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Stretch Profile of a Soft Massive Spring
Lab Report 3

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Aim

- To find the net spring constant of a soft massive spring (i.e. a slinky).
- To determine the stretch profile of a soft massive spring with respect to its unstretched length.

Experimental Setup

List of Apparatus:

- Soft metal slinky
- Measuring tape
- Digital weighing scale
- Masking tape
- Thin thread
- Several small weights
- Fixed suspension platform

Least count of measuring tape = 0.1 cm
Least count of weighing scale = 0.1 g

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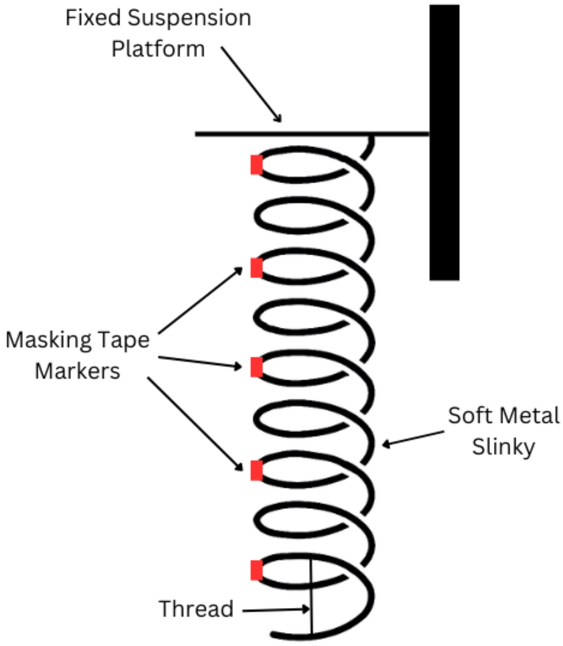

Figure 1: Experimental Setup for a Soft Metal Slinky

Theoretical Background

The slinky is essentially a soft massive spring with mass M and spring constant K, but it stretches non-uniformly. However, it can be approximated by N small massless springs with spring constant k, each suspending a small mass m (Figure 2).

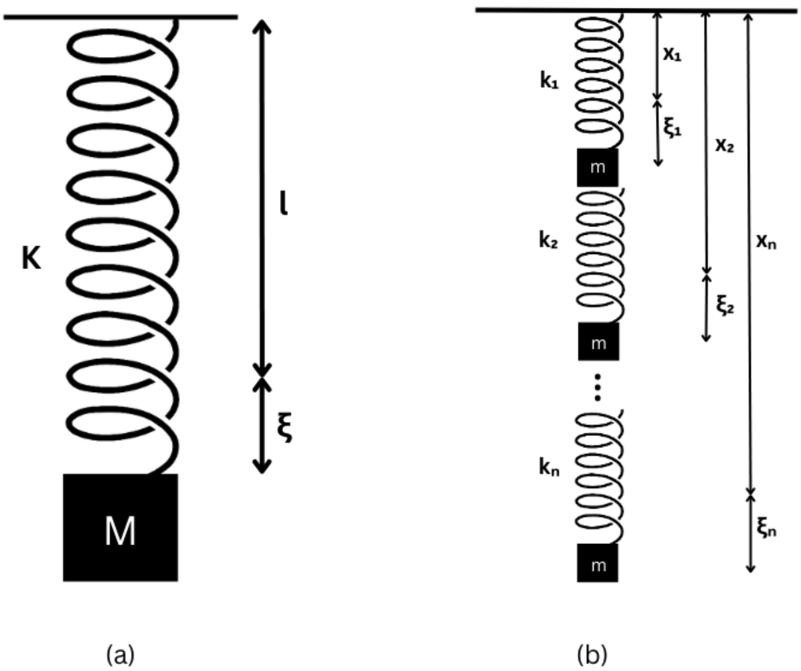

Figure 2: Modelling a soft massive spring with small massless springs and point masses

Let the unstretched length of the original spring be l and its net extension be ξ. Then each segment will have an unstretched length of $\Delta x = l/N$ and a distance $x_n = n\Delta x$, as well its own extension ξ_n. From the spring force formula, we can derive the relationship between the spring constants k_n and K:

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$$\xi = \xi_1 + \xi_2 + \xi_3 \cdots + \xi_n$$
$$\xi = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3} \cdots + \frac{F}{k_n}$$
$$\xi = F\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \cdots + \frac{1}{k_n}\right)$$
$$\implies \frac{1}{K} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_n}$$

Since the spring constants must be equal for each small spring (i.e. $k_1 = k_2 = \cdots = k_n = k$), we find that $k = NK$ and $m = M/N$.

The net potential energy of a given spring segment is the sum of the gravitational potential as well as its spring potential, given by:

$$U_n(\xi_n, \xi_{n-1}) = -mg(x_n - \xi_n) + \frac{1}{2}k(\xi_n - \xi_{n-1})^2$$

Upon minimising this function, we can find the lowest energy state of the system in the form of a differential equation, which can then be solved for with boundary conditions of $d\xi/dx = 0$ at $x = 0, l$. This gives us the following relation:

$$\xi(x) = \frac{Mg}{K}\left[\frac{x}{l} - \frac{1}{2}\left(\frac{x}{l}\right)^2\right] \tag{1}$$

From this equation we know that the stretch profile of a soft massive spring $\xi(x)$ with respect to its unstretched length x must be a parabolic curve.

Procedure

1. Measure the mass and length of the unstretched slinky.
2. Tape the part of the slinky that will not be used and measure the length of the portion that will be used.
3. Using the ratio of spring used from step 1 and 2, find the mass of the segment used.
4. Hang the slinky from a fixed suspension platform with the unused portion resting on the platform. Measure the length of the suspended spring with no mass attached to it.
   [Precaution: Keep the measuring tape parallel to the axis of the slinky to get an accurate length measurement.]
5. Tie a thin thread across the diameter of the last loop of the slinky from which to suspend the masses. Hang various weights from the bottom of the spring and measure the new length of the spring.
6. Find the difference between the lengths measured in step 4 and 5 to find the extension for each weight. Use standard value of g and find spring constant K using Hooke's Law.
   [Precaution: Make sure to hang the weight from the middle of the bottom of the slinky to give it a uniform extension.]
7. Mark out segments of the slinky with masking tape at uniform intervals by counting turns. Measure the unstretched length x_n for each marking.
8. Measure the length from the top of the slinky to each marked loop of the the slinky to find the stretched length. Find the difference between the unstretched and stretched length to find the extension ξ_n for each marking and plot the stretch profile of the slinky.

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Observations

Total mass of spring $= 400.9 \pm 0.1\ g$
Total length of spring $= 10.3 \pm 0.1\ cm$

Length of spring used $(l)= 4.9 \pm 0.1\ cm = 0.049 \pm 0.001\ m$
Mass of spring used $(M)= 400.9 \times \frac{4.9}{10.3} = 190.7 \pm 0.1\ g = 0.1907 \pm 0.0001\ kg$

Spring Constant of a Soft Massive Spring

Length of suspended spring $(L) = 82.5 \pm 0.1\ cm = 0.825 \pm 0.001\ m$

Mass of Weight (g)	Length of Spring $L + \xi$ (m)	Extension ξ (m)
0	0.825	0.000
2.2	0.843	0.018
5.1	0.866	0.041
7.3	0.884	0.059
10.2	0.906	0.081
12.4	0.922	0.097
15.3	0.947	0.122
17.5	0.963	0.138
20.0	0.982	0.157
22.5	1.011	0.186

Table 1: Extension for various masses in a soft massive spring

Stretch Profile of a Soft Massive Spring

The markers were put at every 2 loops or 0.163 cm of the original spring. The entire spring had 60 loops, which gave us 30 readings which are plotted in the Figure 3.

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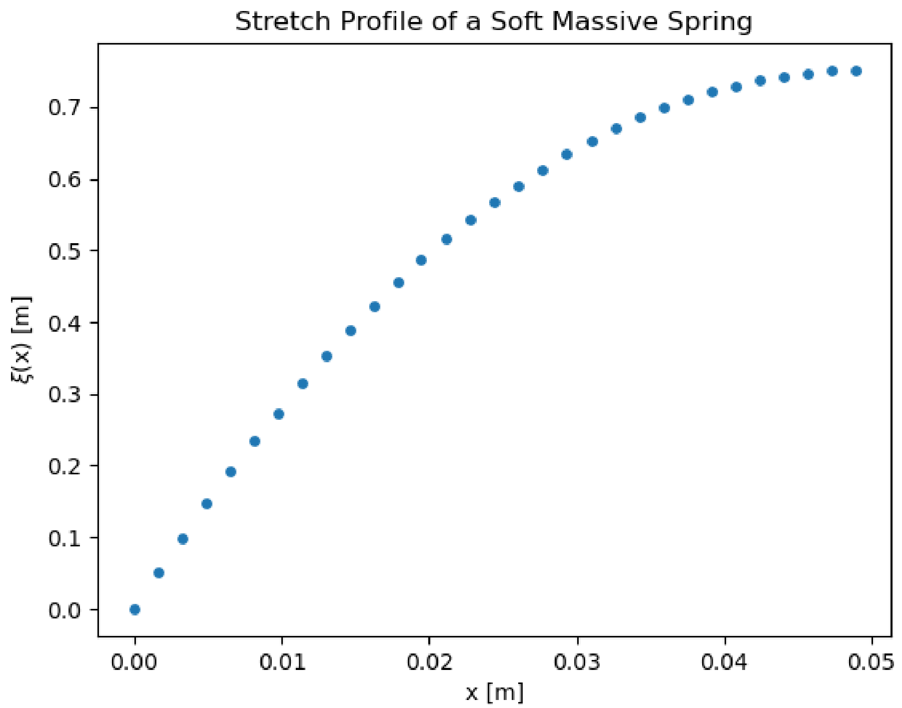

Figure 3: The stretch profile seems to follow a parabolic trend in accordance with our theoretical understanding

The tabulated data for the above graph can be found in the Appendix.

Analysis

Spring Constant of a Soft Massive Spring

The mass and extension data from Table 1 is plotted in Figure 4 and fit with a straight line.

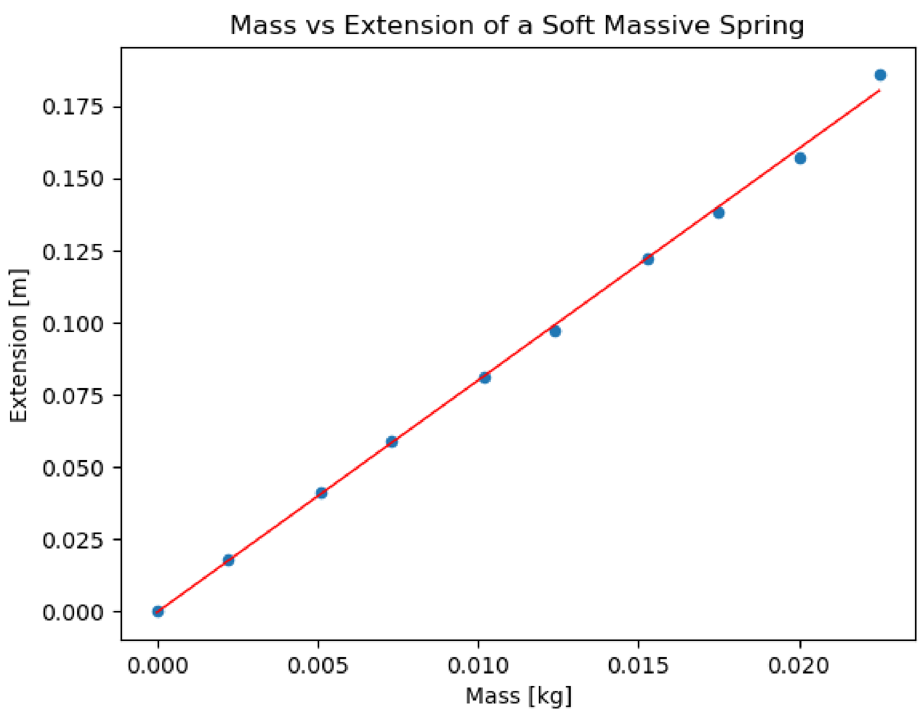

Figure 4: The extension of the spring appears to increase linearly for small masses

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From Figure 4 it is clear that the slinky behaves like a massless spring in terms of extension due to suspended masses. Hence, we can apply Hooke's Law to determine the spring constant of the slinky.

$$Mg = K\xi \tag{2}$$
$$K = \frac{Mg}{\xi}$$

The value of ξ/M is given by the slope of the graph, which is 8.034 m/kg. We used the known value of $g = 9.807\ m/s^2$.

$$K = \frac{9.807}{8.034} = 1.221 N/m$$

Stretch Profile of a Soft Massive Spring

Using Equation 1 as a model, we plotted the function $\xi(x)$ with the following values:

$$\xi(x) = \frac{Mg}{K}\left[\frac{x}{l} - \frac{1}{2}\left(\frac{x}{l}\right)^2\right]$$

$M = 0.1907\ kg$
$g = 9.807\ m/s^2$
$l = 0.049\ m$
$K = 1.221\ N/m$

We also fit a parabolic curve passing through the origin on the plotted data in Figure 3 and compared both curves.

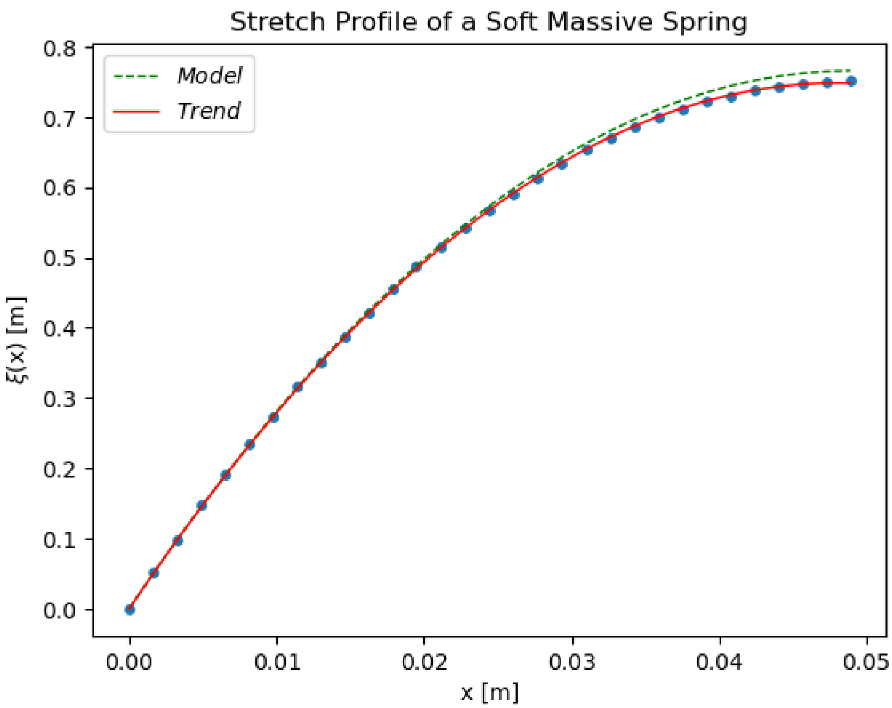

Figure 5: The data trendline (red) closely follows the model (green), only deviating slightly at small values of extension (i.e. the bottom of the suspended spring)

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Error Analysis

The error in the spring constant of the slinky K is derived from the measurement of mass M and extension ξ in Hooke's Law (Equation 2). The relative error in K can be found using the equation:

$$\frac{\Delta K}{K} = \frac{\Delta M}{M} + \frac{\Delta \xi}{\xi} + \frac{\Delta g}{g} \tag{3}$$

$\Delta g/g = 0$ since we used the known value of g in our calculations. For a given data point, we can find our relative error:

$\xi = \xi_m \pm \Delta\xi = 0.041 \pm 0.001\ m$

$M = M_m \pm \Delta M = 0.0051 \pm 0.0001\ kg$

$K_m = 1.221\ N/m$

$\implies \dfrac{\Delta K}{K} = \dfrac{1}{41} + \dfrac{1}{51}$

$\implies \dfrac{\Delta K}{K} = 0.0244 \times 0.0196 = 4.782 \times 10^{-4}$

$\implies \Delta K = \pm 0.054\ N/m$

$\implies K + \Delta K = 1.221 \pm 0.054\ N/m$

$\implies K_{min} = 1.167\ N/m$

and $K_{max} = 1.275\ N/m$

This margin of error is acceptable as its degree of magnitude is less than the measured value of K. If we plot the model for the maximum and minimum values of the spring constant, we can create an error envelope:

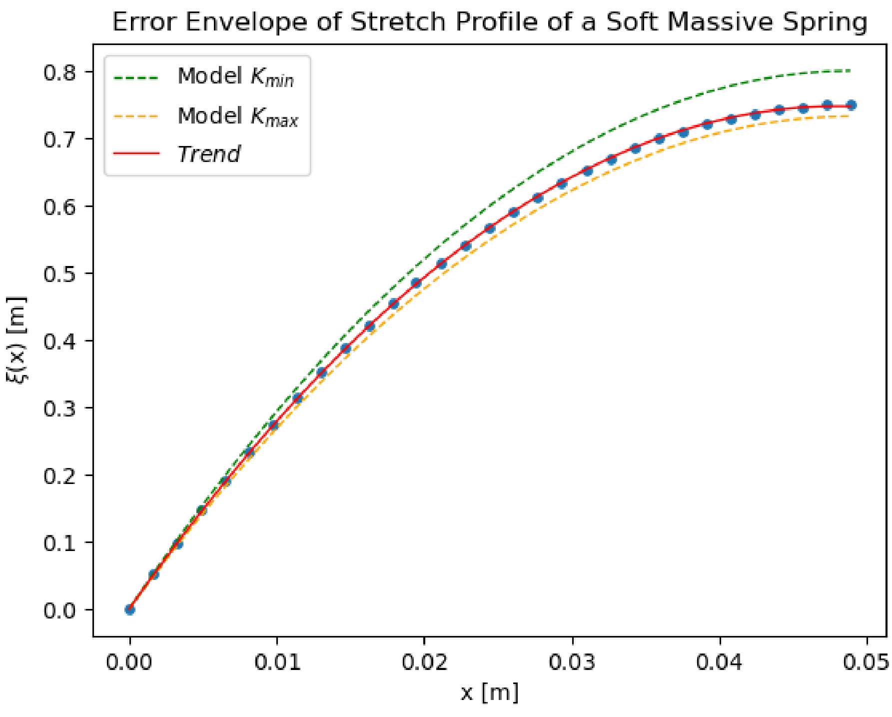

Figure 6: The data lies within the envelope of the error margin of the spring constant

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Since the data lies between the model for K_{min} and K_{max}, it falls within the margin or error. This explains the deviation of the data from the model in Figure 5.

Note that the degree of magnitude of error contributed by each source is the same, which means that the experiment is well balanced.

The deviation of the trend line towards the bottom of the spring is most likely due to the reduced accuracy in measuring the difference in length between intervals as the extension near the end of the spring is close to the least count of the measuring device.

Results

The spring constant of the soft massive spring was found to be $\boxed{K = 1.221\ N/m}$.

The stretch profile of the slinky follows a parabolic trend that is closely modeled by the following equation which is derived from its minimum-energy configuration equation:

$$\boxed{\xi(x) = \frac{Mg}{K}\left[\frac{x}{l} - \frac{1}{2}\left(\frac{x}{l}\right)^2\right]}$$

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Appendix

Length of Section x_n (m)	Length of Stretched Spring $x_n + \xi_n$ (m)	Extension ξ_n (m)
0.000	0.000	0.000
0.00163	0.053	0.05137
0.00326	0.102	0.09874
0.00489	0.152	0.14711
0.00652	0.198	0.19148
0.00815	0.242	0.23385
0.00978	0.283	0.27322
0.01141	0.327	0.31559
0.01304	0.365	0.35196
0.01467	0.403	0.38833
0.0163	0.438	0.4217
0.01793	0.473	0.45507
0.01946	0.506	0.48654
0.02119	0.537	0.51581
0.02282	0.565	0.54218
0.02445	0.592	0.56755
0.02608	0.617	0.59092
0.02771	0.641	0.61329
0.02934	0.663	0.63366
0.03097	0.684	0.65303
0.0326	0.703	0.6704
0.03423	0.72	0.68577
0.03586	0.736	0.70014
0.03749	0.749	0.71151
0.03912	0.761	0.72188
0.04075	0.77	0.72925
0.04238	0.78	0.73762
0.04401	0.787	0.74299
0.04564	0.792	0.74636
0.04727	0.798	0.75073
0.0489	0.8	0.7511

Table 2: Stretch profile of a soft massive spring