# Lab Inference

1. For a normal population with known variance$\sigma^2$, answer the following:
   a) The confidence level for the interval $\bar{x} - \frac{2.14\sigma}{\sigma\sqrt{n}} \leq \mu \leq \bar{x} + \frac{2.14\sigma}{\sigma\sqrt{n}}$
      is determined by the value of Z0 which is 2.5. From the Table,
      Ø(2.5) = P (Z < 2.5) = 0.9938 and
      the confidence level is 2(0.9838 - 0.5) = 96.76%.
   b) The confidence level for the interval $\bar{x} - \frac{2.49\sigma}{\sigma\sqrt{n}} \leq \mu \leq \bar{x} + \frac{2.49\sigma}{\sigma\sqrt{n}}$
      is determined by the by the value of Z0 which is 2.14. From the Table,
      Ø(2.49) = P (Z < 2.49) = 0.9936 and
      the confidence level is 2(0.9936 - 0.5) = 98.72%.
   c) The confidence level for the interval $\bar{x} - \frac{1.85\sigma}{\sigma\sqrt{n}} \leq \mu \leq \bar{x} + \frac{1.85\sigma}{\sigma\sqrt{n}}$
      is determined by the by the value of Z0 which is 2.14. From the Table,
      Ø (1.85) = P (Z < 1.85) = 0.9678 and
      the confidence level is 93.56%
2. A confidence interval estimate is desired for the gain in a circuit semiconductor device. Assume that gain is normally distributed with standard deviation σ = 20
   a) 95% CI for μ when n = 10, σ = 20 and $\bar{x}$= 1000,
      Z= 1.96,
      $\bar{x} - \frac{Z\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{Z\sigma}{\sqrt{n}}$
      = $1000 - 1.96\left(\frac{20}{\sqrt{10}}\right) \leq \mu \leq 1000 + 1.96\left(\frac{20}{\sqrt{10}}\right)$
      = $987.6 \leq \mu \leq 1012.4$
   b) 95% CI for μ when n = 25, σ = 20 and $\bar{x}$= 1000,
      Z= 1.96,
      $\bar{x} - \frac{Z\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{Z\sigma}{\sqrt{n}}$
      = $1000 - 1.96\left(\frac{20}{\sqrt{25}}\right) \leq \mu \leq 1000 + 1.96\left(\frac{20}{\sqrt{25}}\right)$
      = $992.16 \leq \mu \leq 1007.84$
   c) 99% CI for μ when n = 10, σ = 20 and $\bar{x}$= 1000,
      Z= 2.58,
      $\bar{x} - \frac{Z\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{Z\sigma}{\sqrt{n}}$
      = $1000 - 2.58\left(\frac{20}{\sqrt{10}}\right) \leq \mu \leq 1000 + 2.58\left(\frac{20}{\sqrt{10}}\right)$
      = $299.91 \leq \mu \leq 1016.31$
   d) 99% CI for μ when n = 25, σ = 20 and $\bar{x}$= 1000,
      Z= 2.58,
      $\bar{x} - \frac{Z\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{Z\sigma}{\sqrt{n}}$
      = $1000 - 2.58\left(\frac{20}{\sqrt{25}}\right) \leq \mu \leq 1000 + 2.58\left(\frac{20}{\sqrt{25}}\right)$
      = $989.68 \leq \mu \leq 1010.32$

3. Consider the gain estimation in previous problem (problem 2).
   How large must n be if the length of the 95% CI is to be 40?

$\alpha = 1 - 0.95 = 0.05$

$Z\sigma/2 = Z\ (0.05/2) = Z\ (0.025) = 1.96$

½ length = $\frac{(1.96)(20)}{\sqrt{n}} = 15$

$39.2 = 15\sqrt{n}$

$\sqrt{n} = 39.2/15$

$n = (\frac{39.2}{15})^2$

n = 6.82

4. The breaking strength of yarn used in manufacturing drapery material is required to be at least 100 psi, Past experience has indicated that breaking strength is normally distributed and that σ = 2 psi. A random sample of nine specimens is tested and the average breaking strength is found to be 98 psi.
   Find a 95% two sided confidence interval.

$\bar{x} = 98$, n = 9 and σ = 2

Since population standard deviation is known, we use Z-test.

$\alpha = 1 - 0.95 = 0.05$ Using Z-tables,

the critical value is $Z\ (\sigma/2) = Z\ (0.05/2) = Z\ (0.025) = \pm 1.96$.

$$\bar{x} - Z_{0.025}\, \sigma/\sqrt{n} \leq \mu \leq \bar{x} + Z_{0.025}\, \sigma/\sqrt{n}$$

$$= 98 - \frac{(1.96)(2)}{\sqrt{9}} \leq \mu \leq 98 + \frac{(1.96)(2)}{\sqrt{9}}$$

$$= 96.69 \leq \mu \leq 99.30$$

5. The yield of a chemical process is being studied. From previous experience yield is known to be normally distributed and σ = 3. The past five days of plant operation have resulted in the following percent yields: 91.6, 88.75, 90.8, 89.95 and 91.3.
   Find 95% two sided confidence interval.

$\bar{x} = 91.3$, n = 5 and σ = 3

Since population standard deviation is known, we use Z-test.

$\alpha = 1 - 0.95 = 0.05$ Using Z-tables,

the critical value is $Z\ (\sigma/2) = Z\ (0.05/2) = Z\ (0.025) = \pm 1.96$.

$$\bar{x} - Z_{0.025}\, \sigma/\sqrt{n} \leq \mu \leq \bar{x} + Z_{0.025}\, \sigma/\sqrt{n}$$

$$= 91.3 - \frac{(1.96)(3)}{\sqrt{5}} \leq \mu \leq 91.3 + \frac{(1.96)(3)}{\sqrt{5}}$$

$$= 87.85 \leq \mu \leq 93.11$$

6. A civil engineer is analyzing the compressive strength of concrete. Compressive strength is normally distributed with $\sigma^2$= 1000 $psi^2$ . A random sample of 12 specimens has a mean compressive strength of $\bar{x}$= 3250 $psi$

a. Construct a 95% two sided confidence interval on mean compressive strength

α = 1 – 0.95 = 0.05

Z ($\sigma$/2) = Z (0.05/2) = Z (0.025) = ±1.96, and x = 3250, $\sigma^2$= 1000, n = 12

$\bar{x} - Z_{0.025}\, \sigma/\sqrt{n} \leq \mu \leq \bar{x} + Z_{0.025}\, \sigma/\sqrt{n}$

$= 3250 - \frac{(1.96)(\sqrt{1000})}{\sqrt{12}} \leq \mu \leq 3250 + \frac{(1.96)(\sqrt{1000})}{\sqrt{12}}$

$= 3232.11 \leq \mu \leq 3267.89$

b. Construct a 99% two sided confidence interval on mean compressive strength

α = 1 – 0.99 = 0.01

Z ($\sigma$/2) = Z (0.01/2) = Z (0.005) = ±2.58, and x = 3250, $\sigma^2$= 1000, n = 12

$\bar{x} - Z_{0.025}\, \sigma/\sqrt{n} \leq \mu \leq \bar{x} + Z_{0.025}\, \sigma/\sqrt{n}$

$= 3250 - \frac{(2.58)(\sqrt{1000})}{\sqrt{12}} \leq \mu \leq 3250 + \frac{(2.58)(\sqrt{1000})}{\sqrt{12}}$

$= 3226.4 \leq \mu \leq 3273.6$

7. Suppose that in the previous problem (Problem 6) it is desired to estimate the compressive strength with an error that is less than 15 psi at 99% confidence. What sample size is required?

α = 1 – 0.99 = 0.01

Z ($\sigma$/2) = Z (0.01/2) = Z (0.005) = ±2.58, $\sigma$= $\sqrt{1000}$, E= 15

$n = (\frac{Z\,(\sigma/2)\sigma}{E})^2$

$= (\frac{(2.58)(\sqrt{1000})}{15})^2$

$= 29.584 \cong 30$

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