Lecture 7 - EE 359: Wireless Communications - Winter 2020

Doppler from Scattering Function
Shannon Capacity of Wireless Channels

Lecture Outline

- **Doppler Power Spectrum, Doppler Spread, and Coherence Time**
- **Shannon Capacity**
- **Capacity of Flat-Fading Channels**
- **Capacity with Fading Known at Receiver**
- **Capacity with Fading Known at Transmitter and Receiver**
- **Optimal Rate and Power Adaptation (Water Filling)**

1. **Doppler Power Spectrum, Doppler Spread, and Coherence Time:**

   - Doppler power spectrum is defined with respect to $A_c(\Delta f; \Delta t) = \mathcal{F}_\tau[A_c(\tau, \Delta t)]$.
   - Specifically, the Doppler power spectrum is $S_c(\rho) = \mathcal{F}_{\Delta t}[A_c(\Delta f = 0, \Delta t) \stackrel{\triangle}{=} A_c(\Delta t)]$, which measures channel intensity as a function of Doppler frequency.
   - The maximum value of ρ for which $|S_c(\rho)| > 0$ is called the channel Doppler spread, which is denoted by B_d.
   - By the Fourier transform relationship, $A_c(\Delta t) \approx 0$ for $\Delta t > 1/B_d$. Thus, the channel becomes uncorrelated over a time of $1/B_d$ seconds.
   - We define the channel coherance time as $T_c = 1/B_d$. A deep fade lasts approximately T_c seconds. Hence, if the coherence time greatly exceeds a bit time, the signal experiences *error bursts* lasting T_c seconds.

2. **Shannon Capacity**

   - The maximum mutual information of a channel. Its significance comes from Shannon's coding theorem and converse, which show that capacity is the maximum error-free data rate a channel can support.
   - Capacity is a channel characteristic - not dependent on transmission or reception techniques or limitation.
   - In AWGN, $C = B\log_2(1+\gamma)$ bps, where B is the signal bandwdith and $\gamma = P/(N_0 B)$ is the received signal-to-noise power ratio.

3. **Capacity of Flat-Fading Channels:**

   - Depends on what is known about the channel.
   - Three cases: 1) Fading statistics known; 2) Fade value known at receiver; 3) Fade value known at transmitter and receiver.
   - 1 When only fading statistics known, capacity difficult to compute. Only known results are for Finite State Markov channels, Rayleigh fading channels, and block fading.

2 4. **Fading Known at the Receiver:**

- Capacity given by $C = \int_0^\infty B \log_2(1+\gamma)p(\gamma)d\gamma$ bps, where $p(\gamma)$ is the distribution of the fading SNR γ.
- By Jensen's inequality this capacity always less than that of an AWGN channel.
- "Average" capacity formula, but transmission rate is fixed.

3 5. **Capacity with Fading Known at Transmitter and Receiver**

- For fixed transmit power, same capacity as when only receiver knows fading.
- Transmit power as well as rate can be adapted.
- Under variable rate and power $C = \max_{P(\gamma): \int P(\gamma)p(\gamma)d\gamma = \overline{P}} \int_0^\infty B \log_2\left(1 + \frac{P(\gamma)\gamma}{\overline{P}}\right) p(\gamma)d\gamma$, where $P(\gamma)$ is power adaptation

6. **Optimal Power and Rate Adaptation**

- Optimal adaptation found via Lagrangian differentiation.
- Optimal power adaptation is a "water-filling" in time: power $P(\gamma) = \gamma_0^{-1} - \gamma^{-1}$ increases with channel quality γ above an optimized cutoff value γ_0.
- Rate adaptation relative to $\gamma \geq \gamma_0$ is $B \log_2(\gamma/\gamma_0)$: also increases with γ above cutoff.
- Resulting capacity is $C = \int_{\gamma_0}^\infty B \log_2(\gamma/\gamma_0)p(\gamma)d\gamma$.
- Capacity with power and rate adaptation not much larger than when just receiver knows channel, but has lower complexity and yields more insight into practical schemes.
- Capacity in flat-fading can exceed the capacity in AWGN, typically at low SNRs.

Main Points

- Doppler spread defines the channel's maximum nonzero Doppler. Its inverse is the channel coherence time. Signals separated in time by the coherence time have independent fading.

- Capacity of flat-fading channels depends on what is known about the fading at receiver and transmitter.

- Capacity when only the receiver knows the fading is an average of capacity in AWGN, averaged over the fading distribution.

- Capacity when both transmitter and receiver known channel requires optimal adaptation relative to each channel state. Capacity increases when transmitter also knows the channel only when power is adapted.

- Capacity-achieving transmission scheme uses variable-rate variable-power transmission with power water-filling in time.

- Power and rate adaptation does not significantly increase capacity, and rate adaptation alone yields no increase. These results may not carry over to practical schemes.