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Studying the Formation of Rainbows
Lab Report 6

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Aim

- To study different orders of rainbows formed by a drop of water and glycerine.
- To study the second-order rainbows for liquids of different refractive indices and determine the refractive index of an unknown liquid using the second-order rainbow it forms.

Experimental Setup

List of Apparatus:

- A modified spectrometer
- An incandescent lamp
- A syringe holder mounted on a prism table
- Four small syringes
- Four test tubes with water, glycerine, clove oil, and an unknown liquid A (5 ml each)
- Four small petri dishes
- A magnifying reading torch with LED lights
- A spirit level
- A small light blocking screen

The least count (LC) of the Spectrometer was found in the following manner:

$$LC = Main\ Scale\ LC - \frac{Main\ Scale\ Divisions}{Vernier\ Scale\ Divisions} \times Main\ Scale\ LC$$

$$= 30' - \frac{59}{60} \times 30' = 0.5' = 30''$$

Hence, the LC of the Spectrometer = 30 arcsec = 30”

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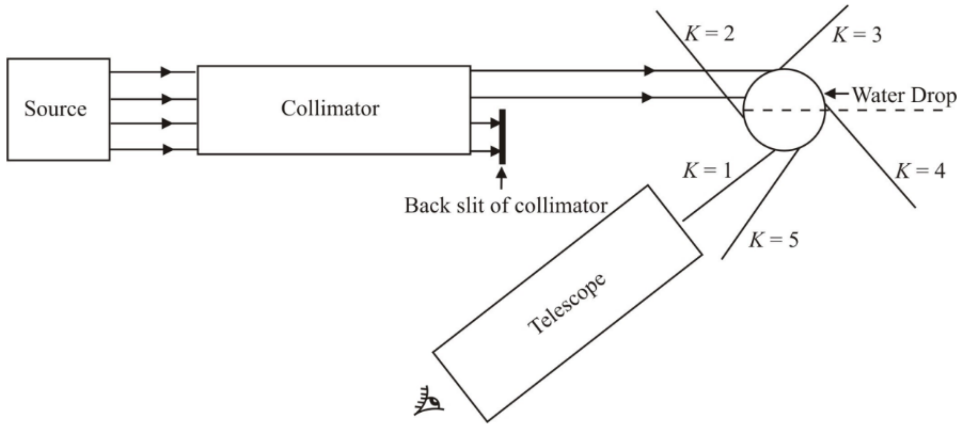

Figure 1: Schematic diagram of the experimental setup to observe the formation of a rainbow using a spectrometer

Theoretical Background

Rainbows are formed by a droplet of a liquid when an incoming ray of light suffers total internal reflection and disperses while emerging, forming a spectrum. The number of internal reflections determines the order K of the rainbow.

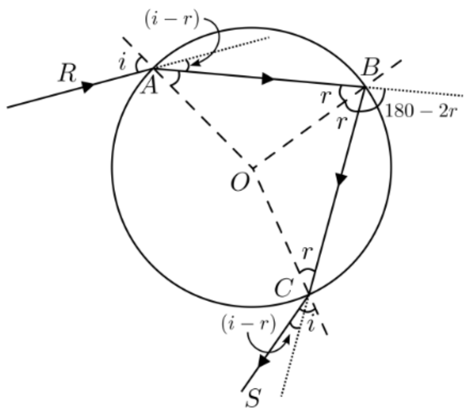

Figure 2: Ray diagram for the formation of a first order rainbow

For a first order rainbow, the total angle of deviation measured between the incident and emergent rays can be written in terms of the angle of incidence i and angle of refraction r (figure 2):

$$\phi_1 = 2(i - r) + 180 - 2r$$

We notice that this pattern will repeat for subsequent internal reflections if the ray is unable to emerge after a single reflection. Hence, we can find a general formula for the deviation of a K^{th} order rainbow:

$$\phi_K = K(180 - 2r) + 2(i - r) \tag{1}$$

Since we know from Snell's Law that:

$$\mu = \frac{\sin i}{\sin r}$$

$$\implies r = \sin^{-1}\left(\frac{\sin i}{\mu}\right)$$

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This allows us to derive the following relation from equation 1:

$$\phi_K = 180K + 2i - 2(K+1)\sin^{-1}\left(\frac{\sin i}{\mu}\right)$$

For $K = 2$, the above expression can be rewritten as:

$$\cos\left(\frac{\phi_2}{6}\right) = \frac{1}{\mu}\left(3\cos\left(\frac{i+180}{3}\right)\cos i + \sin\left(\frac{i+180}{3}\right)\sin i\right) \tag{2}$$

This implies that $\cos(\phi_2/6) = k/\mu$ where k is some constant for a given angle of incidence.

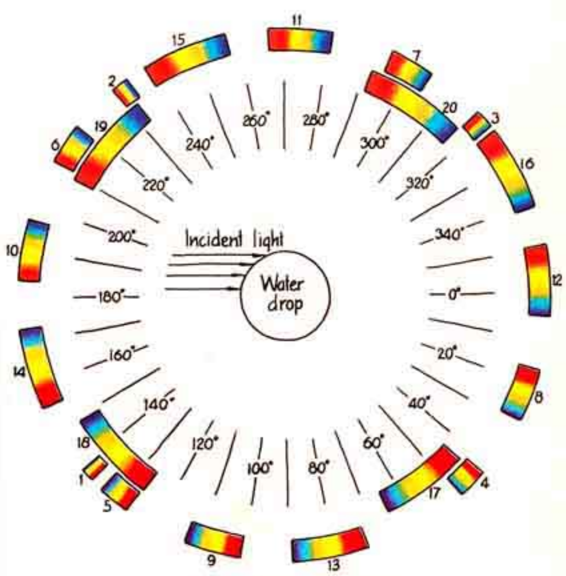

Figure 3: Angles of rainbow formation around a drop of water

The figure above (figure 3) shows the distribution of rainbows around a drop of water illuminated by parallel beams of light as predicted by equation 1.

Procedure

Part A

1. Level the prism table using a spirit level.

2. Fill a syringe with water, securely mount it in the holder and make a large droplet at the nozzle of the syringe. Keep a petri dish underneath to collect excess water drops.
[Precaution: Use the screws in the holder to adjust the size of the droplet by tightening them slowly after making a small droplet]

3. Adjust the lens and collimator so that the beam of light produced by the source is parallel. Use the screen to check that the intensity of the light at all distances is equal to verify that the rays are parallel.
[Precaution: Do not look at the light source directly or through the telescope.]

4. Adjust the alignment of the source, the collimator, the suspended drop, and the telescope, such that the drop is fully illuminated by the parallel beam of light and is visible in the telescope.

5. Keeping the telescope, collimator, and the drop in the straight line, align the vernier scale and main scale zeros such that there is no zero error.
[Precaution: Use only one side of the spectrometer for recording the readings. Use the same side throughout the experiment.]

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6. Close the back slit of the collimator partially, so that only the vertical half of the drop is illuminated by the light.

7. Find the first order rainbow by eye and then by telescope by referring to figure 3. Adjust the telescope such that the bright red line of the spectrum can be seen. Measure the total angle of deviation ϕ_1 by taking both the main scale and vernier scale readings.
[Precaution: Look for rainbows using the screen before finding them through the telescope. If the intensity of the rainbows is not high enough, increase the size of the droplet and ensure only half of it is being illuminated by the light]

8. Repeat step 7 for at least five more orders of rainbows, taking readings for their respective angles of deviation ϕ_K.
[Precaution: Always adjust the telescope such that it is focused on the red line of the spectrum before taking the reading. For certain angles light reflected directly from the external surface of the drop will produce bright white glare spots. Account for these while taking observations.]

9. Repeat the above steps for glycerine.

10. Plot a graph of ϕ_K against K for both water and glycerine.

Part B

1. Replace the syringe with different liquids (clove oil and unknown liquid A) and repeat steps 1-7 for the second order rainbow and measure ϕ_2.
[Precaution: Use different petri dishes, test tubes, and syringes for different liquids to avoid mixing.]

2. Plot a graph of $\cos(\phi_2/6)$ against $1/\mu$. Using the value of $\cos(\phi_2/6)$ of the unknown liquid, find its refractive index from the graph.

Observations

Part A

MSR = Main Scale Reading
VSD = Vernier Scale Divisions
VSR = Vernier Scale Reading
LC = Least Count = 0.5' = 30"

Note that the measurements were taken in the anti-clockwise direction rather than in the clockwise direction. Hence, the angular deviation from the zero deviation alignment is calculated from 360 minus the total reading.

MSR (° ' ")	VSD (div)	VSR=VSD×LC (° ' ")	Total=MSR+VSR (° ' ")	Decimal Total (decimal °)	Angle of Deviation ϕ_K (decimal °)	Order (K)
219° 0'	56	28' 0"	219° 28' 0"	219.467	140.533	1
126° 30'	35	17' 30"	126° 47' 30"	126.792	233.208	2
36° 0'	45	22' 30"	36° 22' 30"	36.375	323.625	3
317° 0'	2	1' 0"	317° 1' 0"	317.017	402.983	4
228° 30'	33	16' 30"	228° 46' 30"	228.775	491.225	5

Table 1: Angular deviation of rainbows formed by a water droplet

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The experiment was repeated for glycerine for which the data is tabulated below (table 2).

MSR (° ' ")	VSD (div)	VSR=VSD×LC (° ' ")	Total=MSR+VSR (° ' ")	Decimal Total (decimal °)	Angle of Deviation ϕ_K (decimal °)	Order (K)
205° 30'	5	2' 30"	205° 32' 30"	205.542	154.458	1
98° 0'	35	17' 30"	98° 17' 30"	98.292	261.708	2
264° 30'	36	18' 0"	264° 48' 0"	264.800	455.200	4
160° 0'	17	8' 30"	160° 8' 30"	160.142	559.858	5
261° 30'	3	1' 30"	261° 31' 30"	261.525	818.475	8
57° 0'	16	8' 0"	57° 8' 0"	57.133	1022.867	10

Table 2: Angular deviation of rainbows formed by a glycerine droplet

Part B

The experiment was repeated for clove oil and unknown liquid A. The second order deviation as well as the value of $\cos(\phi_2/6)$ of each liquid is tabulated below (table 3).

Liquid Used	2^{nd} Order Deviation ϕ_2 (decimal °)	$\cos(\phi_2/6)$	1/Refractive Index ($1/\mu$)
Water	233.208	0.779	0.752
Glycerine	261.708	0.724	0.680
Clove Oil	272.275	0.702	0.654
Unknown Liquid A	264.067	0.719	Unknown

Table 3: Deviation of second order rainbows for liquids of different refractive indexes

The values of refractive index used in table 3 are:

Water (μ_w) = 1.33
Glycerine (μ_g) = 1.47
Clove Oil (μ_o) = 1.53

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Analysis

Part A

We can plot the data in table 1 to find the behaviour of the angle of deviation ϕ_K with respect to the order K for a water droplet.

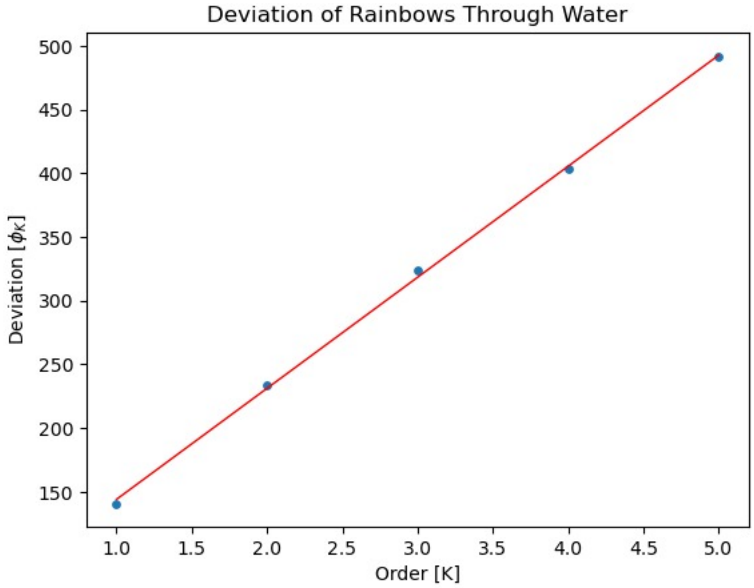

Figure 4: The deviation of a rainbow appears to be linearly related to the order

The linear relationship between ϕ_K and K seen in figure 4 is successfully modelled by equation 1, which if we apply here, we can calculate the angle of incidence (i) and angle of reflection (r) for which we see the rainbows.

$$\phi_K = K(180 - 2r) + 2(i - r)$$

Slope $= 180 - 2r = 87.115$
$\implies r = 46.443°$

Intercept $= 2(i - r) = 56.967$
$\implies i = 74.927°$

Using Snell's Law we can find the refractive index of water:

$$\mu_w = \frac{\sin i}{\sin r} = 1.332$$

Similarly, we can plot the data in table 2 and find the refractive index of glycerine.

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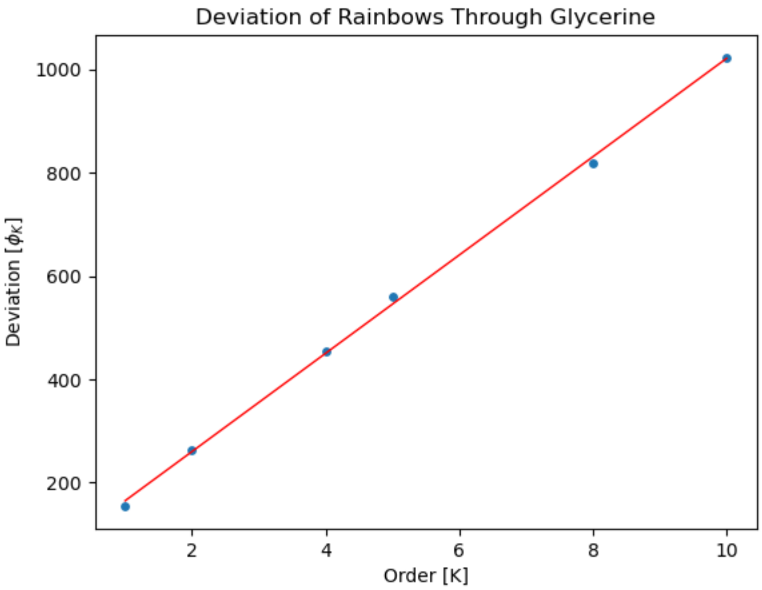

Figure 5: This graph contains extremely high order rainbows which seem to deviate slightly from the trend of the lower orders

$$\text{Slope} = 180 - 2r = 95.193$$
$$\implies r = 42.404°$$

$$\text{Intercept} = 2(i - r) = 69.461$$
$$\implies i = 77.135°$$

$$\mu_g = \frac{\sin i}{\sin r} = 1.446$$

Part B

We can plot the data in table 3 to verify that the relationship between $\cos(\phi_2/6)$ and $1/\mu$ is indeed linear. The trend will allow us to predict the refractive index of unknown liquid using its measurement of ϕ_2.

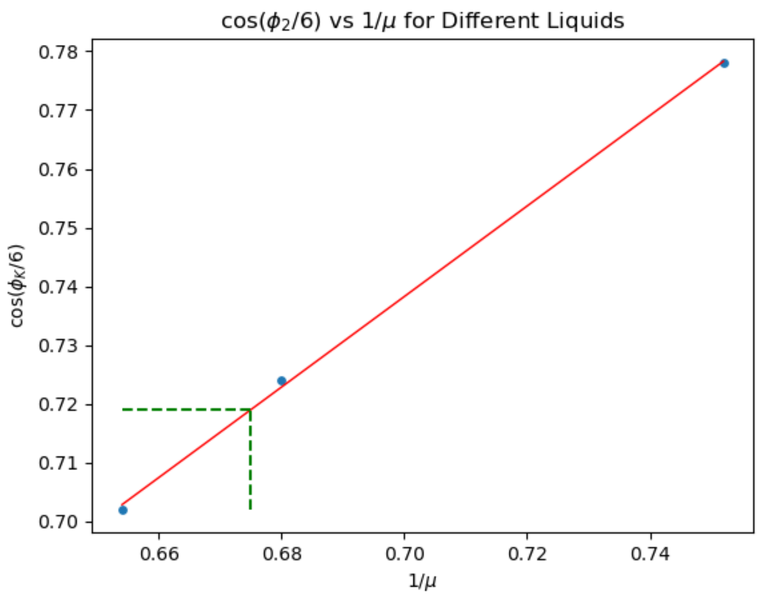

Figure 6: The trend appears to be linear in accordance with the theory

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Figure 6 clearly shows a linear relationship between $\cos(\phi_2/6)$ and $1/\mu$, as predicted by equation 2. Since we now know that the deviation of a second order rainbow is only dependent on the refractive index of the liquid, we can determine the refractive index of the unknown liquid A.

The value of $\cos(\phi_2/6)$ for unknown liquid A is 0.719 (marked by the dotted line in figure 6), which can be put into the following equation that relates $\cos(\phi_2/6)$ and $1/\mu$ using the best fit line's slope and intercept.

$$\mu = \frac{0.77}{\cos(\phi_2/6) - 0.199} \tag{3}$$

This gives us a refractive index of $\mu_l = 1.481$. The liquid could potentially be vegetable oil whose refractive index is around 1.47.

Error Analysis

The percentage error in ϕ_2 for the unknown liquid can be found by:

$$\frac{\Delta\phi_2}{\phi_2} \times 100 = \frac{0.017}{264.067} = 0.006\%$$

We can use the least count error to gives us a range of values for $\cos(\phi_2/6)$:

$$\begin{aligned} \cos(\phi_2/6)_{max} &= 0.71923 \\ \cos(\phi_2/6)_{min} &= 0.71917 \\ \implies \cos(\phi_2/6) &= 0.7192 \pm 0.00003 \end{aligned}$$

Hence, the error in $1/\mu_l$ can be derived from equation 3:

$$\frac{\Delta(\mu_l)}{(\mu_l)} = \frac{\Delta\cos(\phi_2/6)}{\cos(\phi_2/6)}$$

$$\Delta\mu_l = \frac{0.00003}{0.7192} \times 1.481 = 6.177 \times 10^{-5}$$

Clearly, the least count error of the spectrometer contributes negligibly to the error in the refractive index of the unknown liquid. It is more likely that misreading of the vernier scale leads to significantly higher error (upto 0.06% if you consider an error of 10 divisions), in addition to the ambiguous alignment of the telescope for diffused rainbows.

Results

Part A

The deviation of rainbows is linearly related to the order by the following equation:

$$\boxed{\phi_K = K(180 - 2r) + 2(i - r)}$$

Refractive index of water $(\mu_w) = \boxed{1.332}$

Refractive index of glycerine $(\mu_g) = \boxed{1.446}$

Part B

The relationship between μ and $\cos(\phi_2/6)$ is given by:

$$\boxed{\mu = \frac{0.77}{\cos(\phi_2/6) - 0.199}}$$

Refractive index of unknown liquid $(\mu_l) = \boxed{1.481}$