

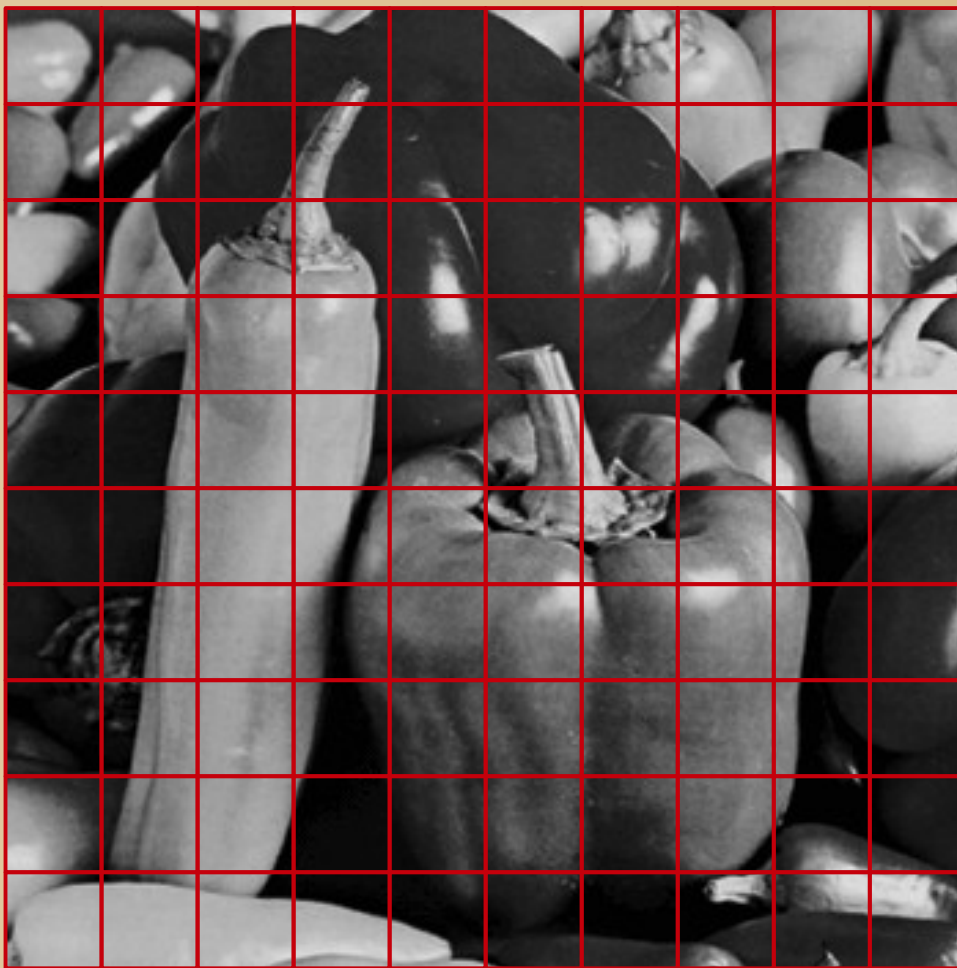
Vector Quantization

(Using LBG Algorithm with Splitting)

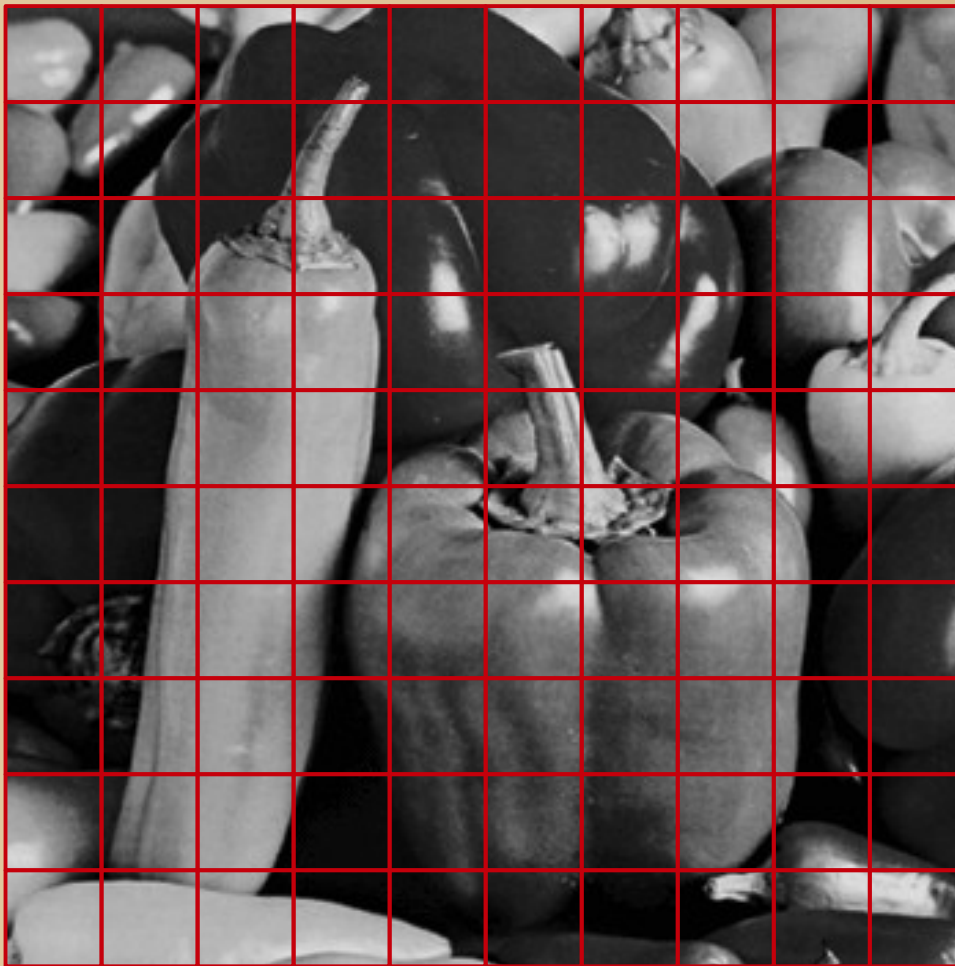
Original Image



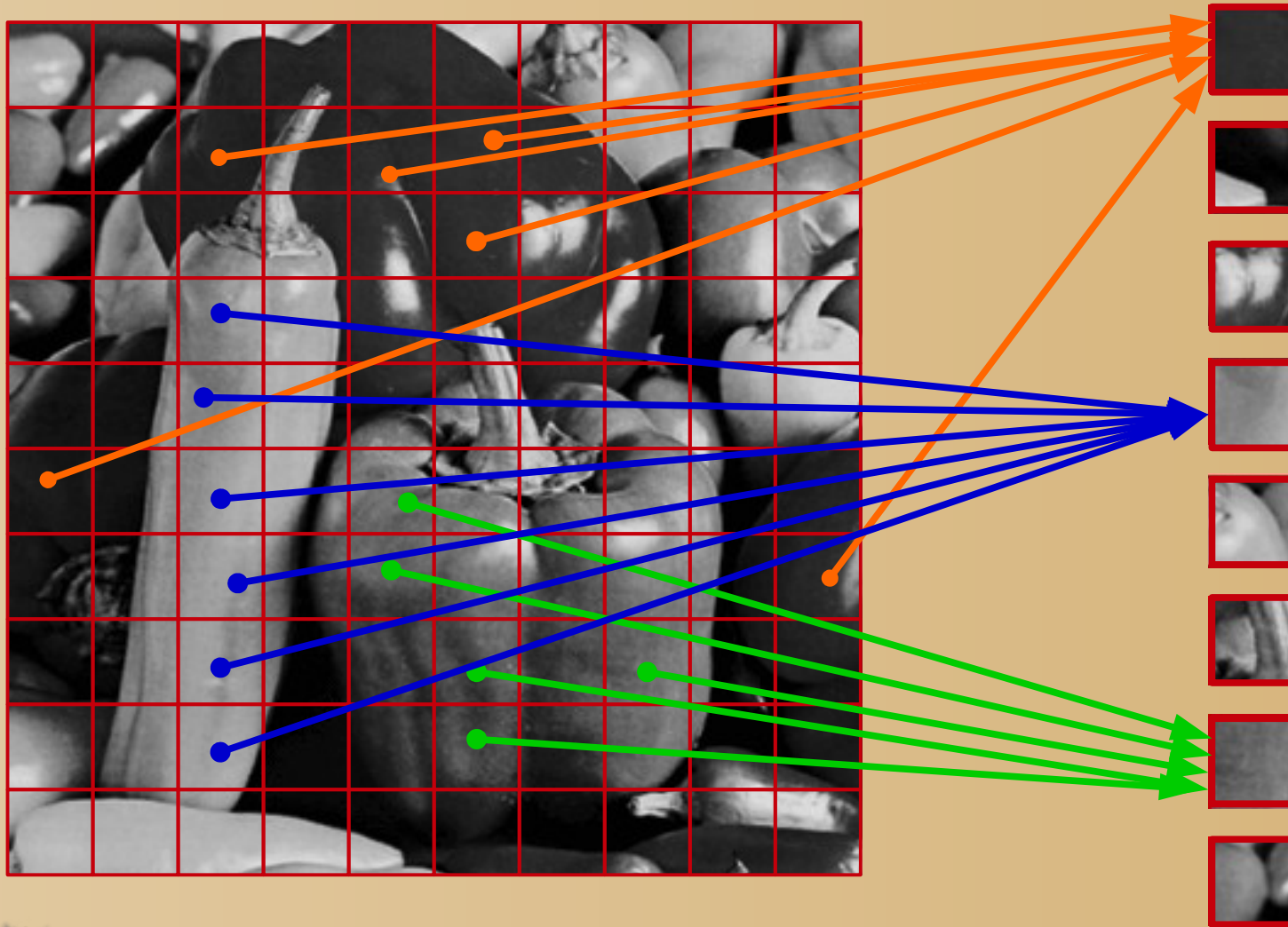
Divide Image into Blocks (Vectors)



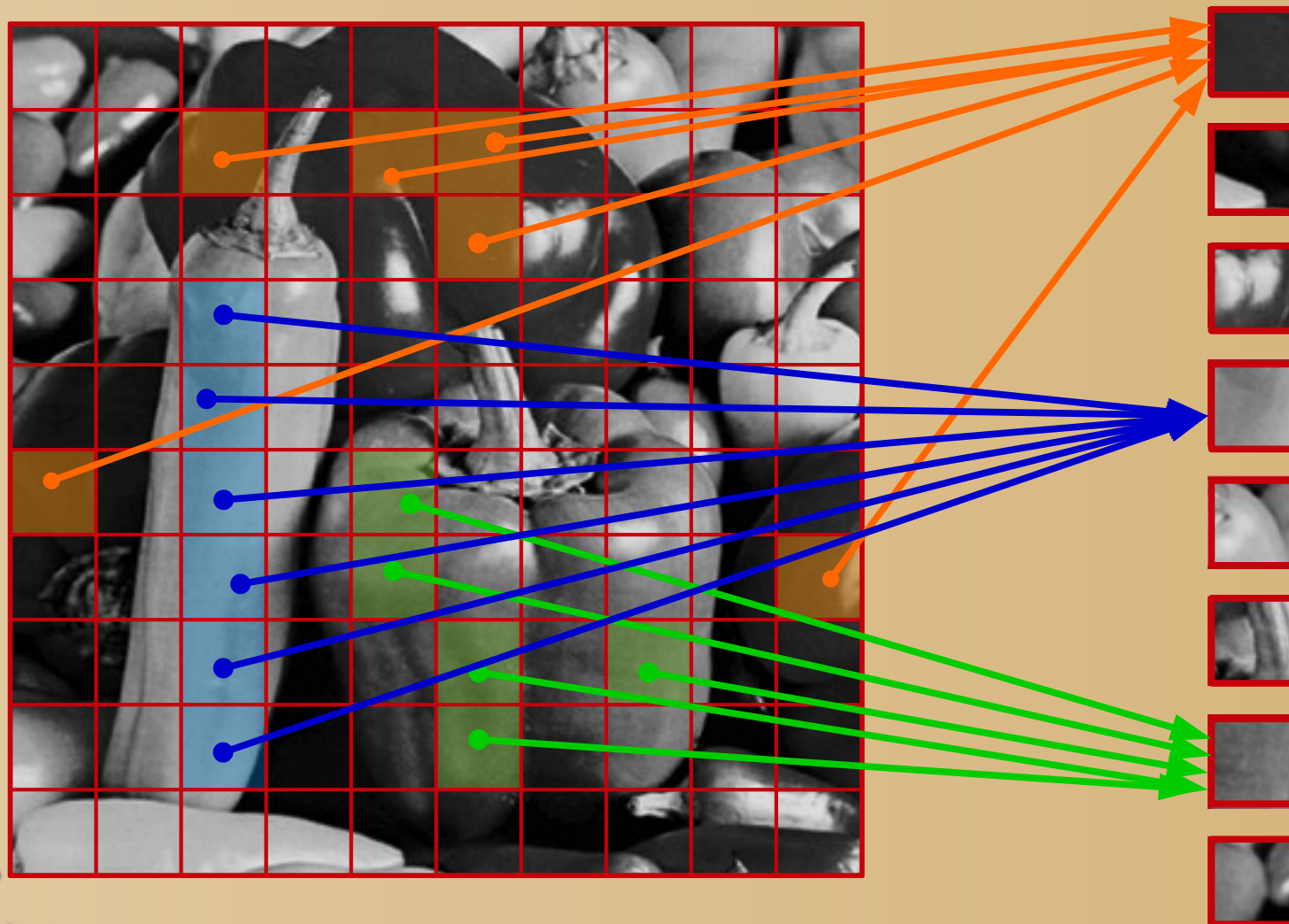
Generate Best “K” Vectors that can be used to Re-Construct Original Image



For Each Block in the Image, Select the Nearest Vector (Using Euclidean Distance)



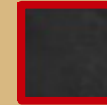
Label each Block in the image with INDEX of Nearest Vector (in the Codebook)



Label each Block in the image with INDEX of Nearest Vector (in the Codebook)



000



001



010



011



100



101



110



111



Compression Ratio

In order to Re-Construct the Image, it is required to have:

- **All Labels** (one label for each BLOCK in the Image)
- **The Codebook itself** which consists of K Vectors, each vector is a small Image with size equal to BLOCK size

Compression Ratio

Example:

The original GRAY image is **600*600 pixels** (each pixel is saved in one byte)

The image is divided into **Blocks each of size 4*4 pixels**

The Codebook (which will be used to Reconsturct the image) consists of 32 Vectors (**32 blocks each of size 4*4**)

Compression Ratio

Example:

Number Blocks in the image = $(600 * 600) / (4 * 4) = 22500$ Blocks

Number of labels = Number of Blocks = 22500 Labels

(Remember: Each label is Index in the Codebook)

As **Number of Vectors in the codebook = 32**, Indexes will range from 0 to 31 (from 00000 to 11111 Binary)

In other words, **each index can be saved in 5 Bits**

(this means **each label is 5 bits**)

Compression Ratio

Exact Compression Ratio:

$$\begin{aligned}\text{Labels Size} &= \text{Number of Labels} * \text{bits/Label} = 22500 * 5 \text{ bits} \\ &= 112500 \text{ bits (14063 Bytes)}\end{aligned}$$

Codebook Storage Size =

$$\begin{aligned}\text{Number of Vectors} * \text{Vector Size (in pixels)} * \text{number of bits to save a pixel} \\ = 32 * (4*4) * 8 \text{ bits} = 4096 \text{ bits (512 bytes)}\end{aligned}$$

$$\begin{aligned}\text{Total Compressed Image Size} &= \text{Label Size} + \text{Code book storage size} \\ &= 112500 + 4096 = 116596 \text{ bits (14575 bytes)}\end{aligned}$$

$$\begin{aligned}\text{Original Image Size} &= 600 * 600 \text{ (pixels)} * 8 \text{ bits/pixel} = 2880000 \text{ bits} \\ &= 360000 \text{ Bytes}\end{aligned}$$

$$\text{Compression Ratio} = \text{Original} / \text{Compressed} = 360000 / 14575 = 24.7:1$$

Compression Ratio

Approximate Compression Ratio:

When Label Size is much greater than Codebook storage size, Codebook Storage size can be neglected during Compression ratio Calculations.

(Remember: Codebook Storage size is independent of Image size, number of Labels depends on Image Size.)

Labels Size = Number of Labels * bits/Label = 22500 * 5 bits = 112500 bits (14063 Bytes)

Codebook Storage Size = $32 * (4*4) * 8 \text{ bits} = 4096 \text{ bits}$ (512 bytes) [can be neglected w.r.t 14063 bytes)

Total Compressed Image Size \approx Label Size = 112500 bits (14063 bytes)

Original Image Size = $600 * 600 \text{ (pixels)} * 8 \text{ bits/pixel} = 2880000 \text{ bits}$ (360000 Bytes)

Compression Ratio = Original / Compressed = $360000/14063 = 25.6:1$

Compression Ratio

Approximate Compression Ratio:

Also, Compression ratio can be calculated approximately on Block bases (not on image bases)

Each block in image is originally stored in $(4*4)$ pixels * 8 bits/pixel

$$= 4*4*8=128 \text{ bits}$$

After compression, Each block is substituted with Label of size 5 bits (as $2^5 = 32$ vectors in the Codebook)

The **compression ratio** = $128: 5 = 25.6:1$ (same as before)

Compression Ratio

What if image is 6000 x 6000, Exact Compression Ratio:

$$\begin{aligned} \text{Labels Size} &= \text{Number of Labels} * \text{bits/Label} = (6000*6000)/(4*4) * 5 \text{ bits} \\ &= 11,250,000 \text{ bits (1,406,300 Bytes)} \end{aligned}$$

Codebook Storage Size =

$$\begin{aligned} &\text{Number of Vectors} * \text{Vector Size (in pixels)} * \text{number of bits to save a pixel} = 32 \\ &* (4*4) * 8 \text{ bits} = 4096 \text{ bits (512 bytes) [No change, Independent of Image Size]} \end{aligned}$$

Total Compressed Image Size = Label Size + Code book storage size

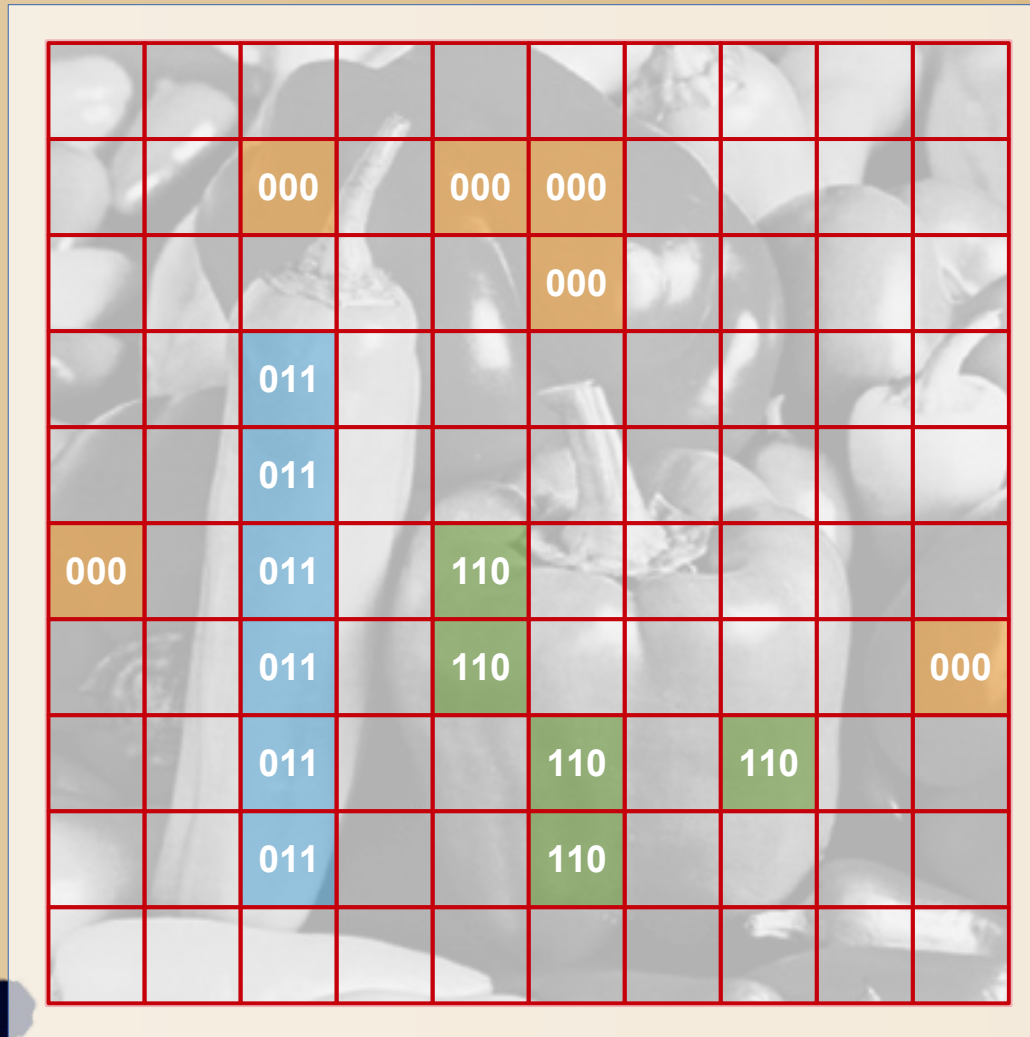
$$= 11250000 + 4096 = 11,254,096 \text{ bits (1,406,762 bytes } \sim \text{1,406,300)}$$

$$\begin{aligned} \text{Original Image Size} &= 6000 * 6000 \text{ (pixels)} * 8 \text{ bits/pixel} = 288000000 \text{ bits} \\ &\text{(36,000,000 Bytes)} \end{aligned}$$

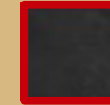
$$\text{Compression Ratio} = \text{Original} / \text{Compressed} = 36,000,000 / 1,406,762 = 25.6:1$$

(Almost as Approximate Compression ratio)

Image Reconsturction



000



001



010



011



100



101



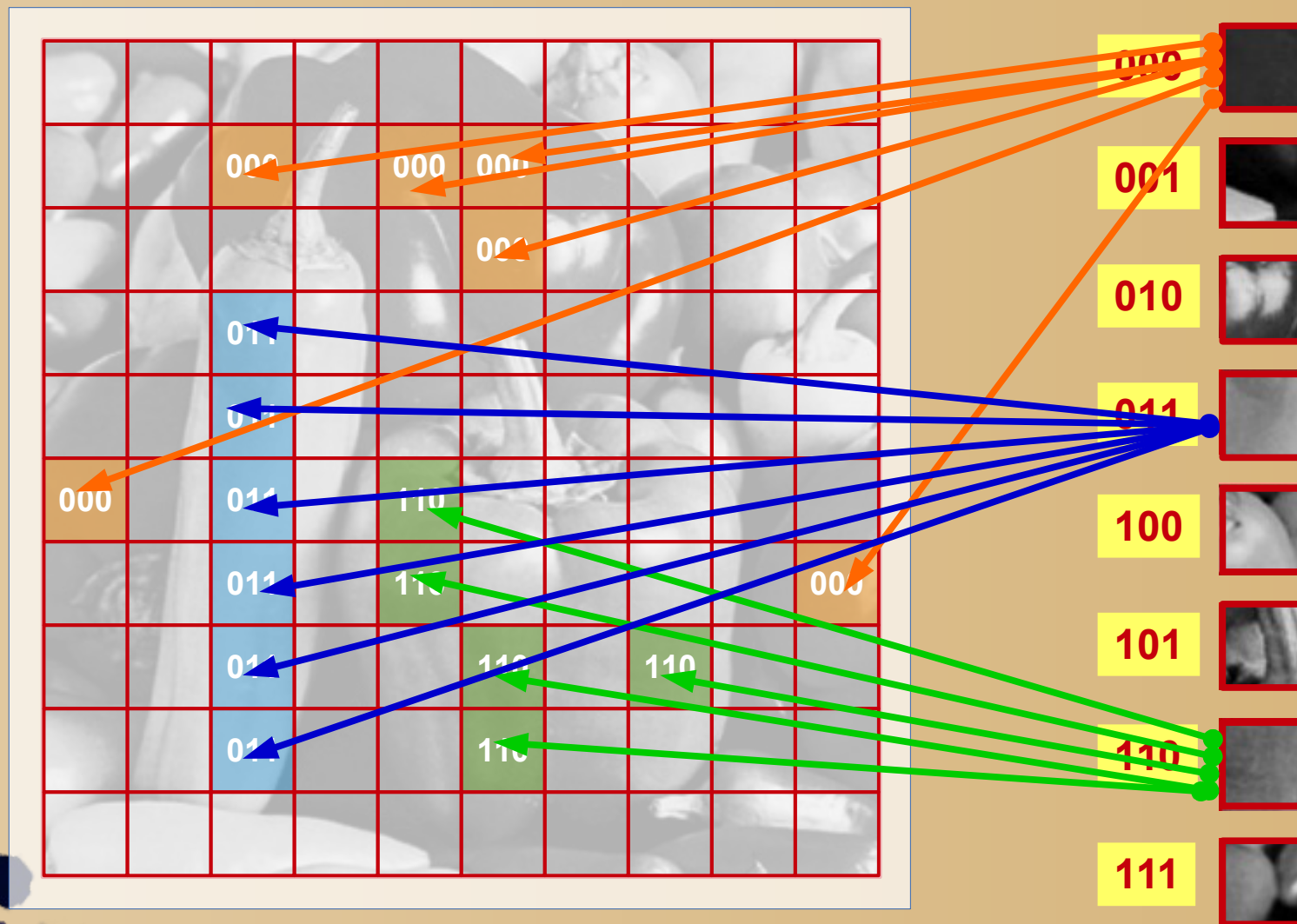
110



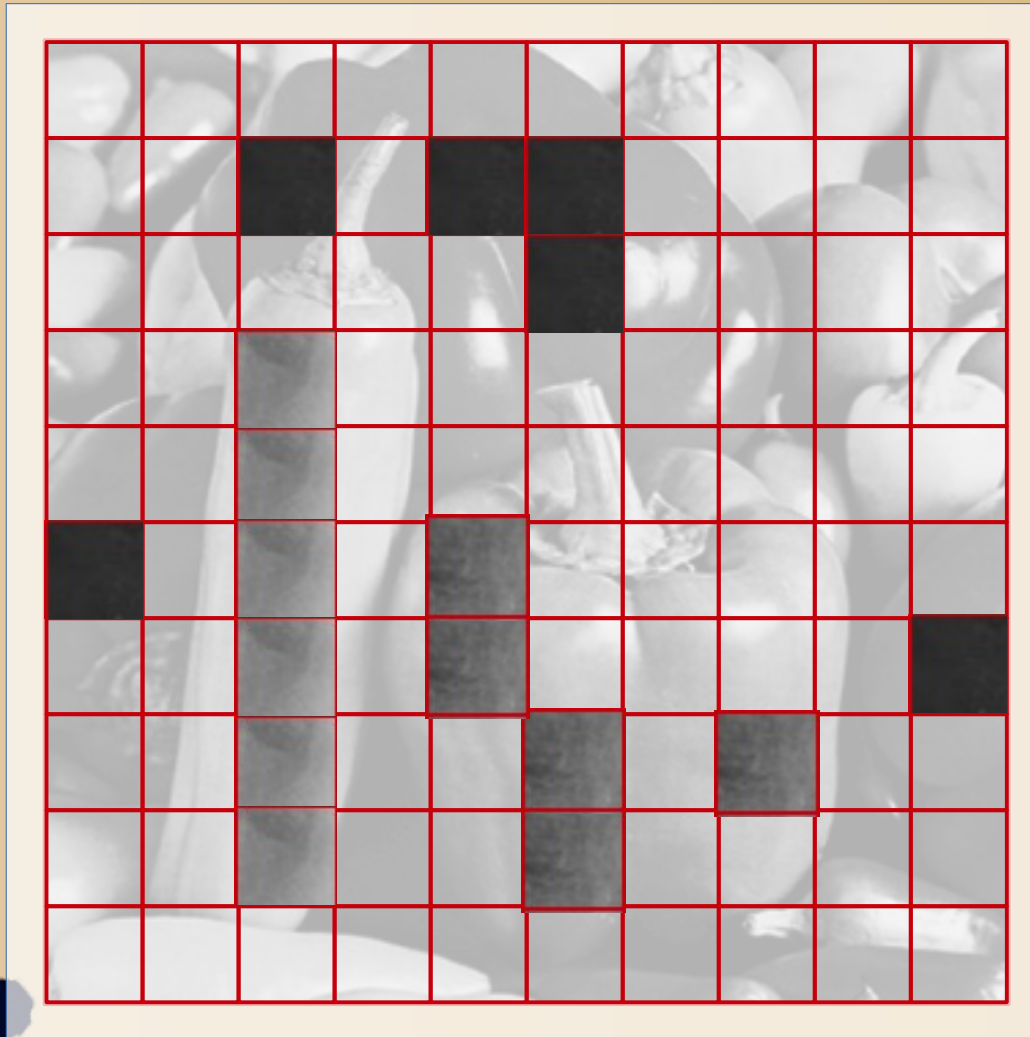
111



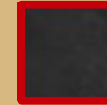
Substitute Each LABEL with Corresponding Vector in the Codebook



Obtained Constructed Image



000



001



010



011



100



101



110



111



Vector Quantization Samples



Original

Blocking Effect

Vector Size is Large

Number of Vectors in codebook is small

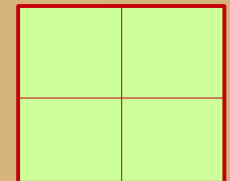
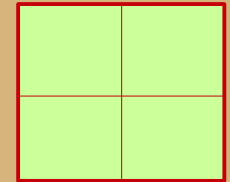
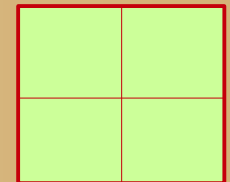
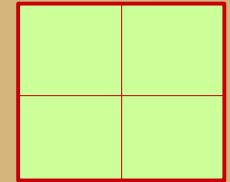
Vector Quantization using Splitting (Example)

*Compress the following Image Using Vector Quantization
(initialize LBG Algorithm using Splitting)
(Each pixel is saved in 8 bits)*

Vector size = 2*2

Number of Vectors in Codebook = 4

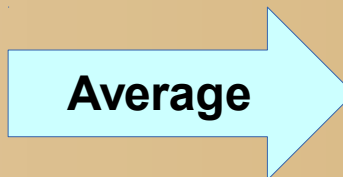
1	2	7	9	4	11
3	4	6	6	12	12
4	9	15	14	9	9
10	10	20	18	8	8
4	3	17	16	1	4
4	5	18	18	5	6



**Reconstruct the Compressed Image,
Calculate Mean Square error between Original and Reconstructed Image
Calculate Compression Ratio
Re-Calculate Compression Ratio if the image is 600*600 pixels**

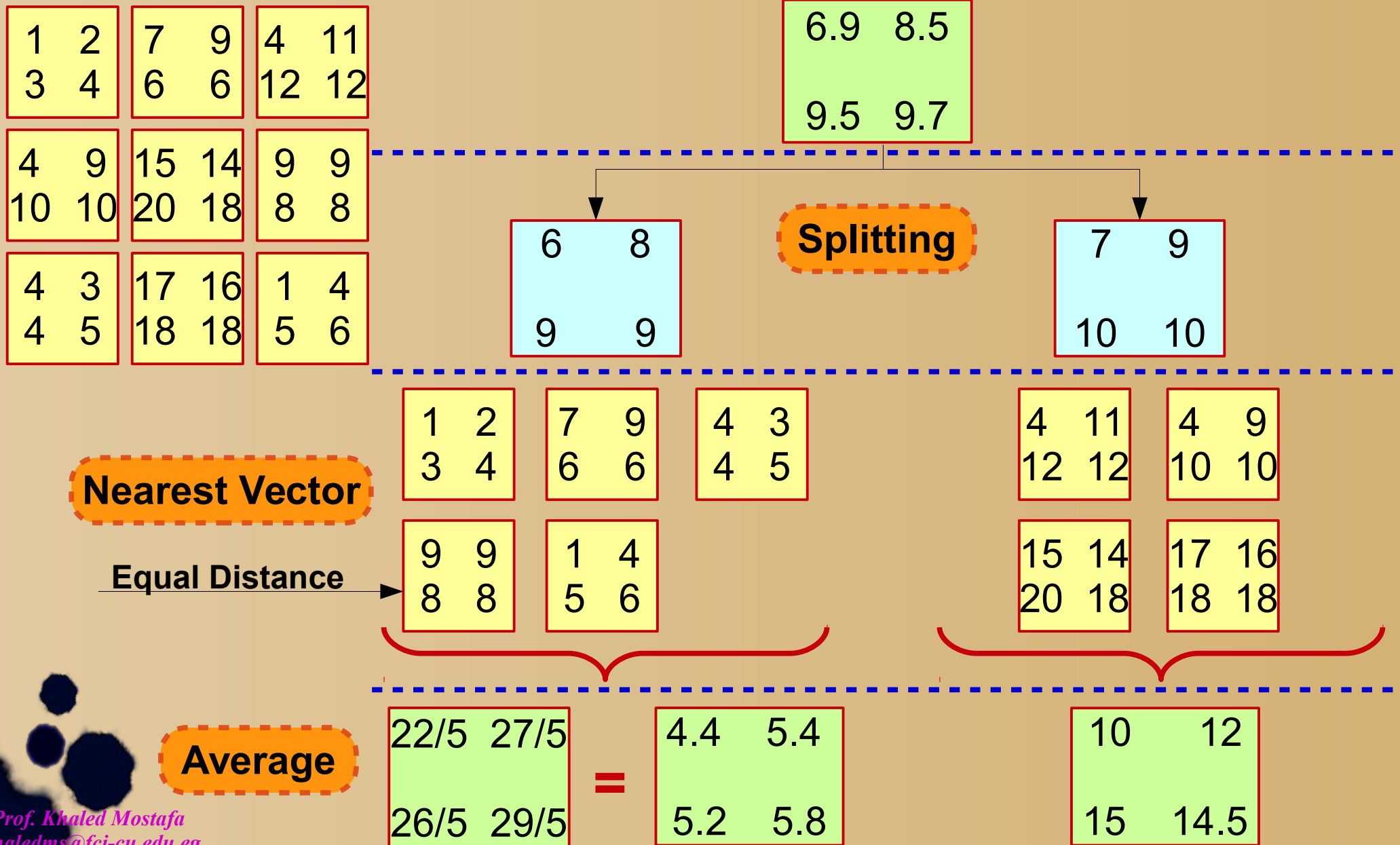
Vector Quantization using Splitting (Apply Splitting)

1 2 3 4	7 9 6 6	4 11 12 12
4 9 10 10	15 14 20 18	9 9 8 8
4 3 4 5	17 16 18 18	1 4 5 6



$$\begin{array}{cc} 62/9 & 77/9 \\ 86/9 & 87/9 \end{array} = \begin{array}{cc} 6.9 & 8.5 \\ 9.5 & 9.7 \end{array}$$

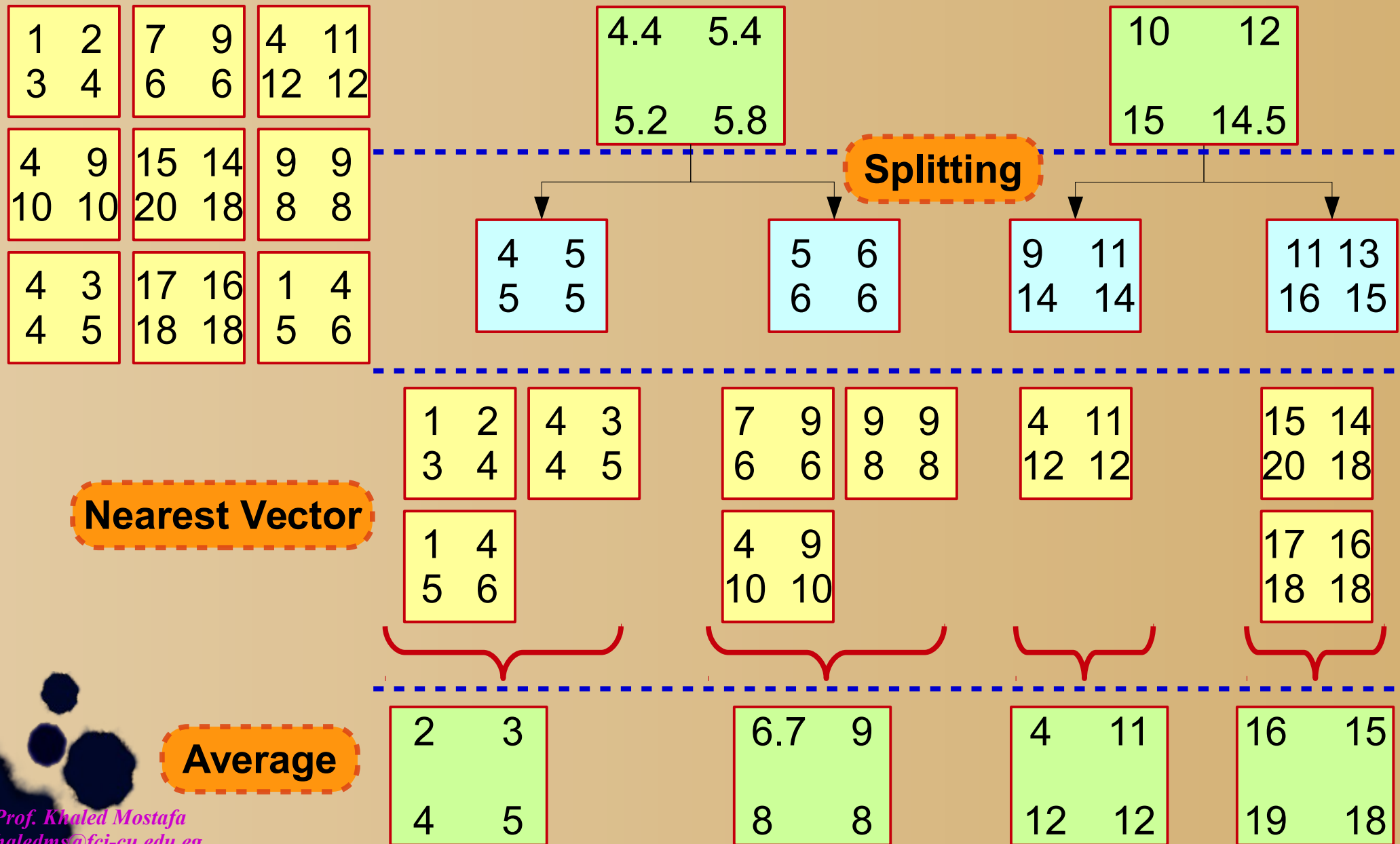
Vector Quantization using Splitting (Apply Splitting)



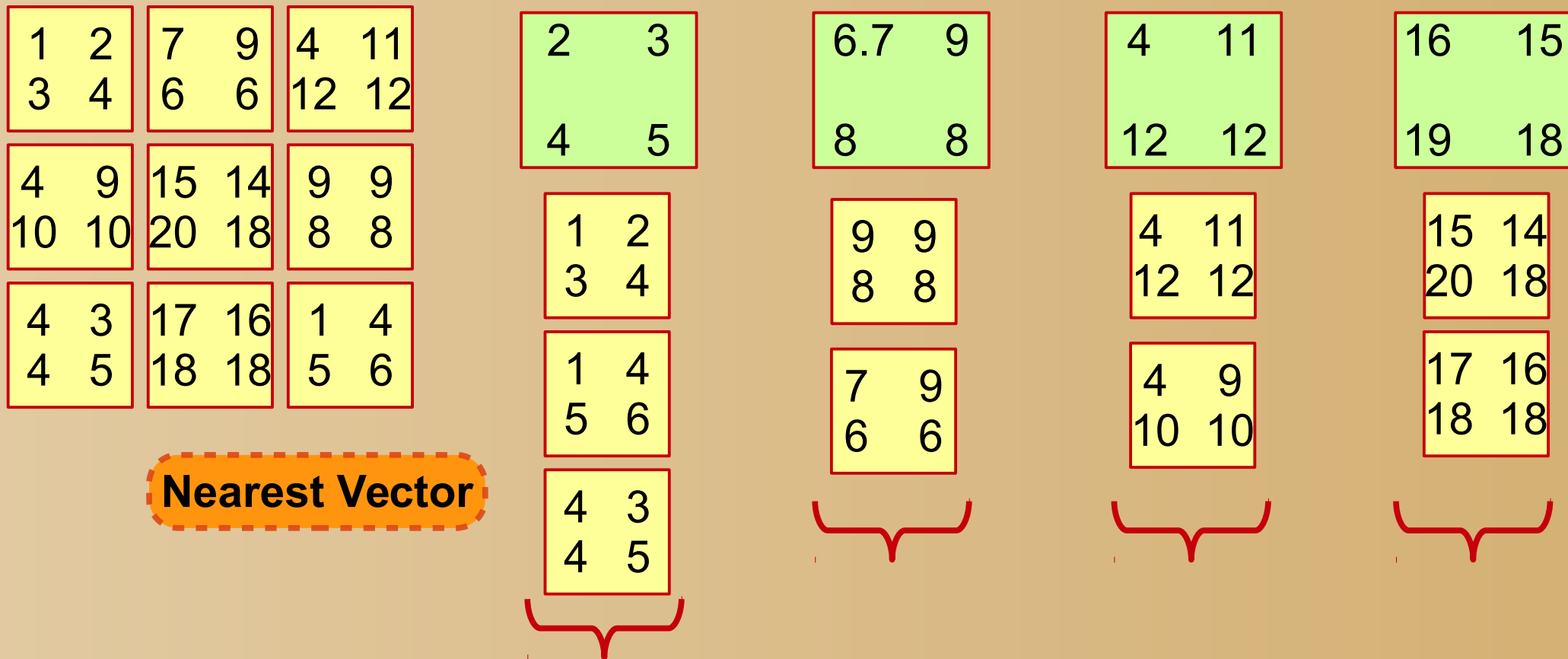
Vector Quantization using Splitting (Apply Splitting)



22



Vector Quantization using Splitting (LBG Algorithm)

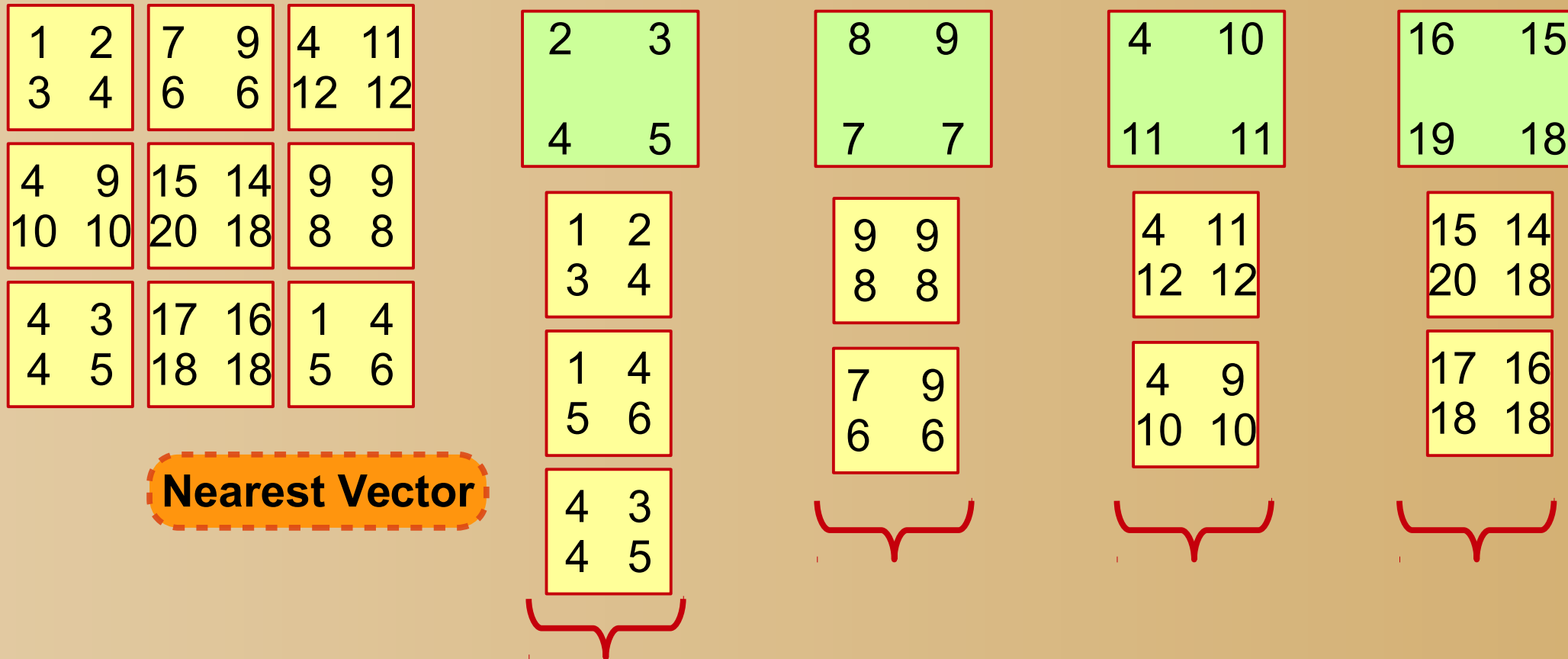


Average

2 3 4 5	8 9 7 7	4 10 11 11	16 15 19 18
------------	------------	---------------	----------------

Changed

Vector Quantization using Splitting (LBG Algorithm)



Average

2 3 4 5	8 9 7 7	4 10 11 11	16 15 19 18
------------	------------	---------------	----------------

No Change (Stop Iteration)

Image Encoding

**Original
Image**

1 2 3 4	7 9 6 6	4 11 12 12
4 9 10 10	15 14 20 18	9 9 8 8
4 3 4 5	17 16 18 18	1 4 5 6

Code Book

00

2 3
4 5

01

8 9
7 7

10

4 10
11 11

11

16 15
19 18

Nearest Vector

**Compressed
Image**

00	01	10
10	11	01
00	11	00

Image Decoding

Compressed Image

00	01	10
10	11	01
00	11	00

Reconstructed Image

2 3 4 5	8 9 7 7	4 10 11 11
4 10 11 11	16 15 19 18	8 9 7 7
2 3 4 5	16 15 19 18	2 3 4 5

Code Book

00

2	3
4	5

01

8	9
7	7

10

4	10
11	11

11

16	15
19	18

Look Up

Mean Squared Error

**Original
Image**

1 2	7 9	4 11
3 4	6 6	12 12
4 9	15 14	9 9
10 10	20 18	8 8
4 3	17 16	1 4
4 5	18 18	5 6

**Reconstructed
Image**

2 3	8 9	4 10
4 5	7 7	11 11
4 10	16 15	8 9
11 11	19 18	7 7
2 3	16 15	2 3
4 5	19 18	4 5

**Squared
Error**

1 1	1 0	0 1
1 1	1 1	1 1
0 1	1 1	1 0
1 1	1 0	1 1
4 0	1 1	1 1
0 0	1 0	1 1

$$\text{Mean Squared Error} = 30/36 = 0.833$$

Compression Ratio

- **Original Image Size**=

$$6*6 \text{ (pixels)} * 8 \text{ bits/pixel} = 6*6*8 = 288 \text{ bits}$$

- **Number of Blocks** (vectors) in Image =

$$(6*6) / (2*2) = 36/4 = 9 \text{ blocks}$$

- Each Block is substituted by 2 Bits Label

- **Labels size** = 9 blocks * 2 bits = 18 bits

- **Codebook size** =

$$4 \text{ Vectors} * (2*2) \text{ pixels/vector} * 8 \text{ bits/pixel} = 4*2*2*8 = 128 \text{ bits}$$

- **Total Compressed size** = Codebook + Labels = 128 + 18 = 146 bits

- **Compression Ratio** = $288/146 = 1.97:1$

Compression Ratio

- **Original Image Size**=

$$600*600 \text{ (pixels)} * 8 \text{ bits/pixel} = 6*6*8 = 2,880,000 \text{ bits}$$

- **Number of Blocks** (vectors) in Image =

$$(600*600) / (2*2) = 360000 / 4 = 90,000 \text{ blocks}$$

- Each Block is substituted by 2 Bit Label
- **Labels size** = 90,000 blocks * 2 bits = 180,000 bits
- **Codebook size** = 128 bits (as before)
- **Total Compressed size** = 128 + 180,000 = ~ 180,000 bits
- **Compression Ratio** = 2,880,000/180,000 = 16:1

(each 4 pixels = 32 bits are substituted with 2 bits label)