ML Assignment #4

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1. (a)(a) The width of the image is $N-K+1$
The height of the image is $M-K+1$

Then, after convolution, the dimensions are:

$$(N-K+1) \times (M-K+1)$$

(b) To compute a single output pixel in the resulting feature map during the forward pass of a convolutional layer, we consider the input image of dimensions $M \times N$ with P channels and a single kernel of size $K \times K$. For each channel, the kernel involves $K \times K$ multiplications. Since, there are P channels, the total number of multiplications across all channels is K^2P. After the multiplications the resulting K^2P elements are summed to compute the final value of the output pixel.

Summing K^2P elements requires K^2P-1 additions.

∴ the total number of elementary operations for a single output pixel is $K^2P + (K^2P-1) = 2K^2P-1$

(c) The image output size
$= (N-K+1)(M-K+1) \times Q$
where Q is number of Kernels

From (b), Operations per pixel = $2k^2P - 1$

The equations for Time complexity are:

$$O(Q(N-K+1)(M-K+1)(2K^2P-1)) - ①$$

$$Min(M, N) >> K - ②$$

From ① & ②

Time complexity: $O(2QNMK^2P - QNM)$

1-b

Assignment step:

In this step, we decide the class membership of the n objects by assigning them to be nearest cluster center

$$\min \sum_{i=1}^{n} |x_i - \mu_{x_i}|^2$$

i is centroid for cluster

Update / Re-estimate Step:

$$\mu_i = \frac{1}{|C_i|} \cdot \sum_{x \in C_i} x$$

We calculate the new centroids by considering the ~~mean~~ mean of all the data points assigned to that cluster.

The loss function for K-means algorithm is :

$$\mathcal{L}(C, \mu) = \min_{\mu} \min_{C} \sum_{i=1}^{K} \sum_{x \in C_i} |x - \mu_i|^2$$

We minimize the objective function iteratively and try to converge to a minimum fixed point.

However, during the iterations, the algorithm might get stuck at a local optimal, where the algorithm would terminate even though we did not reach global optimal.

Hence, we might not always arrive at optimal / global minima with random assignment.