Body 1, ..., Body i, ..., Body n

$$(i=1 \dots n) \quad M_i \ddot{\vec{p}}_i = G\left\{\sum_{j=1}^{n} M_j \cdot \frac{\vec{p}_j - \vec{p}_i}{|\vec{p}_j - \vec{p}_i|^3}\right\}$$

(Note: if $i=j \Rightarrow$ very illegal!)

Force component direction matrix .w

$j \rightarrow$

$i \downarrow$

$$[F_{dir}] = \begin{bmatrix} \vec{0} & \frac{\vec{p}_2 - \vec{p}_1}{|\vec{p}_2 - \vec{p}_1|^3} & \frac{\vec{p}_3 - \vec{p}_1}{|\vec{p}_3 - \vec{p}_1|^3} & \cdots & \frac{\vec{p}_n - \vec{p}_1}{|\vec{p}_n - \vec{p}_1|^3} \\ \frac{\vec{p}_1 - \vec{p}_2}{|\vec{p}_1 - \vec{p}_2|^3} & \vec{0} & \frac{\vec{p}_3 - \vec{p}_2}{|\vec{p}_3 - \vec{p}_2|^3} & \cdots & \frac{\vec{p}_n - \vec{p}_2}{|\vec{p}_n - \vec{p}_2|^3} \\ \frac{\vec{p}_1 - \vec{p}_3}{|\vec{p}_1 - \vec{p}_3|^3} & \frac{\vec{p}_2 - \vec{p}_3}{|\vec{p}_2 - \vec{p}_3|^3} & \vec{0} & \cdots & \frac{\vec{p}_n - \vec{p}_3}{|\vec{p}_n - \vec{p}_3|^3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\vec{p}_1 - \vec{p}_n}{|\vec{p}_1 - \vec{p}_n|^3} & \frac{\vec{p}_2 - \vec{p}_n}{|\vec{p}_2 - \vec{p}_n|^3} & \frac{\vec{p}_3 - \vec{p}_n}{|\vec{p}_3 - \vec{p}_n|^3} & \cdots & \vec{0} \end{bmatrix} \in \mathbb{R}^{3n \times 1}$$

Mass vector

$$\vec{M} = \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_n \end{bmatrix} \in \mathbb{R}^{n \times 1}$$

※ $[F_{dir}] = -[F_{dir}]^T$

Position of each body:

$$\vec{p} = [\vec{p}_1^T \ \vec{p}_2^T \ \cdots \ \vec{p}_n^T]^T$$

Equation of motion:

$$\frac{\partial^2}{\partial t^2}\vec{p} = \frac{\partial^2}{\partial t^2}\begin{bmatrix} \vec{p}_1 \\ \vec{p}_2 \\ \vdots \\ \vec{p}_n \end{bmatrix} = G \cdot [F_{dir}]\vec{M} = G\begin{bmatrix} F_{dir,row1}\vec{M} \\ F_{dir,row2}\vec{M} \\ \vdots \\ F_{dir,rown}\vec{M} \end{bmatrix} = -G\begin{bmatrix} F_{dir,col1}^T\vec{M} \\ F_{dir,col2}^T\vec{M} \\ \vdots \\ F_{dir,coln}^T\vec{M} \end{bmatrix}$$

State space: $\vec{x} = \left[\vec{p}_1^T, \vec{p}_2^T, \dots, \vec{p}_n^T, \frac{\partial}{\partial t}\vec{p}_1^T, \frac{\partial}{\partial t}\vec{p}_2^T, \dots, \frac{\partial}{\partial t}\vec{p}_n^T\right]^T$

$$\frac{\partial}{\partial t}\vec{x} = \begin{bmatrix} \frac{\partial}{\partial t}\vec{p} \\ \frac{\partial^2}{\partial t^2}\vec{p} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial t}\vec{p} \\ G \cdot [F_{dir}]\vec{M} \end{bmatrix}$$