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Diffraction of Light Through a Plane Grating, Single Slit, Wire and Circular Aperture

Lab Report 2

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Aim

- Part A: To determine the wavelength λ of the light emitted by a laser source by studying the diffraction of light due to plane diffraction gratings.
- Part B: To determine the width of the given single slit by studying its diffraction pattern.
- Part C: To determine the diameter of a given wire/hair strand by studying its diffraction pattern.
- Part D: To determine the size of the circular aperture by studying its diffraction pattern.
- Part E: To determine the thickness, pitch and angle of a single helical spring by studying its diffraction pattern.

Experimental Setup

List of apparatus:

- Optical bench
- 10 mW semiconductor red laser source with a mount
- 5 mW DPSS green laser source with a mount
- Measuring tape
- Small plastic scale with least count 0.5 mm
- Graph sheet
- Masking tape
- Set of plane diffraction gratings with grating spacing 100 lines/mm, 200 lines/mm and 600 lines/mm with a holder and mounts
- Single slit of fixed width mounted on a slide
- Thin wire/hair strand mounted on a slide

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- Set of circular apertures mounted on a slide
- Single helix (spring) set in a holder with a mount
- Spirit level

Least count of measuring tape = 0.001 m
Least count of small plastic scale = 0.0005 m

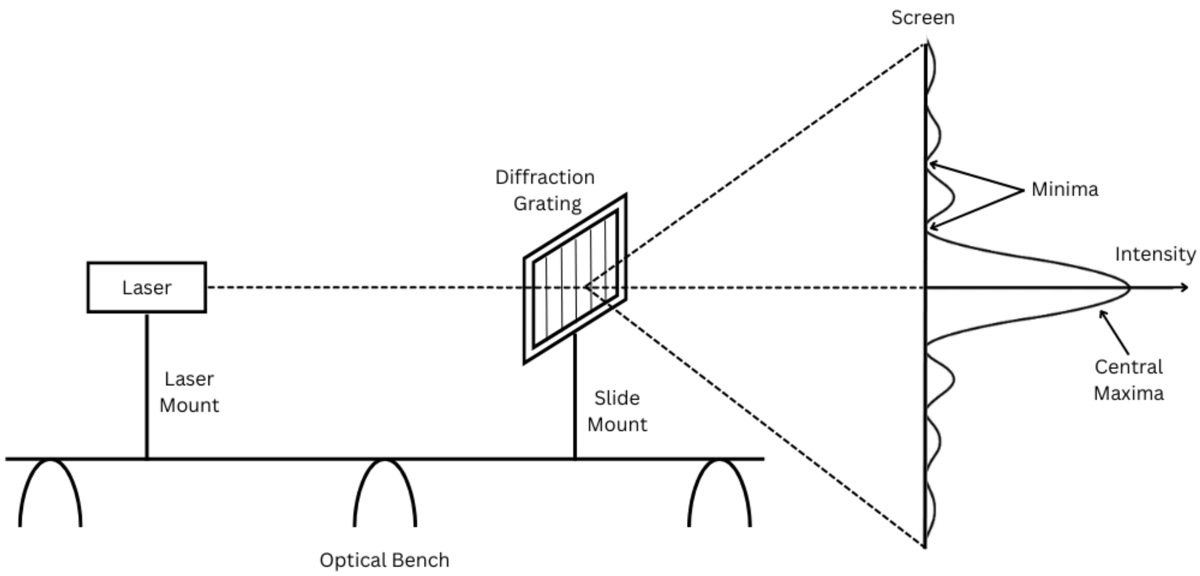

Figure 1: Experimental setup for the diffraction of light

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Theoretical Background

Single Slit (or Thin Wire)

According to Babinet's Principle, complementary objects produce the same diffraction pattern, except for the intensity of the central maxima. This implies that a narrow single slit and a single thin wire/hair strand are complementary since they produce the same diffraction pattern and can be understood by the same theory.

Single slit diffraction (also known as, Fraunhofer diffraction) gives a pattern of varying intensity consisting of a bright central maximum with alternate minima and maxima of decreasing intensity on either side. This is caused by the constructive and destructive interference of light waves passing through the slit.

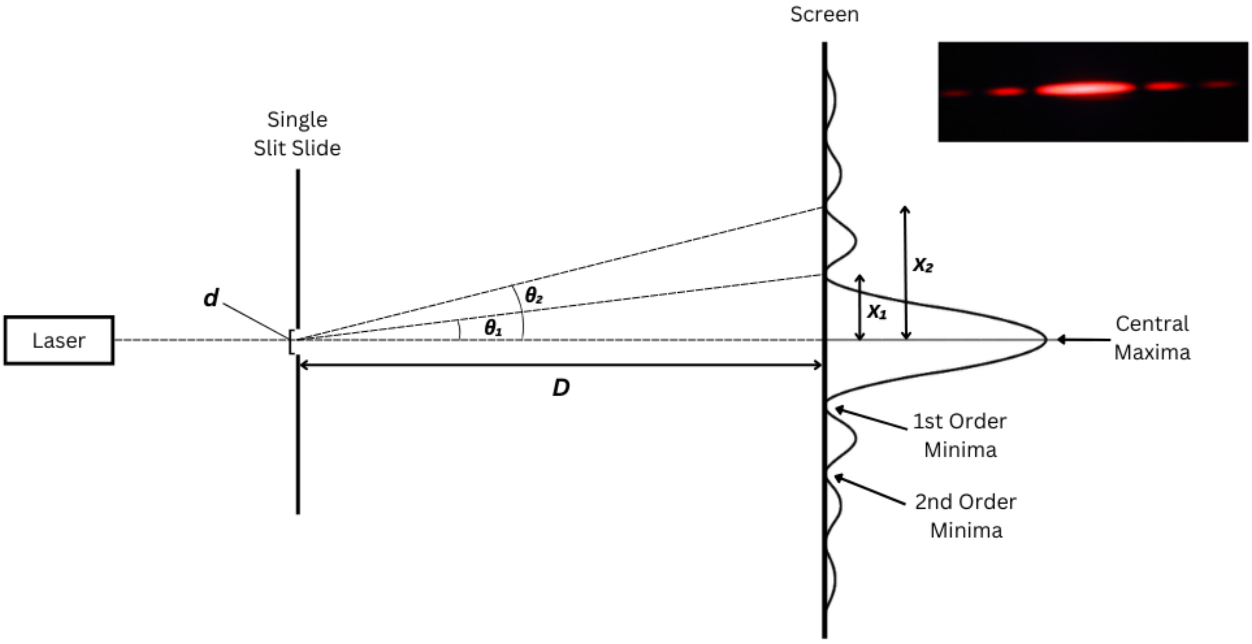

Figure 2: Schematic diagram of diffraction through a single slit (Fraunhofer diffraction)

The position of minimas is given by the equation:

$$d \sin \theta_m = \pm m\lambda \ for \ m = 1, 2, 3 \ldots \tag{1}$$

Where d is the width of the slit, θ_m is the angle corresponding to the mth minima and λ is the wavelength of the incident light. The $\pm$ indicates which side of the central maxima the minima lies on. For small angles we can approximate $\sin \theta_m \approx \tan \theta_m = x_m/D$ where x_m is the distance between the central maxima and the mth minima on the screen and D is the distance between the slit and the screen.

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Plane Diffraction Grating

A transmission diffraction grating consists of a large number of slits separated from one another by an opaque region. The diffraction pattern of a plane diffraction grating consists of widely separated distinct bright intense spots known as principal maxima with faint secondary maxima scattered between them.

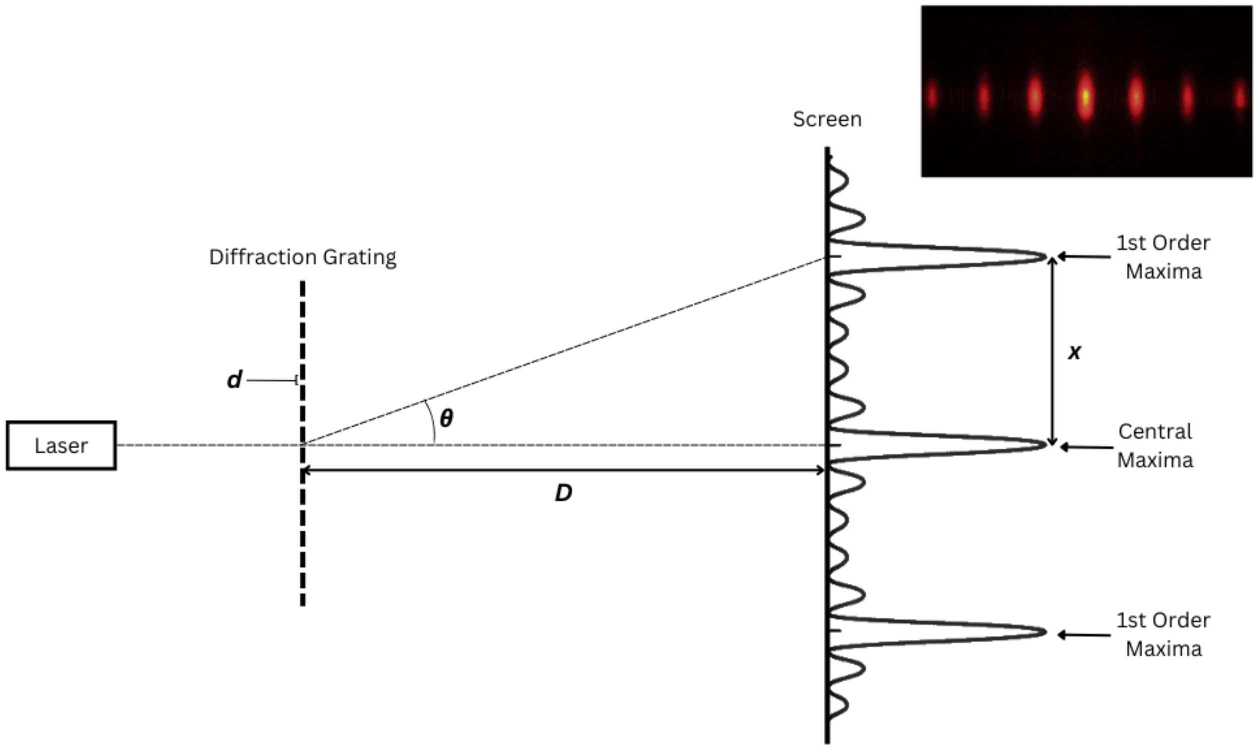

Figure 3: Schematic diagram of diffraction through a plane transmission grating

The location of principal maxima on the screen is given by:

$$d \sin\theta_m = m\lambda \; for \; m = 1, 2, 3 \ldots \tag{2}$$

Where d is the slit width (calculated as 1/number of lines per meter), θ_m is the angle corresponding to the mth principal maxima and λ is the wavelength of the incident light. Here too, we can approximate $\sin\theta_m \approx \tan\theta_m = x_m/D$ where x_m is the distance between the central maxima and the mth minima on the screen and D is the distance between the slit and the screen. Hence, we can modify our equation as:

$$\frac{dx_m}{D} = m\lambda \implies x_m = \frac{m\lambda D}{d} \; for \; m = 1, 2, 3 \ldots \tag{3}$$

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Circular Aperture

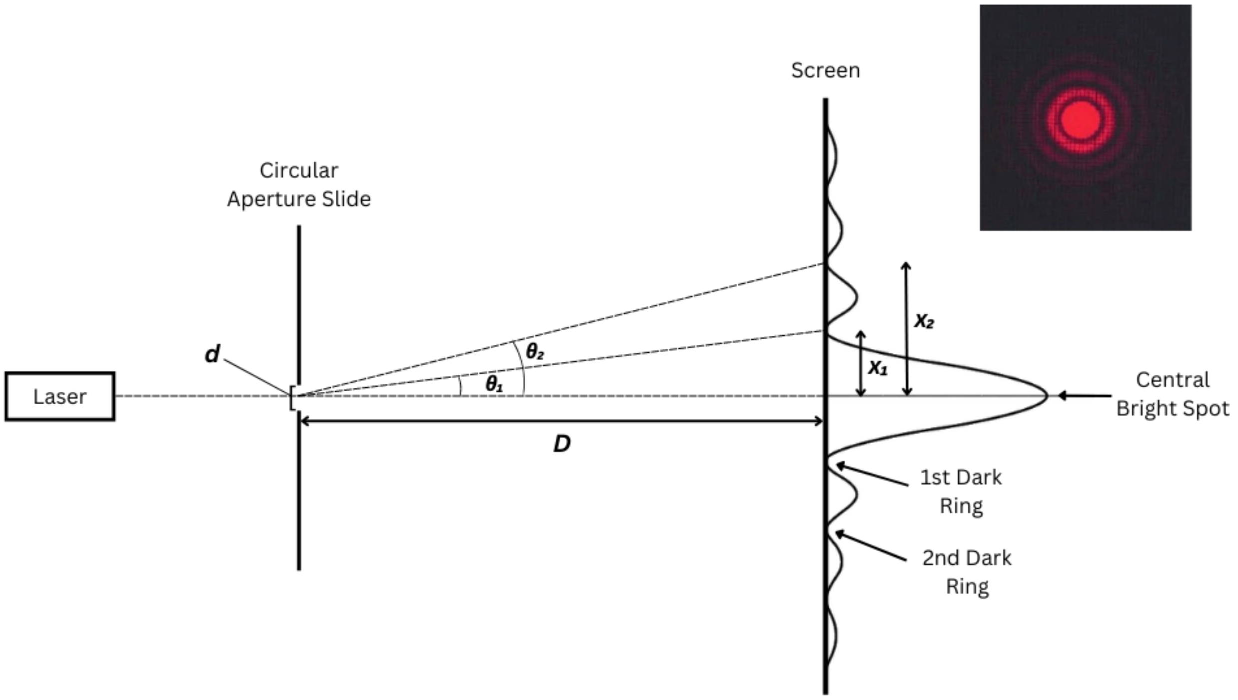

Figure 4: Schematic diagram of diffraction through a circular aperture

The diffraction pattern produced by a circular aperture is known as Airy diffraction, which is similar to Fraunhofer diffraction but is mathematically more complex. However, the same expression can be used with a slight change in the coefficient:

$$d \sin\theta_m = \bar{m}\lambda \ for \ \bar{m} = 1.22, 2.23, 3.23, 4.24\ldots \quad (4)$$

Where d is the diameter of the aperture, θ_m is the angle corresponding to the mth dark ring and λ is the wavelength of the incident light.

Single Helical Spring

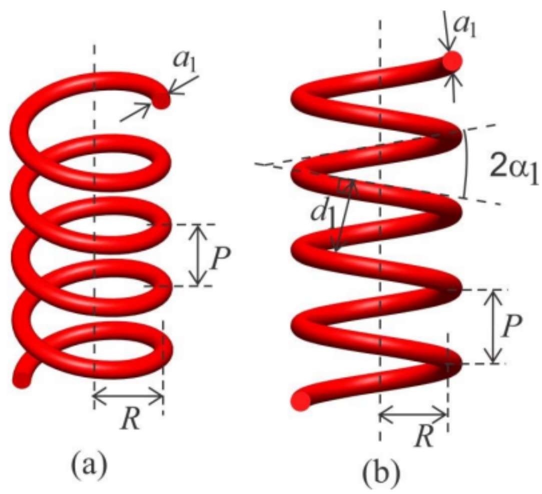

Figure 5: (a) Typical view of helical spring
(b) Schematic diagram when viewed at normal incidence[1]

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A helical spring has 3 measurable parameters - thickness of wire (a_1), distance between loops (d_1) and angle $2\alpha_1$. When viewed at normal incidence its projection is equivalent to two sets of parallel wires of the same thickness at an angle $2\alpha_1$ to each other.

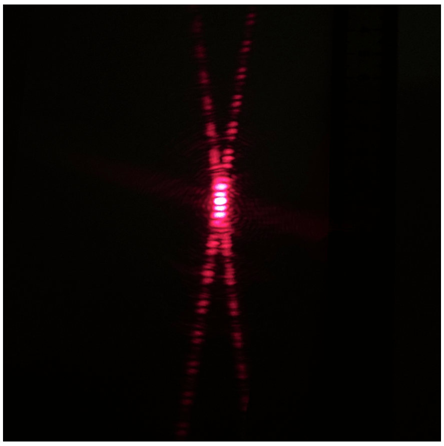

Figure 6: Diffraction pattern produced by a single helical spring[2]

Each line of the X-shaped diffraction pattern corresponds to one set of parallel wires where the broader set of minima are due diffraction from a single wire while the narrow set of minima are formed by the interference due to multiple wires. The angle between the two arms is equal to $2\alpha_1$.

We can use equation (3) to find the distance between two consecutive minima:

$$\beta = x_{m+1} - x_m = \frac{(m+1)\lambda D}{d} - \frac{m\lambda D}{d}$$

$$\implies \beta = \frac{\lambda D}{d}$$

The two length scales in the diffraction pattern can be associated with two fringe widths - β_1 and β_2, such that:

$$a_1 = \frac{\lambda D}{\beta_1} \text{ and } d_1 = \frac{\lambda D}{\beta_2} \tag{5}$$

The pitch of a spring can be found from the distance between loops d_1 and the angle of the spring $2\alpha_1$ using:

$$p = \frac{d_1}{\cos \alpha_1} \tag{6}$$

[1]Praveen Pathak, Charudatt Kadolkar, and Manish Kapoor. *Diffraction due to Helical Structure.* 2015. URL: https://ipho.olimpicos.net/pdf/IPhO$_2$015$_Q$4*.pdf*

[2]Physics Alive. *"Episode 43 - DNA Physics and DNA of Class."* Physics Alive!, 31 May 2022, URL: physicsalive.com/dna.

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Procedure

Step 1: Set up the laser mount and diffraction slide mount on the optical bench such that they are at the same height and aligned horizontally. Check this with the spirit level.

Step 2: Tape a graph sheet on the screen at the central maxima to accurately mark and measure the diffraction pattern. Measure the distance between the diffraction slide and the screen with a tape measure.

[Precaution: The distance between the diffraction slide and screen should be significantly more than the width of the diffraction object (in the order of 10^4-10^6 *times larger) to be able to distinguish a clear diffraction pattern.]*

Step 3: Using a plane diffraction grating with known slit width, measure the distance between principal maxima in the diffraction pattern to determine the wavelength λ of the light source using equation (2).

[Precaution: For proper functioning, keep the laser on throughout the experiment. When not in use, use a light-blocking screen to block the beam.]

Step 4: Using a single slit diffraction slide, measure the distance between minima in the diffraction pattern produced. Calculate the slit width using equation (1).

Step 5: Using a mounted thin wire/hair strand, measure the distance between minima in the diffraction pattern produced. Calculate the thickness of the wire/hair using equation (1).

Step 6: Using a circular aperture diffraction slide, measure the diameter of the dark rings in the diffraction pattern produced. Calculate the aperture diameter using equation (4).

Step 7: Using a mounted single helical spring, measure the primary and secondary fringe widths in the diffraction pattern produced. Also measure the angle between the arms of the X-shaped pattern. Calculate the thickness of the wire, pitch and angle of the spring using equation (5).

Part A: Plane Diffraction Grating

Observations

Grating Number (mm^{-1})	Screen Distance D (m)	Slit Width d (m)	1st Maxima Distance x_1 (m)	2nd Maxima Distance x_2 (m)	3rd Maxima Distance x_3 (m)
100	2.283	1×10^{-5}	0.124	0.252	0.375
200	2.335	5×10^{-6}	0.248	0.514	0.794
600	2.328	1.667×10^{-6}	0.775	1.972	-

Table 1: The distance to principal maxima in the diffraction pattern of green light through a plane grating

To find the approximate wavelength of red light, we took only one reading:

Grating Number = 200 mm^{-1}
Screen Distance (D) = 2.324 m
Slit Width (d) = 5×10^{-6} m
1st Maxima Distance (x_1) = 0.315 m

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Analysis

We can rearrange the values in equation (2) to find the wavelength of light:

$$\lambda = \frac{d \sin\theta}{m} form = 1, 2, 3 \ldots$$

Where,

$$\theta = \arctan\left(\frac{x_m}{D}\right)$$

Grating No.	Theta (rad)			Wavelength λ (m)			Average
(mm^{-1})	θ_1	θ_2	θ_3	λ_1	λ_2	λ_3	Wavelength λ_{avg} (m)
100	0.0543	0.1078	0.1614	$5.427 \cdot 10^{-7}$	$5.381 \cdot 10^{-7}$	$5.356 \cdot 10^{-7}$	$5.388 \cdot 10^{-7}$
200	0.1082	0.2167	0.3277	$5.399 \cdot 10^{-7}$	$5.374 \cdot 10^{-7}$	$5.365 \cdot 10^{-7}$	$5.379 \cdot 10^{-7}$
600	0.3272	0.7027	-	$5.358 \cdot 10^{-7}$	$5.387 \cdot 10^{-7}$	-	$5.373 \cdot 10^{-7}$

Table 2: Wavelength of green light calculated from the diffraction pattern of a plane grating

Average wavelength of green light $(\lambda_g) = 5.380 \cdot 10^{-7}$ m

For red light,

$\theta = 0.1347$

$\lambda_r = \dfrac{d \sin\theta}{m} = 6.715 \cdot 10^{-7}$ m

Part B: Single Slit

Observations

Screen Distance $(D) = 2.374$ m

Wavelength λ (m)	Distance between 1st minima y_1 (cm)	Distance between 2nd minima y_2 (cm)
$5.380 \cdot 10^{-7}$	2.7	5.6
$6.715 \cdot 10^{-7}$	3.6	7.0

Table 3: Distance between minima for single slit diffraction with red and green laser light

Analysis

The width of the slit was found using the equation (1):

$$d = \frac{\pm m\lambda}{\sin\theta_m} \ for \ m = 1, 2, 3 \ldots$$

Where $\theta_m = tan^{-1}(x_m/D)$ and $x_m = y_m/2$ since y_m is the distance between corresponding minima.

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Wavelength λ (m)	Slit Width d_1(mm)	Slit Width d_2 (mm)	Avg Slit Width d_{avg} (mm)
$5.380 \cdot 10^{-7}$	0.0870	0.0908	0.0889
$6.715 \cdot 10^{-7}$	0.0888	0.0913	0.09005

Table 4: Slit width calculated from Fraunhofer diffraction pattern

Average Slit Width (d) = 0.0894 mm

Part C: Thin Wire/Hair Strand

Observations

Screen Distance (D) = 2.374 m

Wavelength λ (m)	Distance between 1st minima y_1 (cm)	Distance between 2nd minima y_2 (cm)
$5.380 \cdot 10^{-7}$	2.9	7.2
$6.715 \cdot 10^{-7}$	3.2	7.0

Table 5: Distance between minima for diffraction of a hair strand with red and green laser light

Analysis

Due to Babinet's Principle, we can use the same equation as single slit diffraction for a hair strand. The diameter of the hair was found using equation (1):

$$d = \frac{\pm m\lambda}{\sin\theta_m} \; for \; m = 1, 2, 3 \ldots$$

Where $\theta_m = tan^{-1}(x_m/D)$ and $x_m = y_m/2$ since y_m is the distance between corresponding minima.

Wavelength λ (m)	Hair Diameter d_1(mm)	Hair Diameter d_2 (mm)	Avg Hair Diameter d_{avg} (mm)
$5.380 \cdot 10^{-7}$	0.0879	0.0708	0.0794
$6.715 \cdot 10^{-7}$	0.0999	0.0914	0.0957

Table 6: Hair diameter calculated from Fraunhofer diffraction pattern

Average Hair Diameter = 0.0876 mm.

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Part D: Circular Aperture

Observations

Screen Distance(D) = 2.374 m

Circle No.	Wavelength λ (m)	Diameter of 1st dark ring y_1(cm)	Diameter of 2nd dark ring y_2(cm)
1	$5.380 \cdot 10^{-7}$	0.9	1.6
1	$6.715 \cdot 10^{-7}$	1.1	2.0
2	$5.380 \cdot 10^{-7}$	0.5	1.8
2	$6.715 \cdot 10^{-7}$	0.6	1.0

Table 7: Diameter of dark rings for diffraction through a circular aperture with red and green laser light

Analysis

The diameter of each circular aperture was determined using equation (4):

$$d = \frac{\bar{m}\lambda}{\sin\theta_m} \ for \ \bar{m} = 1.22, 2.23, 3.23, 4.24\ldots$$

Where $\theta_m = tan^{-1}(x_m/D)$ and $x_m = y_m/2$ since y_m is the diameter and x_m is the radius.

Circle No.	Wavelength λ (m)	Diameter d_1 (mm)	Diameter d_2 (mm)	Avg Diameter d_{avg} (mm)
1	$5.380 \cdot 10^{-7}$	0.355	0.357	0.356
1	$6.715 \cdot 10^{-7}$	0.346	0.356	0.351
2	$5.380 \cdot 10^{-7}$	0.650	0.713	0.682
2	$6.715 \cdot 10^{-7}$	0.622	0.711	0.667

Table 8: Width of aperture 1 using equation (4)

Average diameter of the circular aperture 1 = 0.354 mm

Average diameter of the circular aperture 2 = 0.675 mm

Part E: Single Helical Spring

Observations

Wavelength of green light (λ_g) = $5.380 \cdot 10^{-7}$

Screen Distance (D) = 2.374 m

Primary Fringe Width (β_1) = 0.85 mm = $8.5 \cdot 10^{-4}$

Secondary Fringe Width (β_2) = 0.15 mm = $1.5 \cdot 10^{-4}$

Angle between diffraction arms ($2\alpha_1$) = 24°= 0.4189 rad

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Analysis

Using equations (5), we can determine the thickness of the wire (a_1) and the distance between parallel loops (d_1):

$$a_1 = \frac{\lambda D}{\beta_1} = 1.503 \cdot 10^{-4} m = 0.1503 mm$$
$$d_1 = \frac{\lambda D}{\beta_2} = 8.515 \cdot 10^{-4} m = 0.8515 mm$$

The pitch of the spring can be determined from equation (6):

$$p = \frac{d_1}{\cos \alpha_1} = 8.690 \cdot 10^{-4} m = 0.869 mm$$

Error Analysis

We can do error analysis for the value of wavelength λ for a given value of d if we use equation (3):

$$\lambda = \frac{x_m d}{mD} \; for \; m = 1, 2, 3 \ldots$$

Where we can use the following formula to find the error:

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta x_m}{x_m} + \frac{\Delta d}{d} + \frac{\Delta D}{D}$$

As there is no error in the measurement of d since it is calculated from the given grating number, we can eliminate that term:

$$\frac{\Delta \lambda}{\lambda} = \frac{\Delta x_m}{x_m} + \frac{\Delta D}{D}$$
$$= \frac{10^{-3}}{0.124} + \frac{10^{-3}}{2.283}$$
$$= 8.06 \cdot 10^{-3} + 4.38 \cdot 10^{-4}$$
$$= 8.498 \cdot 10^{-3}$$

$$\Delta \lambda = 4.61 \cdot 10^{-9} m = 4.61 nm$$

Note that the relative error due to x_m is one order of magnitude higher than the relative error due to D, which suggests that we must reduce our Δx_m (i.e. use a more precise measuring instrument) to improve our experiment.

Results

Part A

Wavelength of Green Light (λ_g) = $\boxed{5.380 \cdot 10^{-7} m}$

Wavelength of Red Light (λ_r) = $\boxed{6.715 \cdot 10^{-7} m}$

Part B

Width of Single Slit (a) = $\boxed{0.0894 mm}$

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Part C

Diameter of Hair Strand (d) = $\boxed{0.0876mm}$

Part D

Diameter of the Circular Aperture 1 (d_1) = $\boxed{0.354mm}$

Diameter of the Circular Aperture 2 (d_2) = $\boxed{0.675mm}$

Part E

Thickness of Single Helical Spring Wire (a_1) = $\boxed{0.1503mm}$

Pitch of Single Helical Spring (p) = $\boxed{0.869mm}$

Angle of Single Helical Spring (α_1) = $\boxed{12^\circ = 0.2094rad}$

Discussion

To increase precision in the experiment, the following improvements can be made:

- The entire experimental setup should be put in a dark room so that the diffraction pattern is bright and distinct.
- A digital measuring instrument with a least count of 0.01 mm would enhance the results of the measurements.
- A larger screen to allow for the collection of more data points so that we can graph the data and arrive at a better value for λ and d.