AI Assignment - 2
Theory

Aditya Aggarwal
(2022028)

Answer 1

~~Rule~~ Notations for PL :

R_t signifies that the traffic light is red at given time t.
G_t signifies that the traffic light is green at given time t.
Y_t signifies that the traffic light is yellow at given time t.

Rule 1 :
$$(G_t \vee Y_t \vee R_t) \wedge \neg(G_t \wedge Y_t) \wedge \neg(Y_t \wedge R_t) \wedge \neg(R_t \wedge G_t)$$

Rule 2 :
$$(G_t \Rightarrow Y_{t+1}) \wedge (Y_t \Rightarrow R_{t+1}) \wedge (R_t \Rightarrow G_{t+1})$$

Rule 3 :
$$(\neg(G_t \wedge G_{t+1} \wedge G_{t+2} \wedge G_{t+3})) \wedge (\neg(Y_t \wedge Y_{t+1} \wedge Y_{t+2} \wedge Y_{t+3})) \wedge (\neg(R_t \wedge R_{t+1} \wedge R_{t+2} \wedge R_{t+3}))$$

Answer 2

Defining Predicates :

connected (x, y) : two nodes $x, y \in N$ are connected directly by an edge $\in R$
color (x, c) : a node $x \in N$ has a color $c \in C$
Note that we assume connected $(x, y) \leftrightarrow$ connected (y, x)

1. $\forall x \forall y \forall c \ \text{connected}(x, y) \Rightarrow \neg(\text{colour}(x, c) \wedge \text{color}(y, c))$

$x, y \in N, \ c \in C$

2.

$\exists x \exists y \ \text{color}(x, \text{yellow}) \wedge \text{color}(y, \text{yellow}) \wedge (x \neq y) \wedge$
$\forall z \ (z \neq x \wedge z \neq y) \Rightarrow \neg \text{color}(z, \text{yellow})$

$x, y, z \in N$; yellow is a constant $\in C$.

3.

$\forall x \ \text{color}(x, \text{red}) \Rightarrow \exists y \ \text{color}(y, \text{green}) \wedge (x \neq y) \wedge$
$(\exists z_1 \ \text{connected}(x, z_1) \wedge \text{connected}(z_1, y) \wedge (x \neq z_1 \neq y))$
$\vee (\exists z_1 \exists z_2 \ \text{connected}(x, z_1) \wedge \text{connected}(z_1, z_2) \wedge \text{connected}(z_2, y) \wedge$
$(x \neq z_1 \neq z_2 \neq y)) \vee (\exists z_1 \exists z_2 \exists z_3 \ \text{connected}(x, z_1) \wedge \text{connected}(z_1, z_2)$
$\wedge \ \text{connected}(z_2, z_3) \wedge \text{connected}(z_3, y) \wedge (x \neq z_1 \neq z_2 \neq z_3 \neq y)) \vee$
$\text{connected}(x, y)$ $\quad x, y, z_1, z_2, z_3 \in N$

red, green $\in C$

to shorten this expression, we can define a predicate as follows

path (x, y, R') : There exists a path between 2 nodes x and $y \in N$ consisting of edges $e \in R'$ such that $R' \subset R$.

Now the expression changes to :

$\forall x \ \text{color}(x, \text{red}) \Rightarrow \exists y \ \text{color}(y, \text{green}) \wedge (x \neq y) \wedge \exists R' \subset R$
$\text{path}(x, y, R') \wedge (1 \leq |R'| \leq 4)$

$x, y \in N$; red, green $\in C$

PAGE No
DATE: / 202

4. $\forall c \, \exists x \; color(x, c) \qquad x \in N; \; c \in C$

5. $(\forall c \, \forall x \, \forall y \; color(x,c) \wedge color(y,c) \wedge (x \neq y) \Rightarrow connected(x,y))$
$\wedge \; (\forall c_1 \forall c_2 \, \forall x \, \forall y \; color(x, c_1) \wedge color(y, c_2) \wedge (x \neq y) \wedge (c_1 \neq c_2)$
$\Rightarrow \neg(connected(x,y))$
$x, y \in N; \; c, c_1, c_2 \in C$

Answer 3

Defining propositional conditions:

R : a creature can read
L : a creature is literate
D : the creature is a dolphin
I : a creature is intelligent

Defining predicate conditions:

read (x) : x can read
literate (x) : x is literate
dolphin (x) : x is a dolphin
intelligent (x) : x is intelligent

1. PL : $R \Rightarrow L$
FOL : $\forall x \; read(x) \Rightarrow literate(x)$

2. PL : $D \Rightarrow \neg L$
FOL : $\forall x \; dolphin(x) \Rightarrow \neg(literate(x))$

3. PL : We cannot represent Existential quantifiers in PL
FOL : $\exists x \; dolphin(x) \wedge intelligent(x)$

4. PL : PL does not exist due to above mentioned reason.
FOL : $\exists x \; intelligent(x) \wedge \neg read(x)$

5. $\exists x\,(\text{dolphin}(x) \wedge \text{intelligent}(x) \wedge \text{read}(x)) \wedge$
$\forall y\,(\text{dolfin}(y) \wedge \text{intelligent}(y) \wedge \text{read}(y) \Rightarrow \neg\text{literate}(y))$

The above was FOL. PL does not exist due to use of existential quantifiers.

Checking satisfiability of Statement 4 using statements 1, 2 & 3.

Converting to CNF :

$R1 \equiv \forall x\,(\neg\text{read}(x) \vee \text{literate}(x))$
$R2 \equiv \forall x\,(\neg\text{dolphin}(x) \vee \text{litrate}(x))$
$R3 \equiv \exists x\ \neg(\neg\text{dolphin}(x) \vee \neg\text{intelligent}(x))$

We have the conjunction of these statements as KB.

$KB \wedge \neg\alpha$ ~~should satisfy~~ , $KB \equiv R_1 \wedge R_2 \wedge R_3$

$R_4 \equiv \neg\alpha \equiv \neg\exists x\,(\text{intelligent}(x) \wedge \neg\text{read}(x))$
$\equiv \forall\,(\neg\text{intelligent}(x) \vee \text{read}(x))$

let y be a dolphin that satisfies R_3

$R_5 \equiv \text{dolphin}(y)$
$R_6 \equiv \text{intelligent}(y)$

Resoluting R_2 and R_5 : $R_7 \equiv \neg\text{literate}(y)$
Resoluting R_1 and R_7 : $R_8 \equiv \neg\text{read}(y)$
Resoluting R_4 and R_8 : $R_9 \equiv \neg\text{intelligent}(y)$
Resoluting R_6 and R_9 gives an empty clause.
∴ Thus statement-4 is satisfiable.

Checking satisfiability of statement 5 using statements 1, 2, 3 & 4.

R_1, R_2, R_3 remained as previously defined.

$R_4 \equiv \exists x \neg(\neg intelligent(x) \vee read(x))$

$\alpha : \exists x\ (dolphin(x) \wedge intelligent(x) \wedge read(x)) \wedge$
$\forall y\ (dolphin(y) \wedge intelligent(y) \wedge read(y) \rightarrow \neg literate(y))$

$KB = R_1 \wedge R_2 \wedge R_3 \wedge R_4$

Checking $KB \wedge \neg\alpha$

α is a disjunction, let $\alpha \equiv \alpha_1 \vee \alpha_2$
sub sentence $\neg\alpha_1 \equiv \exists x(\neg dolphin(x) \vee \neg intelligent(x) \vee \neg read(x$
let dolphin y satisfy R_3.

$R_6 \equiv dolphin(y)$
$R_7 \equiv intelligent(y)$

Let z satisfy R_4.
$R_8 \equiv intelligent(z)$
$R_9 \equiv \neg read(z)$

Resoluting R_2 with R_6 : $R_{10} \equiv \neg literate(y)$
Resoluting R_{10} with R_1 : $R_{11} \equiv \neg read(y)$

R_{11} does not resolute with any other clause.
An empty clause could not be achieved through resolution.
∴ Thus, statement 5 is unsatisfiable.