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Assignment 4

(1)

Q1

$\min_{x,y} f(x,y) = 4xy$ subject to $h(x,y) = \frac{x^2}{9} + \frac{y^2}{9} = 1$

$$L(x,y,\lambda) = 4xy - \lambda\left[\frac{1}{9}x^2 + \frac{1}{9}y^2 - 1\right]$$

$$\frac{\partial L}{\partial x} = 4y - \frac{2}{9}x\lambda \Rightarrow 4y = \frac{2}{9}x\lambda \rightarrow [1]$$

$$\frac{\partial L}{\partial y} = 4x - \frac{2}{9}y\lambda \Rightarrow 4x = \frac{2}{9}y\lambda \rightarrow [2]$$

$$\frac{\partial L}{\partial \lambda} = \frac{x^2}{9} + \frac{y^2}{9} - 1 \rightarrow [3]$$

from 1 :- $\lambda = \frac{18y}{x}$

from 2 :-

$$4x = \frac{2y}{9}\left(\frac{18y}{x}\right) \Rightarrow x^2 = \frac{9}{16}y^2$$

from 3

$$\frac{1}{9}\left[\frac{9}{16}y^2\right] + \frac{y^2}{16} = 1$$

$$\frac{y^2}{16} + \frac{y^2}{16} = 1 \quad y^2 = 8 \quad y = \pm 2\sqrt{2}$$

$$x^2 = \left(\frac{9}{16}y^2\right) = \frac{9}{16} * 8 = \frac{9}{2} \quad x = \pm\frac{3\sqrt{2}}{\sqrt{2}}$$

Point $\left[\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right], \left[\frac{-3\sqrt{2}}{2}, 2\sqrt{2}\right], \left[\frac{3\sqrt{2}}{2}, -2\sqrt{2}\right], \left[\frac{-3\sqrt{2}}{2}, -2\sqrt{2}\right]$

$$f\left(\frac{3\sqrt{2}}{2}, 2\sqrt{2}\right) = 4xy = 24$$

Q2) Subject : ..... Date. (2)

$\min_{x,y} f(x,y) = x^2 + y^2 + 2x - 2y + 1$ subject to $h(x,y) = x^2 + y^2 = 2$

$L(x,y,\lambda) = x^2 + y^2 - 2x - 2y + 1 - \lambda x^2 - \lambda y^2 - 2\lambda$

$\frac{\partial L}{\partial x} = 2x - 2 - 2x\lambda \Rightarrow 2x + 2x\lambda = 2$

$2x[1+\lambda) = -2 \rightarrow (1)$

$\frac{\partial L}{\partial y} = 2y - 2 - 2y\lambda \Rightarrow 2y - 2\lambda x = 2$

$2y[1-\lambda] = 2 \rightarrow (2)$

$\frac{\partial L}{\partial \lambda} = x^2 + y^2 - 2 = 0 \rightarrow (3)$

by subs $y = -x$ from (3) $x^2 + (-x)^2 - 2 = 0$

$\frac{2x^2}{2} = 1 \quad \sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$y = -x \quad (x,y) = (-1,1), (1,-1)$

$f(-1,1) = -1 \quad f(1,-1) = 7$

Q3

$$x^2+y^2+z^2-\lambda(xyz-4)$$

$$\frac{\partial L}{\partial x} = 2x - \lambda yz \rightarrow (1) \qquad 2x = \lambda yz$$

$$\lambda = \frac{2x}{yz} \rightarrow (1)$$

$$\frac{\partial L}{\partial y} = 2y - \lambda xz \rightarrow (2) \qquad 2y = \lambda xz$$

$$\frac{\partial L}{\partial z} = 2z - \lambda xy \rightarrow (3) \qquad 2z = \lambda xy$$

$$\frac{\partial L}{\partial \lambda} = xyz - 4 \rightarrow (4)$$

$$2y = \frac{2x}{yz} \times xz \qquad 2y = \frac{2x^2}{y} \qquad \frac{2y^2}{2} = \frac{2x^2}{2}$$

$$y^2 = x^2$$

$$y = x$$

$$2z = \frac{2x}{yz} xy$$

$$2x^2 = 2z^2 \qquad x^2 = z^2$$

$$x = z$$

$$\frac{2z}{1} = \frac{2x^2}{z}$$

Point $F(2^{\frac{2}{3}}, 2^{\frac{2}{3}}, 2^{\frac{2}{3}}) = 6\sqrt[3]{2}$

$F(-2^{\frac{2}{3}}, -2^{\frac{2}{3}}, 2^{\frac{2}{3}}) = 6\sqrt[3]{2}$

$F(-2^{\frac{2}{3}}, 2^{\frac{2}{3}}, -2^{\frac{2}{3}}) = 6\sqrt[3]{2}$

$F(2^{\frac{2}{3}}, -2^{\frac{2}{3}}, -2^{\frac{2}{3}}) = 6\sqrt[3]{2}$

Q1/ $\min_{x,y} f(x,y) = xy + y^2$ subject to $h(x,y) = y^2 + z^2 - 1$

$$L = xy - yz - \lambda_1(xy-1) - \lambda_2(y^2+z^2-1)$$

$\frac{\partial L}{\partial x}$ $y(\lambda+1) \rightarrow (1)$ $\quad y\lambda = y$ $\quad \boxed{\lambda = 1}$

$\frac{\partial L}{\partial y}$ ~~$\lambda x + 2\cdot\lambda_2 y + x + 2$~~ $\rightarrow (2)$ $\quad 2\lambda_2 y = -2$ $\quad \lambda_2 y = -1$ $\quad \boxed{\lambda_2 = -\frac{1}{y}}$

$\frac{\partial L}{\partial z} = 2\lambda_2 z + y \rightarrow (3)$ $\quad -2\lambda_2 z = y$

$\frac{\partial L}{\partial \lambda_1} = xy - 1 \rightarrow (4)$ $\quad xy = 1$ $\quad \frac{2}{y} z = y$

$\frac{\partial L}{\partial \lambda_2} = y^2 + z^2 - 1 \rightarrow (5)$

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Point

$f\left(\sqrt{2}, \frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right) = \frac{1}{2}$

$f\left(\sqrt{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{3}{2}$

$f\left(-\sqrt{2}, \frac{-\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right) = \frac{3}{2}$

$f\left(-\sqrt{2}, \frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) = \frac{1}{2}$