQT 1300	115	792	94	648	23	233
3140	OQT 1000	152	1050	133	920	17	311
3140	OQT 700	220	1520	200	1380	13	461
3140	OQT 400	280	1930	248	1710	11	555
4130	Annealed	81	558	52	359	28	156
4130	WQT 1300	98	676	89	614	28	202
4130	WQT 1000	143	986	132	910	16	302
4130	WQT 700	208	1430	180	1240	13	415
4130	WQT 400	234	1610	197	1360	12	461
4140[1]	Annealed	95	655	54	372	26	197
4140	OQT 1300	117	807	100	690	23	235
4140	OQT 1000	168	1160	152	1050	17	341
4140	OQT 700	231	1590	212	1460	13	461
4140	OQT 400	290	2000	251	1730	11	578
4150	Annealed	106	731	55	379	20	197
4150	OQT 1300	127	880	116	800	20	262
4150	OQT 1000	197	1360	181	1250	11	401
4150	OQT 700	247	1700	229	1580	10	495
4150	OQT 400	300	2070	248	1710	10	578

(Continued)

APPENDIX 3 (*Continued*)

Material designation (SAE number)	Condition	Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 in)	Brinell hardness (HB)
4340[1]	Annealed	108	745	68	469	22	217
4340	OQT 1300	140	965	120	827	23	280
4340	OQT 1000	171	1180	158	1090	16	363
4340	OQT 700	230	1590	206	1420	12	461
4340	OQT 400	283	1950	228	1570	11	555
5140	Annealed	83	572	42	290	29	167
5140	OQT 1300	104	717	83	572	27	207
5140	OQT 1000	145	1000	130	896	18	302
5140	OQT 700	220	1520	200	1380	11	429
5140	OQT 400	276	1900	226	1560	7	534
5150	Annealed	98	676	52	359	22	197
5150	OQT 1300	116	800	102	700	22	241
5150	OQT 1000	160	1100	149	1030	15	321
5150	OQT 700	240	1650	220	1520	10	461
5150	OQT 400	312	2150	250	1720	8	601
5160	Annealed	105	724	40	276	17	197
5160	OQT 1300	115	793	100	690	23	229
5160	OQT 1000	170	1170	151	1040	14	341
5160	OQT 700	263	1810	237	1630	9	514
5160	OQT 400	322	2220	260	1790	4	627
6150[1]	Annealed	96	662	59	407	23	197
6150	OQT 1300	118	814	107	738	21	241
6150	OQT 1000	183	1260	173	1190	12	375
6150	OQT 700	247	1700	223	1540	10	495
6150	OQT 400	315	2170	270	1860	7	601
8650	Annealed	104	717	56	386	22	212
8650	OQT 1300	122	841	113	779	21	255
8650	OQT 1000	176	1210	155	1070	14	363
8650	OQT 700	240	1650	222	1530	12	495
8650	OQT 400	282	1940	250	1720	11	555
8740	Annealed	100	690	60	414	22	201
8740	OQT 1300	119	820	100	690	25	241
8740	OQT 1000	175	1210	167	1150	15	363
8740	OQT 700	228	1570	212	1460	12	461
8740	OQT 400	290	2000	240	1650	10	578
9255	Annealed	113	780	71	490	22	229
9255	Q&T 1300	130	896	102	703	21	262
9255	Q&T 1000	181	1250	160	1100	14	352
9255	Q&T 700	260	1790	240	1650	5	534
9255	Q&T 400	310	2140	287	1980	2	601

Notes: Properties common to all carbon and alloy steels:
Poisson's ratio: 0.27.
Shear modulus: 11.5×10^6 psi; 80 GPa.
Coefficient of thermal expansion: $6.5 \times 10^{-6}\,°F^{-1}$.
Density: 0.283 lb/in^3; 7680 kg/m^3.
Modulus of elasticity: 30×10^6 psi; 207 GPa.
[1]See Appendix 4 for graphs of properties versus heat treatment.

APPENDIX 5 Properties of Carburized Steels

Material designation (SAE number)	Condition	Core properties: Tensile strength (ksi)	Tensile strength (MPa)	Yield strength (ksi)	Yield strength (MPa)	Ductility (percent elongation in 2 in)	Brinell hardness (HB)	Case hardness (HRC)
1015	SWQT 350	106	731	60	414	15	217	62
1020	SWQT 350	129	889	72	496	11	255	62
1022	SWQT 350	135	931	75	517	14	262	62
1117	SWQT 350	125	862	66	455	10	235	65
1118	SWQT 350	144	993	90	621	13	285	61
4118	SOQT 300	143	986	93	641	17	293	62
4118	DOQT 300	126	869	63	434	21	241	62
4118	SOQT 450	138	952	89	614	17	277	56
4118	DOQT 450	120	827	63	434	22	229	56
4320	SOQT 300	218	1500	178	1230	13	429	62
4320	DOQT 300	151	1040	97	669	19	302	62
4320	SOQT 450	211	1450	173	1190	12	415	59
4320	DOQT 450	145	1000	94	648	21	293	59
4620	SOQT 300	119	820	83	572	19	277	62
4620	DOQT 300	122	841	77	531	22	248	62
4620	SOQT 450	115	793	80	552	20	248	59
4620	DOQT 450	115	793	77	531	22	235	59
4820	SOQT 300	207	1430	167	1150	13	415	61
4820	DOQT 300	204	1405	165	1140	13	415	60
4820	SOQT 450	205	1410	184	1270	13	415	57
4820	DOQT 450	196	1350	171	1180	13	401	56
8620	SOQT 300	188	1300	149	1030	11	388	64
8620	DOQT 300	133	917	83	572	20	269	64
8620	SOQT 450	167	1150	120	827	14	341	61
8620	DOQT 450	130	896	77	531	22	262	61
E9310	SOQT 300	173	1190	135	931	15	363	62
E9310	DOQT 300	174	1200	139	958	15	363	60
E9310	SOQT 450	168	1160	137	945	15	341	59
E9310	DOQT 450	169	1170	138	952	15	352	58

Notes: Properties given are for a single set of tests on 1/2-in round bars.
SWQT: single water-quenched and tempered.
SOQT: single oil-quenched and tempered.
DOQT: double oil-quenched and tempered.
300 and 450 are the tempering temperatures in °F. Steel was carburized for 8 h. Case depth ranged from 0.045 to 0.075 in.

Anything that has a hardness over 400 should use flame or induction hardening techniques over through hardening, since they provide the strength required for high contact stress because their inside is still ductile to prevent failure.

TABLE 9–8 Examples of Gear Materials

Heat treatment	Typical alloys (SAE numbers)
Through-hardened or Case-hardened by flame or induction hardening	1045, 4140, 4150, 4340, 4350
Carburizing, case hardened	1020, 4118, 4320, 4820, 8620, 9310

24. **We now calculate our actual safety factor**. Use the formula in figure 9-18 to calculate the allowable bending stress value using the hardness for the material you chose. This formula is also what you will use to calculate your Safety factor in the end:

$$SF = \frac{s_{at} Y_n}{s_t K_r}$$

25. **Now we move to calculating the contact stress** using the following formula:

$$s_c = C_p \sqrt{\frac{W_t K_o K_s K_m K_B K_v}{F D_p I}}$$

We have most of these constants from the bending stress calculations. Key differences are C_p and I, which will be shown below

26. **For the elastic coefficient**, C_p, we generally assume a material of steel which has a C_p of 2300. There are other materials shown here as well:

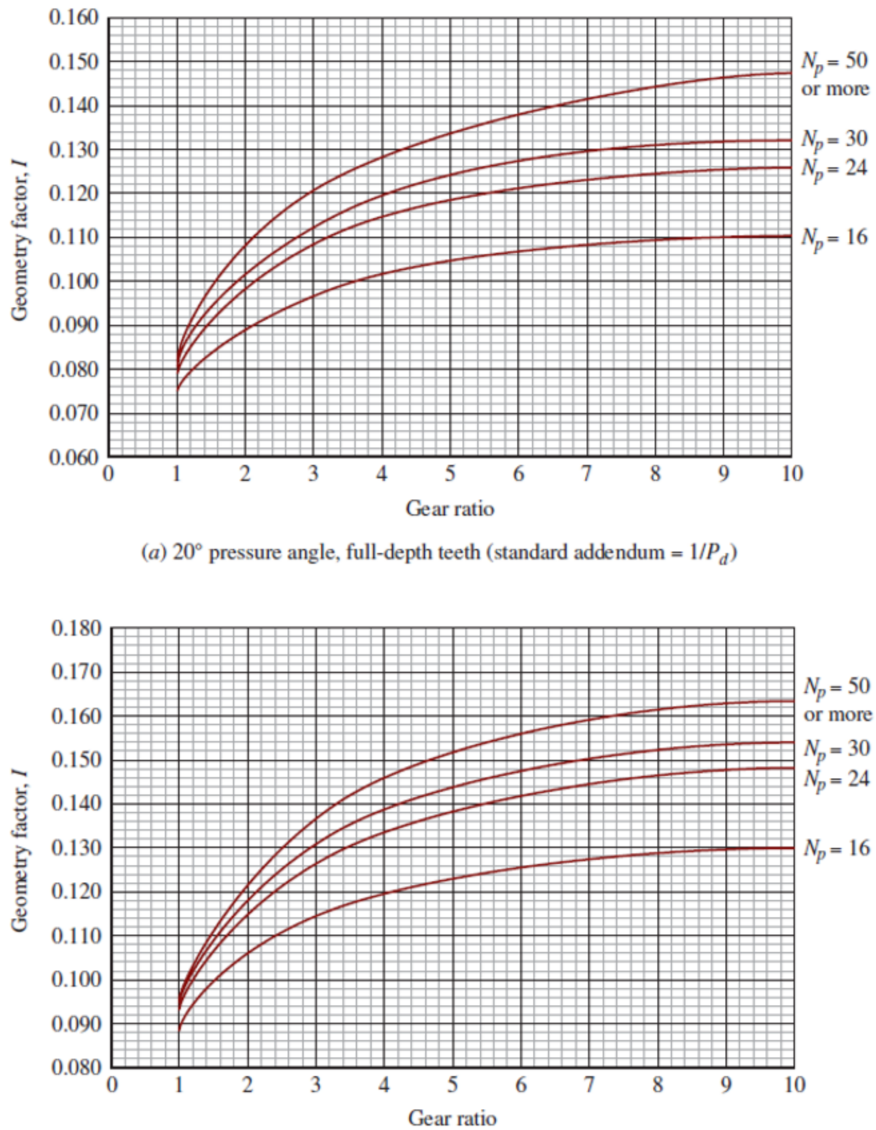
TABLE 9–7 Elastic Coefficient, C_p

		Gear material and modulus of elasticity, E_G, lb/in² (MPa)					
Pinion material	Modulus of elasticity, E_P, lb/in² (MPa)	Steel 30×10^6 (2×10^5)	Malleable iron 25×10^6 (1.7×10^5)	Nodular iron 24×10^6 (1.7×10^5)	Cast iron 22×10^6 (1.5×10^5)	Aluminum bronze 17.5×10^6 (1.2×10^5)	Tin bronze 16×10^6 (1.1×10^5)
Steel	30×10^6	2300	2180	2160	2100	1950	1900
	(2×10^5)	(191)	(181)	(179)	(174)	(162)	(158)
Mall. iron	25×10^6	2180	2090	2070	2020	1900	1850
	(1.7×10^5)	(181)	(174)	(172)	(168)	(158)	(154)
Nod. iron	24×10^6	2160	2070	2050	2000	1880	1830
	(1.7×10^5)	(179)	(172)	(170)	(166)	(156)	(152)
Cast iron	22×10^6	2100	2020	2000	1960	1850	1800
	(1.5×10^5)	(174)	(168)	(166)	(163)	(154)	(149)
Al. bronze	1.75×10^6	1950	1900	1880	1850	1750	1700
	(1.2×10^5)	(162)	(158)	(156)	(154)	(145)	(141)
Tin bronze	16×10^6	1900	1850	1830	1800	1700	1650
	(1.1×10^5)	(158)	(154)	(152)	(149)	(141)	(137)

Source: Extracted from AGMA Standard 2001-D04, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th floor, Alexandria, VA 22314.

Note: Poisson's ratio = 0.30; units for C_p are $(\text{lb/in}^2)^{0.5}$ or $(\text{MPa})^{0.5}$.

27. **Using the actual gear ratio and Np, Find the pitting geometry factor, I**, using the following graphs. Again, assume a 20 degree pressure angle, so use that graph. Figure 9-17



(*a*) 20° pressure angle, full-depth teeth (standard addendum = $1/P_d$)

(*b*) 25° pressure angle, full-depth teeth (standard addendum = $1/P_d$)

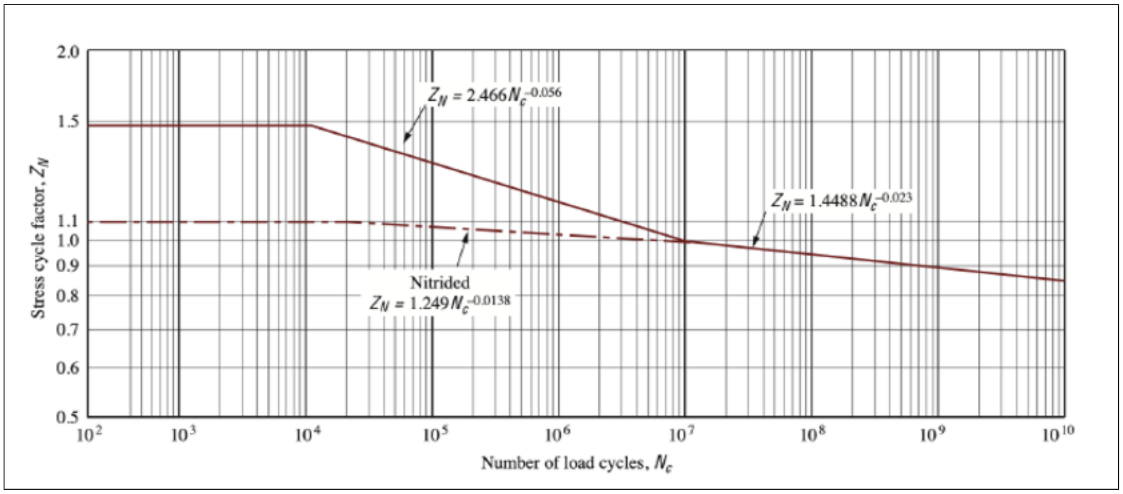
28. **We can now plug everything in to find the contact stress,** s_c

29. **In order to find the allowable contact stress,** s_{ac}**, we use this formula:**

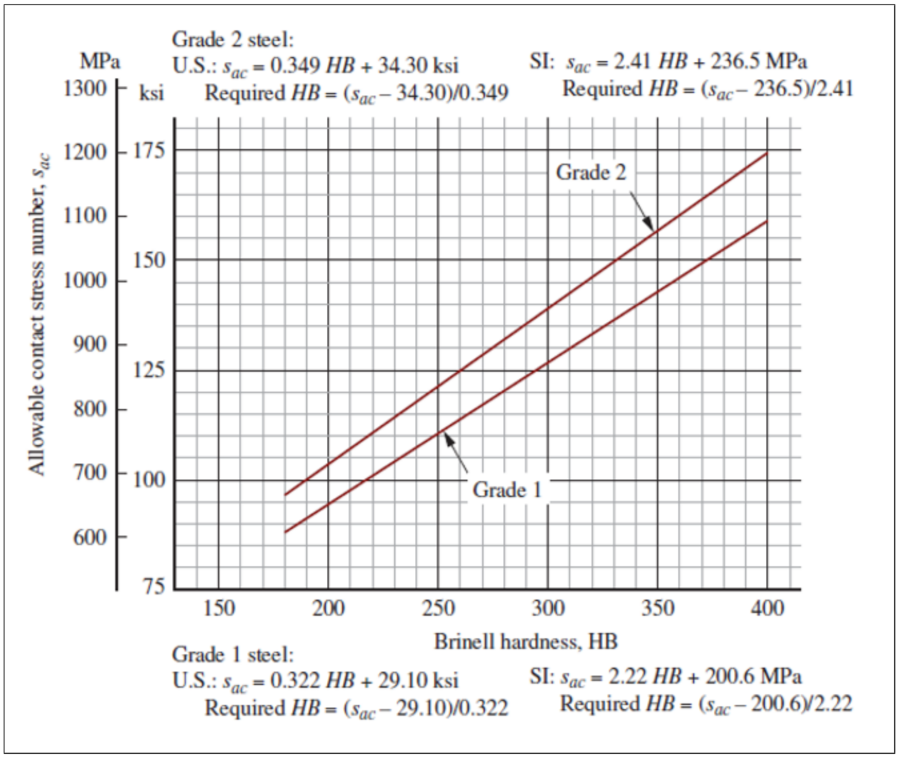
$$s_{ac,P} > s_{cP}\frac{K_R(SF)}{Z_{NP}}$$

$$s_{ac,G} > s_{cG}\frac{K_R(SF)}{Z_{NG}}$$

We already assumed a K_R of 1 and a safety factor of 1 in step 21. Now we need to find Z_N which is the stress cycle factor. We use the following figure 9-22:



30. **We follow a similar procedure to specify the material and to find the safety factor using the contact stresses**. Although the formulas change slightly, use figure 9-19 below



Follow the same procedure for bending stress material specification and safety factor calculations. This is what you will use to calculate your Safety factor in the end:

$$SF = \frac{s_{ac}Z_n}{s_cK_r}$$

9.3.3.2 Helical Gears *using Mott

The process for calculating helical gear stress is very similar to spur gears, with the key differences being that J and I change!. Here's how it goes:

1. **Find your shock for the input and output using the following**:
2. **Find your overload factor (K_o) based on the shocks for the driven machine and the power source using Table 9-1 below**
3. **Using your K_o and your input power, calculate your design power (HP) using the following formula**:

$$P_{des} = PK_o$$