1a

Wednesday, September 28, 2022 12:21 AM

$$f(\theta) = \sum_{i=1}^{n} w_i (\theta - x_i)^2$$

$$f'(\theta) = \sum_{i=1}^{n} w_i \cdot 2(\theta - x_i)$$

Let $f'(\theta) = 0$

$$2\sum_{i=1}^{n} w_i(\theta - x_i) = 0$$

$$\sum_{i=1}^{n} w_i\theta - \sum_{i=1}^{n} w_i x_i = 0$$

$$\sum_{i=1}^{n} w_i\theta = \sum_{i=1}^{n} w_i x_i$$

$$\theta = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

$f''(\theta) = \sum_{i=1}^{n} w_i > 0$, thus it is a minimum.

If some w_i are negative, $f''(\theta)$ may not be >0 and thus there may not be a minimum.

1b

Thursday, September 29, 2022 5:46 PM

f(x) is minimizing across all of the x's and selecting a single value, whereas g(x) is selecting a minimum for each x. g(x) is minimizing over a larger set, therefore f(x) >= g(x) for all x

1c

Sunday, October 2, 2022 2:58 PM

$$V = a\,P(3) - b\,P(6) + (1 - P(1))\,V$$

$$V = \frac{1}{6}a - \frac{1}{6}b + \frac{5}{6}V$$

$$\frac{1}{6}V = \frac{1}{6}a - \frac{1}{6}b$$

$$V = a - b$$

1d

Sunday, October 2, 2022 3:04 PM

$$\log L(p) = \log\left(p^4(1-p)\right) = 4\log p + \log(1-p)$$

$$\nabla \log L(p) = \frac{4}{p} - \frac{1}{1-p} = 0 \Rightarrow$$

$$\frac{4}{p} = \frac{1}{1-p}$$

$$4 - 4p = p$$

$$5p = 4$$

$$p = \frac{4}{5}$$

$$\nabla^2 \log L(p) = \nabla\left(4p^{-1} - (1-p)^{-1}\right) = -\frac{4}{p^2} - \frac{1}{(1-p)^2} < 0$$

thus $p = \frac{4}{5}$ is a maximum

This is intuitively just the proportion of heads obtained.

1e

Sunday, October 2, 2022 3:23 PM

Bayes' Thm: $P(B|A) = \frac{P(A|B)P(B)}{P(A)}$

$$\frac{P(B|A)}{P(A|B)} = \frac{P(B)}{P(A)}$$

$$P(B|A) = P(A|B) \Rightarrow 1 = \frac{P(B)}{P(A)} \Rightarrow P(A) = P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{3} = 2P(A) - P(A \cap B)$$

$$2P(A) = \frac{1}{3} + \underbrace{P(A \cap B)}_{\geq 0}$$

$$2P(A) > \frac{1}{3}$$

$$P(A) > \frac{1}{6}$$

1f

Sunday, October 2, 2022 3:10 PM

$$\nabla f(w) = \nabla\left(\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^T w - b_j^T w)^2\right) + \frac{\lambda}{2}\|w\|^2$$

$$\nabla\|w\|_2^2 = \nabla(w \cdot w) = 2w$$

$$\nabla f(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\nabla(a_i^T w - b_j^T w)^2 + \lambda w$$

$$\nabla f(w) = 2\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^T w - b_j^T w)(a_i - b_j) + \lambda w$$

2a

Sunday, October 2, 2022 3:31 PM

The nxn grid of points can be thought of as a set of n+1 parallel vertical lines intersecting a set of n+1 parallel horizontal lines. To construct a rectangle, we select one line from each set. The number of possible ways to select a line from one set is n(n+1)/2, thus the number of possible ways to place a rectangle is (n(n+1)/2)^2. Thus, the number of ways to place 3 rectangles is 3(n(n+1)/2)^2 which is O(n^4).

2b

Sunday, October 2, 2022 3:52 PM

The cost of touching position (i,j) is c(i,j). We only move down or to the right. Thus, to move to cell (i,j), we moved from either cell (i-1,j) or (i,j-1). To minimize the cost, v(i,j) = c(i,j) + min(v(i-1,j), v(i,j-1)). We will use a dynamic programming approach with memoization to save a 2D array of size nxn where we store the values of v(i,j):

Initialize nxn array A with -1s
If A[i][j] is not -1, return A[i][j]
Else:
    A[i][j] = c(i,j) + min(v(i-1,j), v(i,j-1))
Return A[i][j]

The runtime complexity of this algorithm O(n^2). This is because we are scanning and making updates on a 2d grid where each dimension is O(n), hence O(n^2) overall.

3

Wednesday, September 28, 2022 10:10 PM

a) This model likely encoded, contains, or potentially exacerbates bias against people of a certain race or ethnicity. This is evidenced by the fact that there is a higher false positive rate for black applicants, meaning the model is more likely to incorrectly predict that they will default on the loan. The model perpetuates biases against black and rural applicants as these groups are unfairly neglected by many financial institutions.
b) These algorithms contain information that could be deduced about individuals that they have not consented to share. The authors of these works intended to remain anonymous, however these algorithms do not respect this by removing the anonymity. Thus, personally identifiable information or other sensitive material about the authors may be revealed.
c) This work is problematic regarding the consent to use or share the data used. Clearly, the faces are scraped without the consent of the people in the images, and there also may be a copyright violation even though the URLS are provided.
d) I believe this model may have a damaging effect on the environment. By encouraging people to look for plants to take images of in the app, they may go into and damage the natural environment in which the plants are located or the plants themselves.