

Diffraction and Surface Waves Lab Report 6

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Aim

- Determine the mean size of carrom powder particles from their diffraction pattern.
- Find the surface tension of water from the diffraction pattern created by light reflected off standing surface waves.

Theoretical Background

According to Babinet's principle, complementary objects (such as rods and slits) produce similar diffraction patterns with the exception of the intensity of the central maxima. Hence, point-like objects and circular apertures both produce Airy discs (figure 1) where the size and separation of the fringes contain information about the size of the particle or aperture causing the diffraction.

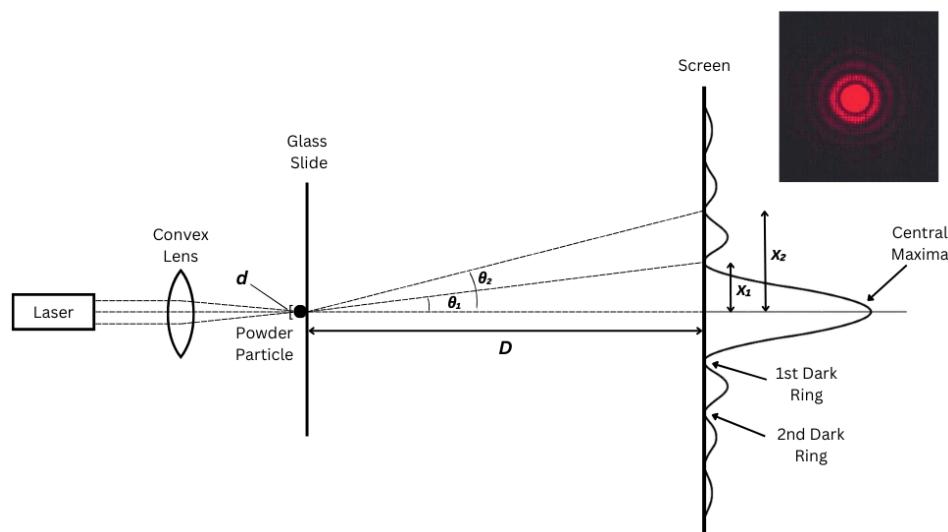


Figure 1: Experimental setup for the diffraction of particles which produce an Airy disc pattern
Source: Made on www.canva.com

Such an Airy disc pattern is characterised by the following equation:

$$\sin \theta = 1.22 \frac{\lambda}{d} \quad (1)$$

Where θ is the angle between the central maxima and the first minima with respect to the obstacle and d is the diameter of the particle causing the diffraction. From figure 1, note that $\tan \theta = x_1/D$ where x_1 is the radius of the central maxima and D is the distance between the screen and glass slide on which the particle rests. Hence, we can find the diameter of the particle:

$$d = \frac{1.22\lambda}{\sin(\tan^{-1}(x_1/D))} \quad (2)$$

Surface Tension Waves

Molecules in the bulk of a liquid experience equal forces in all directions in equilibrium, but molecules at the surface experience a greater force of attraction from the liquid molecules below than the air molecules above. This net force is known as surface tension and it tends to restore the surface of the liquid such that it has the lowest possible surface area (and hence lowest energy). When a liquid in a container is disturbed, standing waves can be generated on its surface, similar to standing waves in a taunt string.

If a laser light is incident on these waves close to the grazing angle, they act like a dynamic diffraction grating, producing an interference pattern similar to an ordinary diffraction grating. However, the dispersion relation for the surface tension waves also contains information about the surface tension and density of the liquid in which the waves are produced. The general dispersion relation for capillary surface waves in liquids is:

$$\omega^2 = gk + \frac{\sigma k^3}{\rho} \quad (3)$$

Where ω is the frequency of the surface wave, k is the wave vector of the surface wave, g is the acceleration due to gravity, σ is the surface tension of the liquid and ρ is the density of the liquid. The gravity wave term can be ignored since the contribution due to the surface tension is significantly higher. Hence, by taking the natural logarithm of the dispersion relation, we find:

$$\ln \omega = \frac{3}{2} \ln k + \frac{1}{2} \ln \frac{\sigma}{\rho} \quad (4)$$

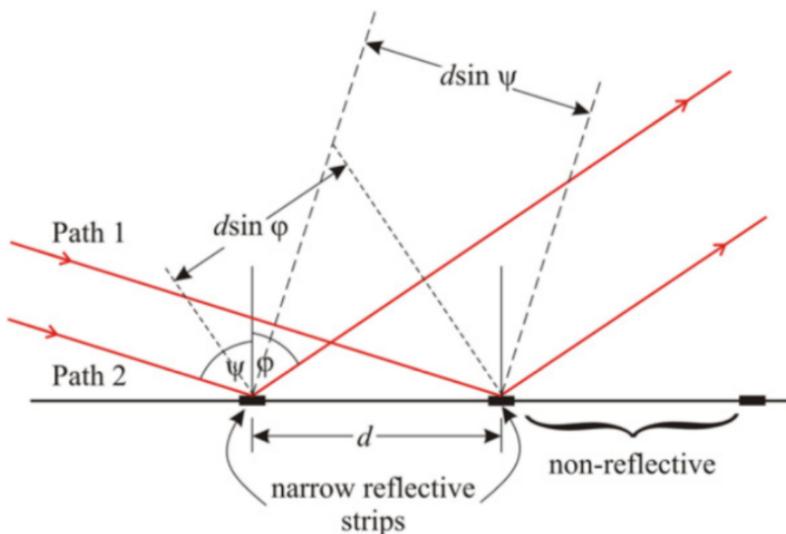


Figure 2: Ray diagram of incident rays on surface waves
 Source: Nikolić & Nešić (2012)

If we assume that the light is only reflected off the peaks of the surface waves, they essentially form a diffraction grating with separation d between the peaks. The path difference between two parallel incident rays is therefore $d(\sin \psi - \sin \phi)$ (figure 2). If the incident light is in phase and this path difference is equal to a whole number of wavelengths of light (λ_l), then constructive interference will occur:

$$d(\sin \psi - \sin \phi_m) = m\lambda_l \quad (5)$$

Let θ be the angle between the central maxima (formed at $m = 0$ or when $\psi = \phi$) and the plane of grating and γ be the angle between the central maxima and the first constructive interference. We find that $\theta = \pi/2 - \psi$ and $\gamma = \psi - \phi_1$.

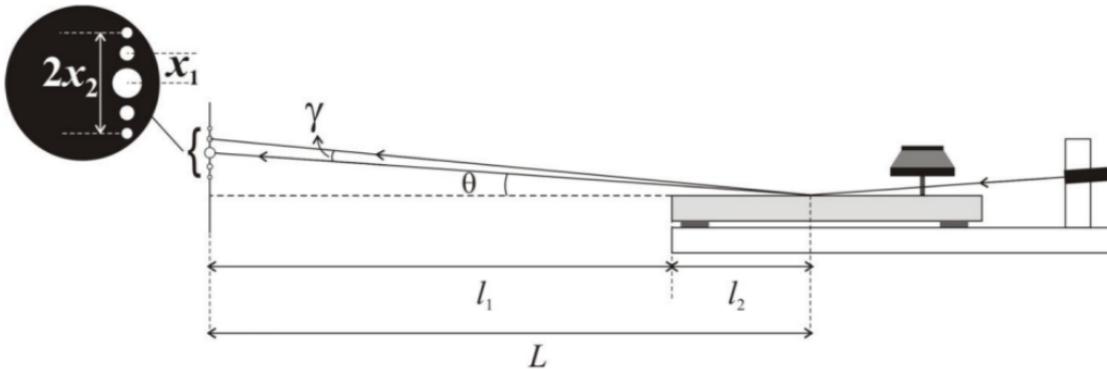


Figure 3: Experimental setup of diffraction of surface waves in a low viscosity fluid
Source: Diffraction Lab 3 Handout

From equation 5 we find:

$$\lambda_l = 2d \sin\left(\frac{\gamma}{2}\right) \sin\left(\theta + \frac{\gamma}{2}\right)$$

If we apply this same reasoning for the constructive maxima formed on the other side of the central maxima, we get a similar relation which we can combine with the above equation to find d :

$$\frac{1}{d} = \frac{1}{\lambda_l} \sin\left(\frac{\gamma}{2}\right) \left[\sin\left(\theta + \frac{\gamma}{2}\right) + \sin\left(\theta - \frac{\gamma}{2}\right) \right]$$

Note that d is essentially the wavelength of the surface capillary waves since it is the distance between consecutive peaks. The wave number of the surface waves will therefore be:

$$k = \frac{2\pi}{d} = \frac{2\pi}{\lambda_l} \sin\left(\frac{\gamma}{2}\right) \left[\sin\left(\theta + \frac{\gamma}{2}\right) + \sin\left(\theta - \frac{\gamma}{2}\right) \right]$$

This expression can be further simplified to:

$$k = \frac{2\pi}{\lambda_l} \sin \gamma \sin \theta \quad (6)$$

This is the expression we use to find the wave number used to plot equation 4. The intercept of this plot gives us the value of surface tension if we know the density of the liquid used in the experiment.

Experimental Setup

- Optical bread board
- Laser source and mount
- Convex lens and mount
- Glass slide and mount
- Carrom powder
- Water tray with tube to measure water height
- Surface vibrator with adjustable amplitude
- Frequency generator app
- Screen and graph paper
- Spirit level
- Measuring tape

Least count of measuring tape = 0.1 cm

Procedure

Diffraction by Particles

1. Adjust the laser on the optical breadboard such that the beam is parallel to its surface. Place the convex lens in the path of the laser light and find its focal length using a movable screen. Fix the glass slide mount on the breadboard such that the light is focused on the same plane as the glass slide.
2. Scatter some carrom powder on the glass slide and mount it vertically such that the focused beam falls directly on some of the particles. Airy disc like diffraction patterns should fall on the distant screen.
3. Measure the distance from the glass slide to the screen. For each Airy disc, measure the diameter of the central maxima.
4. Repeat the measurement of central maxima diameters for at least 100 Airy disc diffraction patterns produced by the powder particles. Adjust the slide or the laser to find additional diffraction patterns if needed.
5. Plot a histogram of the radii of the central maximas and find the mean value by fitting a Gaussian curve to it. Use this mean value to find the average diameter of the carrom powder particles using equation 2.

Surface Wave Diffraction

1. Use a spirit level to level the water tray and fill it with water until the brim. Ensure that no air bubbles are trapped in the tubing and tape the end of the tubing to the screen to act as reference for the height of the water level.
2. Adjust the height of the surface vibrator such that it is immersed in the water and ensure the laser is incident on the surface as close to the grazing angle as possible.

3. Measure the height above the water level that the reflected beam falls on the screen (H) and the distance between the point of reflection and the screen (L). Use this to find $\theta = \tan^{-1}(H/L)$.
4. Using a frequency generator app plugged into the surface vibrator, excite standing waves in the water and allow them to form a stable diffraction pattern on the screen. Use frequencies between 100-200 Hz to produce stable waves with discernible patterns and adjust the amplitude to change the length of the pattern.
5. Measure the distance between the first constructive interference points formed on either side of the central maxima ($2x_1$) and use this to determine $\gamma = \tan^{-1}((H + x_1)/L) - \theta$.
6. Change the frequency of the standing wave and repeat the above measurements to find multiple values of *gamma*.
7. Use equation 6 to find the wave number for each frequency and plot the logarithm of the angular frequency against the logarithm of the wave number. Fit equation 4 to the data and determine the surface tension of water.

Observations

Diffraction by Particles

We observed the diffraction patterns for 100 particles and measured their diameter. The histogram of their radii is shown below, along with the fitted Gaussian curve:

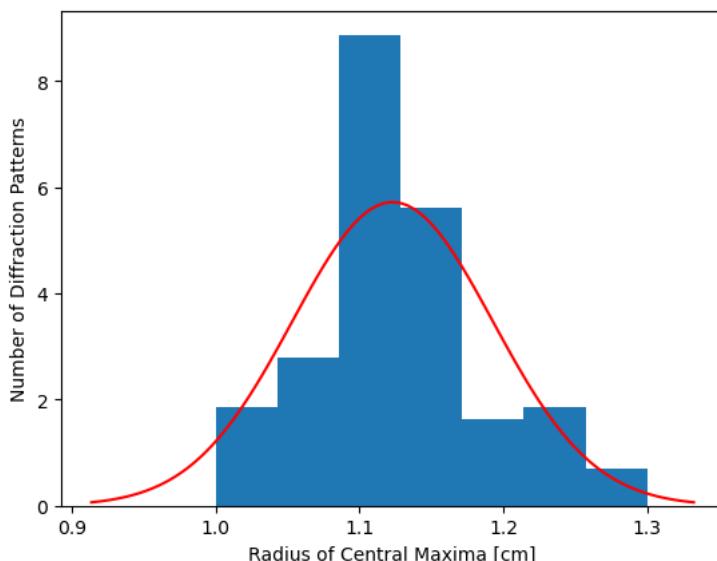


Figure 4: Though the radius of the central maxima varied from particle to particle, they form a roughly Gaussian distribution with a sharp peak close to 1.1 cm

The mean value of the radius of the central maxima was 1.12 cm with a standard deviation of 0.07 cm.

$$x_1 = 1.12 \pm 0.07 \text{ cm}$$

The distance between the screen and the glass slide was $D = 260.7 \pm 0.1 \text{ cm}$.

Surface Wave Diffraction

Distance between the screen and the point of reflection (L) = 277.5 ± 0.1 cm
Height of central maxima above water level (H) = 13.8 ± 0.1 cm

The distance between the central maxima and the first constructive interference (x_1) was measured for different frequencies.

Frequency [Hz]	x_1 [cm]
100	1.00
110	1.05
120	1.15
130	1.25
140	1.30
150	1.30
160	1.35
170	1.45
180	1.50
190	1.50
200	1.60

Table 1: As the frequency of the surface waves increases, the wavelength decreases which effectively reduces the spacing between consecutive reflections. This causes the spacing in the diffraction pattern to increase since the "diffraction grating" is essentially getting finer.

Since the distance between both first maxima (i.e. $2x_1$) was measured, the least count of x_1 is reduced to 0.05 cm.

Data and Error Analysis

Diffraction by Particles

From the equation of the Airy disc diffraction pattern, we can find the average diameter of the carom powder particles.

$$d = \frac{1.22\lambda}{\sin(\tan^{-1}(x_1/D))}$$

Where $\lambda = 6.3 \times 10^{-7}$ m for the red laser light we used. Using the mean value of x_1 and its standard deviation, we find:

$$d = 179 \pm 12 \mu\text{m}$$

Note that the \pm does not indicate the error in d , rather it is the range of diameters that the particles measured lie within. The actual error in d can be approximated by assuming that θ is small (i.e. $x_1 \ll D$), which allows us to write:

$$d = \frac{1.22\lambda D}{x_1}$$

Using quadrature, we find:

$$\frac{\Delta d}{d} = \sqrt{\left(\frac{\Delta x_1}{x_1}\right)^2 + \left(\frac{\Delta D}{D}\right)^2}$$

Putting in the values, we find the error in d .

$$\Delta d = 16 \mu m$$

Since the error is larger than the standard deviation of the particle size, the value of d is reported with the error instead.

Surface Wave Diffraction

The angle between the central maxima and the plane of grating is θ and γ is the angle between the central maxima and the first constructive interference.

$$\theta = \tan^{-1}(H/L) = 0.0496 \pm 0.0004 \text{ rad}$$

Using the fact that $\gamma = \tan^{-1}((H + x_1)/L) - \theta$, we find a list of values of γ and then use equation 6 to find the wave number corresponding to each frequency. We used $\lambda_l = 6.3 \times 10^{-7} \text{ m}$ as the wavelength for the red laser used.

$$k = \frac{2\pi}{\lambda_l} \sin \theta \sin \gamma$$

Frequency f [Hz]	x_1 [cm]	γ [rad]	Wave Number k [m^{-1}]
100	1.00	0.0036	1780.34
110	1.05	0.0038	1869.34
120	1.15	0.0041	2047.33
130	1.25	0.0045	2225.32
140	1.30	0.0047	2314.31
150	1.30	0.0047	2314.31
160	1.35	0.0049	2403.30
170	1.45	0.0052	2581.27
180	1.50	0.0054	2670.25
190	1.50	0.0054	2670.25
200	1.60	0.0057	2848.21

Table 2: The wave number increases as the frequency of the source is increased as theoretically expected

The angular frequency was found using $\omega = 2\pi f$ and $\ln(\omega)$ was plotted against $\ln(k)$ and fitted with a straight line of the form (equation 4):

$$\ln \omega = \frac{3}{2} \ln k + \frac{1}{2} \ln \frac{\sigma}{\rho}$$

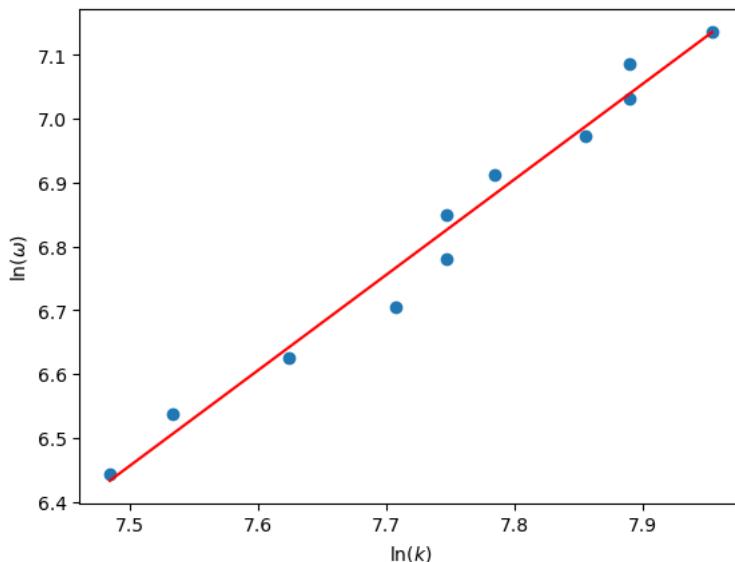


Figure 5: The slope of the log graph of ω vs k should be $3/2$ and the intercept gives us information about the surface tension and density of water

The slope of the fitted line is $p = 1.49$ and the intercept is $q = -4.75$. The experimentally determined slope has an error of only 0.6% since the theoretically expected slope is 1.5. Hence, we can expect the intercept value to be reasonably accurate.

Equating the intercept to the last term in equation 4 and using $\rho = 1000 \text{ kg/m}^3$ as the density of water, we find the surface tension of water:

$$\sigma = \rho e^{2q} = 7.485 \times 10^{-2} \text{ N/m}$$

The known value of the surface tension of water is $7.2 \times 10^{-2} \text{ N/m}$ at room temperature. Hence, the percentage error of the experimentally determined surface tension is 3.9%.

Results

The average diameter of carrom powder particles found using their diffraction pattern is:

$$d = 179 \pm 16 \mu\text{m}$$

The surface tension of water was experimentally determined to be:

$$\sigma = 7.485 \times 10^{-2} \text{ N/m}$$

The percentage error of the experimentally determined surface tension is 3.9%.

Discussion

- It was observed that better standing waves were produced when the vibrator was fed a square wave rather than a sine wave. This makes sense since a square wave will produce a more distinct discontinuous periodic disturbance compared to the continuous motion of a sinusoidal wave.
- The diffraction pattern due to the standing wave "grating" was observed to vibrate slightly on the screen, possibly due to some phase shifts or disturbances that perturbed the standing waves. This led to some imprecision while measuring the distance between the first constructive interferences. It would be better to use a camera or light sensor instead to improve the precision of these measurements, especially since the least count is comparable to the change in the magnitude of the measurement itself.

References

Diffraction (2024) *Lab 3 Handout*

Nikolić, D., Nešić, L. (2012). Determination of surface tension coefficient of liquids by diffraction of light on capillary waves. *European Journal of Physics*, 33(6), 1677–1685. <https://doi.org/10.1088/0143-0807/33/6/1677>