Acoustics and Resonance
Lab Report 1

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Aim

- Use a Melde electrical vibrator to determine the frequency of AC supply.
- Use a Sonometer to determine the frequency of AC supply.
- Find the speed of sound in air using a frequency generator and an air column resonance tube.

Theoretical Background

A wave is a perturbation that transfers energy through space and time. A simple sine wave can be characterised by its time period (T), wavelength (λ), frequency (f), and amplitude (A).

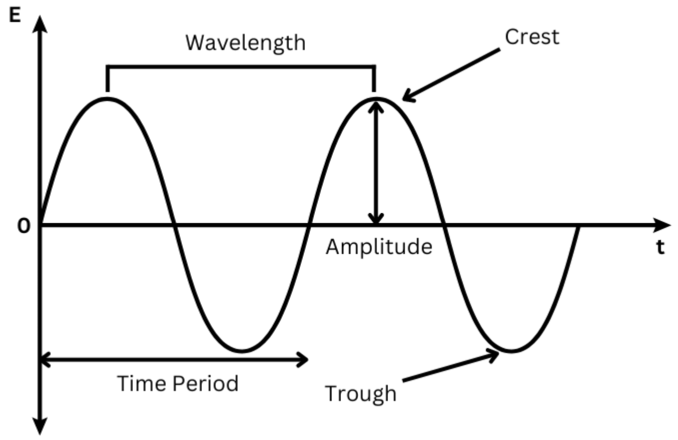

Figure 1: A sine wave can be characterised by its wavelength, time period and amplitude
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The frequency of a wave is the reciprocal of its time period. The wavelength and frequency of a wave share a simple relationship depending on the speed of the wave in the medium through which it propagates:

$$v = f\lambda = \lambda/T \tag{1}$$

Waves can be classified as either transverse or longitudinal depending on the direction of the displacement of the particles in the medium with respect to the direction of the propagation of the wave. Longitudinal waves transfer energy via compressions and rarefactions which occur alternately in the direction of the wave's propagation, such as in air. Transverse waves look like a sine wave and the particles are disturbed in a direction perpendicular to the direction that the wave propagates, like in a vibrating string.

Interference and Standing Waves

When two waves combine, their amplitudes can be arithmetically summed at every corresponding point to give the final waveform. This is known as the *Principle of Superposition*. When the sum of two waves results in a wave with a larger amplitude, it is known as constructive interference whereas if the sum of two waves leads to a smaller amplitude, it is known as destructive interference. The resultant waveform depends on the phase difference and amplitudes of the two waves.

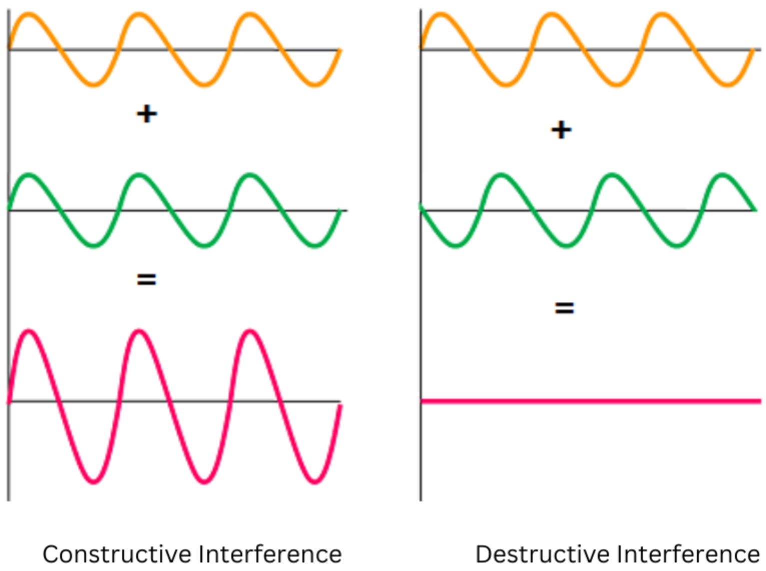

Figure 2: If two identical in-phase waves combine, they interfere constructively resulting in double the amplitude. However, if they are out of phase by a period of π, their amplitudes cancel out completely due to destructive interference.
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When a fixed length of string or an air column is made to vibrate, the waves will reflect off the fixed points and interfere with themselves. While most frequencies will lead to chaotic interference, certain frequencies will allow for a stable wave to form via interference. These waves are known as *standing waves* and are formed when the external vibration matches the natural frequency or harmonics of the natural frequency of the system. This condition is called *resonance*.

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Melde's Electronic Vibrator

An ordinary household 220V AC supply transmits voltage sinusoidally with a frequency of 50Hz. Melde's Electronic Vibrator is a device that converts this frequency into a mechanical oscillation by alternately magnetising a steel rod at 50Hz and placing it in a magnetic field. This causes the steel rod to vibrate in the magnetic field, which in turn causes the string tied to it to vibrate as well.

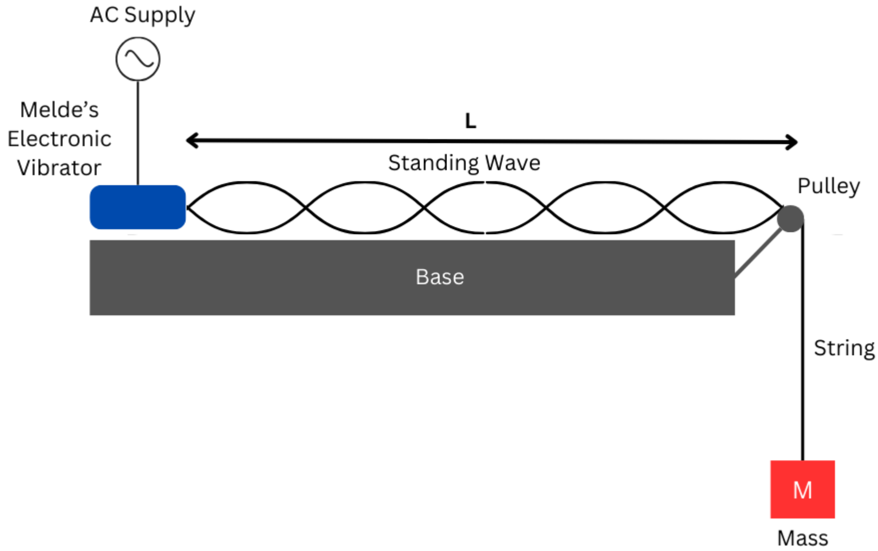

Figure 3: Schematic diagram of Melde's Electronic Vibrator experimental setup
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For a string with a standing wave with length L, tension T, mass per unit length m, and number of loops p, the frequency is given by:

$$f = \frac{p}{2L}\sqrt{\frac{T}{m}} \tag{2}$$

The number of loops in the standing wave correspond to the harmonic of the natural frequency. For instance, when $p = 1$ the string is said to be vibrating at the first harmonic, also known as the fundamental.

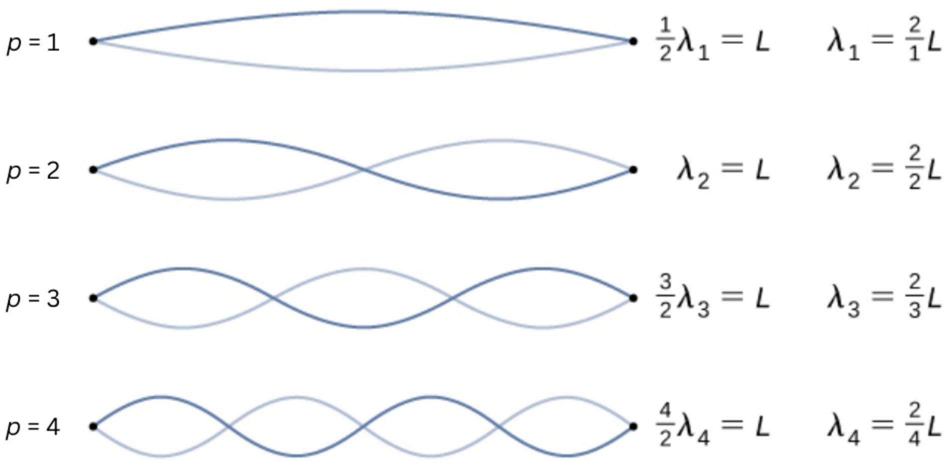

Figure 4: Schematic diagram of standing waves formed in a vibrating string
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The tension of the string can be controlled by changing the mass tied to one end. This allows us to find the wavelength of each harmonic which in turn gives us the fundamental frequency which is the same as the frequency of the AC supply.

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Sonometer

The Sonometer uses a step-down transformer to bring the voltage from 220V to 6V. The secondary coil of the transformer is connected to an electromagnet which is magnetised twice in every cycle, with the polarity switching two times within a single time period. A metal wire is strung close to the magnet is is vibrated by the oscillating magnetic field. Since the magnetic field switches twice in each cycle, its frequency is double the frequency of the AC supply which is 50Hz.

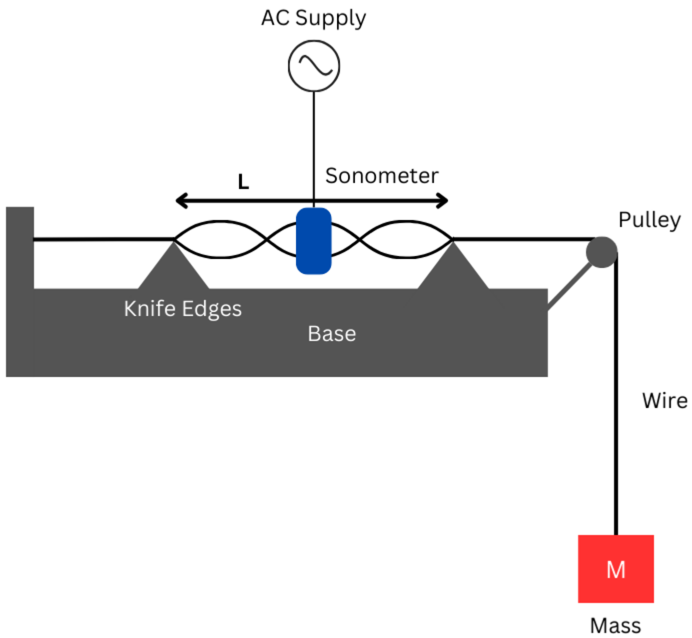

Figure 5: Schematic diagram of a Sonometer experimental setup
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Knife edges can be used to adjust the length of the wire and the mass can be changed to control the tension of the string. The fundamental frequency of the string is given by:

$$f = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

However, since the string vibrates at twice the frequency of the AC supply, the AC supply frequency will be:

$$f_{AC} = \frac{f}{2} = \frac{1}{4L}\sqrt{\frac{T}{m}} \tag{3}$$

Resonance Tube

A resonance tube is a cylindrical tube partially filled with water whose level can be controlled to change the length of the air column in the tube. When the air in the tube is made to vibrate (either using a tuning fork or a frequency generator), standing waves form when the frequency source matches the natural frequency of the tube. As the length of the air column is increased, higher harmonics of the fundamental can resonate.

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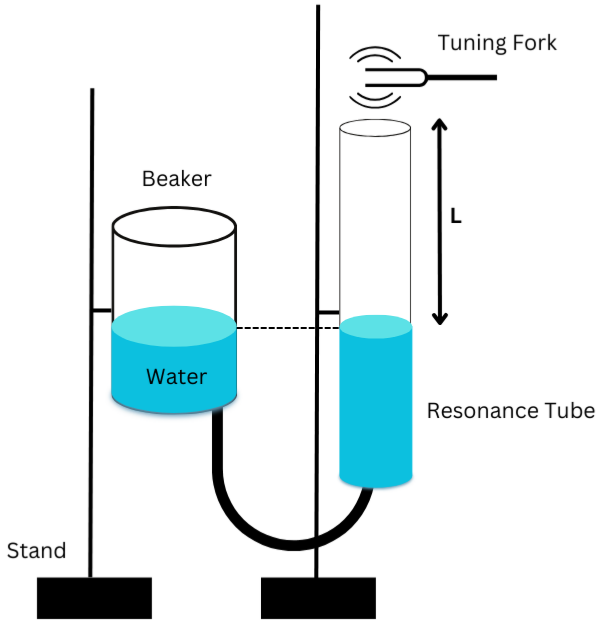

Figure 6: Schematic diagram of a Resonance Tube experimental setup
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Since only one end of the tube is closed, it acts as a fixed node for standing waves while the open end allows for free oscillation which are known as anti-nodes. As a result, the fundamental frequency resonates when the air column is $\lambda/4$. The next harmonics are found when the air column's length is $3\lambda/4$, $5\lambda/4$, and so on.

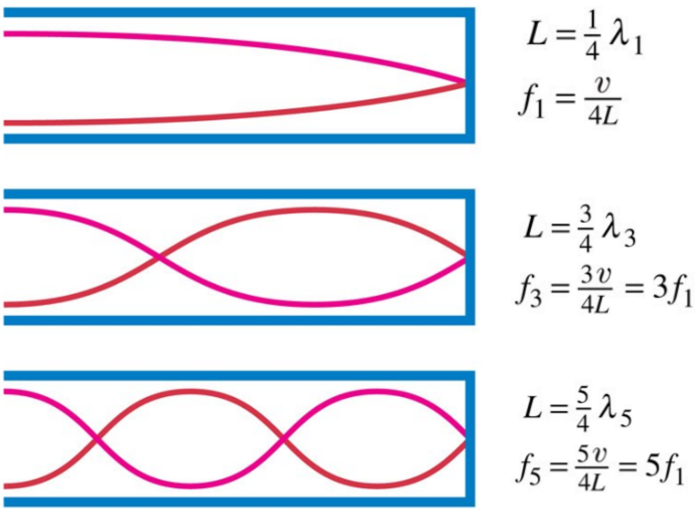

Figure 7: Since one end is a fixed node while the other is an anti-node, only the odd harmonics of the fundamental can be formed
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Experimental Setup

- Melde's Electronic Vibrator
- Sonometer
- Variable water level open-end resonance tube
- Frequency generator
- Oscilloscope
- 50Hz AC power supply
- Permanent magnets
- A set of tuning forks
- Rubber mallet
- String
- Steel Wire
- Fractional and slotted weights
- Sensitive weighing balance
- Measuring tape

Least count of measuring tape = 0.1cm
Least count of weighing balance = 0.0001g

Procedure

Melde's Electronic Vibrator

1. Measure the mass and length of a piece of string and find its mass per unit length. Tie a long piece of the same string to the rod of Melde's Electronic Vibrator and suspend a tray for masses from the other end.
2. Insert permanent magnets on either side of the rod such that they form a magnetic field around it.
3. Keeping the string parallel to the base of the apparatus as seen in figure 3, suspend the tray over the edge with a pulley.
4. Use a measuring tape to measure the distance between the pulley and the end of the string tied to the vibrator's rod.
5. Turn on the electric vibrator and add masses to the tray until maximum amplitude of the standing wave is obtained. Count the number of loops p and the mass M suspended from the string (including the mass of the tray and string hanging from the pulley).
6. Continue to add fractional masses to the tray until further harmonics are observed. Repeat readings for p and M. Use M and the known value of $g = 9.81m/s$ to determine the tension in the string.
7. Plot a graph of p vs $\sqrt{T}$ and use the slope to find the frequency of the AC supply powering the electronic vibrator.
8. Repeat the experiment keeping the string perpendicular to the base of the setup to induce longitudinal waves in the string.

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Sonometer

1. Measure the mass per unit length of the wire and set the knife edges an equal distance apart from the sonometer's electromagnet.

2. Suspend a single slotted mass from pulley end of the wire and turn on the device. Adjust the knife edges until the amplitude of the standing wave is maximised and resonance is achieved. Ensure that the knife edges are equidistant from the vibrator.

3. Note down the length L between the two knife edges and the mass M suspended from the wire when resonance is achieved. Place a small folded piece of paper on the wire and watch for when it falls off to detect when resonance is reached.

4. Increase the amount of mass suspended from the wire and note down the corresponding lengths at which standing waves with large amplitudes are found.

5. Use equation 3 to find the frequency of the AC supply used to power the sonometer.

Resonance Tube

1. Fill the resonance tube with water through the beaker. Ensure that there are no air bubbles anywhere in the setup.

2. Strike a tuning fork with the rubber mallet and hold it over the mouth of the tube such that the prongs are perpendicular to the mouth.

3. Starting from the top, slowly lower the water level until an audible resonating sound can be heard from the tube. Continue to strike the tuning fork and bringing it to the mouth of the tube throughout.

4. Note down the length of the air column L that produces the largest amplitude and the frequency f of the tuning fork used.

5. Continue to lower the water level while striking the same fork until a second resonance condition is reached. Note down this length in a separate column.

6. Repeat the experiment with several tuning forks of different frequencies.

7. Repeat the experiment using a frequency generator attached to an oscilloscope. Use channel 1 for the input wave and channel 2 for the frequency response. The maximum peak-to-peak reading of the channel 2 can be used to determine when resonance is reached.

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Observations

Melde's Electronic Vibrator

Least count of weighing scale = 0.0001g
Least count of measuring tape = 0.001m
Mass per unit length of string (m) = 0.19 ± 0.02 g/m

Transverse Waves

Length of string (L) = 0.522 ± 0.001 m

No. of Loops p	Mass Suspended M [g]
1	52.9214
2	12.9762
3	5.5308
4	3.1468

Table 1: The number of loops formed in a transverse standing wave for different masses which correspond to different amounts of tension in the string

Longitudinal Waves

Length of string $L = 0.728 \pm 0.001$ m

No. of Loops p	Mass Suspended M [g]
2	24.4043
3	11.0342

Table 2: The number of loops formed in a longitudinal standing wave for different masses which correspond to different amounts of tension in the string

Sonometer

Least count of weighing scale = 0.1g
Least count of measuring tape = 0.001m

Mass per unit length of wire (m) = 2.6 ± 0.3 g/m

Length of Wire L [m]	Mass Suspended M [g]
0.384	1634.7
0.442	2147.1
0.512	2657.4
0.558	3172.5

Table 3: The lengths of wire that gave resonance for different masses which correspond to different amounts of tension in the wire

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Resonance Tube

Least count of measuring tape = 0.001m

Tuning Forks

Frequency f [Hz]	$\lambda/4$ [m]	$3\lambda/4$ [m]	$5\lambda/4$ [m]
256	0.322	-	-
320	0.252	0.797	-
384	0.219	0.656	-
426.6	0.189	0.591	-
480	0.168	0.530	0.889
512	0.157	0.486	0.836

Table 4: The air column lengths that correspond to wavelengths of odd harmonics of the fundamental frequency due to the formation of standing waves in a closed tube

Frequency Generator

Least count of oscilloscope = 0.2Hz

Frequency f [Hz]	$\lambda/4$ [m]	$3\lambda/4$ [m]	$5\lambda/4$ [m]
257.8	0.302	0.918	-
300.2	0.266	0.802	-
349.6	0.210	0.689	-
400.0	0.182	0.603	-
452.4	0.157	0.533	0.898
500.0	0.139	0.480	0.802

Table 5: The air column lengths that correspond to wavelengths of odd harmonics of the fundamental frequency due to the formation of standing waves in a closed tube

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Data and Error Analysis

Melde's Electronic Vibrator

Transverse Waves

The tension in the string can be calculated using $T = Mg$ where $g = 9.81\ m/s^2$. Using the data from table 1, we can plot $1/p$ vs $\sqrt{T}$ since we expect this graph to be linear in accordance with equation 2.

$$\sqrt{T} = 2fL\sqrt{m}\left(\frac{1}{p}\right)$$

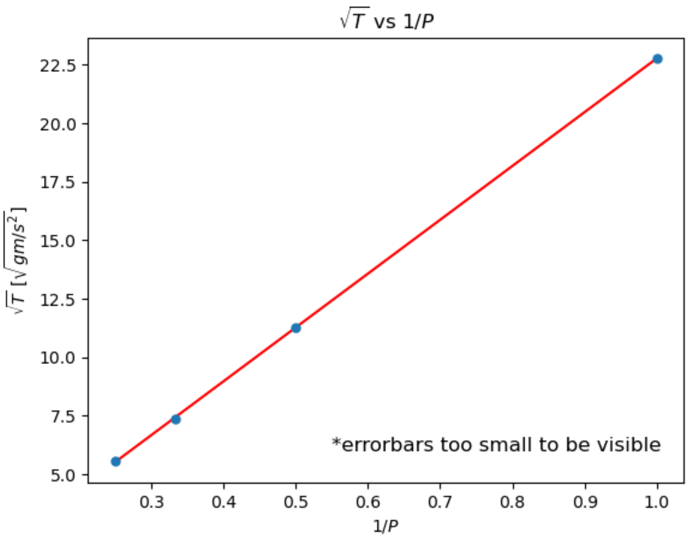

Figure 8: The relationship between $1/p$ and $\sqrt{T}$ is clearly linear in accordance with what we expect from theory

We can use the slope to calculate the frequency f of the AC power supply:

$$f = \frac{Slope}{2L\sqrt{m}} = 50Hz$$

Since the error in f comes from the slope, we can calculate it using:

$$\Delta f = \left(\frac{\Delta L}{L} + \frac{1}{2}\frac{\Delta m}{m}\right) f = 2Hz$$

Hence, the experimentally determined frequency of the AC supply is $f = 50 \pm 2$ Hz, which corresponds to our known value of 50 Hz.

Longitudinal Waves

Since only two standing waves could be found for longitudinal waves, the data cannot be plotted. Instead we can use simply calculate the values using equation 2.

No. of Loops p	Mass Suspended M [g]	Frequency f [Hz]
2	24.4043	48
3	11.0342	49

Table 6: Frequency of AC supply calculated for different masses which correspond to different amounts of tension in the string

These values also lie within an acceptable range of the expected value of 50 Hz.

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Sonometer

As in part 1, we can calculate the tension in the wire using $T = Mg$ where $g = 9.81\ m/s^2$. Using the data from table 3, we can plot L vs $\sqrt{T}$ since we expect this graph to be linear in accordance with equation 3.

$$\sqrt{T} = 4 f_{AC} \sqrt{m}(L)$$

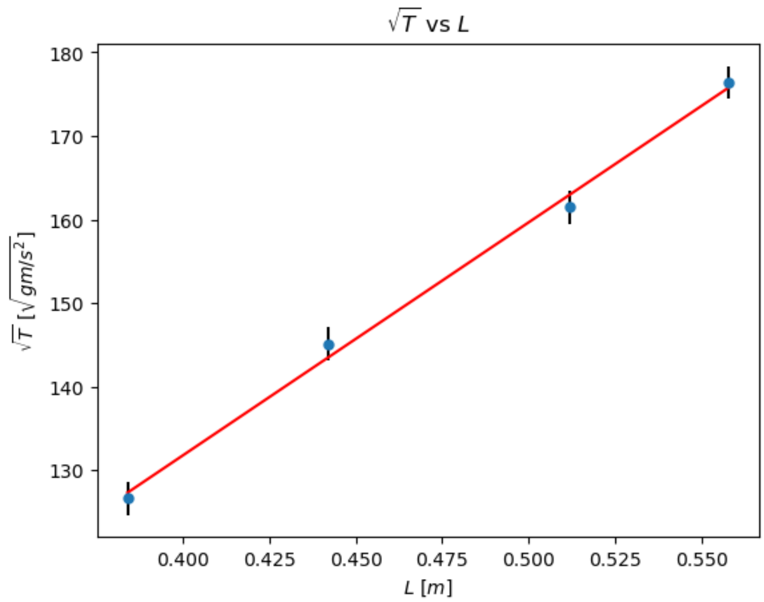

Figure 9: The relationship between L and $\sqrt{T}$ is clearly linear in accordance with what we expect from theory

We can use the slope to calculate the frequency f of the AC power supply:

$$f = \frac{Slope}{2\sqrt{m}} = 48Hz$$

Since the error in f comes from the slope, we can calculate it using:

$$\Delta f = \left(\frac{1}{2}\frac{\Delta m}{m}\right) f = 1Hz$$

Hence, the experimentally determined frequency of the AC supply is $f = 48 \pm 1$ Hz, which is close to the expected value of 50 Hz.

Resonance Tube

Tuning Forks

For each tuning fork (i.e. frequency) several lengths of the air column showed a resonant frequency response. These lengths correspond to $\lambda/4$, $3\lambda/4$, $5\lambda/4$, and so on of the source frequency. Use the harmonic air column lengths to find the same λ allowing us to plot several values for a single f. If we plot a graph of $T = 1/f$ vs λ, we expect a linear graph in accordance with equation 1:

$$v = f\lambda = \frac{\lambda}{T}$$

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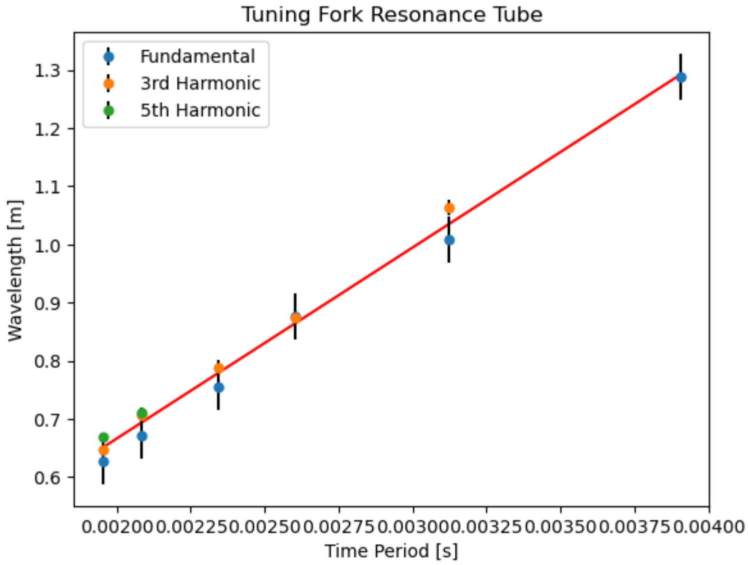

Figure 10: The relationship between T and λ is clearly linear in accordance with what we expect from theory

The slope of this graph gives us the speed of sound in air:

$$v = 328.07 m/s$$

The error in this value comes from the wavelength measurement, however as the errorbars show, the line of best fit lies within the acceptable margin of error.

Frequency Generator

The frequency generator produces a single frequency tone and removes that frequency from the output channel to detect the frequency response of the air column. Both the input and output channel were graphed on an oscilloscope and the maximum peak-to-peak amplitude of the frequency response was used to determine when resonance occurred. We can plot a similar graph of $T = 1/f$ vs λ for this data:

$$v = f\lambda = \frac{\lambda}{T}$$

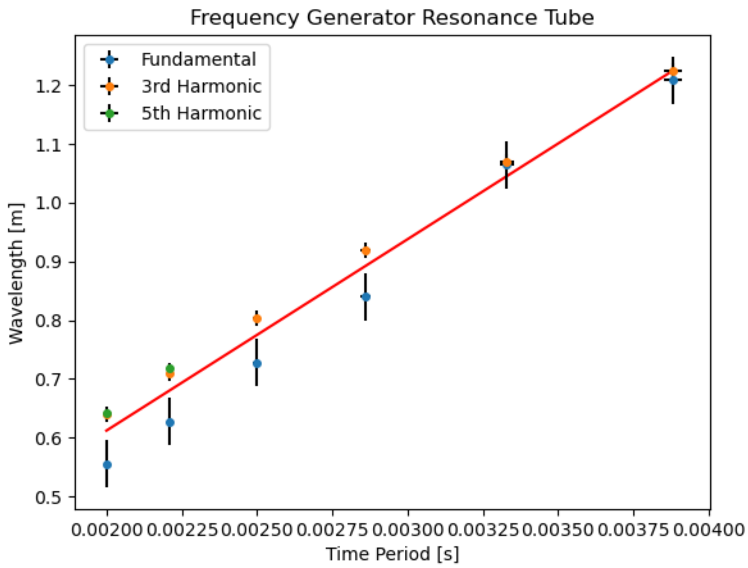

Figure 11: The relationship between T and λ is clearly linear in accordance with what we expect from theory

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The slope of this graph gives us the speed of sound in air:

$$v = 325.32 m/s$$

The error in this value comes from the wavelength measurement and the least count of the oscilloscope, however as the errorbars show, the line of best fit lies within the acceptable margin of error.

Discussion

Melde's Electronic Vibrator

The setup for the longitudinal waves is imperfect since the string is suspended over a movable pulley while allows the string to oscillate along its length, effectively making the pulley end an imperfect node for a standing wave. This means that energy is lost to the suspended length of string and the mass which may introduce unexpected errors in the experiment.

Sonometer

Every resonance condition may not correspond to the fundamental frequency of the length of wire between the two knife edges. It may be possible that the wire is vibrating at its third, fifth or higher harmonic. In order to determine which harmonic is resonating, the knife edges must be moved closer together to check for intermediate nodes. If there are no intermediate nodes, the resonant frequency is the fundamental.

Resonance Tube

It was noticed that a mild frequency response was detected at approximately half the length of $\lambda/4$ for certain frequencies. Furthermore, this frequency response was a distinctly different frequency from the source. The reason for this amplification is not entirely clear.

Results

Melde's Electronic Vibrator

The experimentally determined value of the AC power supply is 50 ± 2 Hz.

Sonometer

The experimentally determined value of the AC power supply is 48 ± 2 Hz.

Resonance Tube

The experimentally determined speed of sound in air found using tuning forks is 328.07 m/s.
The experimentally determined speed of sound in air found using a frequency generator is 325.32 m/s.