

Assignment - 3
Report

Aditya Aggarwal - 2022028 - CSCI

Q1

$$x[n] = \begin{cases} 1 & -N_1 \leq n \leq N_1 \\ 0 & N_1 < |n| \leq \frac{N-1}{2} \end{cases}$$

$$N_1 = 2$$

$$a_k = \frac{1}{N} \sum_{\langle n \rangle} x[n] \cdot e^{j \frac{2\pi n}{N} k}$$

$x[n]$ takes value 1 from only $-N_1$ to N_1 , $N_1 = 2$.

$$\Rightarrow a_k = \frac{1}{N} \sum_{n=-N_1}^{N_1} x[n] \cdot e^{j \frac{2\pi n}{N} k}$$

$$\Rightarrow a_k = \frac{1}{N} \left(e^{-j \frac{4\pi}{N} k} + e^{j \frac{4\pi}{N} k} + e^{-j \frac{2\pi}{N} k} + e^{j \frac{2\pi}{N} k} + 1 \right)$$

$$\Rightarrow a_k = \frac{1}{N} \left(2 \cos\left(\frac{2\pi k}{N}\right) + 2 \cos\left(\frac{4\pi k}{N}\right) + 1 \right)$$

Inference : $x[n]$ has only real coefficients

Q2

$$f(x) = x + \pi \quad -\pi \leq x \leq \pi$$

$$f(x + 2\pi) = f(x)$$

Thus, Time period = 2π

The graph is plotted from -3π to 3π (3 periods)

The fourier representation, which is discrete is plotted from -10 to 10 (roughly the same interval)

$$a_k = \frac{1}{T} \int_T f(t) \cdot e^{-jk(2\pi/T)t} dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (t + \pi) e^{-jkt} dt$$

for $k = 0$,

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (t + \pi) dt$$

$$\Rightarrow \boxed{a_0 = \pi}$$

for $k \neq 0$

$$a_k = j \left(\frac{\cos k\pi}{k} \right)$$

(after solving integral by parts)

$$\therefore a_k = \begin{cases} \pi & k=0 \\ \frac{(-1)^k}{k} \cdot j & k \neq 0 \end{cases}$$

Inference : The fourier series will be ~~not~~ zero on all points except $k=0$ on real axis while it will be zero only on $k=0$ on imaginary axis.

In other words, coefficient is real only for $k=0$ while all other coefficients are imaginary with alternating signs and decreasing magnitude.