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Interferometry and Polarisation

Lab Report 2

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Aim

- Find the wavelength of a laser light source using a Michelson interferometer.
- Determine the refractive index of a glass slide based on how it affects the interference pattern made by a Michelson interferometer.
- Verify Malus' Law by passing a laser light through two polarisers and measuring the change in intensity with respect to the angle between the transmission axes.
- Find Brewster's angle for glass using a photodiode to measure the change in light intensity with respect to the angle of incidence.

Theoretical Background

Michelson Interferometer

The Michelson Interferometer uses a beam splitter angled at 45^o with respect to the incident light to split the amplitude into two equal parts - one of which is refracted and the other is reflected (figure 1). These rays are reflected off two perpendicular mirrors, causing the rays to return to the beam splitter and fall on a screen where they interfere with each other. The path difference due to the positioning of the mirrors and the phase difference caused by the light passing through a denser medium determines whether the rays interfere constructively or destructively.

A laser (light amplification by stimulated emission of radiation) works by exciting atoms in certain materials until they emit light and using feedback through reflection to amplify the emission. This light has roughly the same wavelength which is determined by the material used in the laser and it is concentrated into single narrow beam of coherent light.

To observe an interference pattern over a plane, a beam widener is used to extended source, causing the incident light to be emitted over a range of angles. The pattern formed is radially symmetric since the light is coherent and is therefore in phase when emitted.

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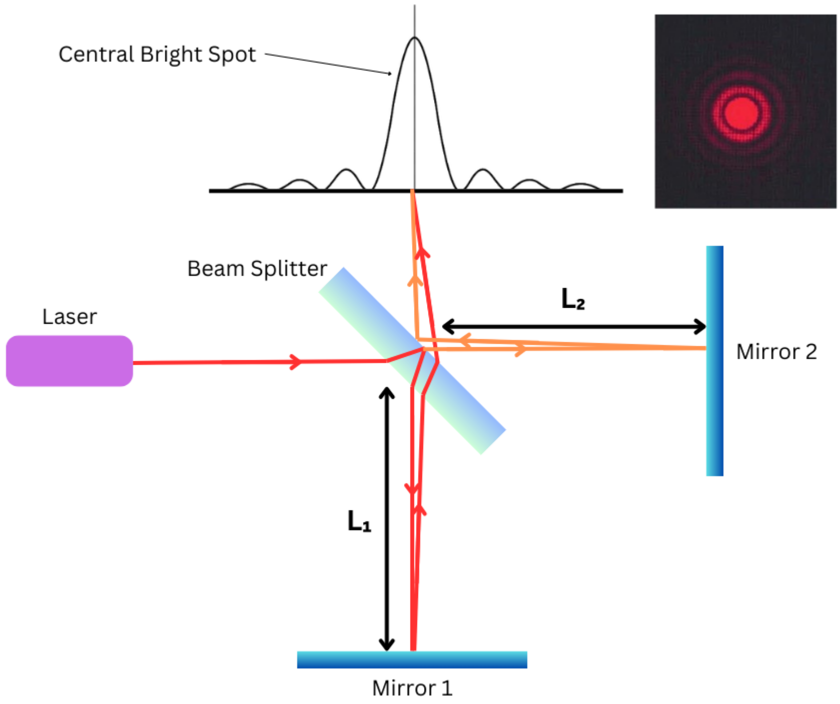

Figure 1: Schematic Diagram of the Michelson Interferometer.
This configuration has a central bright spot, but a change in the
path difference can lead to the formation of a central dark spot as well.
Source: made on www.canva.com

If the difference between L_1 and L_2 is d, then the path difference between the two beams at the center of the screen is $2d$ since the ray is reflected and must travel this distance twice. For a beam at an angle θ from the central axis, the condition for dark fringes becomes:

$$2d\cos\theta = n\lambda$$

This predicts dark fringes for a given value of theta, forming concentric dark and light circles which is what we observe in the interference pattern.

If mirror 2 is moved by a distance Δd, the intensity detected at the center of the screen will increase and decrease as successive interference maxima and minima are experienced, which is what we see as the collapse of each bright or dark ring. Counting the number N of consecutive bright or dark patterns collapsing (or emerging) at the centre allows us to formally write:

$$2\Delta d = N\lambda \tag{1}$$

But since we use a micrometer screw-gauge to move the mirror, we include a calibration constant $C = 0.0241$.

$$2C\Delta d = N\lambda \tag{2}$$

If a glass slide is placed in the path of light between the beam splitter and mirror 2, one of the arms undergoes an additional path change due to refraction and passing through a denser medium. If the glass has a thickness t and refractive index n, the path difference due to the glass is $nt + bc$ when the ray falls perpendicular to the glass (i.e. $i = 0$, see figure 2). If the glass is rotated by an angle θ, the incident angle $i = \theta$ and the path difference changes to $(ad)n + de$. Hence, the change in path difference $\Delta d = (ad)n + de - (nt + bc)$.

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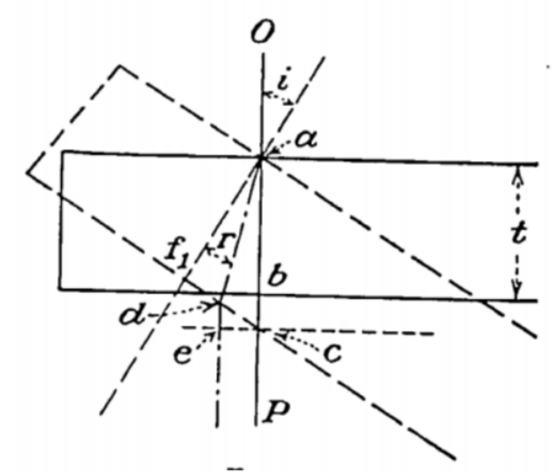

Figure 2: Ray diagram of light passing through a glass slide while changing the angle of incidence on the glass
Source: Lab handout for interferometry

Putting this path difference into equation 1, we find:

$$2\left[(ad)n + de - (nt + bc)\right] = N\lambda$$

$$\implies 2\left[\frac{nt}{\cos(r)} + t\tan(i)\sin(i) - t\tan(r)\sin(i) - nt - \frac{t}{\cos(i)} + t\right] = N\lambda$$

Using Snell's law $\sin(i) = n\sin(r)$, we can rearrange this equation to find the refractive index of glass:

$$n = \frac{(2t - N\lambda)(1 - \cos(i))}{2t(1 - \cos(i)) - N\lambda} \tag{3}$$

Malus' Law

In a linearly polarised electromagnetic wave the electric field E is along the plane perpendicular to the direction of propagation of the wave. The field can be decomposed into two perpendicular components along the x and y axis. If these components are in phase, the light is said to be linearly polarised, but if there is not well-defined phase relationship between the two components it is unpolarised. A polariser is made from a material in which the molecules absorb the parallel electric field components, allowing only the perpendicular component to pass through which effectively linearly polarises the light. The axis along which a polariser allows the electric field to pass is known as the transmission axis of the polariser.

Malus' Law states that if unpolarised light is made to pass through two polarisers, the intensity of the light depends on the angle between the transmission axes of two polarisers (figure 3).

The reason the intensity of the light changes after passing through the second polariser (called the analyser) is because it only allows the component of the linearly polarised light which is parallel to its transmission axis to pass through, while blocking the perpendicular component. Hence, if the two polarisers are parallel, the intensity remains unchanged while if they are perpendicular, the intensity of the light passing through both polarisers is minimised.

$$I = I_o \cos^2\theta \tag{4}$$

Where θ is the angle between the transmission axes and I_o is the initial intensity.

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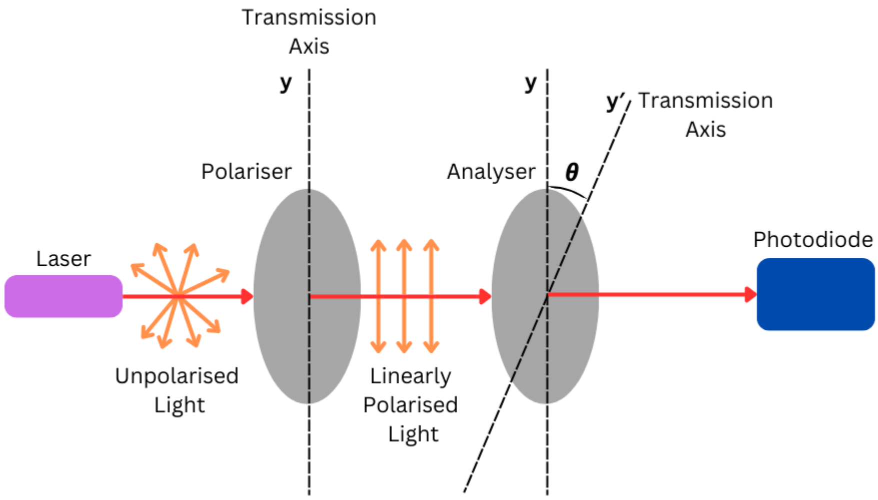

Figure 3: Schematic diagram of experimental setup to verify Malus' Law
Source: made on www.canva.com

The intensity of the light is measured using a reverse biased photo-detector. The incident light excites electrons in the semiconductor, inducing a photocurrent which is proportional to the intensity of the light since intensity is directly related to the number of photons emitted. A reverse biased diode produces a higher current than a forward biased diode for the same amount of incident light, allowing for higher sensitivity.

Brewster's Angle

When light is reflected at a surface of a denser medium, the proportion of light polarised parallel and perpendicular to the plane of incidence changes depending on the angle of incidence. When unpolarised light is incident on a surface, the reflected ray will tend to contain more of one polarisation than the other. At a certain angle, known as Brewster's angle, the reflected ray will entirely consist of light which is polarised perpendicular to the plane of incidence and is therefore linearly polarised. This occurs when the reflected beam and refracted beam are perpendicular and is therefore characterised by the refractive index of the medium.

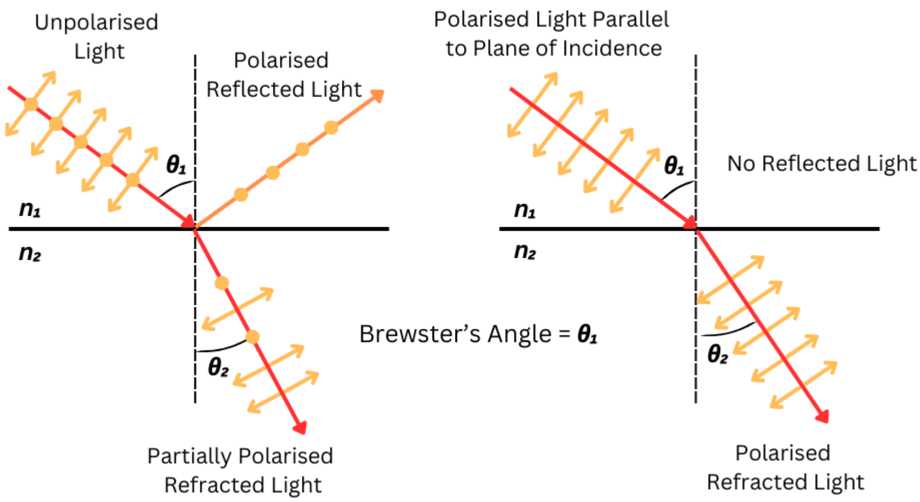

Figure 4: Ray diagram of reflection of unpolarised and polarised light at the surface of a dense medium when the angle of incidence is Brewster's angle
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Since the reflected beam only contains the component perpendicular plane of incidence of the incident light, there will be no reflected beam formed when the incident light is polarised such that it is parallel to the plane of incidence.

Brewster's angle can be derived from the refractive index since we know that $\theta_1 + 90 + \theta_2 = 180$ and $\sin\theta_1 = n\sin\theta_2$:

$$\tan\theta_1 = n \tag{5}$$

Frensel's equations predict that at normal incidence, the parallel and perpendicular polarisation waves to the plane of incidence are physically identical and are reflected equally. As the angle of incidence increases, the parallel component drops and the perpendicular component rises until the Brewster's angle is reached at which point the intensity is expect to reach a minimum. As the angle of incidence approaches 90°, all of the incident light is reflected such that the medium acts as a mirror.

Experimental Setup

- Optical breadboard
- Laser source and mount
- Two mirrors
- Beam splitter
- Holders and mounts for mirrors and beam splitter
- Micrometer screw gauge mount for one mirror
- Glass slide
- Rotatable mount
- Photodiode detector with power supply
- Two rotating polarisers
- Rotation Stage

Least count of micrometer screw gauge = 0.01 mm
Least count of photodetector = 0.1 mA
Least count of rotatable mount = 2°

Procedure

Michelson Interferometer

1. Setup the interferometer as seen in figure 1. Calibrate the laser so that it is parallel to the optical breadboard before adding the other components. The mirrors should be completely perpendicular such that the reflected and refracted beams are coincident on the screen.

2. Adjust the beam splitter till it forms a 45° angle with the incident ray so that the intensity of the reflected and refracted beams is equal. Attach a beam widener to the laser and use the fine adjustment screws on the mirror mounts until a clear interference pattern is visible on the screen.

3. Starting at zero displacement, gradually turn the micrometer screw gauge on the mount of mirror 2 while counting the number of dark/light spots that disappear or emerge at the center of the interference pattern.

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4. Take readings of the mirror's displacement for a large number of fringes and use equation 2 to determine the wavelength of the light emitted by the laser.

5. Repeat the above experiment with a glass slide attatched to a rotatable mount placed between the beam splitter and mirror 2. This time instead of moving the mirror, change the angle of the glass slide with respect to the incident beam and count the number of fringes that disappear or emerge at the center of the interference pattern. Use equation 3 to find the refractive index of glass.

Malus' Law

1. Calibrate the laser such that it is parallel to the optical breadboard. Mount two polarisers and the photodetector in a straight line in front of the laser ensuring that the beam falls on the pinhole of the photodetector.

2. Rotate the analyser with respect to the polariser until maximum intensity is detected at the photodetector and set this angle as $0°$.

3. Gradually rotate the analyser and record the intensity of the light detected at the photodiode. Verify Malus' law by plotting the angle θ against the intensity I.

Brewster's Angle

1. Mount a glass slide on the rotation stage and align it perpendicular to the beam of light such the reflected light coincides with the incident beam.

2. Change the angle of incidence on the glass slide and rotate the photodetector until the reflected beam falls on the pinhole.

3. Gradually rotate the glass slide and keep adjusting the photodetector to measure the intensity of the reflected beam. Find the minima of intensity to find Brewster's angle for glass.

4. Repeat the experiment with a polariser placed between the laser and glass slide that polarises the light parallel to the plane of incidence.

Observations

Michelson Interferometer

Least count of micrometer screw gauge = 0.01 mm
Least count of rotatable mount = $2°$

No. of Fringes N	Δd_1 [mm]	Δd_2 [mm]	Δd_3 [mm]	Δd_4 [mm]	Δd_5 [mm]
25	0.24	0.26	0.36	0.27	0.25
50	0.55	0.60	0.64	0.60	0.56
75	0.87	0.95	0.97	0.93	0.86
100	1.16	1.26	1.29	1.22	1.16
125	1.52	1.61	1.60	1.59	1.51

Table 1: Displacement Δd of the mirror corresponding to the emergence of N fringes
Multiple readings were taken to determine error

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No. of Fringes N	i_1 [deg]	i_2 [deg]	i_3 [deg]	i_4 [deg]	i_5 [deg]
20	10	10	10	10	10
40	16	18	16	18	16
60	20	26	24	24	20
80	24	36	28	30	26
100	30	42	32	34	30

Table 2: Angle of incidence i corresponding to the emergence of N fringes
Multiple readings were taken to determine error

Malus' Law

Least count of photodetector = 0.1 mA

Transmission Axis Angle θ [deg]	I_1 [mA]	I_2 [mA]	I_3 [mA]	I_4 [mA]	I_5 [mA]
0	29.2	29.1	29.1	29.1	29.1
5	28.8	28.9	28.8	28.9	29.0
10	28.7	28.9	28.7	28.8	29.0
15	28.5	28.7	28.4	28.6	28.8
20	28.1	28.2	28.1	28.3	28.6
25	27.9	28.0	27.8	27.9	28.0
30	27.8	27.8	27.8	27.8	27.7
35	27.6	27.6	27.6	27.6	27.5
40	27.4	27.2	27.3	27.3	27.2
45	26.9	26.7	26.7	26.8	26.7
50	26.2	26.2	26.2	26.2	26.1
55	25.6	25.7	25.6	25.6	25.7
60	24.9	25.1	24.8	24.8	25.1
65	23.6	24.0	23.7	23.5	23.9
70	21.5	22.2	21.2	21.3	21.8
75	18.1	18.6	17.5	17.1	18.9
80	16.4	14.7	15.0	14.5	15.4
85	14.2	12.5	12.9	12.4	12.2
90	13.0	11.4	11.3	10.9	10.5
95	12.6	10.9	10.8	10.5	10.2
100	12.7	11.6	11.6	10.8	11.6
105	14.9	13.5	13.2	11.2	13.2

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Transmission Axis Angle θ [deg]	I_1 [mA]	I_2 [mA]	I_3 [mA]	I_4 [mA]	I_5 [mA]
110	16.2	15.4	15.2	13.2	15.3
115	19.9	19.1	18.8	15.1	18.5
120	23.2	22.6	22.9	18.8	22.5
125	24.6	24.3	24.3	22.9	24.3
130	25.3	25.1	25.1	24.3	24.7
135	26.2	26.1	26.2	25.1	25.8
140	27.0	26.8	27.0	26.2	26.6
145	27.5	27.3	27.1	27.0	27.1
150	27.8	27.6	27.5	27.5	27.5
155	28.0	27.8	27.8	27.8	27.9
160	28.1	28.0	28.0	27.9	28.0
165	28.1	28.1	28.1	28.9	28.0
170	28.0	28.1	28.2	28.0	28.0
175	28.3	28.2	28.3	28.3	28.0
180	28.4	28.6	28.5	28.4	28.5

Table 3: Intensity I that varies with respect to the angle of incidence θ
Multiple readings were taken to determine the error

Brewster's Angle

Least count of photodetector = 0.1 mA

Angle of Incidence θ [deg]	I_1 [mA]	I_1 [mA]	I_3 [mA]
12	7.3	8.5	6.9
14	7.2	7.1	7.8
16	7.4	8.3	7.4
18	6.3	6.8	6.6
20	4.8	6.7	6.8
22	6.6	6.9	6.4
24	5.7	6.6	6.6
26	5.4	6.4	6.3
28	6.5	6.5	6.3
30	7.3	6.7	6.5
32	7.5	6.6	6.1
34	6.8	5.7	5.7
36	4.6	5.0	5.4

Angle of Incidence θ [deg]	I_1 [mA]	I_1 [mA]	I_3 [mA]
38	5.6	5.3	5.1
40	4.6	4.7	4.8
42	3.5	4.4	4.4
44	3.9	4.2	3.8
46	5.4	3.6	3.4
48	4.4	3.5	2.9
50	2.9	2.6	2.7
52	2.0	2.1	2.2
54	1.8	2.1	1.9
56	2.0	1.6	1.4
58	4.4	2.1	1.8
60	4.4	4.0	1.8
62	5.7	5.8	2.4
64	6.7	6.7	3.9
66	10.9	7.0	5.0
68	13.0	8.9	5.9
70	8.2	10.6	7.5
72	14.3	11.7	9.0
74	13.6	14.4	11.5
76	22.4	16.0	13.1
78	20.5	17.8	15.8
80	23.4	22.5	17.4

Table 4: Intensity of the reflected beam corresponding to various angles of incidence
Multiple readings were taken to determine error

Angle of Incidence θ [deg]	I [mA]
12	5.6
14	5.6
16	6.1
18	5.4
20	5.9
22	5.1
24	4.7
26	5.0

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Angle of Incidence θ [deg]	I [mA]
28	4.5
30	4.8
32	4.1
34	4.0
36	3.4
38	3.6
40	3.1
42	2.8
44	2.7
46	1.8
48	1.2
50	1.1
52	0.8
54	0.4
56	0.2
58	0.9
60	1.1
62	1.2
64	2.1
66	3.7
68	4.7
70	5.8
72	7.6
74	10.0
76	11.2
78	13.7
80	15.4

Table 5: Intensity of the reflected beam corresponding to various angles of incidence with a polariser placed between the laser and glass slide

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Data and Error Analysis

Michelson Interferometer

Plotting the data from table 1 after taking the mean displacement of the mirror's displacement Δd gives us the following graph:

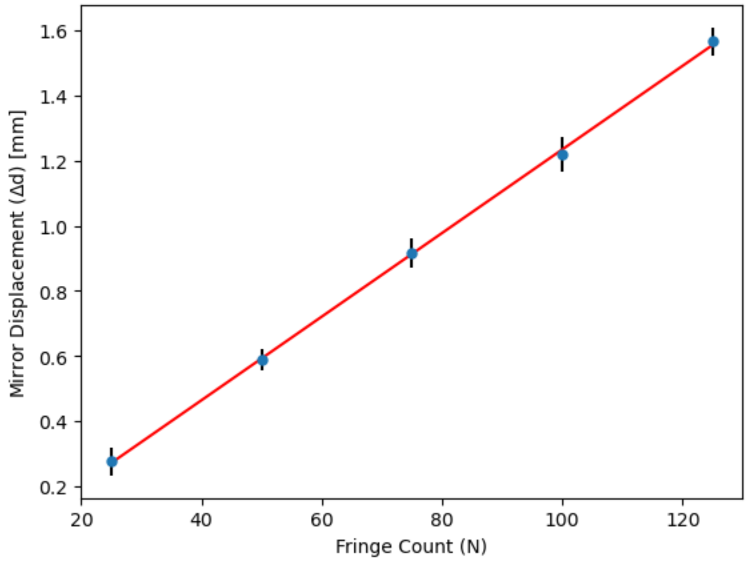

Figure 5: The number of bright fringes that emerge in the interference pattern varies linearly with respect to the displacement of the mirror, which is effectively the change in path difference of the interfering rays

The error was determined by finding the standard deviation of the readings which is a measure of the inherent experimental error while taking data.

$$\sigma = \sqrt{\frac{\sum(x_i - \mu)^2}{N}}$$

Where x_i refers to each data point for a given fringe count, μ is their mean, and N is the total number of data points (i.e. 5).

Looking at equation 2, we know the slope of this graph is proportional to the wavelength of the light source:

$$\frac{\Delta d}{N} = \frac{\lambda}{2C}$$

From this we can find the wavelength of the laser light:

$$\lambda = 2C\left(\frac{\Delta d}{N}\right) = 618.5\ nm$$

For the second part of the experiment, we inserted a glass slide in the path of the light, further altering the path difference. Equation 2 predicts that the refractive index of the material (i.e. glass) is given by:

$$n = \frac{(2t - N\lambda)(1 - \cos(i))}{2t(1 - \cos(i)) - N\lambda}$$

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We plotted the numerator of this expression against the denominator for different values of N and i. The value of λ used was the experimental value found in the last part of the experiment.

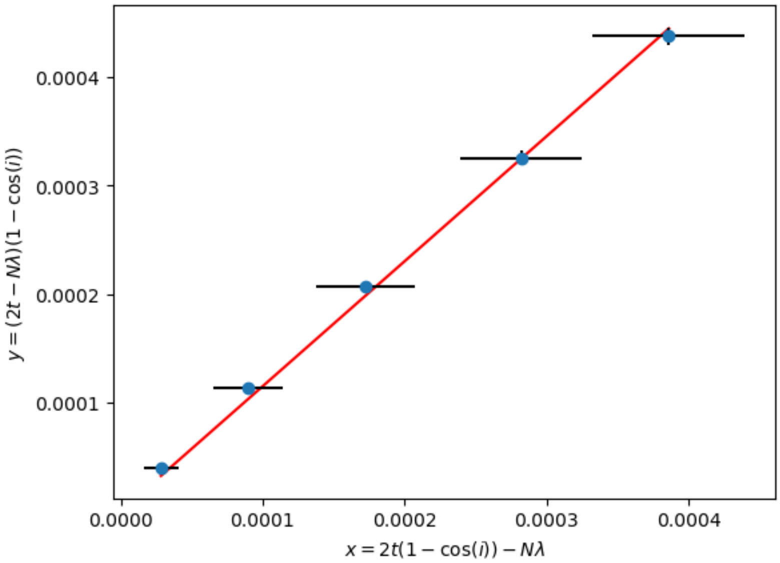

Figure 6: The slope of this graph gives us the refractive index of the material used to change the path difference

The error in both the x and y data comes from the error in the measurement of the angle of incidence which was found by taking the standard deviation of each set of readings associated with a given fringe count. The standard deviation values were put through the same function to determine their error along each axis.

The slope of the graph is equivalent to the refractive index of glass:

$$n = 1.15$$

Malus' Law

Plotting the data from table 3, we get the following graph after taking the mean intensities for each reading:

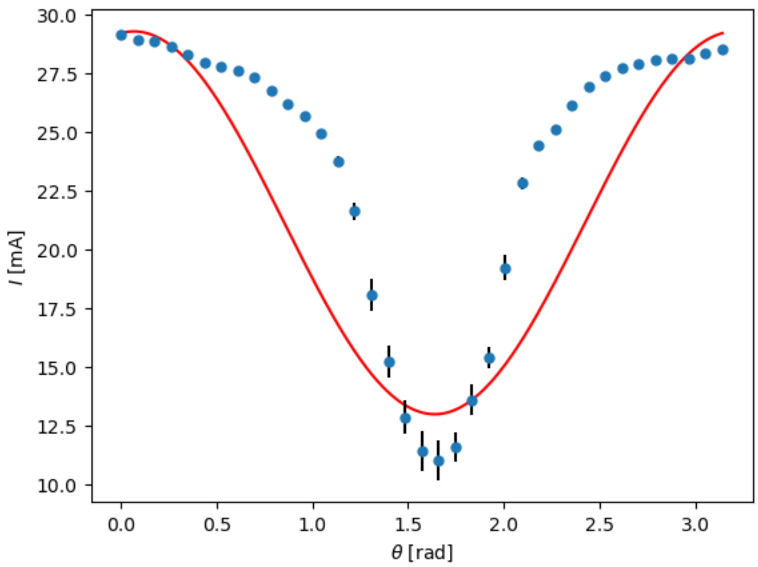

Figure 7: The intensity appears to vary sinusoidally with respect to the transmission axis angle

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The error was determined for each data point by finding the standard deviation of the readings. The best fit had to be manually adjusted by a phase and amplitude factor since the minimum amplitude was not zero. The model function was a modified version of equation 4 which accounted for these shifts:

$$I = I_o \cos^2(\theta + \phi) + \delta$$

The parameters found were:
$I_o = 16.3\ mA$
$\delta = 29.3\ mA$
$\phi = -4.78°$

Brewster's Angle

The data from table 2 was plotted to get the following graph. The mean values of the intensity were found for each reading and their standard deviation was the error:

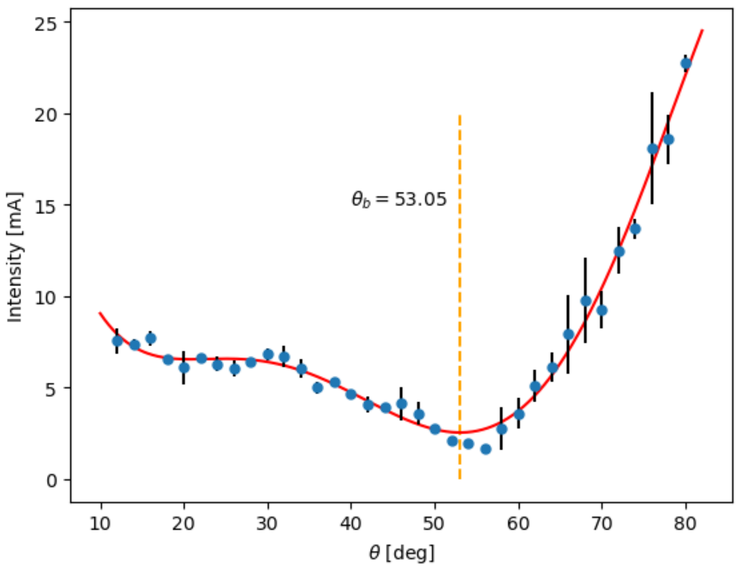

Figure 8: The intensity reached a minimum value before rising sharply as predicted by Frensel's equations

A 5th degree polynominal function was used to fit the data and find the local minima. Brewster's angle was found to be 53.05°, which can be used to determine the refractive index of glass using equation 5:

$$n = \tan \theta_b = 1.33$$

The expected value of n is 1.5 and Brewster's angle of glass is 56.3°.

We repeated the experiment with a polariser that linearly polarised the light parallel to the plane of incidence. The data from table 5 is plotted below:

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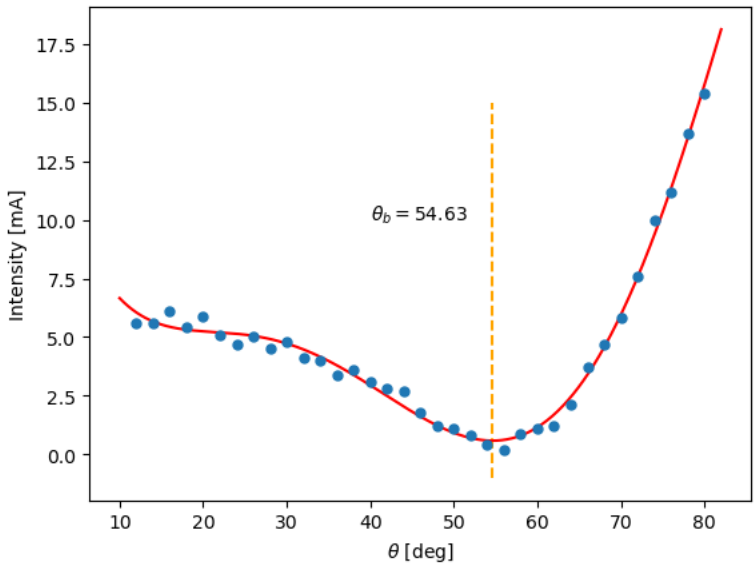

Figure 9: The minimum intensity approaches zero since the incident light has no component perpendicular to the plane of incidence

Error analysis was not possible for this data set since only one set of readings could be taken for this part of the experiment.

A 5th degree polynominal function was used to fit the data and find the local minima. Brewster's angle was found to be 54.63°, which can be used to determine the refractive index of glass using equation 5:

$$n = \tan \theta_b = 1.40$$

Discussion

The emergence of fringes in the second part of the Michelson Interferometer experiment is highly sensitive to changes in θ for angles approaching 90°. A more precise tool for changing the angle of the glass slide should be used in a narrower range to determine the refractive index.

The drop in intensity in the verification of Malus' law is much steeper than theoretically expected. This may be because a strong undiffused laser was used which caused the photodetector to get saturated, reducing its sensitivity for higher intensities. Using a diffuser or weaker laser may improve the results.

The use of a diffuser in the determination of Brewster's angle adds additional error to the data since the intensity varies significantly across the reflected beam and the placement of the photodiode with respect to the rotating glass slide cannot be controlled precisely. Hence, it is better to use a laser with a single point beam rather than an extended beam.

Results

Michelson Interferometer

Wavelength of laser light = 618.5 nm
Refractive index of glass = 1.15

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Malus' Law

The intensity of light varied with respect to the transmission axis angle with the following function:

$$I = I_o \cos^2(\theta + \phi) + \delta$$

Where the parameters found were:
$I_o = 16.3\ mA$
$\delta = 29.3\ mA$
$\phi = -4.78°$

Brewster's Angle

Brewster's Angle (without a polariser) = 53.05°
Brewster's Angle (with a polariser) = 54.63°