Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Time Period of a Simple Pendulum
Lab Report 0

Ayaan Dutt
Lab partner: Raheem Manoj

Professor Bikram Phookun
TAs: Ankit Shrestha, Chandra shekhar Mishra, Kamal Nayan
TF: Uzma Khan

Date of Experiment: January 23rd, 2023
Date of Submission: February 13th, 2023

Aim

To determine the relationship between the time period of a simple pendulum with the following adjustable physical parameters of the system:

- Mass of the bob
- Length of the pendulum
- Amplitude of release

Experimental Setup

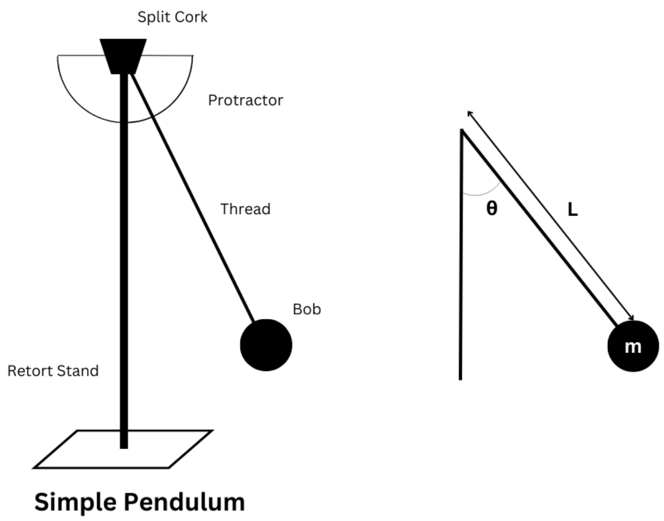

Figure 1: Experimental setup and schematic diagram of a simple pendulum

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

List of Apparatus:

- Retort Lab Stand
- Split Cork
- Thread
- Metallic Bob
- Measuring Tape
- Protractor
- Stopwatch
- Weighing Scale

Least Count of Tape Measure = 0.1 cm
Least Count of Weighing Scale = 0.1 g
Least Count of Stopwatch = 0.01 s
Least Count of Protractor = 1°

Theoretical Background

Though it is commonly known that the time period of a pendulum is only determined by its length, amplitude, and gravitational acceleration acting on the bob, as described in equation (1), the purpose of this experiment is to verify this theoretical result.

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_o^2 + \frac{11}{3072}\theta_o^4 + \dots\right) \tag{1}$$

Given this information, we should expect to see no change in time period with respect to the mass of the bob but a non-linear relationship with the length of the pendulum. We should also expect to see a deviation from the mean time period at large amplitudes of release.

Procedure

Each variable (i.e. mass of the bob, length of the pendulum, and amplitude of release) was individually varied while keeping the rest of the parameters constant in order to determine their specific effect on time period.

Step 1: Measure the mass of the bob with the weighing scale.
[Precaution: Keep the bob in the center of the scale to ensure equal distribution of weight]

Step 2: Measure the length of the pendulum with the tape measure by lining the tape vertically with the thread. Take readings at the top and the bottom of the bob and average these values to determine the length till the center of mass of the bob (Figure 2).

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

$$L = \frac{L_1 + L_2}{2} \tag{2}$$

[Precaution: Ensure that the thread is taunt and vertically suspended while measuring. Use a flat surface to mark the length from the top and bottom of the bob.]

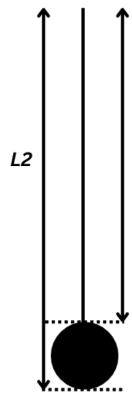

Figure 2: Determining the length of the pendulum to the center of mass of the bob

Step 3: Hold the bob lightly and bring it up until the thread aligns with the desired amplitude of release on the protractor. Release the bob quickly and allow for 2 oscillations before starting to count the oscillations (This is to reduce the chance of error between release and starting the stop watch). Take 4 readings for the time period of 10 oscillations for a given set of parameters before changing the independent variable.
[Precaution: Keep the thread taunt before release and ensure that the bob does not bounce or swing elliptically after release. Start counting the oscillations from 0, not 1, to avoid miscounting. Also, start and stop counting when the bob reaches zero velocity to allow for ease in measurement.]

Step 4: Divide each reading by 10 to obtain the average time period of 1 oscillation and then average across readings to get a mean time period. Compared the mean time periods while the independent variable changes to determine its relationship with the time period of a simple pendulum.
[Precaution: Keep in mind that the number of digits after processing the data is not necessarily indicative of the degree of accuracy of the data.]

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Observations

0.1 Variation with Mass (Part A)

Constant parameters:
Length of Pendulum = 40.8 cm
Amplitude of Release = 30°

Mass of Bob (g)	Time of 10 oscillations (s)				Time of 1 oscillation (s)				Avg. Time (s)
M	T_1	T_2	T_3	T_4	t_1	t_2	t_3	t_4	t_{avg}
11.8	12.88	13.00	12.91	12.94	1.288	1.300	1.291	1.294	1.293
22.7	12.97	12.91	12.94	13.00	1.297	1.291	1.294	1.300	1.297
25.5	12.90	12.88	12.84	12.91	1.290	1.288	1.284	1.291	1.288
26.8	13.00	12.97	12.99	12.94	1.300	1.297	1.299	1.294	1.298
31.8	13.00	13.00	12.96	12.97	1.300	1.300	1.296	1.297	1.298

Table 1: Time period of a simple pendulum with different masses

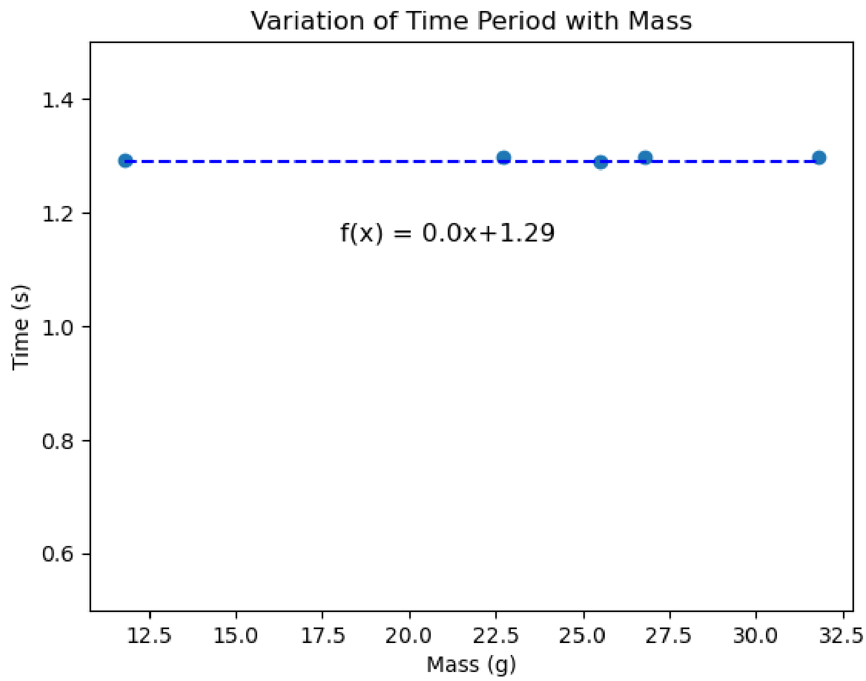

Figure 3: The zero slope of the graph appears to indicate that the time period of the pendulum is independent of the mass of the bob

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

0.2 Variation with Length (Part B)

Constant parameters:
Mass of Bob = 26.8g
Amplitude of Release = 30°

Length (cm)	Time of 10 oscillations (s)				Time of 1 oscillation (s)				Avg. Time (s)
L	T_1	T_2	T_3	T_4	t_1	t_2	t_3	t_4	t_{avg}
40.8	13.00	12.97	12.99	12.94	1.300	1.297	1.299	1.294	1.298
35.8	12.25	12.28	12.25	12.22	1.225	1.228	1.225	1.222	1.225
30.8	11.29	11.22	11.22	11.20	1.129	1.122	1.122	1.120	1.123
25.8	10.41	10.38	10.28	10.34	1.041	1.038	1.028	1.034	1.035
20.8	9.25	9.16	9.25	9.25	0.925	0.916	0.925	0.925	0.923
15.8	8.19	8.28	8.15	8.15	0.819	0.828	0.815	0.815	0.819

Table 2: Time period of a simple pendulum with different lengths

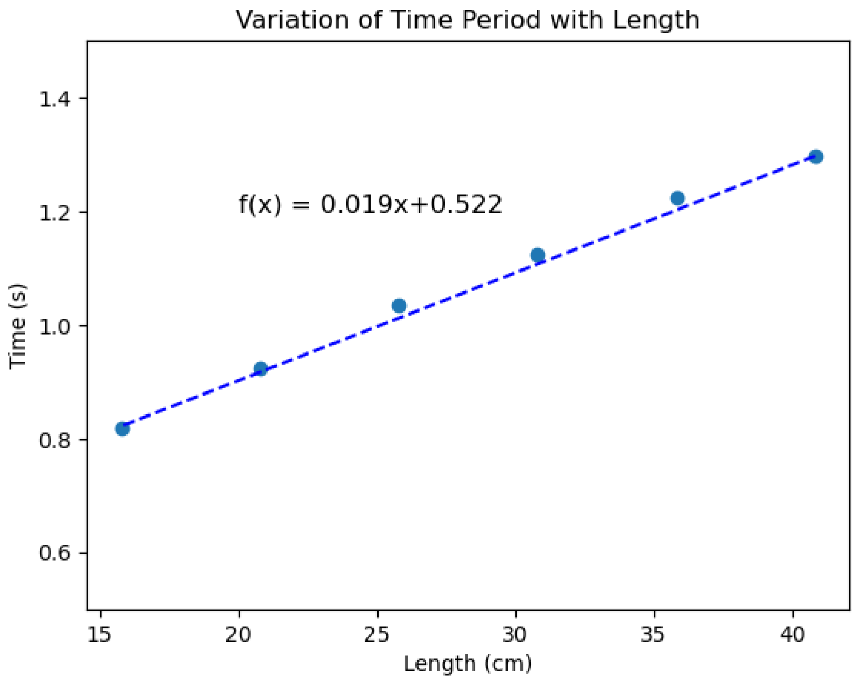

Figure 4: If we observe closely, the data is not accurately approximated by a straight line. This suggests that the time period and the length of the pendulum have a non-linear relationship.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

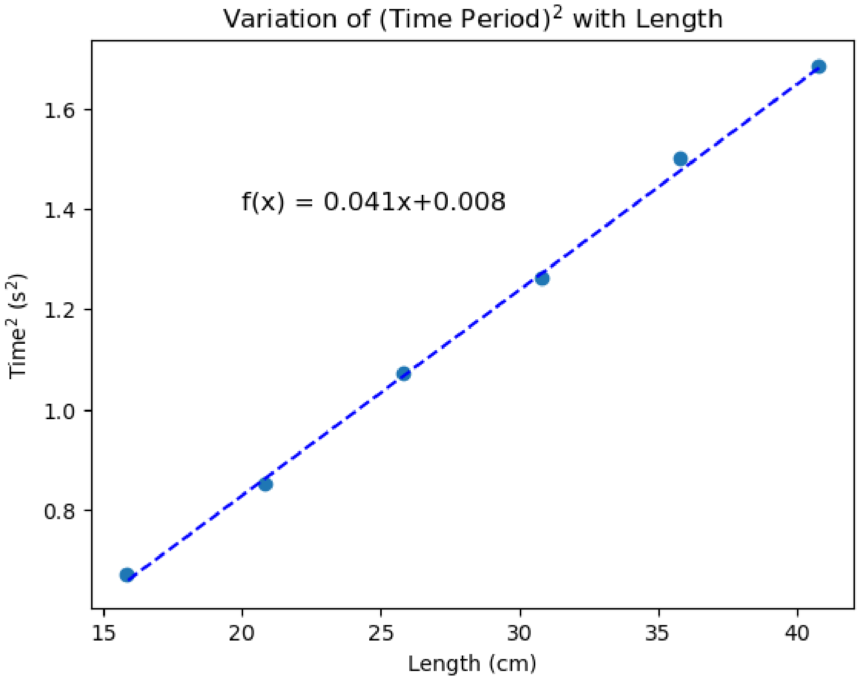

Figure 5: This data is accurately modelled by a straight line, suggesting that the square of the time period is directly proportional to the length of the pendulum.

0.3 Variation with Amplitude (Part C)

Constant parameters:
Mass of Bob = 26.8g
Length of Pendulum = 30.8 cm

Amplitude (°/rad)	Time of 10 oscillations (s)				Time of 1 oscillation (s)				Avg. Time (s)
θ	T_1	T_2	T_3	T_4	t_1	t_2	t_3	t_4	t_{avg}
$15°/\frac{\pi}{12}$ rad	11.13	11.13	11.12	11.15	1.113	1.113	1.112	1.115	1.113
$30°/\frac{\pi}{6}$ rad	11.29	11.22	11.22	11.20	1.129	1.122	1.122	1.120	1.123
$45°/\frac{\pi}{4}$ rad	11.44	11.44	11.47	11.31	1.144	1.144	1.147	1.131	1.142
$60°/\frac{\pi}{3}$ rad	11.78	11.68	11.69	11.72	1.178	1.168	1.169	1.172	1.172
$75°/\frac{5\pi}{12}$ rad	12.06	12.06	12.00	12.09	1.206	1.206	1.200	1.209	1.205

Table 3: Time period of a simple pendulum with different amplitudes

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

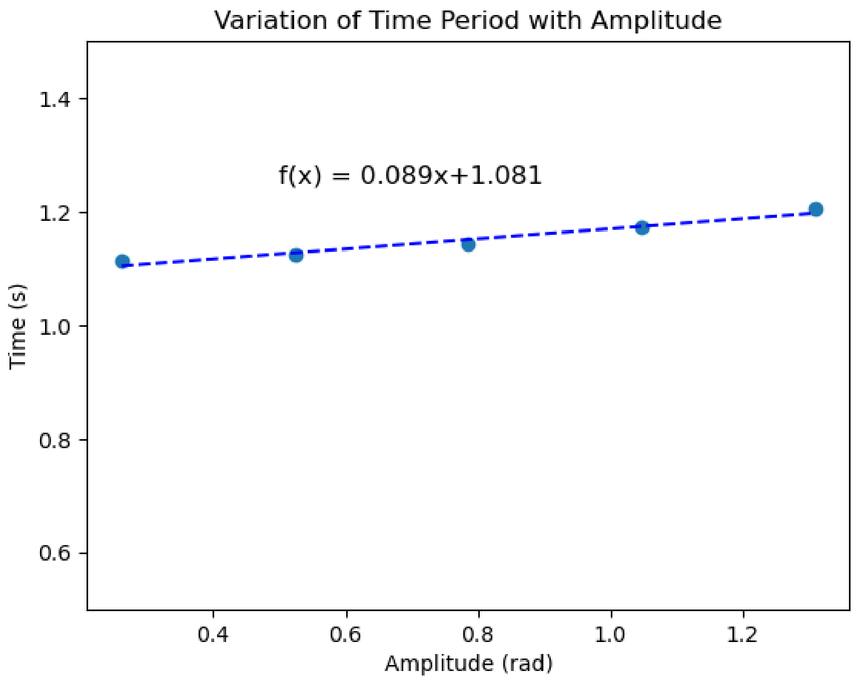

Figure 6: The time period appears to vary slightly due to larger amplitudes, however the low slope indicates a negligible rate of increase.

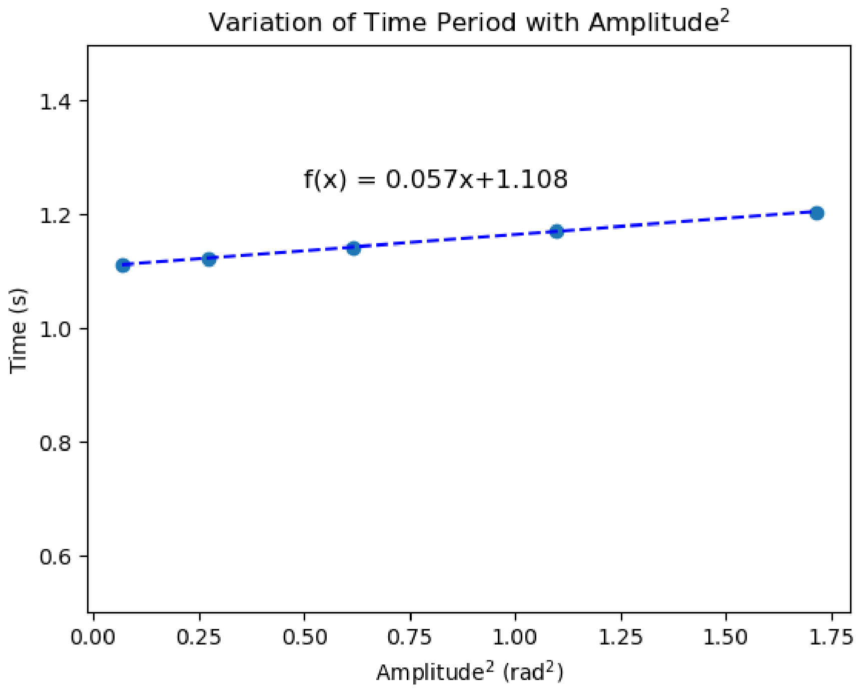

Figure 7: A straight line fits this data even better than the graph of time period versus amplitude. This suggests that time period varying with the square of the amplitude of release in radians is a better approximation of the relationship between the two parameters.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Analysis

Variation with Mass (Part A):

As seen in Figure 3, the time period shows negligible variation with respect to mass, resulting in a zero slope line. This is in accordance with our theoretical understanding of a simple pendulum whose time period is independent of the mass of the bob.

Variation with Length (Part B):

Though the graph between time period and length of the pendulum (Figure 4) appears nearly linear, we know from our theoretical understanding that it must be non-linear. This is evidenced by the greater deviation of central data points from the best fit straight line, suggesting a somewhat parabolic trend instead.

The time period^2 vs length of the pendulum graph (Figure 5) displays a more uniform linear trend, with a slope of $0.041s^2/cm = 4.1s^2/m$. If we use this value as the proportionality constant between T^2 and L, we can predict the value of g *[Note: we do not include amplitude in our calculations since the effect of small amplitudes is negligible]*:

$$\frac{T^2}{L} = \frac{4\pi^2}{g}$$
$$\implies \frac{4\pi^2}{g} = 4.1$$
$$\implies g = \frac{4\pi^2}{4.1}$$
$$\implies g = \boxed{9.629m/s^2}$$

This value is approximately equal to the true value of $g = 9.807m/s^2$. Hence, we can conclude that $T^2 \propto L$.

Variation with Amplitude (Part C):

The graph between amplitude of release and time period (Figure 6) shows a slight increase in time period as the amplitude is raised. This appears to correspond to our understanding that only large amplitudes have a significant impact on the time period of a pendulum.

In Figure 7, the data appears to fit the straight line more accurately, suggesting that $T \propto \theta^2$. However, more data must be collected to verify the true mathematical relationship between time period and amplitude.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Error Analysis

For small amplitudes, the factors that could contribute to the error in the value of g are the length (L) and time period (T) of the pendulum. The relative error in g can be found by the following formula:

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T} \tag{3}$$

Length error can be estimated to be equal to the least count of the measuring device (i.e. $1mm$). However, the error in time period may arise from both the least count of the stopwatch and the human reaction time involved in the measurement. To determine which error is more significant, an experiment was devised (see Appendix 2) and the human error was found to be around $0.05s$, which is greater than the least count error. Hence, for a given length:

$$L = L_m \pm \Delta L = 0.308 \pm 0.001m$$

$$T = T_m \pm \Delta T = 1.12 \pm 0.05s$$

$$g_m = 9.629$$

$$\implies \frac{\Delta g}{g_m} = \frac{0.001}{0.308} + 2\frac{0.05}{1.12} = 0.093$$

$$\implies \Delta g = 0.093 \times 9.629 = 0.895m/s^2$$

$$\boxed{g = 9.629 \pm 0.895m/s^2}$$

The true value of $g = 9.807m/s^2$ lies within this range. This means that the margin of error in our experiment is acceptable.

Results

- The time period of a simple pendulum was found to be independent of the mass of the bob (M).
- The time period is directly proportional to the square root of length of the pendulum ($T^2 \propto L \implies T \propto \sqrt{L}$).
- The time period also appears to be directly proportional to the square of the amplitude of release ($T \propto \theta^2$).
- The value of g was found to be $9.629 \pm 0.895m/s^2$.

Discussion

Note that the contribution of length error is approximately 1 order of magnitude larger than the time period error. To improve our experiment we can reduce the error in time period by taking measurements for 50-100 oscillations instead of 10 oscillations.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Furthermore, a greater number of readings for all variables would be required to see the general trend of amplitude and length versus time period. The given data sets suggest nearly linear trends, which we know to be theoretically false.

Appendix 1

The data for the time period against the various parameters can be found below:

- Mass vs Time Period.txt
- Length vs Time Period.txt
- Amplitude vs Time Period.txt

The graphs of this data were plotted using the following code - Simple Pendulum Graphs.ipynb.

Appendix 2

Aim: To determine the average human error involved in the measurement of time using a stopwatch.

Procedure:

- A program that displays a coloured box for a definite amount of time after a random number of seconds was run. A stopwatch with least count 0.01s was used to time the time between the appearance and disappearance of the box.
- This was repeated 22 times to gather sufficient data points and the mean and standard deviation of the data was found.

Observations:

Sr. No.	Time (s)
1	4.88
2	4.84
3	4.79
4	4.72
5	4.81
6	4.75
7	4.72
8	4.84
9	4.88

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Sr. No.	Time (s)
10	4.82
11	4.75
12	4.81
13	4.82
14	4.81
15	4.78
16	4.72
17	4.84
18	-
19	4.81
20	4.78
21	4.88
22	4.75

Table 4: Time measurements of the duration of display of the coloured box

Mean = 4.8s

Standard Deviation = 0.051s

The standard deviation of the measurements has been assumed as the approximate human error.

The data and calculations for the reaction time experiment to determine human error and the planetary data graphs can be found at Reaction Time Experiment.ipynb.

This sheet also contains an exercise in graphing planetary data and determining the relationship between time period and the semi-major axis of a body orbiting the sun. From the log graph we can conclude that the relationship is $T^2 \propto R^3$ since the slope is $1.5 = \frac{3}{2}$.