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Free Fall and Terminal Velocity
Lab Report 5

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Aim

- To determine the acceleration due to gravity experienced by a freely falling object.
- To study the determine the terminal velocity of a free-falling object and its dependence on the mass of the object.

Theoretical Background

From Newton's second law, we know that the acceleration of an object falling vertically is proportional to the force it experiences due to gravity.

$$a_y = \frac{F}{m} = \frac{-mg}{m} = -g$$

By integrating this equation with respect to time, we can find the velocity along the y-direction:

$$\int dv = -g \int dt$$

$$\implies v_y(t) = v_y(0) - gt$$

Integrating once more with respect to time, we find the position-time relation:

$$\int dy = \int (v_y(0) - gt)dt$$

$$\implies y(t) = y(0) + v_y(0)t - \frac{1}{2}gt^2 \tag{1}$$

Hence, for a body that only experiences gravitational acceleration (neglecting the effect of air resistance), equation 1 describes its motion.

If we introduce drag to this picture, we must consider an additional drag force (F_d). This drag force is produced by the air that pushes against the object as it falls. The mass of the air pushed by the object travelling at velocity v in time Δt is $m_a = \rho_a A v \Delta t$, where A is the cross sectional area of

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object and ρ_a is the density of air. After being pushed, the air gains a momentum of $m_a v = \rho_a A v^2 \Delta t$, which is equivalent to experiencing a force $F_a = \rho_a A v^2$. Due to Newton's third law, the force causing the air to accelerate induces an equal an opposite force that causes the object to decelerate. However, we have not yet considered the effect of the shape of the object, so we incorporate this information by adding a *drag coefficient* C to our equation. Hence, the drag force becomes:

$$F_d = -C\rho_a A v^2 = -\alpha v^2$$

Hence, our equation for motion is:

$$ma = F_g - F_d = mg - \alpha v^2$$

Note that if $F_g = F_d$, the body will reach a steady state in which it will attain a terminal velocity (v_t) given by:

$$v_t = \sqrt{\frac{mg}{\alpha}} \tag{2}$$

Solving our equation of motion in terms of v gives us the following equation:

$$m\frac{dv}{dt} = mg - \alpha v^2$$

$$\implies \int \frac{1}{(g - \frac{\alpha v^2}{m})} dv = \int dt$$

$$\implies v(t) = v_t \tanh\left(\frac{gt}{v_t}\right) \tag{3}$$

Integrating once more, we can get the position-time equation:

$$\int dy = \int v_t \tanh\left(\frac{gt}{v_t}\right) dt$$

$$\implies y(t) = \frac{v_t^2}{g} \ln\left(\cosh\left(\frac{gt}{v_t}\right)\right) \tag{4}$$

Experimental Setup

- A DSLR Camera (Nikon D5600) which can take videos of up to 60 fps
- A tripod with appropriate attachments to hold the cameras
- A computer with Tracker Video Analysis Software
- A meter scale to calibrate the videos
- A set of small balls of different materials
- A large cupcake liner
- 16 small magnets to be placed on the cupcake liners to increase their mass
- A ladder
- A weighing scale
- A spirit level

Least count of weighing scale = 0.1 g

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Procedure

Part A

1. Set up the tripod and camera a suitable distance from a plain white wall such that a height of at least 2 meters is in the frame. Place the meter scale in the frame of the video.

2. Ensure that the tripod is levelled with the spirit level and set the camera to 60 fps recording.

3. Choosing appropriately massive balls, stick a brightly coloured sticker on them for reference and mark another reference point on the wall as your drop point.

4. Record 20-30 videos of different balls falling from the same point. Drop the balls such that they fall vertically and the sticker is visible throughout the fall.

5. Using the sticker as a reference, track the balls in the videos and find the position-time data for each ball. Use equation 1 to determine their acceleration.

6. Plot a histogram of the accelerations and fit a Gaussian curve on it. Determine the value of g from the mean of the Gaussian curve.

Part B

1. Measure the mass of the large cupcake liner and the entire set of 16 magnets. Divide the weight of the magnets by 16 to determine the weight of each individual magnet.

2. Repeat the steps in part A for the cupcake liner, adding one magnet to increase its mass with each subsequent drop.

3. Use tracker to determine the velocity-time data of the cupcake liners and fit a tan hyperbolic function similar to equation 3 on the data. Determine both v_t and g for each mass.

4. Plot a histogram of the g values and fit a Gaussian curve on it. Determine the best value of g from the mean of the Gaussian curve.

5. Plot a log graph of v_t vs m to determine the power law relationship between them. Verify the theoretically expected result from equation 2.

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Observations and Analysis

Part A

We used three different balls for this part of the experiment - a table tennis (TT) ball, a small black squash ball and a large red rubber ball. Out of the 27 different videos, 23 could be successfully tracked and an acceleration could be determined (raw data in Appendix). We plotted a histogram of the accelerations and fit a Gaussian curve on top of it:

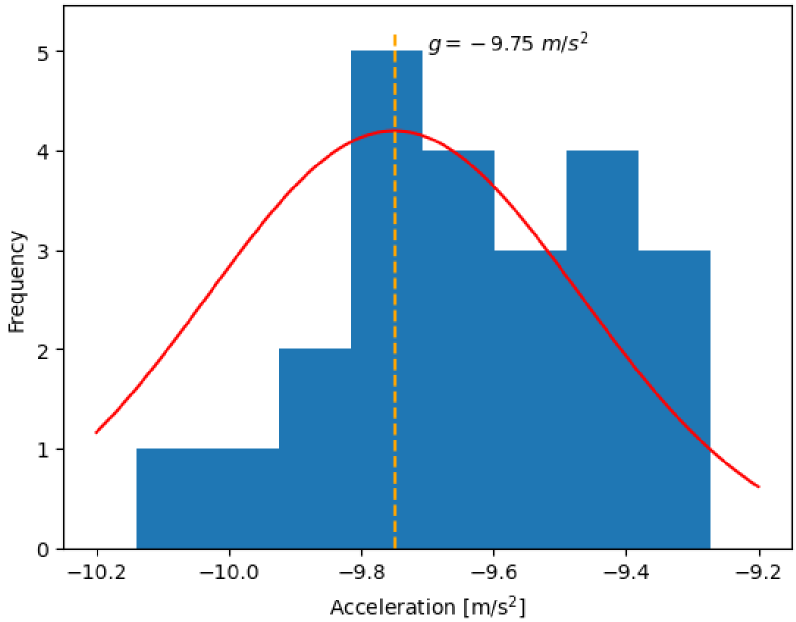

Figure 1: Since the material of the ball has no bearing on its acceleration, we should expect a normal distribution for the data since it is only subject to random variation

The mean of the Gaussian fit is $-9.75\ m/s^2$, with a standard deviation of $0.28\ m/s^2$. This is an acceptable value of g since the true value of $g = -9.81\ m/s^2$ lies within one standard deviation of the experimentally determined value.

Part B

Least count of weighing scale = 0.1 g
Mass of 16 magnets = 1.6 ± 0.1 g
Mas of 1 magnet = 0.1 ± 0.006 g

The terminal velocity and gravitational acceleration for each mass was found by plotting the velocity-time data and curvefitting a tanh function of the form $v(y) = v_t \tanh(gt/v_t)$ to it.

Mass of Liner (m) [g]	Gravitational Acceleration (g) [m/s^2]	Terminal Velocity (v_t) [m/s]
0.8	-7.746	-1.519
1.0	-7.804	-1.643
1.1	-11.4	-1.632
1.2	-6.726	-1.986
1.3	-9.471	-2.036
1.4	-7.859	-2.089

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Mass of Liner (m) [g]	Gravitational Acceleration (g) [m/s^2]	Terminal Velocity (v_t) [m/s]
1.5	-7.628	-2.279
1.6	-7.195	-2.236
1.7	-9.0	-2.293
1.8	-8.73	-2.319
1.9	-7.484	-2.601
2.0	-7.605	-2.514
2.1	-8.248	-2.51
2.2	-8.181	-2.528
2.3	-9.693	-2.519
2.4	-7.247	-2.83

Table 1: The terminal velocity and gravitational acceleration for different masses of cupcake liners

The graphs for the tan hyperbolic fits of the v-t data are below:

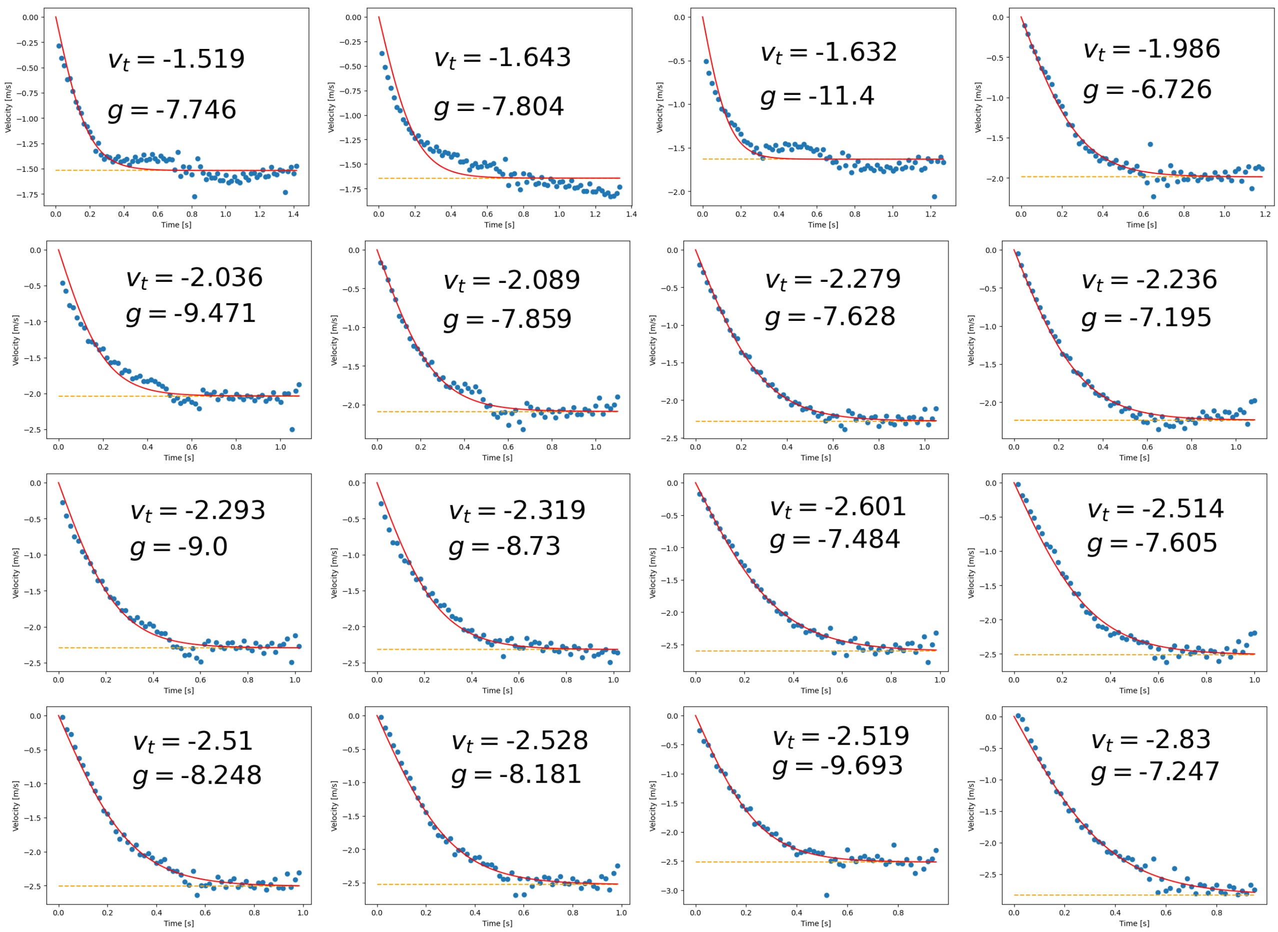

Figure 2: The tan hyperbolic fit of each v-t data set yielded
a corresponding terminal velocity and gravitational acceleration

Clearly, objects with sufficiently high cross-sectional surface area to mass ratio experience signifi-

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cant drag force and attain a terminal velocity described by equation 2.

$$v(t) = v_t \tanh\left(\frac{gt}{v_t}\right)$$

We plotted a histogram of the g values found from this analysis and determined the best value for g.

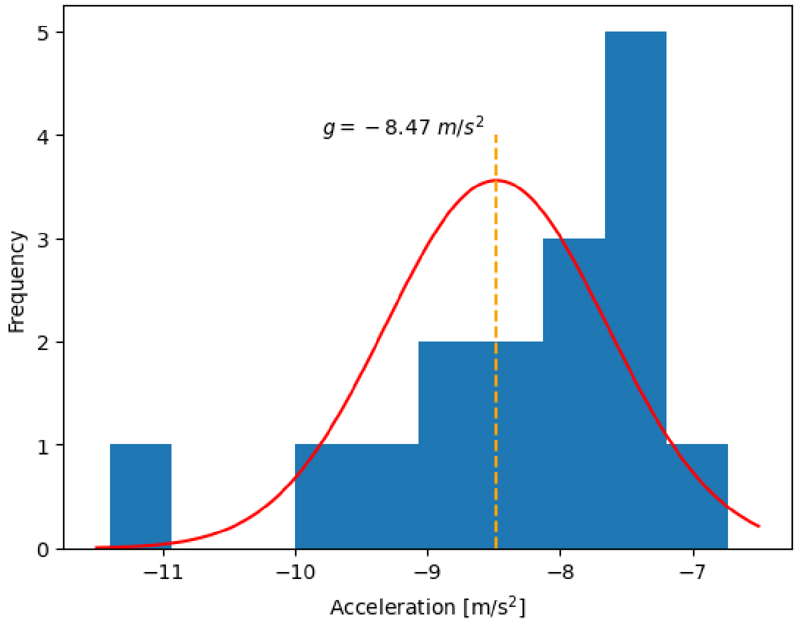

Figure 3: The value of g is lower in this histogram, most likely because of energy lost through heat and oscillations, which have not been accounted for

The mean of the Gaussian fit is $-8.47\ m/s^2$, with a standard deviation of $0.84\ m/s^2$. Though this value is worse than the part A result, it is an acceptable value of g since the true value of $g = -9.81\ m/s^2$ lies within two standard deviation of the experimentally determined value.

Finally, plotting a log graph of the terminal velocity against the mass of the liner (table 1), we determined the power law relationship between the two quantities.

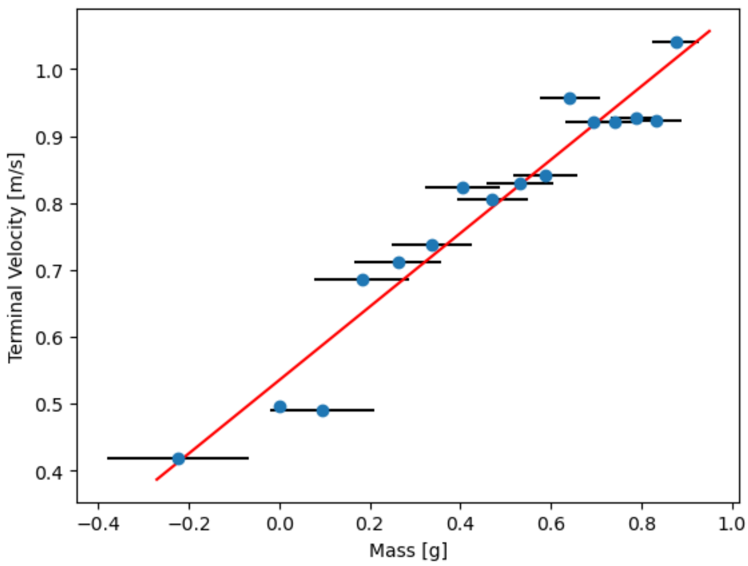

Figure 4: The slope of the log graph between v_t and m determines the power law relationship between the two quantities

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Assuming $v_t \propto m^p \implies v_t = km^p \implies \ln(v_t) = p\ln(m) + \ln(k)$. The slope of the graph is $p = 0.549$, which is approximately 0.5. The theoretically expected result is 0.5, since equation 2 tell us:

$$v_t = \sqrt{\frac{mg}{\alpha}} \implies v_t \propto m^{1/2}$$

Hence, we have experimentally verified that the terminal velocity depends on the square root of the mass of the object.

Error Analysis and Discussion

The error in the slope of the $\ln(v_t)$ vs $\ln(m)$ graph is due to the error in the mass measurement. Since each mass has an error of $\pm 0.006\ g$, the errorbar for each data point varies in the log graph. The error for each data point was calculated in the following manner:

$$\Delta m = ||\ln(m + 0.006)| - |\ln(m - 0.006)||$$

Since this gives the maximum deviation for each data point given the least count error in mass.

For the histogram, the value of g is within an acceptable margin of error if it is within one standard deviation. Hence, for part A:

$$g = -9.75 \pm 0.28\ m/s^2$$

Additional sources of error may be:

- The cupcake liners tend to oscillate as they fall, leading to loss of energy as heat or kinetic energy in the x-direction. This could lead to error in the results since we have only analysed the y-direction motion and assumed each object behaves like a point mass.
- If the shape of the cupcake liner changed during the experiment due to small deformations, the value of α which contains information about the shape of the object would different. This would effectively change the object being analysed and lead to deviation from the behaviour expected from an unchanging object.
- The Tracker software is only capable of capturing granular data in fixed time intervals, from which we extrapolate the continuous motion of the mass. The time intervals over which Tracker can capture data is limited by the frame rate of our camera (60 fps in our case). A higher frame rate camera would allow for more accurate data collection and results.

Results

Part A

The gravitation acceleration found by analysing the fall of massive spheres which experience negligible drag is:

$$g = -9.75 \pm 0.28\ m/s^2$$

Part B

Light-weight objects with a large surface area such as a cupcake liner experience a significant drag force and their motion can be described by a tan hyperbolic function:

$$v(t) = v_t \tanh\left(\frac{gt}{v_t}\right)$$

The terminal velocity was found be proportional to the square root of the mass of the object.

$$v_t = \sqrt{\frac{mg}{\alpha}} \implies v_t \propto m^{1/2}$$

Appendix

The raw data for the experiment can be found in the following drive file:
$https://drive.google.com/drive/folders/1li0jmJYqYzbbX0zKmdqVP1bxkfEX6dru?usp=sharing$