Statistical Fading Models. Narrowband Models

Lecture Outline

- **Statistical Multipath Model.**

- **Time Varying Channel Impulse Response.**

- **Narrowband Fading Approximation.**

- **In-Phase and Quad Signal Components under CLT.**

- **Mean, Autocorrelation, and Cross Correlation in Narrowband Fading.**

1. **Statistical Multipath Model:**

   - At each time instant there are a random number $N(t)$ of multipath signal components.

   - At time t the nth component has a random amplitude $\alpha_n(t)$, angle of arrival $\theta_n(t)$, Doppler shift $f_{D_n}(t) = \frac{v}{\lambda} \cos \theta_n(t)$ and associated phase shift $\phi_{D_n}(t) = \int_\nu f_{D_n}(\nu)d\nu$, and path delay relative to the LOS (the 0th) component $\tau_n(t) = (d_n(t) - d_0(t))/c$.

   - Thus, the received signal is given by the following expression, which implies the channel has a time-varying impulse response.

   $$r(t) = \Re \left\{ \sum_{n=0}^{N(t)} \alpha_n(t)u(t - \tau_n(t))e^{j(2\pi f_c(t - \tau_n(t)) + \phi_{D_n}(t))} \right\}$$

2. **Time-Varying Channel Impulse Response:**

   - The received signal $r(t)$ is the convolution of the input signal with the equivalent low-pass channel impulse response $c(\tau, t)$:

   $$r(t) = \Re \left\{ \left(\int_{-\infty}^{\infty} c(\tau, t)u(t - \tau)d\tau \right) e^{j2\pi f_c t} \right\}.$$

   - The channel can thus be modeled as a time-varying linear filter:

   $$c(\tau, t) = \sum_{n=1}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\delta(\tau - \tau_n(t)),$$

   where $c(\tau, t)$ is the channel response at time t to an impulse at time $t - \tau$.

   - Note there are two time parameters in this expression. The parameter t denotes the time when the impulse response is observed. The parameter $t - \tau$ denotes the time when the impulse was put into the channel relative to the observation time t.

   - In other words, τ denotes how long ago the impulse was put into the channel for the current observation.

3. **Properties of Received Signal:**

   - The received signal consists of N multipath components scaled by amplitude $\alpha_n(t)$ and phase $\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}(t)$.

- The amplitude term $\alpha_n(t)$ varies slowly (due to shadowing).
- The phase $\phi_n(t)$ varies rapidly due to small shifts in the signal component path delays.
- The phase variation causes rapid variation in the received signal amplitude due to constructive and destructive interference of the multipath components.

4. **Narrowband Approximation:**

   - Define the multipath delay spread as $T_m(t) = \max_n \tau_n(t) - \min_n \tau_n(t)$. For random multipath the delay spread is defined relative to its mean or standard deviation.
   - Assume $T_m(t) \ll 1/B$ for all t (or the equivalent for random multipath). Then $u(t) \approx u(t - \tau_n(t))$ for all n and t.
   - Received signal simplifies to

   $$r(t) = \Re\left\{ u(t)e^{j2\pi f_c t} \left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)} \right] \right\}$$

   - No signal distortion: multipath only affects complex scale factor in brackets.
   - Characterize scale factor by assuming $u(t) = e^{j\phi_0}$ so $s(t) = \cos(2\pi f_c t + \phi_0)$

5. **In-Phase and Quad Signal Components under CLT**

   - Received signal can be written in terms of in-phase and quadrature components as $r(t) = r_I(t)\cos(2\pi f_c t) - r_Q(t)\sin(2\pi f_c t)$ where $r_I(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\cos(\phi_n(t))$ and $r_Q(t) = \sum_{n=0}^{N(t)} \alpha_n(t)\sin(\phi_n(t))$ where $\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}(t) - \phi_0$ now also incorporates phase ϕ_0 of the transmitted signal.
   - If $N(t)$ large then in-phase and quadrature signal components are jointly Gaussian (amplitude of scale factor is Rayleigh).
   - Thus, received signal characterized by mean, autocorrelation, and cross correlation.
   - Assuming $\phi_n(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$.

6. **Mean, Autocorrelation, and Cross Correlation in Narrowband Fading**

   - Assuming $\phi_n(t)$ uniform, $E[r_I(t)] = E[r_Q(t)] = 0$ and $E[r_I(t)r_Q(t)] = 0$. Thus, $r_I(t)$ and $r_Q(t)$ are uncorrelated, hence independent. Moreover, $E[r(t)] = 0$
   - $A_{r_I}(t, t+\tau) = A_{r_I}(\tau) = A_{r_Q}(\tau) = .5\sum_{n=0}^{N-1} E[\alpha_n^2]E_{\theta_n}[\cos(2\pi v\tau/\lambda)\cos\theta_n]$, so the processes are WSS and hence stationary. Note that $.5\sum_n E[\alpha_n^2] = P_r$, the total average received power.
   - Using a similar analysis, get $A_{r_I, r_Q}(t, t+\tau) = E[r_I(t)r_Q(t+\tau) = P_r E_{\theta_n}[\sin(2\pi v\tau/\lambda)\cos\theta_n] = A_{r_I, r_Q}(\tau)$.
   - Using these derivations, we get the autocorrelation for the received signal as $A_r(t, t+\tau) = A_{r_I}(\tau)\cos(2\pi f_c \tau) + A_{r_I, r_Q}(\tau)\sin(2\pi f_c \tau) = A_r(\tau)$. Since this only depends on τ, the received signal is WSS and hence stationary.

Main Points

- Statistical multipath model leads to a time varying channel impulse response

- The resulting received signal has rapidly varying amplitude due to constructive and destructive multipath combining

- Narrowband model and CLT lead to inphase, quadrature, and received signals that are stationary Gaussian processes with zero mean and an autocorrelation function that depends on the AOAs of the multipath components.