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Low-Pass/High-Pass Filters and LCR Circuits
Lab Report 2

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Aim

- To demonstrate the use of an RC circuit as a differentiator and integrator.
- To study the frequency response of an RC circuit as a low-pass filter and a high-pass filter.
- To study the frequency response of an LCR circuit in series and tank configurations.

Theoretical Background

For a time varying current and voltage input, we are going to assume the following forms to simplify calculations (Note: We can only use these forms as long as the operations on $V(t)$ and $I(t)$ are linear):

$$V(t) = \widetilde{V} e^{i\omega t}$$
$$I(t) = \widetilde{I} e^{i\omega t}$$

We have 3 types of components in our circuit which are discussed in detail below.

1. Resistors (R)

The current that flows through an ideal resistor is proportional to the potential difference applied across it.

$$V = IR$$

Where R is the resistance. The unit of resistance is Ohms (Ω). The ratio of voltage to current (z_R) in a time time-varying system is:

$$z_R = \frac{V}{I} = R$$

From this we can find the expression of current for a simple time varying voltage $V = V_o e^{i\omega t}$ where V_o is real.

$$I = \frac{V_o e^{i\omega t}}{R}$$

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$$\implies I = \frac{V_o}{R}\cos(\omega t) + \frac{iV_o}{R}\sin(\omega t)$$

Considering only the real part,

$$I_R = \frac{V_o}{R}\cos(\omega t)$$
$$V_R = V_o\cos(\omega t)$$

Notice that voltage (V_R) and current (I_R) are in phase.

2. Capacitors (C)

The charge built up on the plates of an ideal capacitor is proportional to the voltage applied across it.

$$V = \frac{Q}{C}$$

Where C is the capacitance. The unit of capacitance is Farads (F), or more commonly, micro-Farads (μF). The ratio of voltage to current (z_C) in a time time-varying system is:

$$\frac{d}{dt}\widetilde{V}e^{i\omega t} = \frac{1}{C}\frac{dQ}{dt}$$
$$\implies i\omega V = \frac{I}{C}$$
$$\implies z_C = \frac{V}{I} = \frac{1}{i\omega C}$$

From this we can find the expression of current for a simple time varying voltage $V = V_o e^{i\omega t}$ where V_o is real.

$$I = i\omega C V_o e^{i\omega t}$$
$$\implies I = i\omega C V_o\cos(\omega t) - \omega C V_o\sin(\omega t)$$

Considering only the real part,

$$I_C = -\omega C V_o\sin(\omega t)$$
$$\implies I_C = \omega C V_o\cos(\omega t + \pi/2)$$
$$V_C = V_o\cos(\omega t)$$

Notice that the current (I_C) leads the voltage (V_C) by a phase of $\pi/2$.

3. Inductors (L)

When a variable current is passed through an ideal inductor, the magnetic field produced causes an electromotive force which is given by:

$$V = L\frac{dI}{dt}$$

Where L is the inductance. The unit of inductance is Henrys (H), or more commonly, milli-Henrys (mH). The ratio of voltage to current (z_L) in a time time-varying system is:

$$\int \widetilde{V}e^{i\omega t}dt = L\int\frac{dI}{dt}dt$$
$$\implies \frac{V}{i\omega} = LI$$
$$\implies z_L = \frac{V}{I} = i\omega L$$

From this we can find the expression of current for a simple time varying voltage $V = V_o e^{i\omega t}$ where V_o is real.

$$I = \frac{V_o e^{i\omega t}}{i\omega L}$$

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$$\implies I = \frac{V_o}{i\omega L}\cos(\omega t) + \frac{V_o}{\omega L}\sin(\omega t)$$

Considering only the real part,

$$I_L = \frac{V_o}{\omega L}\sin(\omega t)$$

$$\implies I_L = \frac{V_o}{\omega L}\cos(\omega t - \pi/2)$$

$$V_L = V_o\cos(\omega t)$$

Notice that the current (I_L) lags behind the voltage (V_L) by a phase of $\pi/2$.

The phase difference between the current and voltage of each component can be visualised in the following graph:

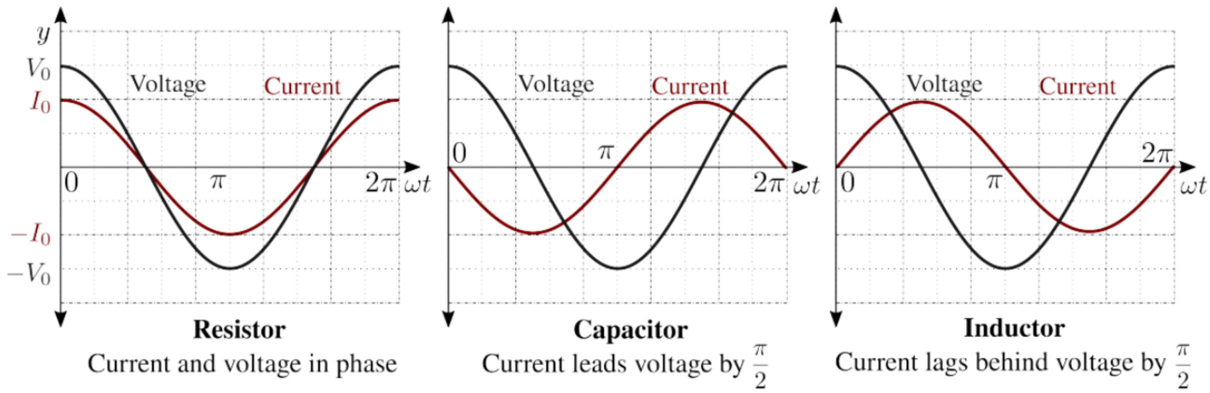

Figure 1: The phase difference between the current and voltage depends on the component (source: Lab 2 Electronics handout)

The RC Circuit: Low-Pass/High-Pass Filter

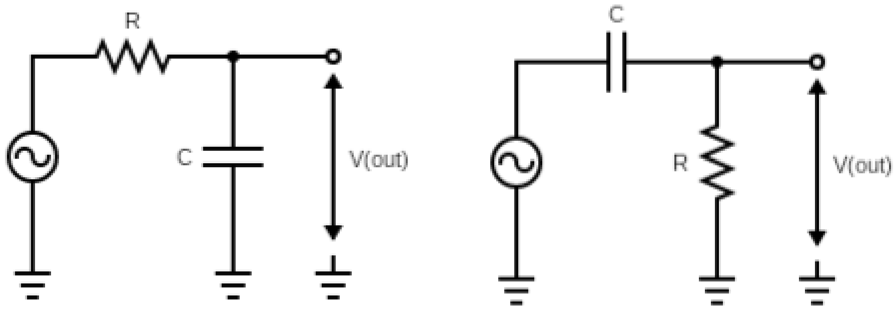

Figure 2: (left) RC integrator and low-pass filter circuit
(right) RC differentiator and high-pass filter circuit
(source: made on https://www.circuit-diagram.org)

RC Integrator and Low-Pass Filter

Consider the circuit on the left in figure 2 in which a resistor and capacitor are combined in series and the output voltage (V_{out}) is taken across the resistor. The applied voltage (V_{in}) must be equal to the sum of potential difference across each component.

$$V_{in} = V_C + V_R$$

$$V_{in} = Iz_C + Iz_R$$

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$$\implies V_{in} = I\left(\frac{1}{i\omega C} + R\right)$$

$$\implies I = \frac{V_{in}}{1/i\omega C + R}$$

When $\omega >> 1/RC$, the capacitive reactance is negligible and the capacitor effectively acts as a short circuit. This means:

$$I \approx \frac{V_{in}}{R}$$

The output voltage is measured across the capacitor so $V_C = V_{out}$:

$$Q = CV_{out}$$

$$\implies I = C\frac{dV_{out}}{dt}$$

Since the current in the circuit is the same in both components since they are in series, we can equate them:

$$\frac{V_{in}}{R} \approx C\frac{dV_{out}}{dt}$$

$$\implies V_{out} \approx \frac{1}{RC}\int_0^t V_{in}dt \tag{1}$$

Hence, the output voltage taken over the capacitor is proportional to the integral of the input voltage when $\omega >> 1/RC$. We call this reference frequency the cutoff frequency $\omega_c = 1/RC$. Note that in the high frequency regime, the capacitor acts as a short circuit causing the output voltage to essentially be zero. Therefore, high frequencies are blocked by the circuit while low frequencies pass relatively undisturbed, which is why this circuit is known as a low-pass filter.

RC Differentiator and High-Pass Filter

Consider the circuit on the right in figure 2 in which a resistor and capacitor are combined in series and the output voltage (V_{out}) is taken across the resistor. Once again, we get the same expression for current in the circuit:

$$I = \frac{V_{in}}{1/i\omega C + R}$$

Assuming $\omega << 1/RC$, the capacitive reactance is significantly larger than the resistance allowing us to make the following approximation:

$$V_{in} \approx \frac{I}{i\omega C} = V_C$$

Hence the input voltage is approximately equal to the voltage across the capacitor. This allows us to make the following statement about the current through the capacitor:

$$Q = CV_C$$

$$\implies I = C\frac{dV_C}{dt} \approx C\frac{dV_{in}}{dt}$$

Since the output voltage is measured across the resistor, we can say:

$$I = \frac{V_{out}}{R}$$

Equating the two expressions for current:

$$\frac{V_{out}}{R} \approx C\frac{dV_{in}}{dt}$$

$$\implies V_{out} \approx RC\frac{dV_{in}}{dt} \tag{2}$$

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Hence, the output voltage taken over the resistor is proportional to the derivative of the input voltage when $\omega << 1/RC$. We call this reference frequency the cutoff frequency $\omega_c = 1/RC$. Note that in the low frequency regime $V_C \approx V_{in}$ and the output voltage across the resistor is essentially zero. Therefore, low frequencies are blocked by the circuit while high frequencies pass relatively undisturbed, which is why this circuit is known as a high-pass filter.

The LCR Circuit

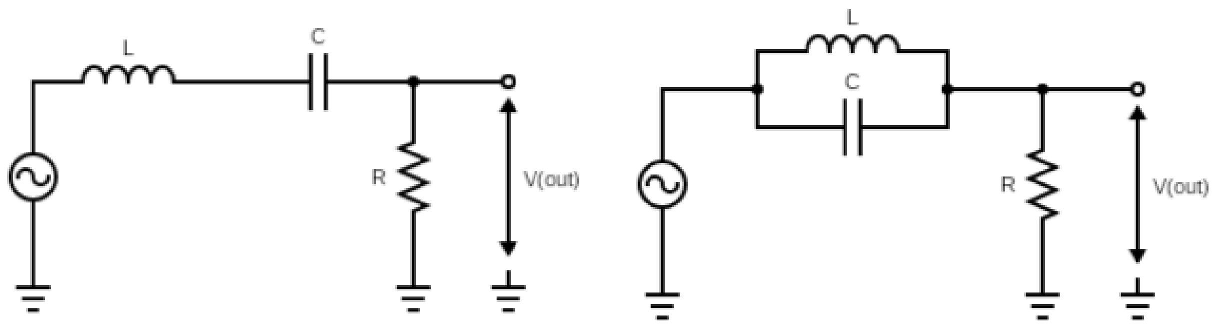

Figure 3: (left) A LCR circuit in series configuration (right) A LCR circuit in tank configuration (source: made on https://www.circuit-diagram.org)

Applying Kirchoff's loop rule to a LCR circuit in series configuration (fig 3, left), we can show that the sum of the potential drop across all the components in a closed loop are equal to zero:

$$V_L + V_R + V_C = 0$$
$$\implies L\frac{dI}{dt} + IR + \frac{Q}{C} = 0$$
$$\implies \frac{d^2I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC}I = 0$$
$$\implies \frac{d^2I}{dt^2} + \gamma\frac{dI}{dt} + \omega_o^2 I = 0$$

Where $\gamma = R/L$ and $\omega_o = 1/\sqrt{LC}$. We can solve this differential equation by assuming a solution of the form $I = Ae^{\lambda t}$:

$$Ae^{\lambda t}\left(\lambda^2 + \gamma\lambda + \omega_o^2\right) = 0$$
$$\implies \lambda^2 + \gamma\lambda + \omega_o^2 = 0$$
$$\implies \lambda = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_o^2}}{2}$$

Underdamped Oscillations

If $\gamma < 2\omega_o$ or $R < 2\sqrt{L/C}$, the solutions of the equation are imaginary, giving us the following equation:

$$I = Ae^{-\frac{\gamma t}{2}}e^{\pm i\omega_1 t}$$
$$\implies I = C_1\cos(\omega_1 t)e^{-\frac{\gamma t}{2}} + C_2\sin(\omega_1 t)e^{-\frac{\gamma t}{2}}$$

Where $\omega_1 = \sqrt{\omega_o^2 - \gamma^2/4}$ and C_1 and C_2 are constants set by the initial conditions. This behaviour is considered underdamped and the current executes damped simple harmonic oscillations.

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Overdamped Oscillations

If $\gamma > 2\omega_o$ or $R > 2\sqrt{L/C}$, we get two real solutions for the equation giving us the following equation:

$$I = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Where λ_1 and λ_2 are the two real solutions and C_1 and C_2 are constants set by the initial conditions. This behaviour is considered overdamped and the amplitude of the current quickly falls to zero.

Resonance and Bandwidth

In a series LCR circuit the current is given by:

$$I = \frac{V_{in}}{z_R + z_C + z_L}$$

$$\implies I = \frac{V_{in}}{R + \frac{1}{i\omega C} + i\omega L}$$

$$\implies I = \frac{V_{in}}{R + i\left(\omega L - \frac{1}{\omega C}\right)}$$

From the above expression it is clear that the current is maximised when $\omega L = 1/\omega C$:

$$\omega = \frac{1}{\sqrt{LC}} = \omega_o$$

This is known as the resonant frequency. If we plot the frequency response of a series LCR circuit we would expect the peak to be at $\omega = \omega_o$.

Similarly, for a LCR circuit in tank configuration (fig 3, right):

$$I = \frac{V_{in}}{z_R + \left(\frac{1}{\frac{1}{z_C} + \frac{1}{z_L}}\right)}$$

$$\implies I = \frac{V_{in}}{R + \left(\frac{1}{i\omega C + \frac{1}{i\omega L}}\right)}$$

$$\implies I = \frac{V_{in}}{R + \left(\frac{1}{i(\omega C - \frac{1}{\omega L})}\right)}$$

From the above expression it is clear that current is minimized when $\omega C = 1/\omega L$ which gives us the same resonant frequency $\omega_o = 1/\sqrt{LC}$. If we plot the frequency response of a tank LCR circuit we would expect the current to reach a minimum value at $\omega = \omega_o$.

The width of the peak in the bode plot can be quantified in terms of the bandwidth of the circuit defined as:

$$\Delta f = \frac{\gamma}{2\pi}$$

Hence, an underdamped series LCR circuit in which $\gamma < 2\omega_o$ will have a low bandwidth and a correspondingly narrow peak. On the other hand, an overdamped series LCR circuit in which $\gamma > 2\omega_o$ will have a high bandwidth and a correspondingly wide peak.

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Experimental Setup

- A 10 $k\Omega$ and 100 Ω resistor
- A 0.047 μF and 0.01 μF capacitor
- A 30 mH inductor
- A breadboard with four connection points at each terminal
- A function generator to generate sine, square, and triangle waves (Tektronix AFG1022)
- A Digital Storage Oscilloscope (DSO) with two channels (GDS-1102-U)
- BNC terminal cables and one BNC-to-BNC cable
- A BNC T-connector
- A pendrive

Procedure

Part A

1. Connect the 0.047 μF capacitor and 10 $k\Omega$ resistor in series to form a low-pass filter (fig 2, left). Connect the output across the capacitor and ensure that the grounding cables are all in the same terminal.
2. Supply an input signal to the circuit using the function generator. Use the BNC T-connector to split the input and use the BNC-to-BNC cable to supply the input signal to channel 1 of the DSO while using channel 2 for the output signal. Keep the input voltage fixed throughout the experiment.
3. Calculate the cutoff frequency of the circuit ($f_c = 1/2\pi RC$) and set the frequency of the input signal such that $f >> f_c$.
4. Feed a sinusoidal wave into the circuit, and observe the waveform of the output voltage. Repeat this for a triangular wave and square wave.
5. Repeat the same process with the circuit in the high-pass configuration (fig 2, right).
6. Use a pendrive to store the waveforms of the input and output signals.

Part B

1. Connect the 0.01 μF capacitor and 10 $k\Omega$ resistor in series to form a low-pass filter. Connect the output across the capacitor and ensure that the grounding cables are all in the same terminal.
2. Connect the function generator and DSO as in Part A.
3. Feed a sinusoidal signal into the circuit. Calculate the cutoff frequency of the circuit ($f_c = 1/2\pi RC$) and decide an appropriate range of frequencies from $f << f_c$ to $f >> f_c$ to test (try to stay within 100-50,000 Hz as the function generator may not produce a stable signal outside of this range).
4. Measure the input and output voltages of the circuit for each frequency as you vary the frequency from $f << f_c$ to $f >> f_c$.
5. Compute the gain in dB for each set of voltages and plot a graph of gain vs frequency (i.e. a Bode plot).
6. Repeat the same process for the high-pass filter configuration.

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Part C

1. Connect the 30 mH inductor, 0.01 μF capacitor and 10 $k\Omega$ resistor in series to form a series LCR circuit. Connect the output across the capacitor and ensure that the grounding cables are all in the same terminal.

2. Feed a sinusoidal signal into the circuit. Calculate the resonant frequency of the circuit ($f_o = 1/2\pi\sqrt{LC}$) and decide an appropriate range of frequencies from $f << f_o$ to $f >> f_o$ to test.

3. Measure the input and output voltages of the circuit for each frequency as you vary the frequency from $f << f_c$ to $f >> f_c$.

4. Compute the gain in dB for each set of voltages and plot a graph of gain vs frequency (i.e. a Bode plot).

5. Repeat the same process for the high-pass filter configuration.

6. Use the XY function on the DSO to find the Lissajous curve formed when $\omega << \omega_o$, $\omega = \omega_o$ and $\omega >> \omega_o$.

Observations

Note: The yellow waveform is the input signal and blue waveform is the output waveform.

Part A

RC Differentiator

Cutoff Frequency $(\omega_c/2\pi) = 339\ Hz$
Input Frequency $(\omega/2\pi << \omega_c/2\pi) = 30\ Hz$

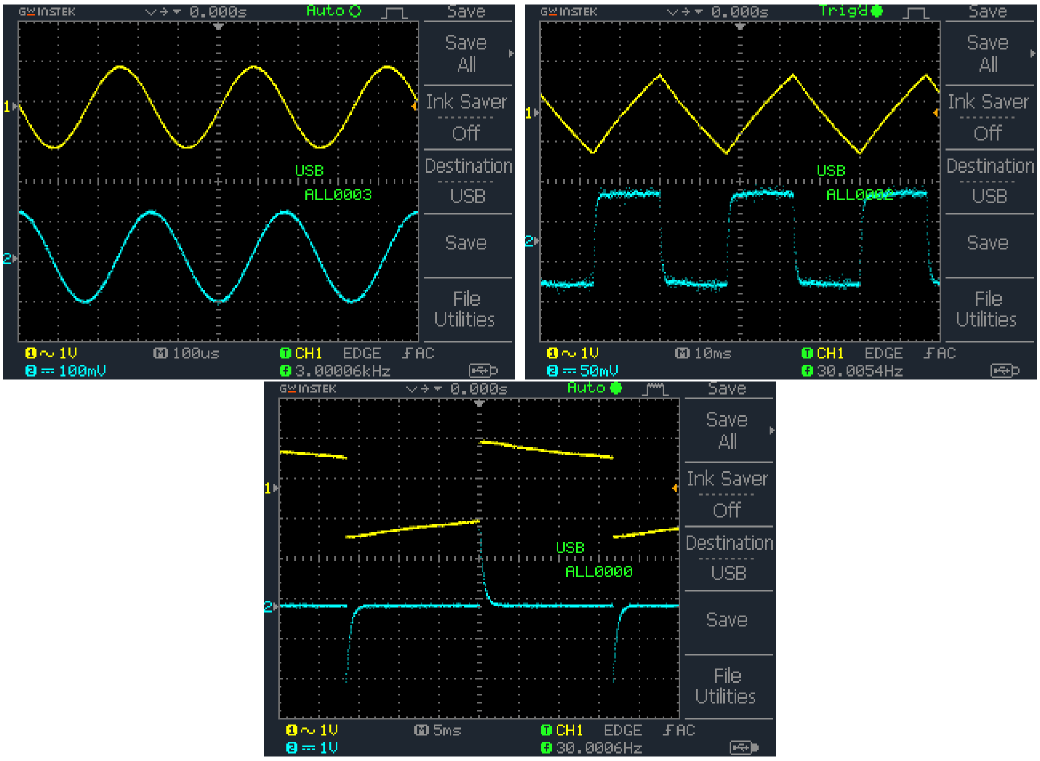

Figure 4: (top left) Sinusoidal wave input gives a sinusoidal wave output with a phase shift
(top right) Triangle wave input gives a square wave output
(bottom) Square wave input gives a linear output

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RC Integrator

Cutoff Frequency $(\omega_c/2\pi) = 339\ Hz$
Input Frequency $(\omega/2\pi >> \omega_c/2\pi) = 30,000\ Hz$

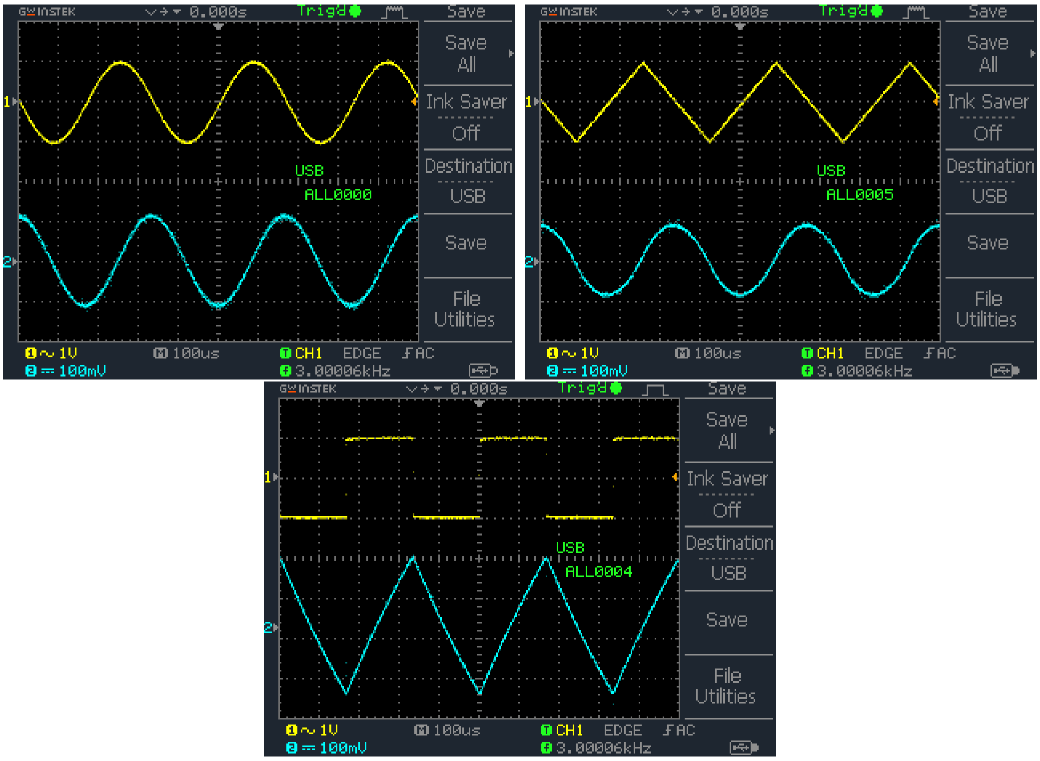

Figure 5: (top left) Sinusoidal wave input gives a sinusoidal wave output with a phase shift
(top right) Triangle wave input gives a quadratic wave output
(bottom) Square wave input gives a triangle wave output

Part B

Least Count of Voltage $= 0.02\ V$
Input Voltage $(V_{in}) = 2.00 \pm 0.02\ V$
Cutoff Frequency $(\omega_c/2\pi) = 1591\ Hz$

High-Pass Filter

Frequency (f) $[kHz]$	Voltage (V_{out}) $[V]$	log(f) $[\log(kHz)]$	Gain $[dB]$
0.10	0.12	-2.30	-56.3
0.15	0.18	-1.90	-48.2
0.20	0.24	-1.61	-42.4
0.20	0.24	-1.61	-42.4
0.30	0.36	-1.20	-34.3
0.40	0.46	-0.916	-29.4
0.60	0.68	-0.511	-21.6
0.80	0.86	-0.223	-16.9
1.20	1.18	0.182	-10.6

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Frequency (f) [kHz]	Voltage (V_{out}) [V]	log(f) [log(kHz)]	Gain [dB]
1.60	1.38	0.470	-7.42
2.40	1.66	0.875	-3.73
3.20	1.78	1.160	-2.33
4.80	1.92	1.860	-0.816
6.40	1.96	1.860	-0.404
9.60	2.00	2.260	0
12.80	2.00	2.550	0
19.20	2.00	2.950	0
25.60	2.00	3.240	0
38.40	2.00	3.640	0
51.20	2.00	3.940	0

Table 1: Output voltage and gain for a frequency range of 0.1-51.2 kHz

Low-Pass Filter

Frequency (f) [kHz]	Voltage (V_{out}) [V]	log(f) [log(kHz)]	Gain [dB]
0.10	2.00	-2.30	0.000
0.15	2.00	-1.90	0.000
0.20	2.00	-1.61	0.000
0.30	2.00	-1.20	0.000
0.40	1.96	-0.916	-0.404
0.60	1.92	-0.511	-0.816
0.80	1.84	-0.223	-1.668
1.20	1.68	0.182	-3.487
1.60	1.44	0.470	-6.570
2.40	1.18	0.875	-10.553
3.20	0.96	1.163	-14.679
4.80	0.68	1.568	-21.813
6.40	0.52	1.856	-26.941
9.60	0.36	2.262	-34.296
12.8	0.26	2.549	-40.804

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Frequency (f) [kHz]	Voltage (V_{out}) [V]	log(f) [log(kHz)]	Gain [dB]
19.2	0.18	2.955	-48.159
25.6	0.14	3.243	-53.185
38.4	0.09	3.648	-62.022
51.2	0.06	3.936	-70.131

Table 2: Output voltage and gain for a frequency range of 0.1-51.2 kHz

Part C

Input Voltage (V_{in}) $= 2.00 \pm 0.02\ V$
Resonant Frequency ($\omega_o/2\pi$) $= 9189\ Hz$
Least Count of Voltage$_1$ ($V(1)_{out}$) $= 0.02\ V$
Least Count of Voltage$_2$ ($V(2)_{out}$) $= 0.008\ V$
Damping Factor (γ_1) [10 $k\Omega$] $= 3.34 \times 10^5\ \Omega/H$
Damping Factor (γ_2) [100 Ω] $= 3.34 \times 10^3\ \Omega/H$

LCR Series Circuit

Raw data for the frequency and voltages can be found in the Appendix.

log(f) [log(kHz)]	Gain$_1$ [dB]	Gain$_2$ [dB]
-0.223	-15.5	-96.5
0.182	-9.88	-96.5
0.470	-6.57	-88.4
0.875	-3.25	-82.7
1.163	-1.67	-74.5
1.569	-0.40	-64.3
1.856	0.00	-53.7
2.079	0.00	-40.4
2.262	0.00	-19.5
2.549	0.00	-46.0
2.955	-0.404	-64.3
3.243	-1.45	-71.5
3.648	-3.73	-78.2
3.936	-6.29	-88.5
4.341	-11.2	-88.5

Table 3: Gain for a series LCR circuit with a 10 $k\Omega$ and 100 Ω resistor, respectively for a frequency range of 0.8-76.8 kHz

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LCR Tank Circuit

Raw data for the frequency and voltages can be found in the Appendix.

log(f) [log(kHz)]	Gain$_1$ [dB]	Gain$_2$ [dB]
-0.223	0.00	-15.7
0.182	0.00	-19.9
0.470	0.00	-24.3
0.875	0.00	-31.7
1.163	-0.201	-37.6
1.569	-0.404	-48.6
1.856	-0.816	-60.7
2.079	-3.02	-74.5
2.128	-4.96	-82.7
2.175	-7.71	-82.7
2.219	-13.0	-96.5
2.262	-21.5	-96.5
2.303	-20.4	-96.5
2.342	-13.0	-96.5
2.380	-8.01	-82.7
2.549	-1.66	-68.8
2.955	-0.404	-50.5
3.243	-0.201	-41.7
3.648	-0.201	-32.1
3.936	0.00	-26.0
4.341	0.00	-18.7

Table 4: Gain for a tank LCR circuit with a 10 $k\Omega$ and 100 Ω resistor, respectively for a frequency range of 0.8-76.8 kHz

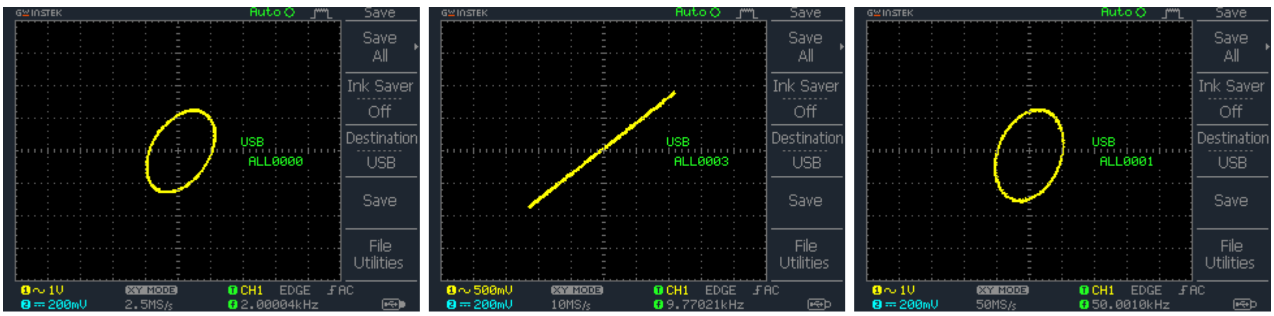

Figure 6: Lissajous figures of input and output voltage in a tank LCR circuit when (left) $\omega << \omega_o$, (middle) $\omega = \omega_o$, (right) $\omega >> \omega_o$

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Analysis

Part A

RC Differentiator

When we operate in the low frequency regime while using a high-pass filter, the RC circuit appears to behave like a differentiator.

From fig 4 we can see that the output signal is proportional to the derivative of the input signal. For instance, the derivative of the input sine wave will be a cosine wave, which is simply a sine wave with a phase shift of $\pi/2$, which is visible in fig 4 (top left).

$$\frac{d}{dx}\sin x = \cos x = \sin(x + \pi/2)$$

Similarly, in a triangle wave each half cycle is a linear function which when differentiated corresponds to a constant function in the half cycle of the square wave in the output signal. Assuming the function for a half cycle is given by $mx + c$:

$$\frac{d}{dx}(mx + c) = m$$

When the slope (m) is negative, the output function takes a negative value as well, thereby forming a periodic square wave (fig 4, top right). Hence, the square wave is proportional to the derivative of the triangle wave.

Likewise, when the input is a square wave (which is a constant for a each half cycle), the output is simply zero, which is the derivative of a constant.

$$\frac{d}{dx}(k) = 0$$

Irrespective of whether k is positive or negative, the output $V_{out} = 0$ (fig 4, bottom).

RC Integrator

When we operate in the high frequency regime while using a low-pass filter, the RC circuit appears to behave like an integrator.

From fig 5 we can see that the output signal is proportional to the integral of the input signal. For instance, the derivative of the input sine wave will be a negative cosine wave, which is simply a sine wave with a phase shift of $-\pi/2$, which is visible in fig 5 (top left).

$$\int \sin x dx = -\cos x + C = \sin(x - \pi/2) + C$$

Similarly, in a triangle wave each half cycle is a linear function which when integrated corresponds to a parabolic function in the half cycle of the output signal. Assuming the function for a half cycle of the input is given by $mx + c$:

$$\int (mx + c)dx = \frac{mx^2}{2} + cx + C$$

When the slope (m) is negative, the output function forms an inverted parabola while a positive slope forms an upright parabola, thereby forming a periodic parabolic wave (fig 5, top right). Hence, the parabolic wave is proportional to the integral of the triangle wave.

Finally, for a square wave input, we get a triangle wave output signal. We have already shown that the derivative of a triangle wave is a square wave, so it follows that the integral of a square wave

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will be a triangle wave. We can verify this by assuming the function of a square wave is a constant k for a half cycle:

$$\int k dx = kx + C$$

When k is positive, the integral will be a linear function with positive slope and when k is negative, the integral will be a linear function with negative slope. This effectively forms a triangle waveform.

Part B

High-Pass Filter

Plotting the bode plot of log(frequency) vs gain from the data in table 1, we obtain the following graph:

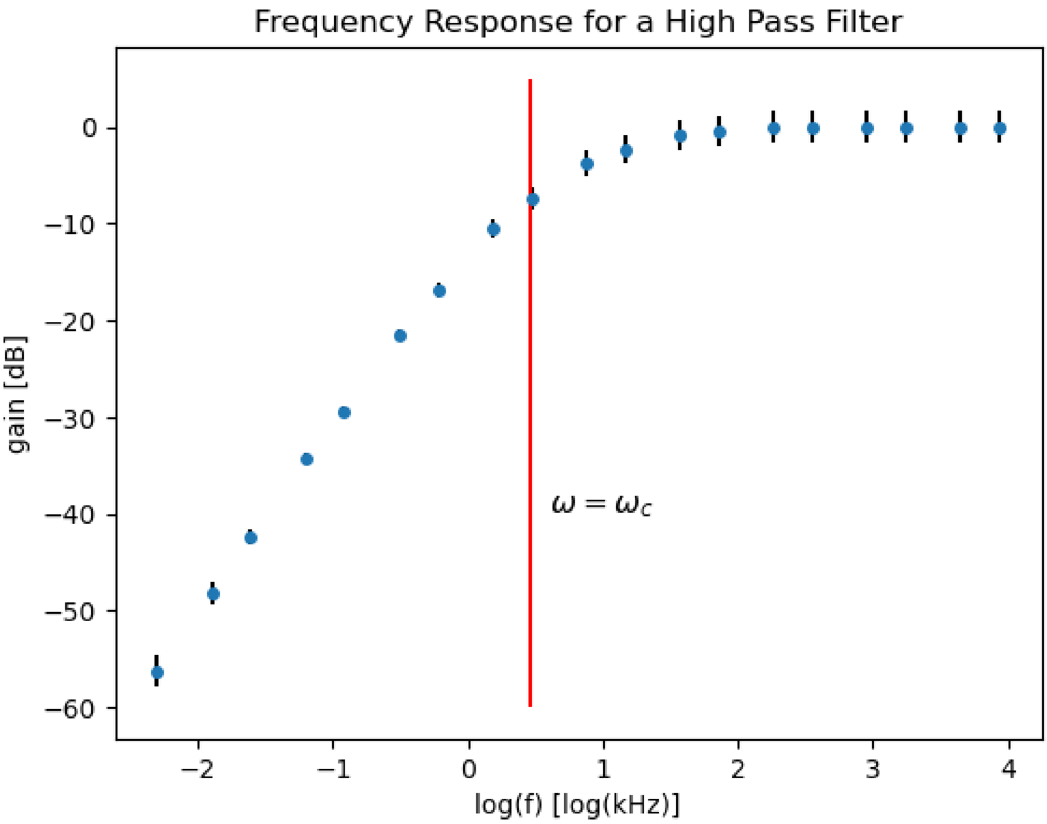

Figure 7: The gain increases rapidly for $\omega < \omega_c$ until the cutoff frequency ω_c after which it begins to plateau for $\omega > \omega_c$

This corresponds with our theoretical understanding of a high-pass filter in which low frequencies are blocked by the circuit (i.e. negative gain) while high frequencies pass relatively undisturbed (i.e. gain close to zero).

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Low-Pass Filter

Plotting the bode plot of log(frequency) vs gain from the data in table 2, we obtain the following graph:

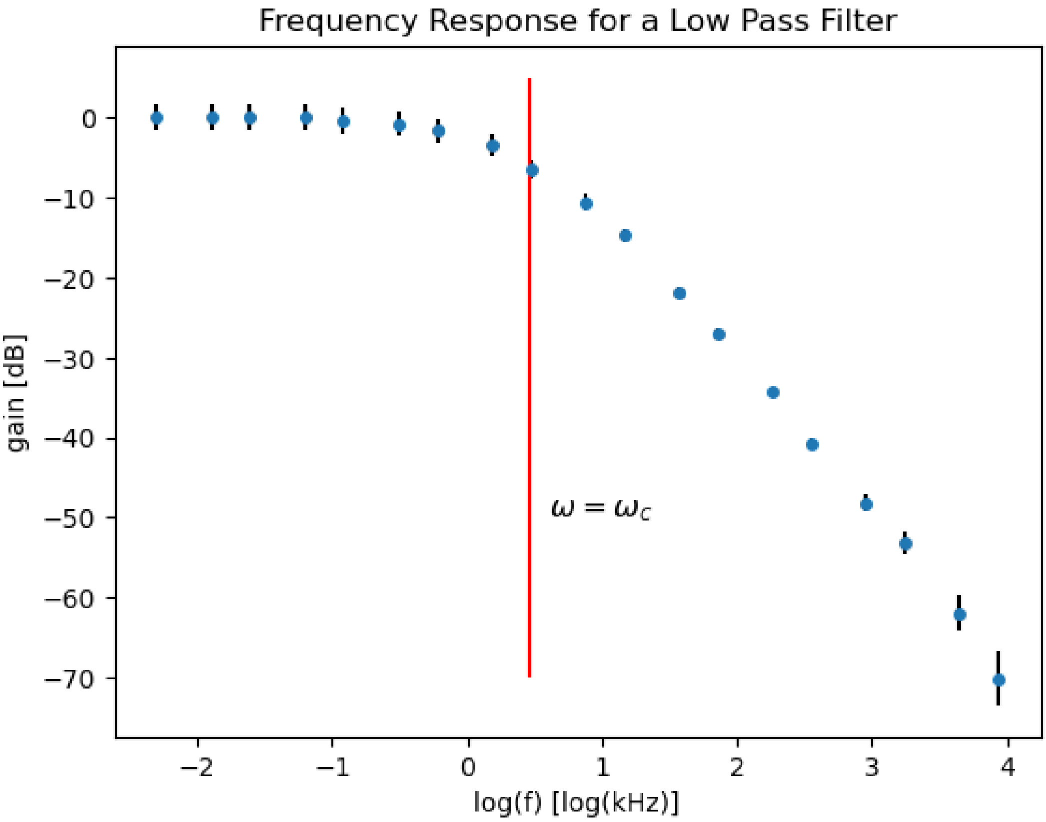

Figure 8: For $\omega < \omega_c$ the gain remains constant and low until the cutoff frequency ω_c after which it begins to rapidly decrease for $\omega > \omega_c$

This corresponds with our theoretical understanding of a low-pass filter in which low frequencies pass relatively undisturbed (i.e. gain close to zero) whereas high frequencies are blocked by the circuit (i.e. negative gain) since the capacitor acts as a short circuit causing the output voltage to essentially be zero.

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Part C

Plotting the bode plot of log(frequency) vs gain from the data in table 3, we obtain the following graph:

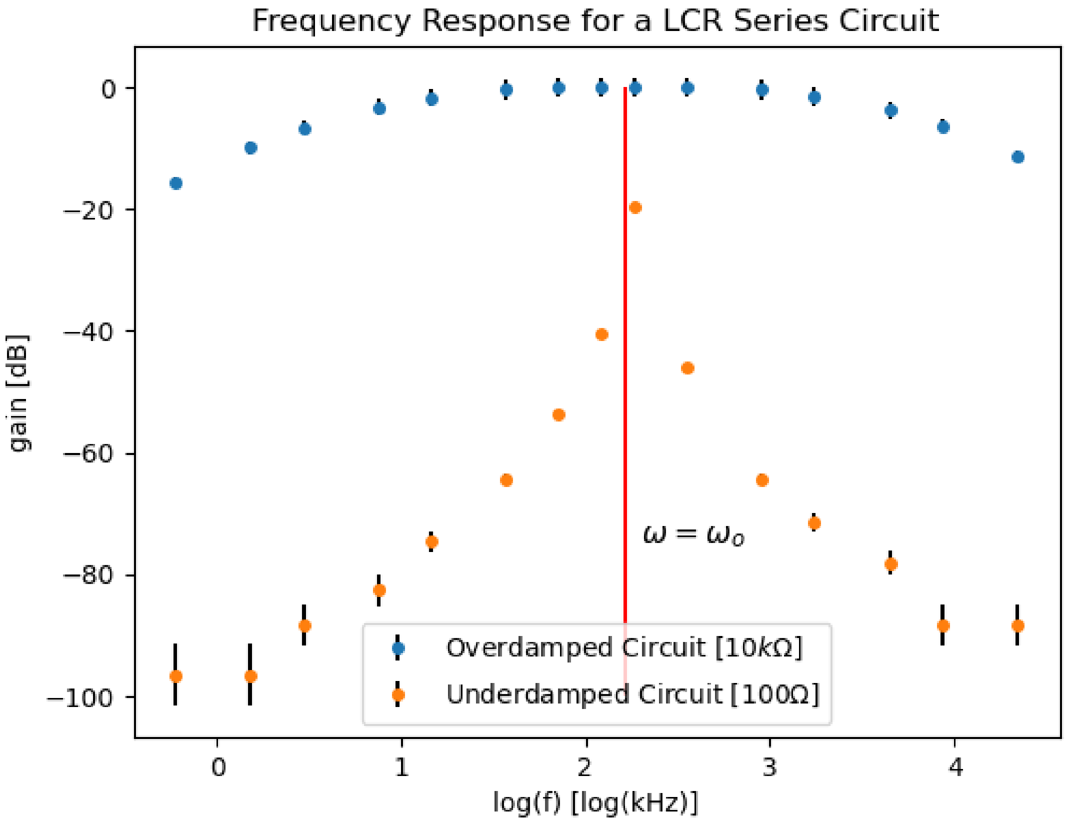

Figure 9: The overdamped circuit forms a broad peak whereas the underdamped circuit forms a narrow peak

The nature of the two circuits can be found by comparing their respective damping factors with the resonant frequency:

$$\gamma_1 = R/L = 3.34 \times 10^5 \ \Omega/H$$

$$\gamma_2 = R/L = 3.34 \times 10^3 \ \Omega/H$$

$$2\omega_o = 1/\sqrt{LC} = 1.82 \times 10^4 \ Hz$$

$\gamma_1 > 2\omega_o$, which means that the circuit with the 10 $k\Omega$ resistor is overdamped. This large resistance leads to a larger bandwidth (given by $\Delta f = \gamma/2\pi$) and a correspondingly broad peak in the bode plot. However in the circuit with the 100 Ω resistor, $\gamma_2 < 2\omega_o$ which means that it is an underdamped system. The low resistance causes a lower bandwidth and a correspondingly narrow peak.

It can also be noticed that the peak does not lie exactly at $\omega = \omega_o$. This may be due to the non-ideal nature of the system which causes the natural frequency of circuit to shift sightly, thereby deviating from the theoretically expected result.

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Plotting the bode plot of log(frequency) vs gain from the data in table 4, we obtain the following graph:

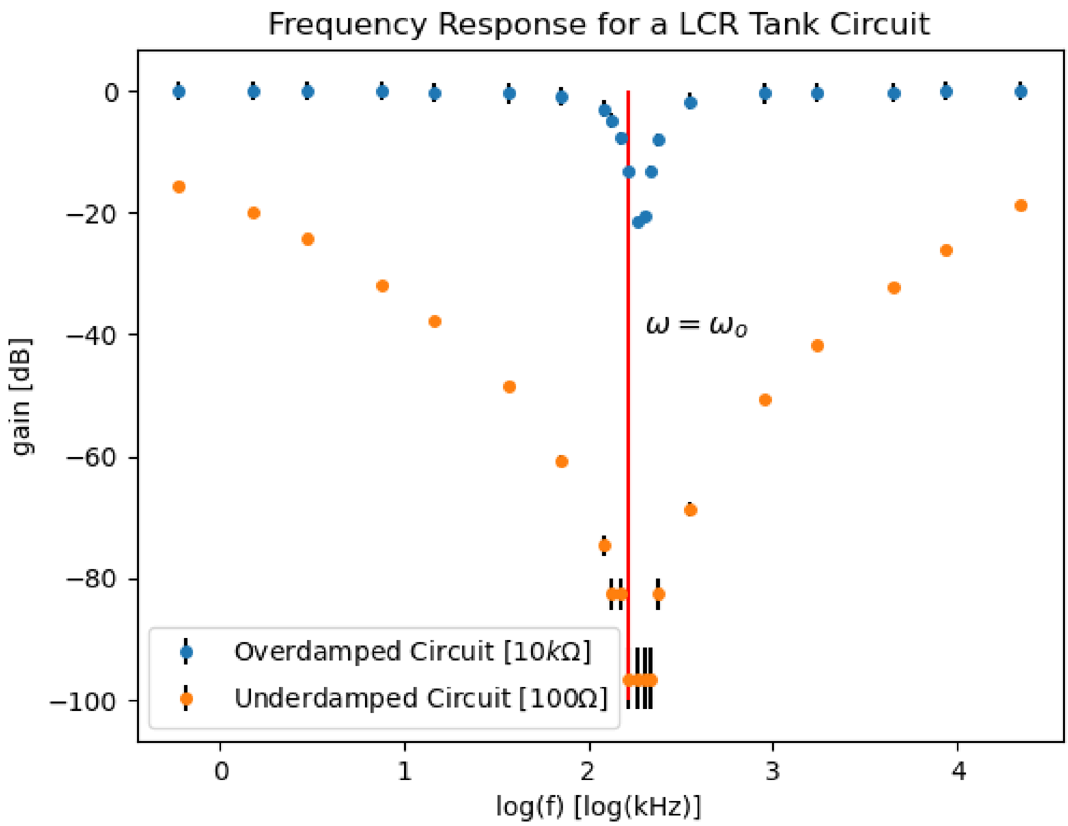

Figure 10: Both circuits reach a minimum gain close to the resonant frequency of the system

As expected from our theoretical understanding of a tank LCR circuit, the current is minimised when $\omega = \omega_o = 1/\sqrt{LC}$. This is reflected in the frequency response graph as a minima peak near the resonant frequency. This is an invertion of the result observed in a series LCR circuit.

When we observe the Lissajous figures formed by the input and output frequencies in an LCR circuit (fig 6), we notice that when $\omega << \omega_o$ or $\omega >> \omega_o$, an ellipse is formed, implying that there is a phase difference between the two signals. However, when $\omega = \omega_o$ and the system is in resonance, the signals are perfectly in phase, as represented by a straight line (fig 6, middle). This is because at the resonant frequency, the effects of the capacitor and inductor cancel each other out and the circuit behaves as a purely resistive circuit in which the input and output are in phase.

Error Analysis

The error in the gain originates from the least count error from the measured voltages. Applying the general formula for the propagation of error in a multi-variable function:

$$\delta f = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 \delta x_i^2}$$

Here our f is our gain and the dependent variables are V_{out} and V_{in} with their respective least count errors. We know that gain in dB is given by:

$$gain = 20 \log_{10}(V_{out}/V_{in})$$

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Hence,

$$\delta(gain) = \sqrt{\left(\frac{\partial(gain)}{\partial V_{out}}\right)^2 \delta V_{out}^2 + \left(\frac{\partial(gain)}{\partial V_{in}}\right)^2 \delta V_{in}^2}$$

$$\implies \delta(gain) = \sqrt{\left(\frac{20}{\ln(10)V_{out}}\right)^2 \delta V_{out}^2 + \left(-\frac{20}{\ln(10)V_{in}}\right)^2 \delta V_{in}^2}$$

$$\implies \delta(gain) = \sqrt{\left(\frac{75.444}{V_{out}^2}\right) \delta V_{out}^2 + \left(\frac{75.444}{V_{in}^2}\right) \delta V_{in}^2}$$

$$\implies \delta(gain) = 8.685\sqrt{\left(\frac{\delta V_{out}}{V_{out}}\right)^2 + \left(\frac{\delta V_{in}}{V_{in}}\right)^2}$$

The above expression was used to find the error in gain for each data point and plot its corresponding error bar. Additional sources of error may have been present due to loose connections or energy loss in the circuit. However, our data lies within an acceptable margin of error to draw the necessary conclusions.

Results

Part A

When we operate in the low frequency regime while using a high-pass filter, the RC circuit behaves like a differentiator such that the output signal is proportional to the derivative of the input signal.

Similarly, when we operate in the high frequency regime while using a low-pass filter, the RC circuit behaves like an integrator such that the output signal is proportional to the integral of the input signal.

Part B

In a high-pass filter low frequencies are blocked by the circuit (i.e. negative gain) while high frequencies pass relatively undisturbed (i.e. gain close to zero). In a low-pass filter in which low frequencies pass relatively undisturbed (i.e. gain close to zero) whereas high frequencies are blocked by the circuit (i.e. negative gain).

Part C

In an LCR series circuit the maximum gain is achieved at the resonant frequency. The bandwidth of the peak is dependent on the damping factor (γ) of the system and a higher resistance will cause overdamping and therefore a broader bandwidth. Conversely, low resistance will cause underdamping and therefore a narrow peak. The point of critical damping is when $\gamma = 2\omega_o$.

The frequency response in a tank LCR circuit is an inversion of the frequency response in a series LCR circuit. The gain reaches a minimum value at $\omega = \omega_o$ in both underdamped and overdamped cases.

When a LCR circuit is at resonant frequency, the input and output signal are in phase and the circuit behaves like a pure resistive circuit.

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Appendix

Frequency (f) [kHz]	Voltage$_1$ ($V(1)_{out}$) [V]	Voltage$_2$ ($V(2)_{out}$) [V]
0.80	0.92	0.016
1.20	1.22	0.016
1.60	1.44	0.024
2.40	1.70	0.032
3.20	1.84	0.048
4.80	1.96	0.080
6.40	2.00	0.136
8.00	2.00	0.264
9.60	2.00	0.752
12.8	2.00	0.200
19.2	1.96	0.080
25.6	1.86	0.056
38.4	1.66	0.040
51.2	1.46	0.024
76.8	1.14	0.024

Table 5: Output voltages for a series LCR circuit with a 10 $k\Omega$ and 100 Ω resistor, respectively for a frequency range of 0.8-76.8 kHz

Frequency (f) [kHz]	Voltage$_1$ ($V(1)_{out}$) [V]	Voltage$_2$ ($V(2)_{out}$) [V]
0.80	2.00	0.912
1.20	2.00	0.736
1.60	2.00	0.592
2.40	2.00	0.408
3.20	1.98	0.304
4.80	1.96	0.176
6.40	1.92	0.096
8.00	1.72	0.048
8.40	1.56	0.032
8.80	1.36	0.032
9.20	1.04	0.016
9.60	0.68	0.016
10.0	0.72	0.016

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Frequency (f) [kHz]	Voltage$_1$ ($V(1)_{out}$) [V]	Voltage$_2$ ($V(2)_{out}$) [V]
10.4	1.04	0.016
10.8	1.34	0.032
12.8	1.84	0.064
19.2	1.96	0.160
25.6	1.98	0.248
38.4	1.98	0.400
51.2	2.00	0.544
76.8	2.00	0.784

Table 6: Output voltages for a tank LCR circuit with a 10 $k\Omega$ and 100 Ω resistor, respectively for a frequency range of 0.8-76.8 kHz