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Electromagnetic Damping and Eddy Currents
Lab Report 6

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Date of Experiment: November 14th, 2023

Date of Submission: November 28th, 2023

Aim

- To measure the frictional torque and moment of inertia for a rotating flywheel.
- To study the effects of electromagnetic damping produced by eddy currents in a rotating flywheel.

Theoretical Background

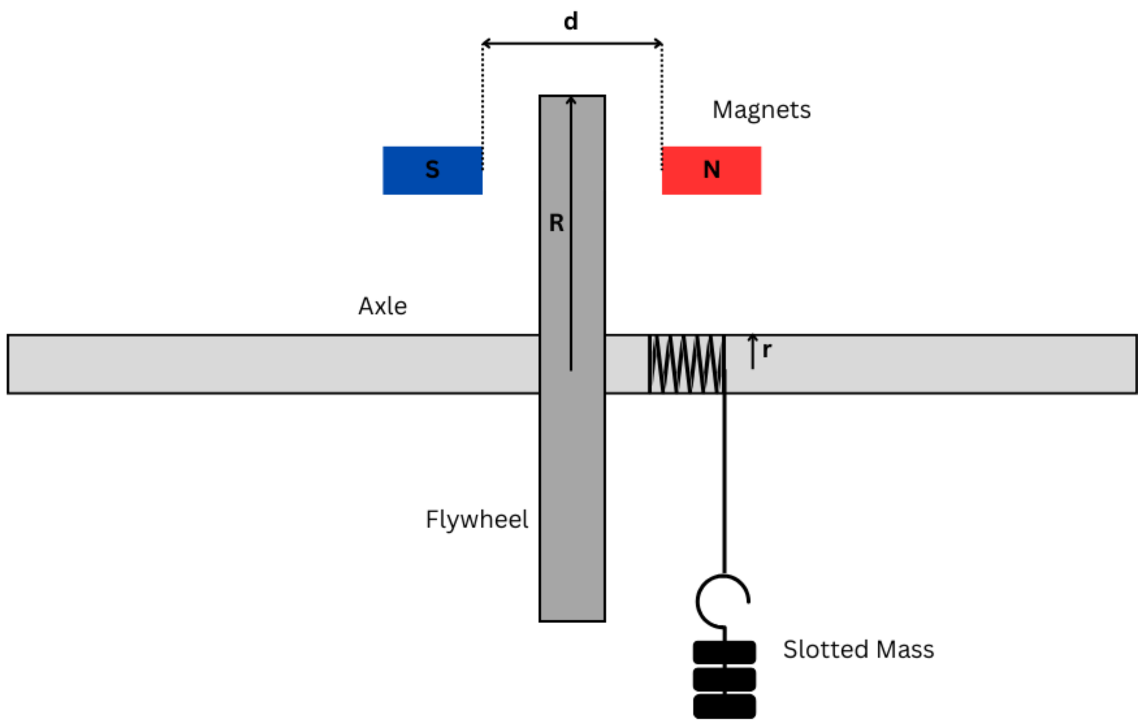

Figure 1: Experimental Setup for a electromagnetic damping of a rotating flywheel
(Source: Made on www.canva.com)

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When a conductor moves through a magnetic field, it experiences a change in magnetic flux inducing current in the conductor. These current loops - known as *eddy currents* - are formed such that the magnetic field induced by them opposes the change in magnetic flux. The interaction between the external and induced magnetic fields exerts a force on the conductor that damps its motion.

In the experimental setup in figure 1, a thick aluminium flywheel is rotated by a falling mass. Two bar magnets on either side of the disc produce a magnetic field passing through the disc, which induces a damping force as the wheel begins to rotate.

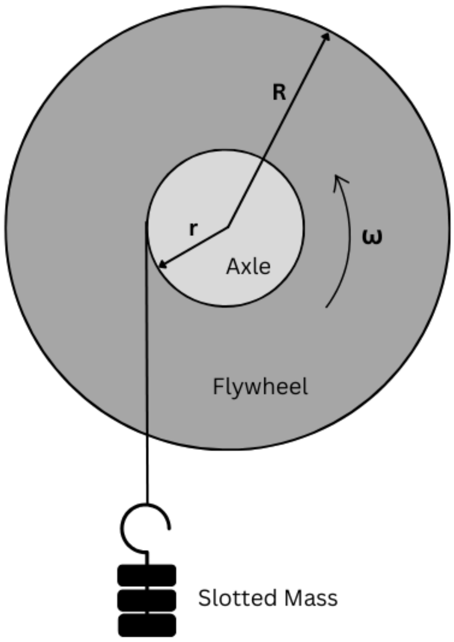

Figure 2: Schematic diagram of the rotating flywheel without a magnetic field passing through the disc (Source: Made on www.canva.com)

Newton's second law when applied to rotational dynamics tells us that the torque on a body is proportional to its angular acceleration. The constant of proportionality is called the moment of inertia of the body and it is characterised by the mass distribution in the body.

$$\tau = I\alpha$$

In figure 2, the axle experiences two torques - a torque from the falling mass (τ_r) and a frictional torque (τ_f). Balancing these, we find:

$$I\alpha = \tau_r - \tau_f$$

$$\implies \frac{Ia}{r} = m(g-a)r - \tau_f$$

$$\implies a = \frac{mgr^2}{(I+mr^2)} - \frac{r\tau_f}{(I+mr^2)}$$

Where m is the falling mass, a is its acceleration and I is the moment of inertia of the flywheel-axle system. Note that $I >> mr^2$ since the moment of inertia for the entire system will be much greater than the moment of inertia of just the axle ($\sim mr^2/2$). This allows us to make the approximation:

$$a \approx m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right) \tag{1}$$

If we add electromagnetic damping to this picture, we find that there is an additional component of torque which we must consider - induced magnetic torque (τ_B). We know that τ_B must act such

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that it opposes the change in magnetic flux, which implies that it acts opposite to the torque applied by the falling mass. Our torque equation therefore becomes:

$$I\alpha = \tau_r - \tau_f - \tau_B$$

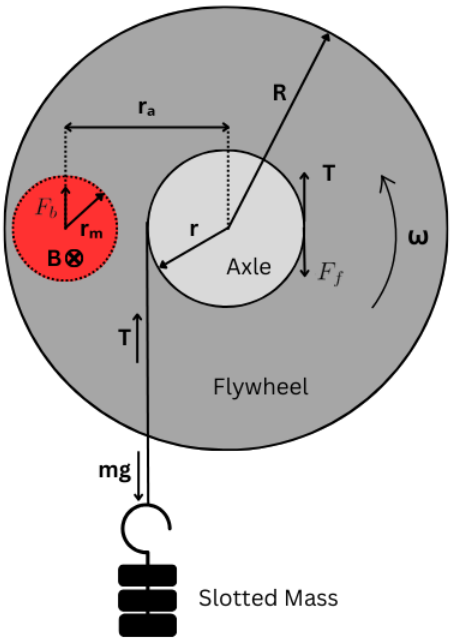

Figure 3: Schematic diagram of the rotating flywheel with a magnetic field passing through the disc (Source: Made on www.canva.com)

If we assume that the magnetic field is restricted to a circle of radius r_m whose center is a distance r_a from the axle, we can consider the force on a conducting path along the diameter of this circle of length $2r_m$. Since this path travels at a velocity of $v = \omega r_a$, the motional emf produced is:

$$\epsilon_B = vlB = (\omega r_a)(2r_m)B$$

Assuming that the resistance over this path is R^*, the current due to the motional emf is given by:

$$i = \frac{\epsilon_B}{R^*} = \frac{2\omega r_a 2 r_m B}{R^*}$$

This current will in turn experience a force since it runs across a magnetic field. This magnetic damping force (F_B) opposes the motion of the flywheel (figure 3) and is given by:

$$F_b = iBl = \left(\frac{4r_a r_m^2}{R^*}\right)\omega B^2$$

Finally, the torque produced by this force is:

$$\tau_B = r_a \times F_B = \left(\frac{4r_a^2 r_m^2}{R^*}\right)\omega B^2 = K\omega B^2$$

Where K is a constant for a given system configuration. The modified equation of motion is therefore:

$$I\alpha = \tau_r - \tau_f - \tau_B$$

$$\implies \frac{Ia}{r} = m(g-a)r - \tau_f - K\omega B^2$$

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$$\implies a = \frac{mgr^2}{(I+mr^2)} - \frac{r\tau_f}{(I+mr^2)} - \frac{rK\omega B^2}{(I+mr^2)}$$

Since the damping torque is proportional to the angular velocity of the disk, the disk will slow down until it reaches a steady state where the flywheel spins at a "terminal" angular velocity. Hence, the falling mass will also achieve a terminal velocity $v_t = r\omega$. In this steady state, $a = 0$, which gives our condition for terminal velocity:

$$mgr^2 - r\tau_f - rK\omega B^2 = 0$$
$$\implies (r\omega)KB^2 = mgr^2 - r\tau_f$$
$$\implies r\omega = v_t = \frac{gr^2}{KB^2}\left(m - \frac{\tau_f}{gr}\right) \tag{2}$$

Hence, the terminal velocity is inversely proportional to the square of the magnetic field and varies linearly with respect to the mass of attached to the axle.

Experimental Setup

- A flywheel disc assembly, mounted on the wall
- A pair of magnets of opposite polarity mounted with screw gauges
- A string tied to a hook attachment for slotted masses
- A set of slotted masses
- A Gauss meter to measure magnetic fields
- A meter scale
- A pair of vernier calipers
- A weighing scale
- A high fps camera
- A computer with Tracker software
- A spirit level

Least count of meter scale = 1 mm
Least count of vernier calipers = 0.02 mm
Least count of screw gauge = 0.01 mm
Least count of weighing scale = 0.1 g
Least count of Gauss meter = 10^{-4} T
Frame rate of camera = 60 fps

Procedure

Part A

1. Ensure that the magnets are removed from the setup for this part of the experiment.
2. Attach the string to the axle of the flywheel and wind it such that the string does not cross or overlap. Successive turns should touch each other, and should not extend beyond the length or diameter of the axle.
3. Allow the mass to drop gently, taking care that it does not oscillate as it falls.
4. Record a video of the mass as it falls and analyse it with Tracker to find its acceleration.
5. Repeat this procedure for different masses. Plot a graph between acceleration and mass and use it to find the moment of inertia and frictional torque of the system.

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Part B

1. Insert a pair of magnets in their holders, and place them on the assembly as shown in figure 1.
2. Make sure that the magnets are equally spaced from the disc. In order to do this, bring them as close to the disc as possible, and note down the readings on the screw gauges on either side. Using this as your reference, rotate the screw gauge by the same amount on either side keeping a small distance between the magnets.
3. Repeat the procedure given in Part A for different masses.
4. Record a video of each mass as it falls and analyse it with Tracker to find its terminal velocity.
5. Plot a graph between the terminal velocity and the mass of the object, and verify the theoretically expected relationship between them.

Part C

1. Find the initial separation between the magnets using vernier calipers and then use the screw gauge attachments on the magnets to vary the separation.
2. Measure the magnetic field halfway between the magnets using a Gauss meter. Plot a graph of magnetic field versus the magnets' separation. Use this graph to find the magnetic field at the centre of the disk as you vary the separation between the magnets.
3. Keep the magnets some known distance apart and find the terminal velocity for a given mass.
4. Using the same mass, change the separation between the magnets (effectively varying magnetic field) and find the terminal. Repeat this for different separations between the magnets.
5. Plot a graph of the terminal velocity v_t against magnetic field B. Verify the theoretically expected relationship between them.

Observations and Analysis

Part A

Radius of axle = $7.5 \pm 0.01\ mm$
Least count of weighing scale = $0.1\ g$

Mass [kg]	Acceleration [m/s^2]
5.11×10^{-2}	-1.477×10^{-3}
1.016×10^{-1}	-5.326×10^{-3}
1.522×10^{-1}	-9.245×10^{-3}
2.021×10^{-1}	-1.299×10^{-2}
2.523×10^{-1}	-1.671×10^{-2}
3.027×10^{-1}	-2.051×10^{-2}
3.536×10^{-1}	-2.428×10^{-2}
4.036×10^{-1}	-2.818×10^{-2}

Table 1: The acceleration of different masses attached to a free-moving flywheel

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The raw data position-time data can be found in the Appendix. If the data in table 1 is plotted, we get the following graph:

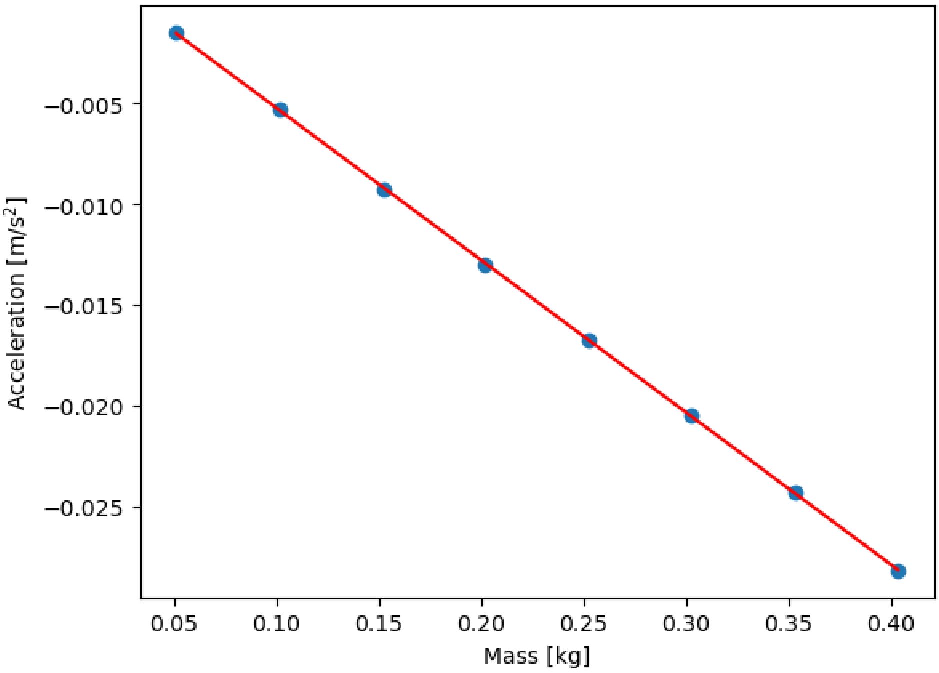

Figure 4: The mass and acceleration appear to be linearly related as predicted by equation 1

Equation 1 tells us that the theoretically expected relationship between acceleration and mass is

$$a \approx m\left(\frac{gr^2}{I}\right) - \left(\frac{\tau_f r}{I}\right)$$

Hence, the slope and intercept of graph can be used to find the moment of inertia (I) and the frictional torque (τ_f) of the system. The values we find from the linear fit of the graph in figure 1 are:

Moment of Inertia (I) $= 7.31 \times 10^{-3}\ kgm^2$

Frictional Torque (τ_f) $= 2.26 \times 10^{-3}\ Nm$

Part B

Separation between magnets (d) $= 30.26 \pm 0.01\ cm$
Least count of weighing scale $= 0.1\ g$

Mass [kg]	Terminal Velocity [m/s]
1.016×10^{-1}	-7.496×10^{-2}
1.522×10^{-1}	-9.837×10^{-2}
2.021×10^{-1}	-1.155×10^{-1}
2.523×10^{-1}	-1.273×10^{-1}
3.027×10^{-1}	-1.407×10^{-1}
3.536×10^{-1}	-1.541×10^{-1}
4.036×10^{-1}	-1.622×10^{-1}

Table 2: The terminal velocity of different masses attached to a magnetically damped flywheel

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The raw data position-time data can be found in the Appendix. If the data in table 2 is plotted, we get the following graph:

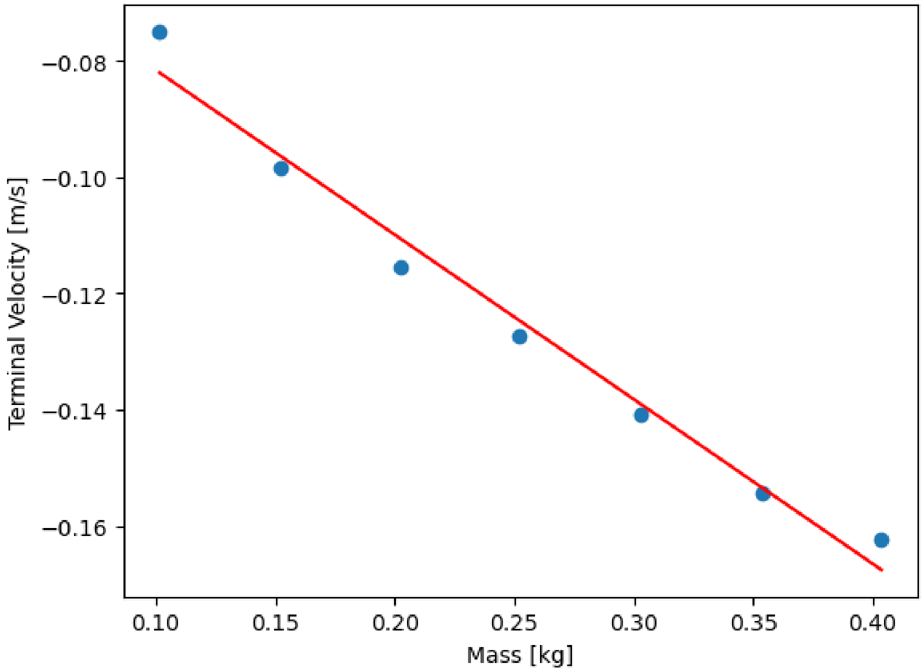

Figure 5: The mass and terminal velocity appear to be linearly related as predicted by equation 2

The theoretically expected relationship between terminal velocity and mass from equation 2 is:

$$v_t = \left(\frac{gr^2}{KB^2}\right)m - \left(\frac{\tau_f r}{KB^2}\right)$$

The slope is negative since g is negative and the slope is negative since the frictional torque opposes the direction of the rotation. Both the slope and intercept of the linear fit of the graph in figure 5 are negative. Therefore, it satisfies an equation of the above form.

$$v_t \propto m$$

Part C

Mass used $= 101.6 \pm 0.1\ g$
Least count of screw gauge = 0.001 cm
Least count of Gauss meter = 10^{-4} T

Magnet Separation [cm]	Magnetic Field [T]	Terminal Velocity [m/s]
2.026×10^{-2}	7.34×10^{-2}	-2.143×10^{-2}
2.126×10^{-2}	6.67×10^{-2}	-2.157×10^{-2}
2.226×10^{-2}	6.09×10^{-2}	-2.620×10^{-2}
2.326×10^{-2}	5.56×10^{-2}	-3.036×10^{-2}
2.426×10^{-2}	5.09×10^{-2}	-3.184×10^{-2}
2.526×10^{-2}	4.66×10^{-2}	-3.752×10^{-2}

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Magnet Separation [cm]	Magnetic Field [T]	Terminal Velocity [m/s]
2.626×10^{-2}	4.29×10^{-2}	-4.088×10^{-2}
2.726×10^{-2}	3.95×10^{-2}	-4.645×10^{-2}
2.826×10^{-2}	3.65×10^{-2}	-4.816×10^{-2}
2.926×10^{-2}	3.37×10^{-2}	-5.189×10^{-2}
3.026×10^{-2}	3.12×10^{-2}	-5.425×10^{-2}

Table 3: The terminal velocity for different magnet separations and therefore different magnetic field strengths

The raw data position-time data can be found in the Appendix. If plot a log graph of the magnetic field and magnet separation data in table 3, we get the following graph:

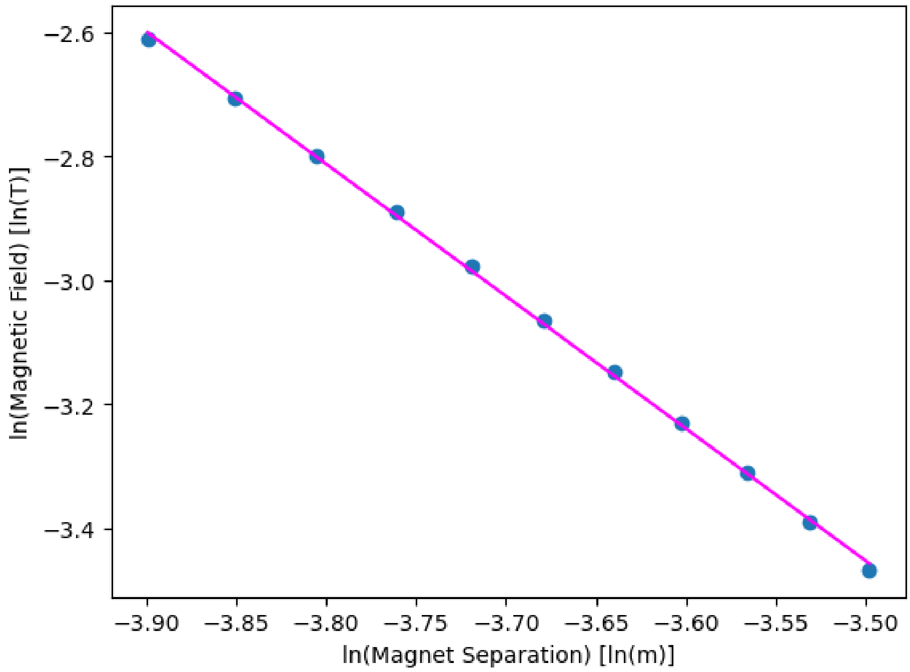

Figure 6: The log graph between B and d has a linear trend, implying a power rule relationship between the two quantities

We know that the magnetic field is given by the equation:

$$B = \frac{\mu_o}{4\pi}\frac{m}{x^2}$$

$$\implies \ln(B) = -2\ln(x) + \ln\left(\frac{\mu_o m}{4\pi}\right)$$

Hence, it follows that the magnetic field should have an inverse square relationship with the separation between the magnets. In a log graph, this would be equivalent to a slope of -2. The slope of the linear fit of figure 6 is -2.13, which suggests an inverse square law.

$$B \propto \frac{1}{d^2}$$

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If we use the data in table 3 to plot the magnetic field against the terminal velocity, we get the following graph:

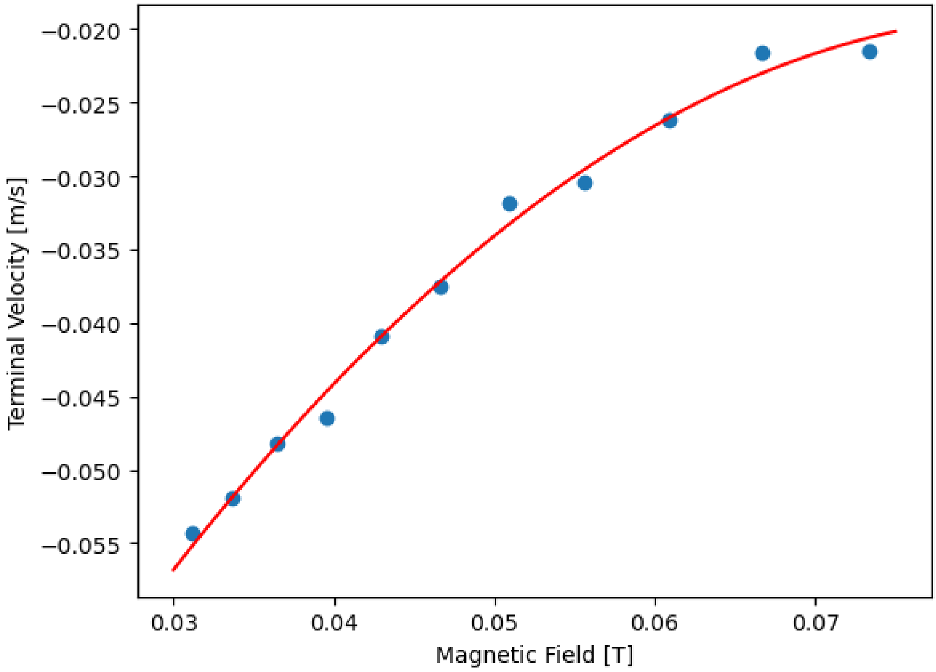

Figure 7: The relationship between terminal velocity and magnetic field is clearly non-linear

Equation 2 tells us that the theoretically expected relationship between terminal velocity and magnetic field is given by:

$$v_t = \frac{1}{B^2}\left(\frac{mgr^2}{K} - \frac{\tau_f r}{K}\right)$$

The data can be fitted with a quadratic function, therefore verifying the inverse square relationship between B and v_t. The constant of proportionality is negative since both terms in the above equation are negative (g and τ_f are negative).

$$v_t \propto \frac{1}{B^2}$$

Discussion and Error Analysis

The error in the moment of inertia originates from the least count error in the measurement of the radius of the axle since it is derived from the slope in equation 1. The least count of the Vernier calipers is 0.02 mm which was used to measure the diameter of the axle. When this value is halved to get the radius, the error also gets halved, making $\Delta r = 0.01\ mm$.

$$\Delta I = \left(2\frac{\Delta r}{r}\right) I$$

$$\implies \Delta I = \left(2 \times \frac{0.01}{7.5}\right) 7.31 \times 10^{-3} = 1.95 \times 10^{-5}\ kgm^2$$

Hence, the moment of inertia $(I) = 7.31 \times 10^{-3} \pm 1.95 \times 10^{-5}\ kgm^2$. This is equivalent to a percentage error of 0.3%.

The error in the frictional torque comes from both the error in moment of inertia and the error in radius:

$$\Delta\tau_f = \left(\frac{\Delta r}{r} + \frac{\Delta I}{I}\right)\tau_f$$

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$$\implies \Delta\tau_f = \left(\frac{0.01}{7.5} + \frac{\Delta 1.95}{731}\right) 2.26 \times 10^{-3} = 9.04 \times 10^{-6}\ Nm$$

Hence, the frictional torque $(\tau_f) = 2.26\times10^{-3}\pm9.04\times10^{-6}\ Nm$. This is equivalent to a percentage error of 0.4%.

Additional sources of error may be:

- The mass drifts in the x-direction as the cord unwinds from the axle, causing it to follow a diagonal path rather than a vertical path. Hence, some of the velocity information contained in its x-motion is lost in this analysis since we have only analysed y-position data. This may cause a slight deviation from the theoretically expected results.
- Small oscillations of the mass while falling could be a source of energy loss, thereby reducing the amount of energy converted to vertical kinetic energy which accelerates the flywheel.
- The Tracker software is only capable of capturing granular data in fixed time intervals, from which we extrapolate the continuous motion of the mass. The time intervals over which Tracker can capture data is limited by the frame rate of our camera (60 fps in our case). A higher frame rate camera would allow for more accurate data collection and results.

Results

Part A

The acceleration of a rotating disc is linearly proportional to the mass attached to its axle. The graph between a and m gives us the moment of inertia and frictional torque of the system.

Moment of Inertia $(I) = 7.31 \times 10^{-3} \pm 1.95 \times 10^{-5}\ kgm^2$ (0.3% error)

Frictional Torque $(\tau_f) = 2.26 \times 10^{-3} \pm 9.04 \times 10^{-6}\ Nm$ (0.4% error)

Part B

The magnetic field has an inverse square relationship with the separation between the magnets. In a log graph, the expected slope is -2 and the experimentally found slope is -2.13.

$$B \propto \frac{1}{d^2}$$

The terminal velocity of a rotating flywheel is linearly proportional to the mass attached to its axle. The relationship between v_t and m for a fixed magnetic field is:

$$v_t = \left(\frac{gr^2}{KB^2}\right) m - \left(\frac{\tau_f r}{KB^2}\right)$$

Where r is the radius of the axle, g is gravitational acceleration, K is a constant characterised by the geometry and material of the system and B is the magnetic field. Simply put, $v_t \propto m$.

Part C

The terminal velocity of a rotating flywheel is inversely proportional to the square of the strength of the magnetic field acting on the disc. The relationship between v_t and B for a given mass is:

$$v_t = \frac{1}{B^2}\left(\frac{mgr^2}{K} - \frac{\tau_f r}{K}\right)$$

Where r is the radius of the axle, g is gravitational acceleration, K is a constant characterised by the geometry and material of the system and m is the magnetic field. Simply put, $v_t \propto 1/B^2$.

Appendix

The raw data for the experiment can be found in the following drive file:
$https://drive.google.com/drive/folders/18rr9ectioSgf1zbBMQVjlwUlpYm03jB4?usp=sharing$