Wien's Displacement Law
Lab Report 4

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Aim

- Verify Wien's displacement law for a tungsten filament lamp and determine Wien's constant.

- Find Planck's constant from the black-body curve, given the value of the Boltzmann constant.

Theoretical Background

Objects which absorb all wavelengths of incident thermal radiation are known as black-bodies. Wien's displacement law states that the black-body radiation curve for different temperatures will peak at different wavelengths that are inversely proportional to the temperature of the body. This is a consequence of the Planck's radiation law which expresses the spectral radiance of a black-body as a function of the temperature of the body and the wavelengths it emits:

$$I(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{1}$$

Where λ is wavelength, T is temperature, $h = 6.626 \times 10^{-34} Js$ is Planck's constant, $c = 3 \times 10^8 m/s$ is the speed of light and $k = 1.38 \times 10^{-23} J/K$ is the Boltzmann constant. By maximising the spectral radiance with respect to wavelength while keeping the temperature constant, we can determine the peak wavelength:

$$\left(\frac{\partial I}{\partial \lambda}\right)_T = \frac{hc}{\lambda kT} \frac{e^{hc/\lambda kT}}{e^{hc/\lambda kT} - 1} - 5 = 0$$

Let $x = hc/\lambda kT$,

$$x = 5(1 - e^{-x})$$

Upon solving this, we find that the peak wavelength is given by:

$$\lambda_{max} = \frac{b}{T} \tag{2}$$

Where $b = 2.8977 \times 10^{-3} mK$, which is called the Wien's constant.

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In our experimental setup we use a Thor Lab F2 50 mm equilateral glass prism to disperse the light from the tungsten filament bulb passed through a collimator (figure 1).
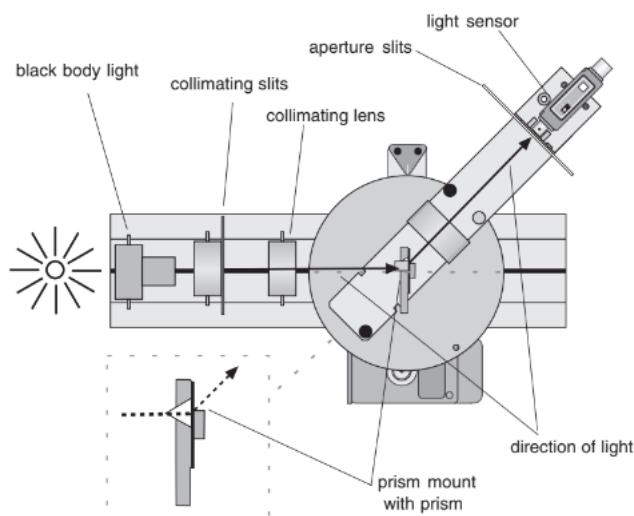

Figure 1: Experimental setup for verifying Wien's displacement law
Source: Instruction Manual and Experiment Guide for the PASCO scientific Model OS-8542

The angle of minimum deviation is found be varying the angle of incidence on the prism and finding the point at which the dispersed rainbow "bounces" or reverses the direction of movement as the angle of incidence is changed. In the minimum deviation configuration, the refractive index associated with each wavelength is given by:

$$n(\delta) = \sqrt{\left(\frac{2}{\sqrt{3}}\sin\delta + \frac{1}{2}\right)^2 + \frac{3}{4}} \qquad (3)$$

The Sellmeier equation describes how the wavelength transforms to refractive index. The three-term form is often used to characterise glass:

$$n(\lambda) = \sqrt{1 + \frac{A_1\lambda^2}{\lambda^2 - B_1} + \frac{A_2\lambda^2}{\lambda^2 - B_2} + \frac{A_3\lambda^2}{\lambda^2 - B_3}} \qquad (4)$$

The constants of this equation are characteristic for a given prism and the constants for an F2 50 mm prism were found from the Thor lab website:

$A_1 = 1.345$
$B_1 = 9.977 \times 10^{-3}$
$A_2 = 2.091 \times 10^{-1}$
$B_2 = 4.705 \times 10^{-2}$
$A_3 = 9.374 \times 10^{-1}$
$B_3 = 1.119 \times 10^2$

Using equations 3 and 4 it is possible to find the wavelengths corresponding to the angles of deviation. This allows for us to plot the intensity versus wavelength graph for various temperatures, which have the same trend as predicted by Planck's law (equation 1).

If we compare the intensities between two curves for a given wavelength, we find:

$$\frac{I_1}{I_2} = \frac{e^{hc/\lambda k T_2} - 1}{e^{hc/\lambda k T_1} - 1} \approx \frac{e^{hc/\lambda k T_2}}{e^{hc/\lambda k T_1}}$$

This approximation is true when λ lies close to the visible/infrared range. Hence, Planck's constant

can be found from the intensity-wavelength data using:

$$h = \frac{k\lambda}{c} \ln \frac{I_1}{I_2} \left(\frac{T_1 T_2}{T_1 - T_2} \right) \tag{5}$$

Finally, the temperature of the tungsten filament bulb is given by:

$$T = T_o + \frac{R/R_o - 1}{\alpha_o} \tag{6}$$

Where T_o is the room temperature in Kelvin, R is the resistance of the material at temperature T, R_o is the temperature of the material at room temperature and α_o is the temperature coefficient of resistance of the material. For tungsten, $R_o = 37\Omega$ and $\alpha_o = 4.5 \times 10^{-3} K^{-1}$.

Experimental Setup

- PASCO Model OS-8542 Light Sensor

- Laptop with PASCO-software installed

- Optical spectrometer with collimator

- Tungsten filament lamp with variable voltage

- Thor lab F2 50 mm prism

- Screen with a mount

- Torch

Least count of spectrometer $= 0.34°$
Least count of light sensor $= 0.01$ W/m^2
Least count of lamp voltmeter $= 1$ V
Least count of lamp ammeter $= 0.01$ A

Procedure

Angle of Minimum Deviation

1. The light source is placed in line with a collimator such that the beam falls directly on the screen and sensor. The light is switched on and the position of the screen is noted. This is our reference position corresponding to zero deviation.

2. The prism is placed on the rotating table and the dispersed light is observed by moving the angle of the screen. The prism is adjusted until a clear spectrum with distinct colours is visible.

3. The prism table is gradually rotated clockwise and the position of the screen is changed such that the same colour always falls on the sensor (in our case, our reference colour was red). The angle of the screen was noted for every 2° of change in the angle of incidence.

4. The angle of incidence is plotted against the angle of deviation and the minima is used to find the angle of minimum deviation. The prism is kept in minimum deviation configuration for the rest of the experiment.

Determining Wien's Constant

1. The current and voltage of the lamp is noted and the resistance of the tungsten filament is determined using Ohm's law (R=V/I) and equation 6 is used to find the temperature of the bulb.

2. Connect the PASCO light sensor with the laptop and tare the initial value when the sensor is in complete darkness to get a reference.

3. Gradually move the screen across the entire spectrum emerging from the prism, taking intensity readings with the PASCO software every $0.67°$. Find the angle of deviation for each screen position by subtracting the zero deviation angle measured in part 1.

4. Using equation 3 and 4 (Sellmeier equation), tranform the angle of deviation data into wavelengths and plot a wavelength versus intensity graph.

5. Fit Planck's law (equation 1) onto the data and determine the peak wavelength λ_{max}. Multiply this with the temperature of the bulb found in step 1 to determine Wien's constant.

6. Repeat this experiment for different voltages (and therefore different bulb temperatures).

Determining Planck's Constant

1. For a fixed value of wavelength, determine the intensity of the radiation for two different temperatures using the wavelength-intensity graph from the previous part.

2. Put the intensities and temperatures into equation 5 to find Planck's constant.

3. Repeat this for several different wavelengths and pairs of temperature curves.

Observations

Angle of Minimum Deviation

Least count of light sensor = 0.01 W/m^2
Least count of spectrometer = $0.34°$
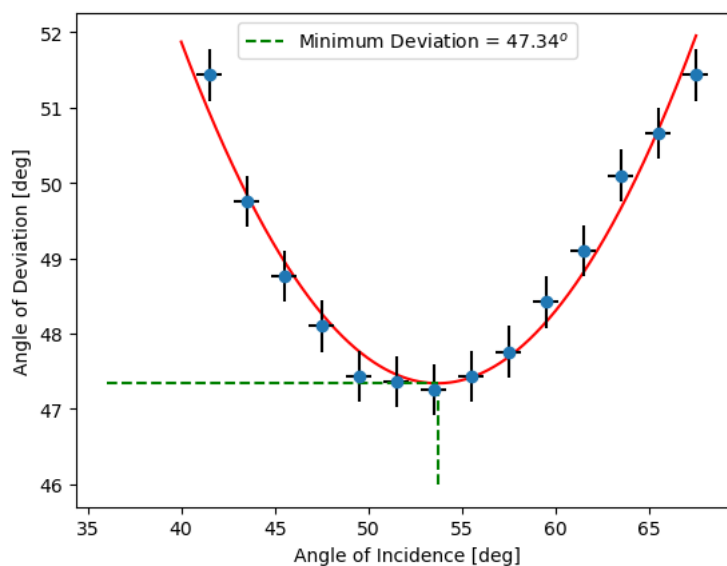
Angle of Zero Deviation = $262.67°$

Figure 2: The angle of deviation attained a minimum value for a certain angle of incidence

Determining Wien's Constant

Least count of lamp voltmeter = 1 V
Least count of lamp ammeter = 0.01 A

Voltage [V]	Current [A]	Resistance [Ω]
100	0.27	370.37
150	0.34	441.17
200	0.39	512.82

Table 1: The resistance for various voltages supplied to the lamp found using Ohm's Law (V=IR)

Data and Error Analysis

Angle of Minimum Deviation

As the angle of incidence on the prism was changed, the angle of deviation for red light decreased and then increased as seen in figure 2. This data was fit with a quadratic function and the minimum value was found at 47.34°. According to the Thor lab website, the angle of minimum deviation for 633 nm light (which is red) for an F2 50 mm prism is 47.9°. Hence, the experimentally determined value is reasonably close to the known value from the manufacture's website. The percentage error of the experimental value is 1.16%

Both the angle of deviation and angle of incidence are the difference of two spectrometer readings and hence the error in each value is twice the least count error of the spectrometer, i.e. the error in both variables is 0.67°.

Determining Wien's Constant

Using equation 6 and the data from table 1, we can determine the temperature of the tungsten filament which acts as a black-body for our experiment.

$$T = T_o + \frac{R/R_o - 1}{\alpha_o}$$

For tungsten, $R_o = 37\Omega$ and $\alpha_o = 4.5 \times 10^{-3} K^{-1}$.

Voltage [V]	Current [A]	Resistance [Ω]	Temperature [K]
100	0.27	370.37	2302.224
150	0.34	441.17	2727.486
200	0.39	512.82	3157.781

Table 2: The temperature of the filament decreases as the voltage of the lamp is reduced

The angles of deviation were put through equation 3 to find the corresponding refractive indices. These were then put through the Sellmeier equation to find the wavelength associated with each refractive index. The data for correlating the wavelength and refractive index was found on the Thor lab website and fit with the appropriate Sellmeier equation:

$$n(\lambda) = \sqrt{1 + \frac{A_1\lambda^2}{\lambda^2 - B_1} + \frac{A_2\lambda^2}{\lambda^2 - B_2} + \frac{A_3\lambda^2}{\lambda^2 - B_3}}$$
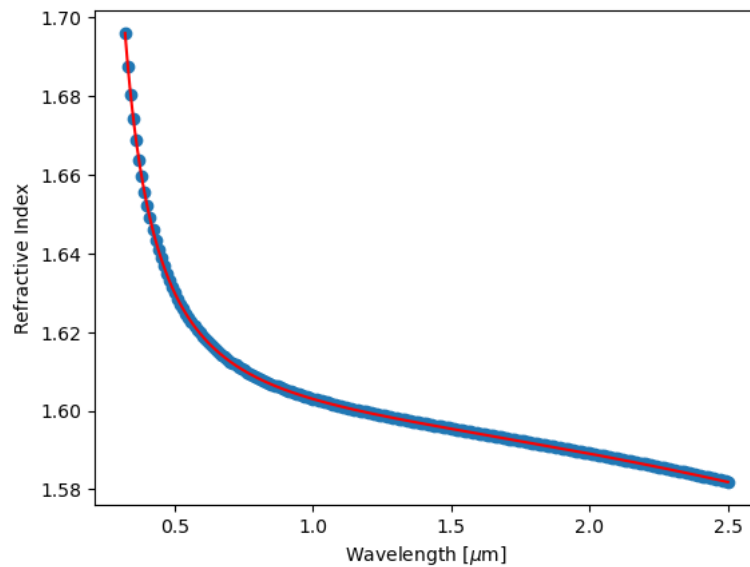
Figure 3: The coefficients of the Sellmeier equation were found on the Thor lab website

The wavelength data was used to plot a graph against the intensity measured on the PASCO light intensity sensor.
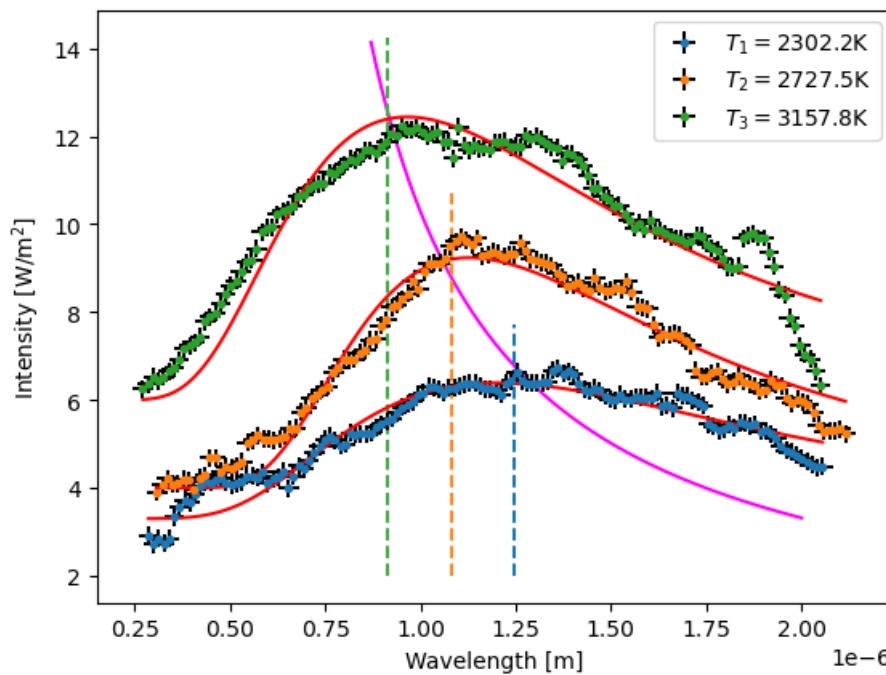

Figure 4: Higher temperatures peaked at lower wavelengths as predicted by Wien's displacement law

The data was fit with a function with the form of Planck's law (equation 1) and the peak value of each function was determined by taking its derivative and equating it to zero. The peak wavelength of is predicted to be inversely proportional to the temperature of the bulb, given by:

$$\lambda_{max} = \frac{b}{T}$$

Where $b = 2.8977 \times 10^{-3} mK$ is Wien's constant.

λ_{max} [μm]	Temperature [K]	Wien's Constant [mK]
1.248	2302.224	2.872×10^{-3}
1.080	2727.486	2.947×10^{-3}
0.913	3157.781	2.885×10^{-3}

Table 3: The Wien's constant found from each peak wavelength

The experimentally found Wien's constants have a standard deviation of $3.2 \times 10^{-5} mK$ and the mean value is $2.90 \times 10^{-3} mK$. Hence, the experimentally determined Wein's constant is:

$$b = 2.90 \times 10^{-3} \pm 0.03 \times 10^{-3} \ mK$$

The known value of Wien's constant ($b = 2.8977 \times 10^{-3} mK$) lies within this margin of error. The percentage error of the experimental value is 0.1%.

Determining Planck's Constant

We used the peak values of the wavelengths as the fixed wavelengths over which to find the intensities and determine Planck's constant using equation 5:

$$h = \frac{k\lambda}{c} \ln \frac{I_1}{I_2} \left(\frac{T_1 T_2}{T_1 - T_2} \right)$$

The intensities of every temperature for each peak wavelength are tabulated below:

Wavelength	T_1	T_2	T_3
λ_1	6.379	10.861	17.488
λ_2	6.306	11.192	19.582
λ_3	5.834	9.150	20.935

Table 4: The intensity (in W/m^2) for every temperature for each peak value of wavelength found from the intensity-wavelength curves

For each pair of temperatures, we can find a value of Planck's constant from the above equation and data:

Temperature	λ_1	λ_2	λ_3
T_1 and T_2	4.763×10^{-34}	3.989×10^{-34}	2.792×10^{-34}
T_2 and T_3	5.126×10^{-34}	5.863×10^{-34}	6.963×10^{-34}
T_1 and T_3	4.917×10^{-34}	4.785×10^{-34}	4.563×10^{-34}

Table 5: The Planck's constant (in Js) determined from every pair of temperatures

The pair of T_2 and T_3 give us our best value of Planck's constant:

$$h = 5.9 \times 10^{-34} \pm 0.8 \times 10^{-34} \ Js$$

The known value of Planck's constant ($h = 6.626 \times 10^{-34} Js$) lies within this margin of error. The percentage errror of the experimentally determined result is 9.6%.

Discussion

- The PASCO light sensor is extremely sensitive to small changes in ambient heat since it detects infrared radiation as well. This means that the presence and movement of humans in the vicinity of the sensor while taking readings may have introduced additional noise into the data which has not been accounted for in the error analysis.

- The Sellmeier equation used to model the relationship between refractive index and wavelength only has 3 terms, however, the true expanded form of the equation has infinite terms and therefore the form used is merely an approximation.

- We were unable to create a completely distinct spectrum despite multiple adjustments of the lamp, collimator and spectroscope apparatus. The mixing of colours may have contributed to slight differences in the spectral radiance data.

Results

Angle of Minimum Deviation

The angle of minimum deviation (using red light as a reference), for an equilateral F2 50 mm glass prism was found to be $47.34°$. The percentage error is 1.16%.

Determining Wien's Constant

Wien's constant was experimentally found to be $2.9 \times 10^{-3} \pm 0.03 \times 10^{-3} \ mK$. The percentage error is 0.1%.

Determining Planck's Constant

Planck's constant was experimentally found to be $5.9 \times 10^{-34} \pm 0.8 \times 10^{-34} \ Js$. The percentage error is 9.6%.

References

- *Wein's Displacement Law* (2021) Lab 3 Handout

- Dryzek, J., Ruebenbauer, K. (1992). *Planck's constant determination from black-body radiation.* American Journal of Physics, 60(3), 251–253. https://doi.org/10.1119/1.16904

- PASCO scientific (1999) *Instruction Manual and Experiment Guide for the PASCO scientific Model OS-8542*