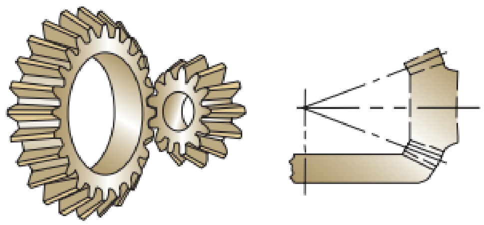
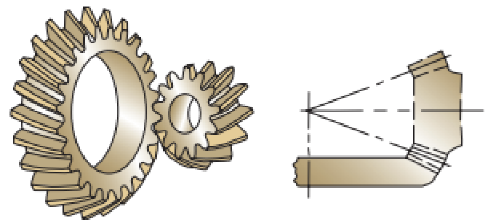
2.3 Bevel Gears

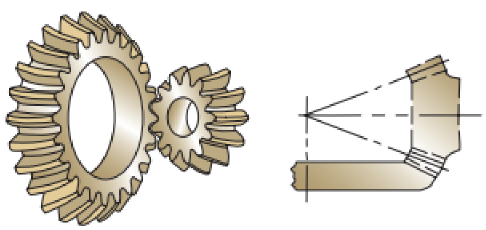
2.3.1 Anatomy



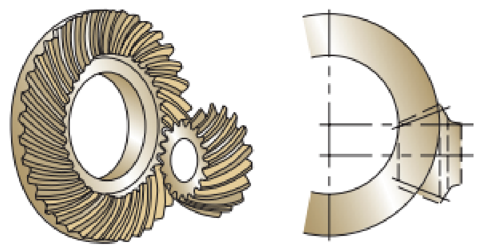
(a) Straight bevel



(b) Spiral bevel

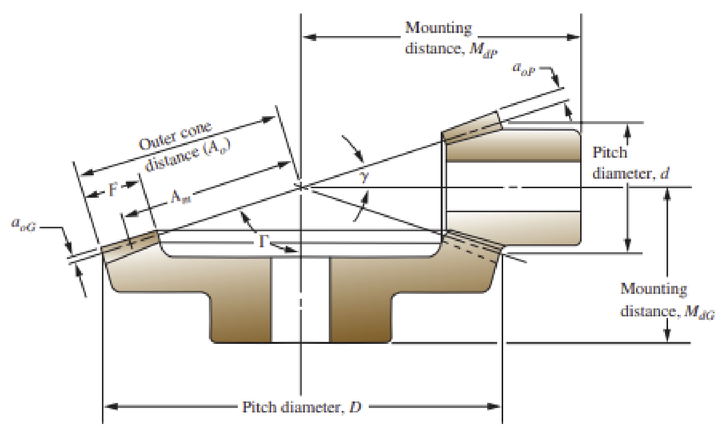


(c) Zero spiral bevel



(d) Hypoid

(e) Straight bevel gear pair



2.3.2 Formulae

N_P = number of teeth on pinion (driving)
N_G = number of teeth on gear (driven)
P_d = diametral pitch (teeth/in)
d = diameter of pinion (in)
D = diameter of gear (in)
γ = cone angle of pinion
Γ = cone angle of gear
ϕ = pressure angle
A_o = outer cone distance (in)

F = face width (in)
A_m = mean cone distance (in)
p_m = mean circular path (in)
h = mean working depth (in)
c = clearance (in)
h_m = mean whole depth (in)
c_1 = mean addendum factor
a_G = gear mean addendum (in)
a_P = pinion mean addendum (in)
b_G = gear mean dedendum (in)
b_P = pinion mean dedendum (in)
δ_G = gear dedendum angle
δ_P = pinion dedendum angle
a_oG = gear outer addendum (in)
a_oP = pinion outer addendum (in)
D_o = gear outside diameter (in)
d_o = pinion outside diameter (in)
W_t = transmitted load (lbf)
W_x = axial load (lbf)
W_r = radial load (lbf)

2.3.3 Formulae

Geometry:

$$\begin{aligned}
\text{diametral pitch: } & P_d = \frac{N_P}{d} = \frac{N_G}{D} \\
\text{gear ratio: } & m_G = \frac{N_G}{N_P} \\
\text{pinion cone angle: } & \tan\gamma = \frac{N_P}{N_G} \\
\text{gear cone angle: } & \tan\Gamma = \frac{N_G}{N_P} \\
\text{outer cone distance: } & A_{oG} = \frac{D}{2\sin\Gamma},\ A_{oP} = \frac{d}{2\sin\gamma} \\
\text{nominal face width: } & F_{\text{nom}} = 0.3A_o \\
\text{max face width: } & F_{\max} = \min\left\{\frac{A_o}{3}, \frac{10}{P_d}\right\} \\
\text{mean cone distance: } & A_m = A_o - 0.5F \\
\text{mean circular pitch: } & p_m = \frac{\pi A_m}{P_d A_o} \\
\text{mean working depth: } & h = \frac{2A_m}{P_d A_o} \\
\text{clearance: } & c = 0.125h
\end{aligned}$$

$$\text{mean whole depth: } h_m = h + c$$
$$\text{mean addendum factor: } c_1 = 0.21 + \frac{0.29}{m_G^2}$$
$$\text{gear mean addendum: } a_G = c_1 h$$
$$\text{pinion mean addendum: } a_P = h - a_G$$
$$\text{gear mean dedendum: } b_G = h_m - a_G$$
$$\text{pinion mean dedendum: } b_P = h_m - a_P$$
$$\text{gear dedendum angle: } \tan \delta_G = \frac{b_G}{A_{mG}}$$
$$\text{pinion dedendum angle: } \tan \delta_P = \frac{b_P}{A_{mP}}$$
$$\text{gear outer addendum: } a_{oG} = a_G + 0.5F \tan \delta_P$$
$$\text{pinion outer addendum: } a_{oP} = a_P + 0.5F \tan \delta_G$$
$$\text{gear outside diameter: } D_o = D + 2a_{oG} \cos \Gamma$$
$$\text{pinion outside diameter: } d_o = d + 2a_{oP} \cos \gamma$$

Forces and motion:

$$\text{pitch line speed: } v_t = \frac{\pi D n_G}{12} = \frac{\pi d n_P}{12}$$
$$\text{torque: } T = \frac{63000P}{n}$$
$$\text{mean radius: } r_m = \frac{d}{2} - \frac{F}{2} \sin \gamma$$
$$R_m = \frac{D}{2} - \frac{F}{2} \sin \Gamma$$
$$\text{transmitted load: } W_t = \frac{T_P}{r_m} = \frac{T_G}{R_m}$$
$$\text{radial load: } W_r = W_t \tan \phi \cos \Gamma = W_t \tan \phi \cos \gamma$$
$$\text{axial load: } W_x = W_t \tan \phi \sin \Gamma = W_t \tan \phi \sin \gamma$$
$$\text{also transmitted load: } W_t = \frac{126000P}{Dn} = \frac{33000P}{v_t}$$

You may notice that the formulas for transmitted load are different. I don't know why. Use the top one for force analysis and the bottom one for design selection.

2.3.4 Force Analysis

1. Find the transmitted, axial, and radial loads, W_t, W_r, W_x.

2. Draw a free body diagram of the forces acting on the gear. Include $\vec{W}$, the two forces on the bearing and the torque on the shaft (these directions will all be arbitrary except for $\vec{W}$). The force $\vec{W}$ will act at a distance of R_m away from the axle of the gear.
   W_t will point in the direction of the motion of the gear at that point
   W_r will point toward the shaft

W_a will point in the direction of the angular velocity vector of the gear (using the right hand rule with the rotation of the gear)

3. Find the sum of moments about one of the bearings and the sum of forces to solve for the unknowns.
   Recall

   $$\vec{M} = \vec{r} \times \vec{F}$$

   Note that the axial force of the bearings will only take place on one bearing. Choose which bearing to select for either compressive or tensile force.

2.3.5 Design Selection

1. Find the shock and get K_O from the spur gear guide.

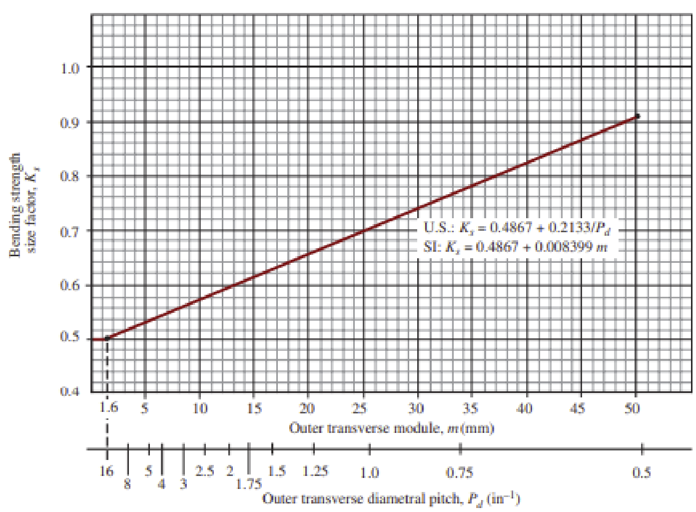
2. compute or guess any of the basic mechanical values missing, such as N or F, using the spur gear guide as reference.

3. Compute v_t and W_t

   $$v_t = \frac{\pi D n_G}{12}$$
   $$W_t = \frac{33000P}{v_t}$$

4. Find the size factor K_s from this equation or the table

   $$K_s = \begin{cases} 0.5 & P_d \geq 16 \\ 0.4867 + \frac{0.2133}{P_d} & P_d < 16 \end{cases}$$



5. Get K_{mb} where

   - $K_{mb} = 1$ for both gears straddle mounted

- $K_{mb} = 1.1$ for one gear straddle mounted
- $K_{mb} = 1.25$ for neither gear straddle mounted

6. Compute $K_m = K_{mb} + 0.0036F^2$
7. Find the quality number A_v from the application or the pitch line speed. This table is shit so just guess what looks right.

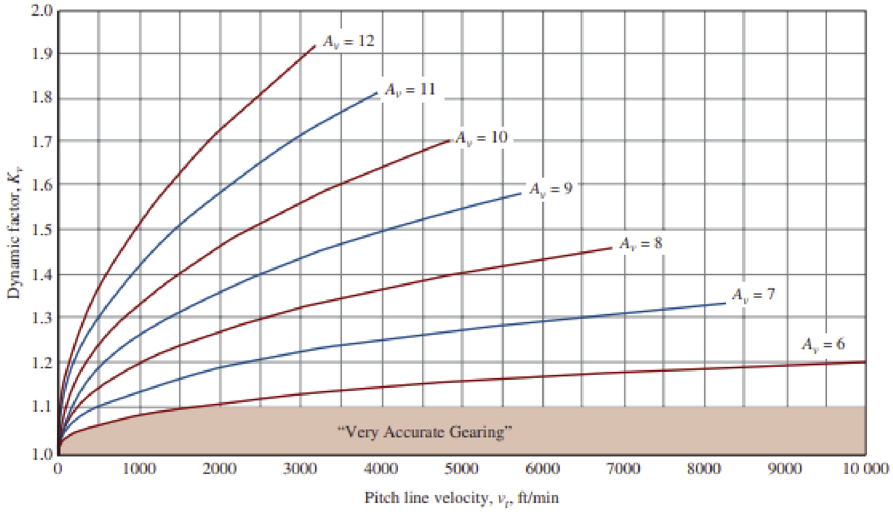
TABLE 9–5 Recommended AGMA Quality Numbers

Application	Quality number	Application	Quality number
Cement mixer drum drive	A11	Small power drill	A9
Cement kiln	A11	Clothes washing machine	A8
Steel mill drives	A11	Printing press	A7
Grain harvester	A10	Computing mechanism	A6
Cranes	A10	Automotive transmission	A6
Punch press	A10	Radar antenna drive	A5
Mining conveyor	A10	Marine propulsion drive	A5
Paper-box-making machine	A9	Aircraft engine drive	A4
Gas meter mechanism	A9	Gyroscope	A2

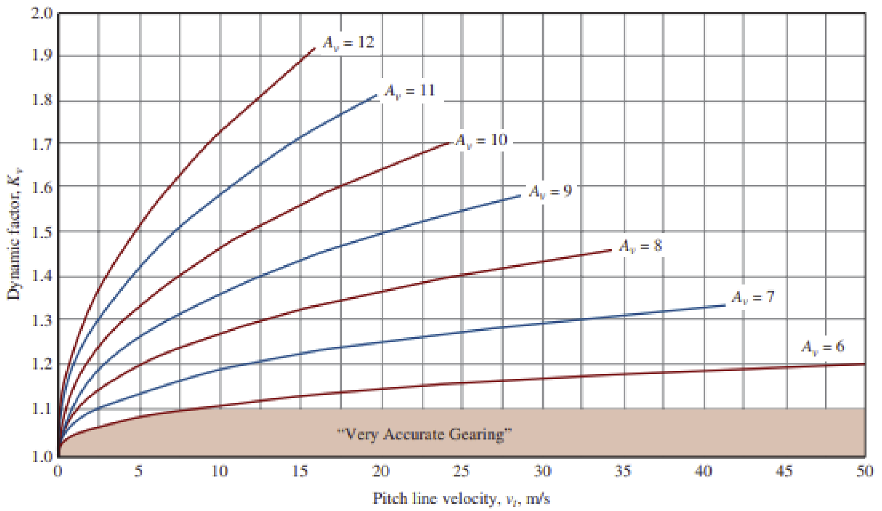
Machine tool drives and drives for other high-quality mechanical systems

Pitch line speed (fpm)	Quality number	Pitch line speed (m/s)
0–800	A10	0–4
800–2000	A8	4–11
2000–4000	A6	11–22
Over 4000	A4	Over 22

8. Get K_v from here

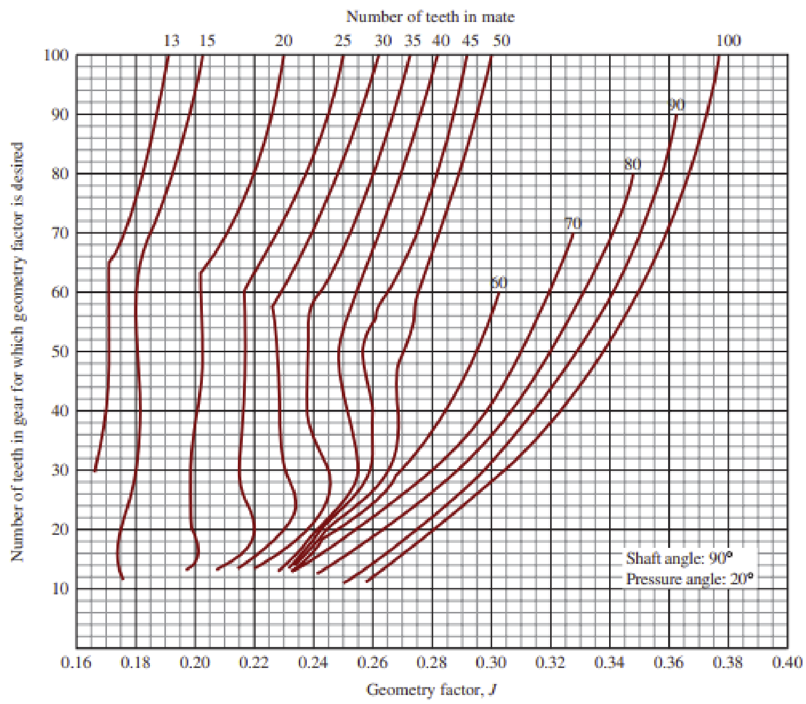


(*a*) U.S. Customary units



(*b*) SI Metric units

9. Get J from the mutant octopus graph you see below



10. Calculate the bending stress number

$$s_t = \frac{W_t P_d K_O K_s K_m K_v}{FJ}$$

11. Specify a service factor, SF between 1.00 and 1.50. Usually pick 1.00 but if your data is uncertain then ramp that shit up.

12. Gander a guess at how reliable your system will be. Let's assume for most cases that you're not that shit of an Engineer and it works 99% of the time.

13. Use your rigorously calculated reliability to get K_R and C_R from this table

TABLE 10–3 Reliability Factors for Allowable Bending and Contact Stresses

Reliability R	Interpretation	Reliability factors: Bending K_R	Reliability factors: Contact C_R
0.9	Fewer than one failure in 10	0.85	0.92
0.99	Fewer than one failure in 100	1.00	1.00
0.999	Fewer than one failure in 1000	1.25	1.12
0.9999	Fewer than one failure in 10 000	1.50	1.22

Source: Adapted from AGMA 2003-C10, *Rating the Pitting Resistance and Bending Strength of Generated Straight Bevel, Zerol Bevel and Spiral Bevel Gear Teeth,* with the permission of the publisher, American Gear Manufacturers Association, 1001 North Fairfax Street, 5th Floor, Alexandria, VA.

14. Guess what the lifetime of your machine will be. Don't worry, there's a shitty table to help you.

TABLE 9–12 Recommended Design Life

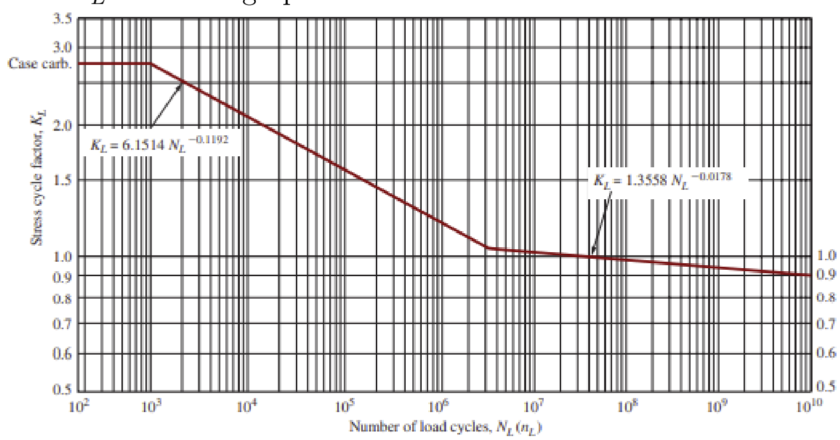
Application	Design life (h)
Domestic appliances	1000–2000
Aircraft engines	1000–4000
Automotive	1500–5000
Agricultural equipment	3000–6000
Elevators, industrial fans, multipurpose gearing	8000–15 000
Electric motors, industrial blowers, general industrial machines	20 000–30 000
Pumps and compressors	40 000–60 000
Critical equipment in continuous 24-h operation	100 000–200 000

Source: Eugene A. Avallone and Theodore Baumeister III, eds. *Marks' Standard Handbook for Mechanical Engineers.* 9th ed. New York: McGraw-Hill, 1986.

15. Find the number of loading cycles using these formulas

$$N_{LP} = (60)(\text{lifetime})n_P$$
$$N_{LG} = (60)(\text{lifetime})n_G$$

16. Find K_L from this graph



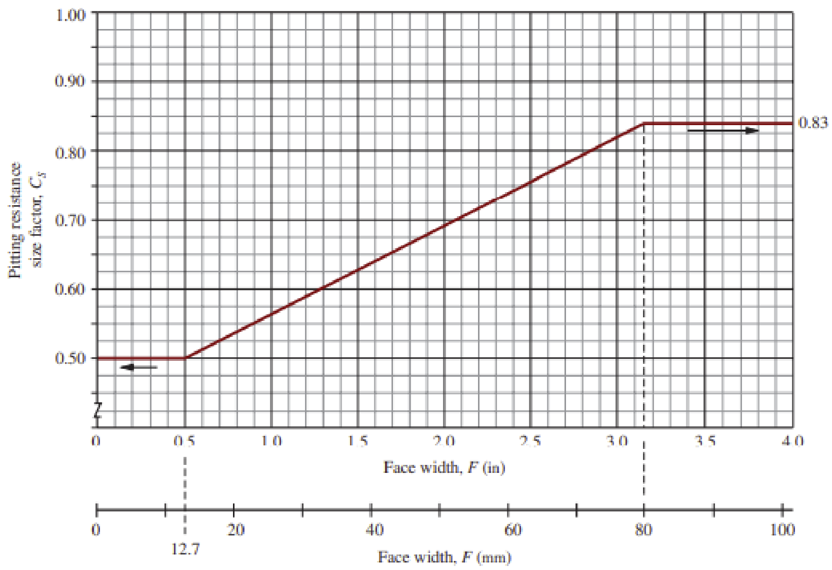
17. Take the safety factor SF to be anywhere between 1 and 1.5. We always just assume $SF = 1$ because fuck safety.

18. find the max allowable bending strength

$$s_{at} = \frac{s_t(SF)K_R}{K_L}$$

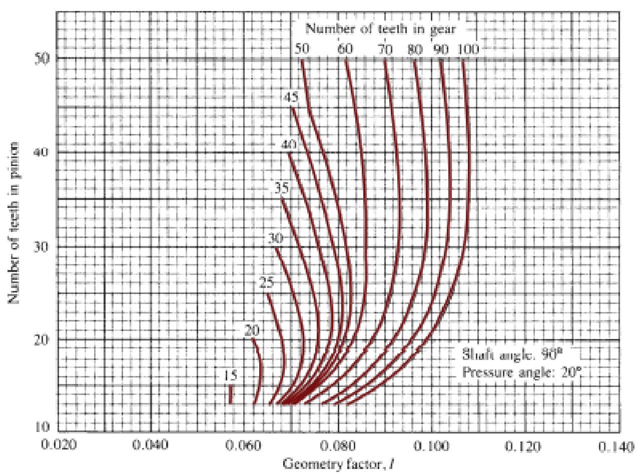
19. Take $C_p = 2300$ for steel

20. Compute $C_s = 0.125F + 0.4375$



21. Get $C_{xc} = 1.5$ for properly crowned teeth (what we usually assume)
    $C_{xc} = 2$ for non-crowned teeth.

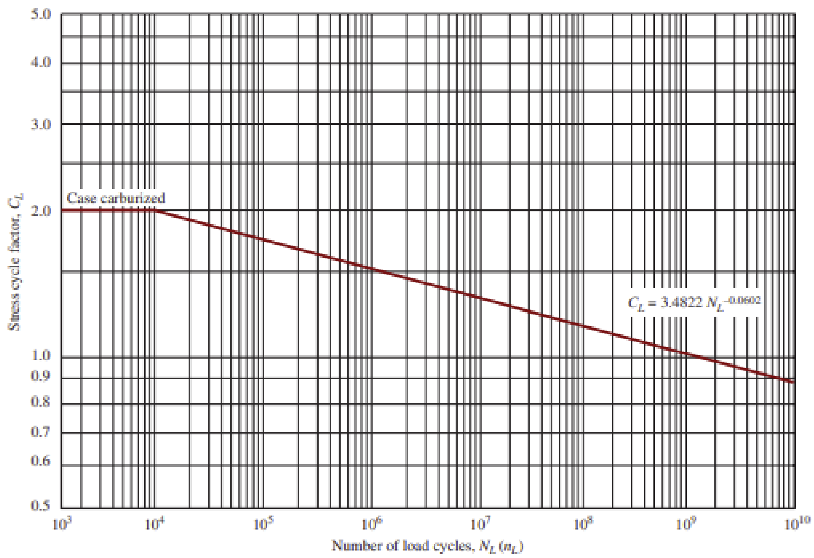
22. Get I from this graph



23. Compute the contact stress number

$$s_c = C_p \sqrt{\frac{W_t K_O K_m K_v C_s C_{xc}}{F D_p I}}$$

24. Get C_L from here



25. Get the max allowable contact stress number

$$s_{ac} = \frac{s_c(SF)C_R}{C_L}$$

26. Follow the remaining steps for material selection from spur gear guide.