Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Thermometry
Lab Report 3

Ayaan Dutt
Lab partner: Drishana Kundu

Professor Susmita Saha
GA: Archana
TAs: Anjali Madangarli, Satwik Wats

Lab Supervisor: Sudarshana Banerjee
Lab Technician: Pradip Chaudhari

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Aim

- Calibrate a
  1. Alcohol Thermometer
  2. Platinum Resistance Thermometer (Pt100)
  3. Thermistor
  4. Thermocouple

  using the melting and boiling point of water and a pre-calibrated Mercury thermometer.
- Determine the cooling curves for water, using containers of different surface areas and verify Newton's Law of Cooling.
- Use one of the above thermometers to measure the melting point of paraffin wax.
- Use Newton's Law of Cooling on a Lees Disc apparatus to find the thermal conductivity of a glass slab.

Theoretical Background

Liquid Thermometers

A liquid thermometer consists of a glass bulb that holds the liquid. The bulb is made larger than the capillary to enhance its sensitivity. The thin glass surrounding the bulb ensures rapid heat transfer. When the liquid in the bulb expands due to temperature changes, it causes a significant shift in the length of the liquid column within the narrow capillary tube. The narrower the tube, the more sensitive the thermometer becomes. Additionally, the thick round glass stem around the capillary tube acts like a magnifying glass. Alcohol and mercury are commonly used liquids for such thermometers since they expand rapidly and linearly in the functional range.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Resistance Thermometers

Resistance thermometers exploit the temperature response of the resistance of certain materials to determine the temperature. For instance, platinum resistance thermometers use on the fact that the electrical resistance of Platinum increases linearly with temperature, and that the coefficient of change of resistance with temperature is large. Platinum also has a high melting point, making it ideal for thermometry. Pt100 thermometers are calibrated such that at 0° they have a resistance of exactly 100Ω. Hence, the resistance of a Pt100 thermometers varies according to:

$$R_T = mT + R_0 \tag{1}$$

Where T is the temperature, m is the coefficient of change of resistance with temperature and R_0 is the temperature at 0° which is 100Ω.

A thermistor on the other hand has a non-linear temperature response since it is made of a semiconductor whose resistance decreases as the temperature increases. This happens because higher temperatures excite more electrons and push them into the conduction band, allowing current to flow more readily through the material. The resistance-temperature response of a thermistor is modelled by the Steinhart-Hart equation:

$$\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3 \tag{2}$$

Where a, b and c are constants.

Thermocouple

A thermocouple measures the electromotive force generated by the temperature gradient induced in two metal wires, one of which is kept in a reference temperature of 0° ice and the other is kept in the substance whose temperature is being measured. The hotter end of the wire has more excited electrons that tend to drift towards the cooler low-energy end, which induces the voltage across the ends of the thermocouple. As the temperature at the junction increases, so does the emf and hence the voltage can be used to determine the temperature.

Melting and Boiling Points

When a substance undergoes a phase transition from solid to liquid or liquid to gas, it maintains a constant temperature while both states coexist in thermal equilibrium. This occurs because the energy supplied to the system is entirely consumed in changing the state of the substance and there is therefore no energy left to raise the temperature. Hence, the temperature-heat curve of a substance going through two phase transitions while being supplied with a constant power is shown below:

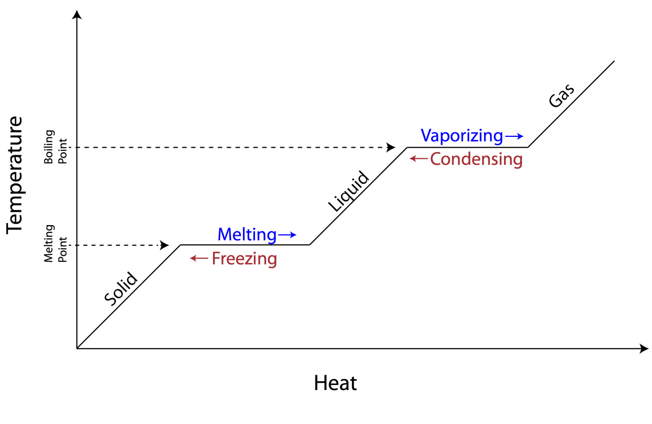

Figure 1: Each phase transition corresponds to a period of levelling during which two states coexist at the same temperature
Source: $https://www.aplusphysics.com/courses/honors/thermo/phase_changes.html$

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Newton's Law of Cooling

Newton's law of cooling states that the rate of heat loss by a body is proportional to the difference between the temperature of the body and its surrounding environment. We know this to be true from direct observation - a hot body will initially cool down quickly, but as it approaches room temperature it will take longer to cool down by the same amount. The rate of cooling is also dependent on the area over which the heat is lost since a larger surface area will allow heat to dissipate more quickly.

$$\frac{dQ}{dt} \propto A(T - T_s)$$

From the heat transfer equation we can determine the relationship between heat and temperature change:

$$dQ = mcdT$$

Where m is the mass of the body and c is its specific heat capacity. Taking the rates of change with respect to time:

$$\frac{dQ}{dt} = mc\frac{dT}{dt}$$

This allows us to write Newton's law of cooling in terms of rate of temperature drop:

$$\frac{dT}{dt} = -\frac{hA}{mc}(T - T_s) \tag{3}$$

Where h is a constant of proportionality which we call the heat coefficient of the body. This is a separable first-order differential equation which gives can be solved with the intial conditions $T(0) = T_0$ and $T(t) = T$:

$$\ln\left(\frac{T - T_s}{T_0 - T_s}\right) = \frac{-hA}{mc}t$$

$$\implies T = e^{\frac{-hAt}{mc} + \ln(T_0 - T_s)} + T_s \tag{4}$$

Hence, the temperature drops exponentially with respect to time.

Fourier's Law of Heat Transfer and Thermal Conductivity

Fourier's law of heat transfer describes the rate of heat flow in substance whose faces are maintained at different temperatures T_1 and T_2 ($T_2 > T_1$):

$$\frac{dQ}{dt} = \frac{kA(T_2 - T_1)}{x}$$

Where A is the area of the body which is in contact with the each temperature surface and x is the distance between them, or the thickness of the body. k is a constant for a given material and it is known as the thermal conductivity of the material. The larger the value of k, the greater the amount of heat that is allowed to pass through it per unit time, hence the name.

Consider a situation in which a metallic disc heats up another disc of another material. At steady state, the heat radiated by the exposed portion of the metallic disc will be equal to the heat conducted to through the of the unknown material disc. The heat radiated by such a disc is given by:

$$\frac{dQ}{dt} = Mc\frac{dT}{dt}\frac{(r + 2h)}{2(r + h)}$$

Where M is the mass of the metallic disc, c is its specific heat capacity, r is its radius and h is its height. At steady state, we can equate these equations and rearrange to determine the thermal conductivity of the unknown material:

$$k = \frac{Mc\frac{dT}{dt}}{\pi r^2(T_2 - T_1)} \times \frac{(r + 2h)x}{2(r + h)} \tag{5}$$

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Experimental Setup

- Hot Plate
- Mercury Thermometer
- Alcohol Thermometer
- Pt100 Resistance Thermometer
- Thermistor
- Thermocouple
- Multimeters capable of measuring voltage and resistance
- Vernier Calipers
- Stopwatch
- Glass Beakers (400ml, 1000ml and 2000ml)
- Test Tubes
- Retort stands and clamps
- Lee's disc apparatus
- Ice and Distilled Water
- Paraffin Wax
- Cotton
- Gloves

Least Count of Voltmeter = 0.1 V
Least Count of Ohmmeter = 0.1 Ω
Least Count of Mercury Thermometer = 0.2°C
Least Count of Alcohol Thermometer = 1°C
Least Count of Lee's Disc Apparatus = 0.01°C
Least Count of Vernier Calipers = 0.02 mm
Least Count of Stopwatch = 0.01 s

Procedure

Calibration

1. Take a large (2000ml) beaker filled with an ice-water mixture and place it on the hot plate but do not switch it on. Insert all five thermometers (mercury, alcohol, Pt100, thermistor and thermocouple) in the mixture such that they are suspended at the same depth (to ensure uniformity of heating). Remember to insert one end of the thermocouple in a small beaker of ice throughout the experiment to maintain a reference termperature.

2. Take a reading at 0°C of temperature/voltage/resistance for all the thermometers and switch the hot plate on. The Pt100 thermometer should give a reading close to 100Ω at 0°C.

3. As the mixture begins to melt and heat up, at fixed intervals of temperature on the mercury thermometers, take corresponding readings on the other instruments.

4. Continue to take readings until the water reaches its boiling point (100°C). Plot the calibration curves for each thermometer.

Ayaan Dutt
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Newton's Law of Cooling

1. Measure 100ml of distilled water and boil it in a 2000ml beaker. Once it reaches 100°C, insert a Pt100 thermometer in the liquid and remove the beaker from the hot plate.

2. Every 30 seconds, take a measurement of the resistance in the Pt100 thermometer. Allow the liquid to cool until it is close to room temperature.

3. Using the data from the calibration curve, convert the resistance into temperature. Plot a temperature-time curve of the data and fit an exponential function to it.

4. Repeat the experiment with a 400ml and 1000ml beaker but with the same volume of water (100ml). Measure the radius of each beaker with a vernier caliper and determine the surface area over which heat is lost.

5. Use the exponential fit of each cooling curve and Fourier's law of heat transfer to determine the heat transfer coefficient of each beaker.

Melting and Boiling Point of Paraffin Wax

1. Put the paraffin wax in a test tube and insert a Pt100 thermometer inside the test tube. Plug the open end with cotton to prevent any loss via evaporation.

2. Immerse the test tube in a beaker of water which is placed on the hot plate and switch it on. Wait until the solid has entirely melted and the thermometer is within the liquid.

3. Switch off the heat and remove the test tube from the water. Record the resistance at 30 second intervals until the wax has completely solidified and its temperature stabilises.

4. Return the test tube to the beaker and heat it again, this time with the thermometer inserted within the solid. Take readings at 30 second intervals until the solid completely melts and the temperature begins to rise.

5. Use the calibration data to convert the resistance to temperature and plot the cooling and melting curves for the wax. Use the periods of stable temperature to determine its melting and cooling point.

Lee's Disc and Thermal Conductivity

1. Insert the sensors into the metal discs D_2 (which will be exposed on one side) and D_3 (which will conduct the heat from the heater). Place the glass disc between the metallic discs D_2 and D_3.

2. Setting the target temperature at 50°C, turn on the heater and allow the system to reach steady state. Once the temperatures have stabilised, note down the temperatures T_2 (exposed disc) and T_3 (heated disc).

3. Remove the glass disc and heat up the exposed metallic disc D_2 by a further 5°C. Once it achieves this temperature, remove the disc from the heater and track the time taken for it to fall every 1°C until it drops by 10°C.

4. Plot the temperature-time curve and determine the slope at the steady state temperature. Use this value for dT/dt in equation 5.

5. Measure the radius and thickness of the metallic disc and glass disc with a pair of vernier calipers. Also measure the mass of the metallic disc and find its specific heat capacity.

6. Use this data with equation 5 to determine the thermal conductivity of glass.

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Observations and Analysis

Alcohol Thermometer

Alcohol expands linearly with respect to temperature and should give identical readings to the pre-calibrated mercury thermometer. The temperature data of both thermometers is plotted below:

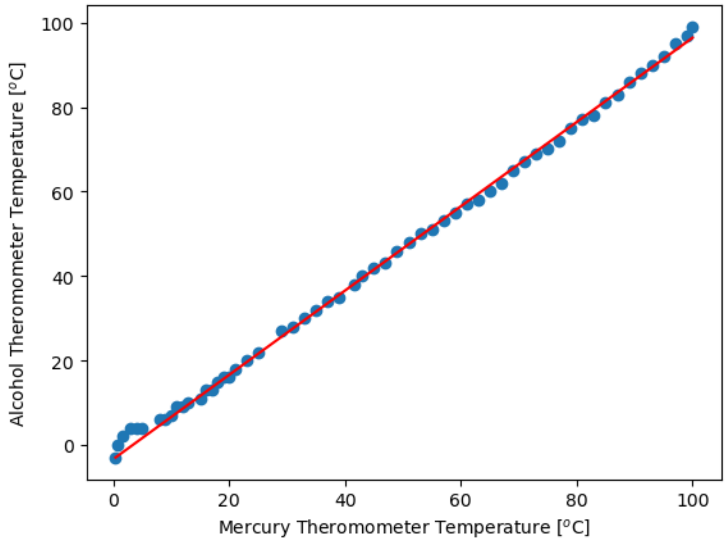

Figure 2: The alcohol thermometer measures temperature linearly as seen by the trend

The slope of the above graph is $0.99 \approx 1$ since 1°C rise on the mercury thermometer must correspond to a 1°C rise on the alcohol thermometer. However, the non-zero intercept of -3.35°C indicates a systemic error in the setup. This error may be due to non-uniformity in the temperatures detected at each bulb or due to an inherent calibration error within the thermometer.

Pt100 Thermometer

The resistance of platinum responds linearly with respect to temperature. Pt100 thermometers are calibrated such that at 0°C their resistance is 100Ω. The temperature-resistance curve is plotted below:

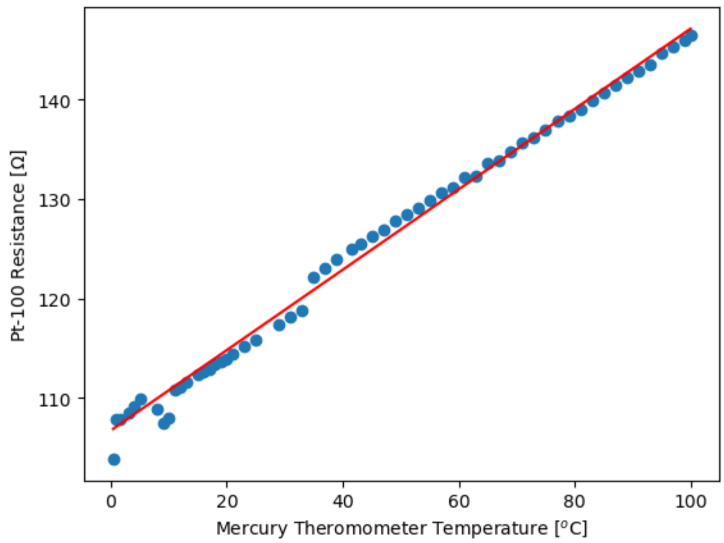

Figure 3: The resistance of a Pt100 thermometer varies linearly with temperature as theoretically expected

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Using a first order polyfit function to fit the data, we found the temperature detected by a Pt100 thermometer is given by:

$$T = \frac{R - R_0}{m} \tag{6}$$

Where $m = 0.404\Omega/°C$ and $R_0 = 106.7\Omega$. The percentage error from the expected value of 100Ω is 6.7%.

Thermistor

The thermistor's resistance varies non-linearly with respect to temperature due to its properties as a semiconductor. Its temperature response curve is plotted below:

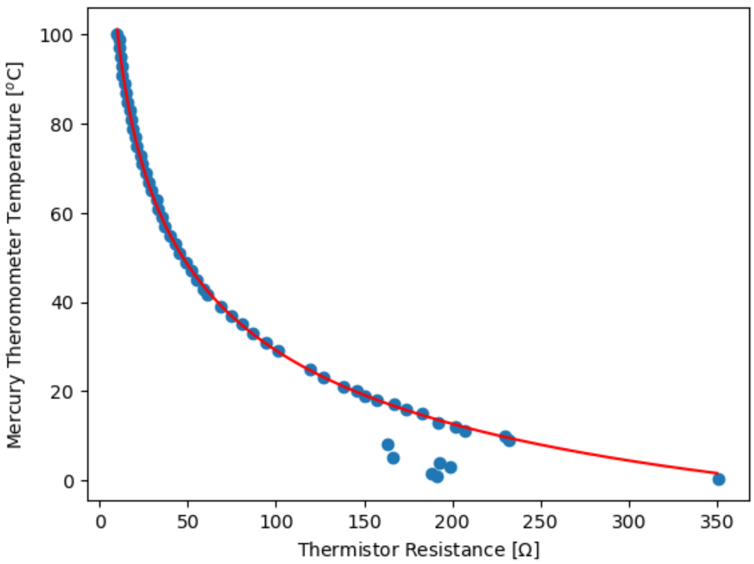

Figure 4: As the temperature increases, the resistance of the thermistor falls as expected by a semiconductor

The data was fit with model function with the form:

$$\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3$$

Where a, b and c were constants that were determined by the curvefit function.

It was noticed that at low temperatures, the thermistor was particularly sensitive, causing significant fluctuations during the melting process.

Ayaan Dutt
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Thermocouple

The potential difference between the ends of a thermocouple is expected to rise linearly with respect to temperature. The voltage temperature graph is plotted below:

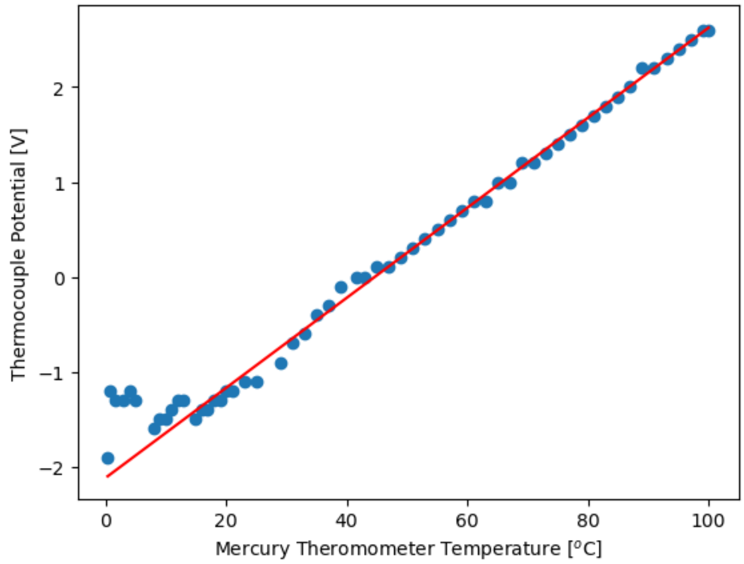

Figure 5: The voltage rose linearly with respect to temperature as expected

Note that the voltage began from a negative value at 0°C and varied linearly with the temperature. This contradicts our expected intercept of 0V at 0°C since the reference beaker and experimental beaker must be at the same temperature. This discrepancy can be explained by a systemic error inherent to the multi-meter used for the experiment which may have reported a consistently lower voltage than the true value. This would cause the entire graph to be vertically shifted by a constant, as seen in the above graph.

Newton's Law of Cooling

Mass of Water Used (m) = 100 ± 0.1 g
Specific Heat Capacity of Water (c) = 4186 J/kg °C
Least Count of Vernier Calipers = 0.02 mm

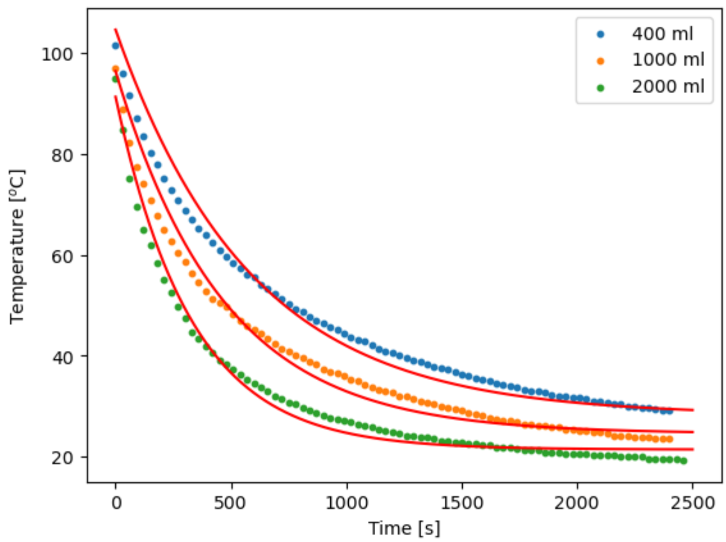

Figure 6: The rate of cooling is greater for larger beakers since they have a larger surface area over which heat is dissipated

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Volume [ml]	Diameter [mm]	Area (A) [mm^2]
400	82.66	5366.37
1000	105.58	8754.94
2000	132.02	13688.92

Table 1: The dimensions and volumes of each of the beakers used

The temperature falls as a negative exponential of time as predicted by theory. The above cooling curves were fit with an exponential fit of the same form as equation 4:

$$T = (T_0 - T_s)e^{-kt} + T_s \tag{7}$$

Where k is a constant, T_0 is the initial temperature from which the cooling began and T_s is the temperature of the surrounding environment. In each case T_0 has been taken as the first temperature reading ($\approx$ 100°C) and T_s = 25°C. Comparing with the formula, we find that the value of k is equal to:

$$k = \frac{hA}{mc}$$
$$\implies h = \frac{kmc}{A} \tag{8}$$

Using the values of k found from the curvefits of the data and the data from table 1, the heat coefficient for each beaker was determined and plotted:

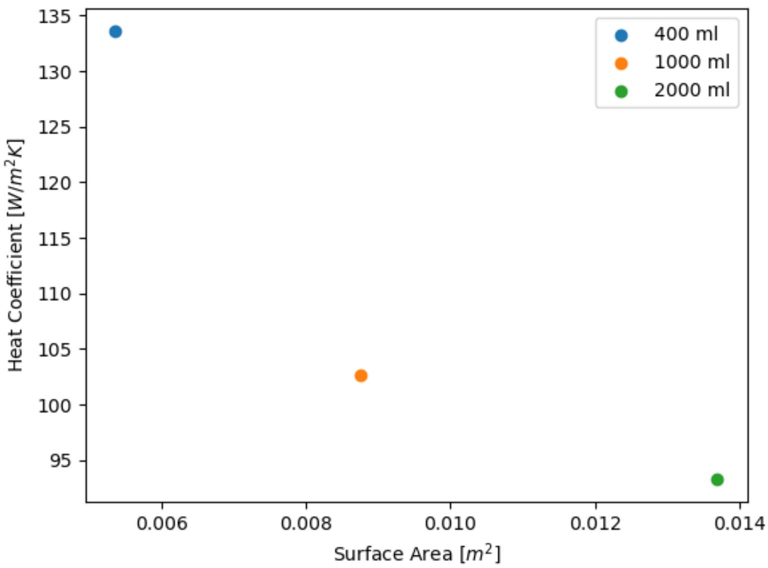

Figure 7: The heat coefficient decreases as the size of the beaker increases

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Heating and Cooling of Paraffin Wax

The calibration curve (equation 6) for the Pt100 thermometer was used to convert the resistance data into temperature which has been plotted against time to get the heating and cooling curves for the wax. The heating curve for paraffin wax is shown below:

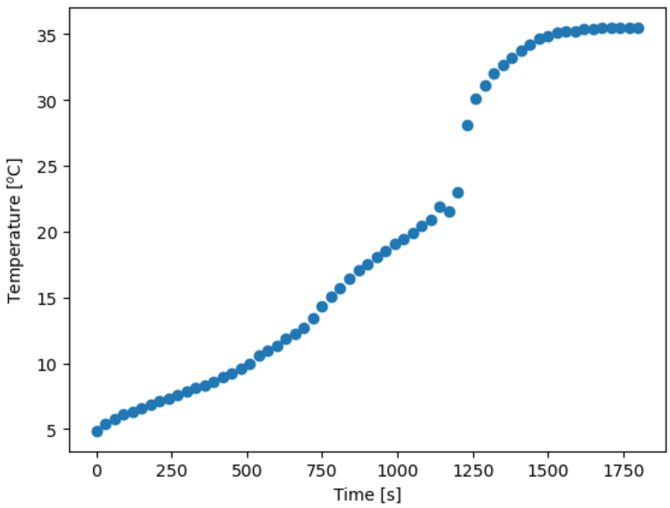

Figure 8: The temperature stabilises close to 35°C

As expected, the temperature rises until it reaches a phase transition point, in this case boiling, at which point the temperature of the mixture stabilises at around 35°C. This is within 5% of the expected value of the known melting point of paraffin wax which is 37°C.

The abrupt spike in the temperature close to 1250 seconds is unexpected, but it may be explained by an accidental change in the heating rate of the wax during the experiment which may have briefly raised its temperature by a large amount.

The cooling curve for the wax is shown below:

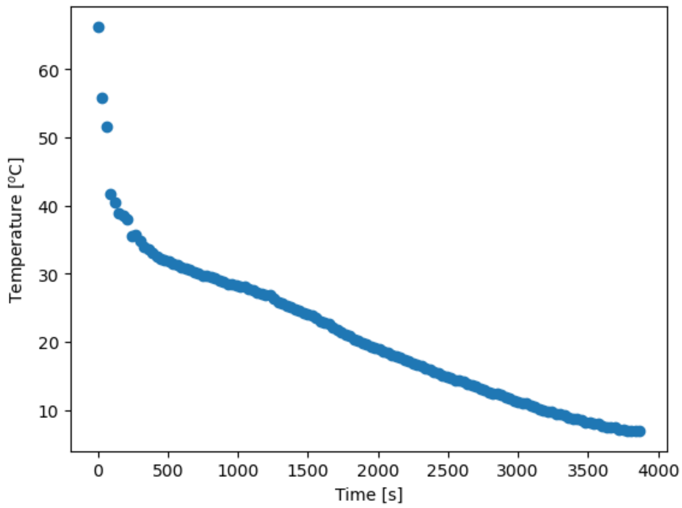

Figure 9: The cooling rate significantly slows down after a threshold temperature close to 35°C

The wax also begins to cool down significantly slower below 35°C which would indicate that the substance is solidifying or condensing. However, in this case the temperature does not stabilise while undergoing the phase transition. This could be due to excess loss of energy in the form of heat since the test tube is not insulated.

 Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Lees Disc and Thermal Conductivity

Least Count of Lees Disc Apparatus = 0.01°C
Least Count of Vernier Calipers = 0.02 mm
Least Count of Weighing Scale = 0.1 g

Steady State Temperature of Heat Supply Disc (T_3) = 48.2 ± 0.01°C
Steady State Temperature of Exposed Disc (T_2) = 47.5 ± 0.01°C

Radius of Metal Disc (r_m) = 38.04 ± 0.01 mm
Radius of Glass Disc (r_g) = 38.07 ± 0.01 mm
Thickness of Metal Disc (h_m) = 23.18 ± 0.02 mm
Thickness of Glass Disc (h_g) = 4.40 ± 0.02 mm

Mass of Metal Disc (M) = 434.2 ± 0.1 g
Specific Heat Capacity of Brass (c) = 380 J/kg °C

The cooling curve for the metal disc gives us the value of dT/dt by finding the best fit function and determining the slope at the steady state temperature.

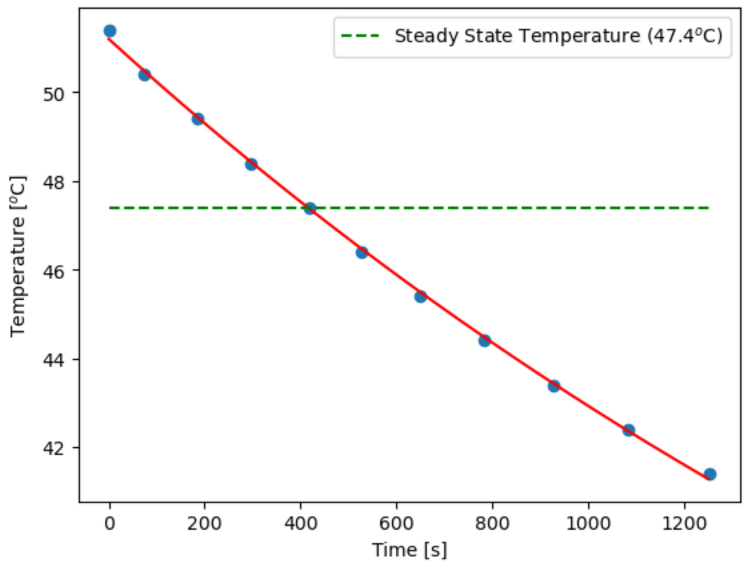

Figure 10: Cooling curve for the exposed bass disc for a range of 10°C. Though the trend looks nearly linear, it is in fact an exponentially decaying temperature but the decay is imperceptible at this scale

At the steady state temperature of 47.4°C, the value of $|dT/dt| = 0.008°C/s$. Putting the above data into the following equation, we can find the thermal conductivity of glass:

$$k = \frac{Mc(dT/dt)}{\pi r_g^2 (T_3 - T_2)} \cdot \frac{(r_m + 2h_m)h_g}{2(r_m + h_m)} = 1.25\ W/mK$$

The error in the value of k can be determined by taking the error in quadrature:

$$\delta k = k\sqrt{\left(\frac{\delta M}{M}\right)^2 + \left(\frac{\delta(dT/dt)}{(dT/dt)}\right)^2 + \left(\frac{\delta r_m}{r_m}\right)^2 + \left(\frac{\delta h_m}{h_m}\right)^2 + \left(2\frac{\delta r_g}{r_g}\right)^2 + \left(\frac{\delta h_g}{h_g}\right)^2 + \left(\frac{\delta T_3 + \delta T_2}{T_3 - T_2}\right)^2}$$

Putting the least count errors known from the various instruments used, we find that the value of k with error is:

$$k = 1.25 \pm 0.04\ W/mK$$

 Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Discussion

- During the calibration process, note that for low temperatures the data is inconsistent with the models. This inconsistency can be attributed to large non-uniformities during the initial melting process since the ice-water mixture was highly heterogeneous in the beginning. Furthermore, the sensitivity of certain thermometers may have varied with respect to temperature. This data has been ignored while fitting the best fit curves to avoid spurious results.
- The values of heat coefficient which depend on the value of k determined from the best fit of equation 7 is highly sensitive to the value of T_s chosen. We have chosen $T_s = 25°\text{C}$ since this was the temperature shown on the lab ambient thermometers on the day that the analysis was done, but the true value during the experiment was not measured and the results therefore may be subject to some deviation.
- The sudden rise of temperature during the heating of the paraffin wax is likely due to an accidental alteration of the heating rate via the hot plate during the experiment which caused a brief unexpected spike in the temperature. This error was not detected until analysis and therefore the experiment could no longer be repeated. However, this spike does not affect the results of the experiment.
- The cooling curve for the paraffin wax does not stabilise at a fixed temperature but instead it continues to drop slowly but steadily. This may be due to excess energy loss via heat during the solidifying process which is leading to a simultaneous loss of energy via dissipation and condensation.

Results

Alcohol Thermometer

The height of the alcohol level in the thermometer varies linearly with temperature and when plotted against the temperature readings of a pre-calibrated mercury thermometer, the slope is 1.

Pt-100 Thermometer

The resistance of platinum responds linearly with respect to temperature. The resistance is given by:

$$R_T = mT + R_0$$

Where $m = 0.404\Omega/°\text{C}$ and $R_0 = 106.7\Omega$ (Percentage error of 6.7%).

Thermistor

The resistance of a thermistor decreases as temperature increases. The variation can be fit by the following equation:

$$\frac{1}{T} = a + b\ln R + c(\ln R)^3$$

Where a, b and c are constants.

Thermocouple

The voltage detected by a thermocouple varies linearly with respect to temperature with higher voltage generated at higher temperatures.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Newton's Law of Cooling

The temperature of a freely cooling substance is given by:

$$T = (T_0 - T_s)e^{-kt} + T_s$$

Where $k = hA/mc$ in which h is the heat coefficient, A is the surface area over which heat is dissipated, m is the mass of the substance and c is its specific heat capacity.

The heat coefficient for 100 ml of water in 400ml, 1000ml and 2000ml beakers are 133.60 W/m^{2}K, 102.67 W/m^{2}K and 93.28 W/m^{2}K, respectively.

Heating and Cooling of Paraffin Wax

The melting and condensing point of paraffin wax is 35°C ($< 5\%$ percentage error).

Lees Disc and Thermal Conductivity

The rate of change of temperature (dT/dt) at the steady state temperature of 47.5°C is 0.008°C/s for a brass metal disc. The thermal conductivity of glass is therefore:

$$k = 1.25 \pm 0.04 \ W/mK$$