Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Electron Charge-to-Mass Ratio (e/m) Using the Thompson Method

Lab Report 2

Ayaan Dutt
Lab partner: Mansi Bisht

Professor Pramoda Kumar
TF: Bharti Chachra
TAs: Sanjana Gupta, Satwik Wats

Lab Supervisor: Sudarshana Banerjee
Lab Technician: Pradip Chaudhari

Date of Experiment: September 10th, 2024

Date of Submission: September 24th, 2024

Aim

- Use an e/m apparatus with a Helmholtz coil to find the charge to mass ratio of an electron.
- Use the Thompson Method to find the charge to mass ratio of an electron.

Theoretical Background

In 1897, J.J. Thompson measured the ratio of an electron's electric charge (e) to its mass (m) by observing the effect of mutually perpendicular magnetic and electric fields on the path of a beam of electrons. Thompson's method involved accelerating a beam of electrons through a known potential and then applying a magnetic field perpendicular to their path, causing the beam to deflect. By measuring the deflection of the beam and the strength of the electric and magnetic fields, he was able to deduce the charge-to-mass ratio of electrons. In this experiment, we used two different experimental setups that use the same principles as Thompson's method but operate differently to determine the e/m ratio.

Helmholtz Coil Apparatus

The Helmholtz coil apparatus consists of an e/m-tube which is a bulb-like structure that contains a filament, a cathode, an anode, a grid and a pair of deflection plates (fig 1). The filament heats the cathode, causing it to emit electrons which are then accelerated by a known potential difference applied between the cathode and anode. The grid is at a positive potential with respect to the cathode and at a negative potential with respect to the anode, helping the beam focus into a narrower stream. Both the grid and the anode have holes that allow electrons to pass through. The tube itself is filled with helium gas at low pressure, and as some electrons emitted by the cathode collide with helium atoms, the helium atoms are excited and emit visible light. This causes the electron beam to leave a visible trail in the tube, making it possible to observe any changes in the path of the beam.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

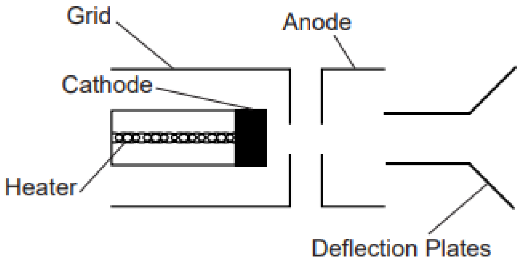

Figure 1: Schematic diagram of the e/m apparatus electron gun
Source: Instruction manual and experiment guide for the PASCO scientific model SE-9638

As the electrons are accelerated through a potential of V, they gain kinetic energy which is equal to their charge times the accelerating potential. Assuming that the speeds it reaches are non-relativistic, we find:

$$eV = \frac{1}{2}mv^2$$

$$\implies v = \sqrt{\frac{2eV}{m}} \tag{1}$$

The e/m tube is placed between a pair of fixed Helmholtz coils, which generate a uniform and known magnetic field. The socket holding the tube can be rotated so that the electron beam is perpendicular to the magnetic field. When electrons travel through a magnetic field with a velocity v, they experience the Lorentz force, given by $\vec{F_m} = q(\vec{v} \times \vec{B})$. Since the magnetic field and velocity vectors are perpendicular to each other in the e/m tube, the electrons experience a force of magnitude $F_m = evB$, which acts perpendicular to both the magnetic field and velocity. Since this force is perpendicular to the velocity vector, it causes a centripetal acceleration, making the electrons trace a circular path with radius r. Since centripetal force is given by mv^2/r and the magnetic force is the only force acting on the electrons, they can be equated:

$$evB = \frac{mv^2}{r}$$

$$\implies v = \frac{eBr}{m} \tag{2}$$

Substituting this in the expression of v which we derived in equation 1, we find:

$$\frac{eBr}{m} = \sqrt{\frac{2eV}{m}}$$

$$\implies \frac{e}{m} = \frac{2V}{B^2r^2} = \frac{8V}{B^2D^2} \tag{3}$$

Where $D = 2r$ (i.e. the diameter of the circle traced by the electron beam). The magnetic field produced between a pair of Helmholtz coils is given by:

$$B = \frac{\mu_0 NI}{(5/4)^{3/2}a} \tag{4}$$

Where μ_0 is the permeability of free space, N is the number of turns in the coil, I is the current passed through the coil and a is the radius of the coil. Upon substituting this into equation 3, we get:

$$\frac{e}{m} = \left(\frac{125a^2}{128\pi^2N^2} \times 10^{14}\right)\frac{V}{I^2D^2}$$

For our apparatus, $N = 140$ and $a = 13.5$ cm. Putting these values in, the e/m ratio is:

$$\frac{e}{m} = (9.2 \times 10^6)\frac{V}{I^2D^2} \tag{5}$$

Cathode Ray Tube (CRT) Apparatus

A cathode ray tube (CRT) consists of a similar mechanism as the e/m tube - it is a vacuum tube with a filament, a cathode, an anode, a focusing coil, a pair of deflection plates and a fluorescent screen. The filament is heated up and excite electrons from the cathode which are focused into a beam by the anode and focusing coil. A vertical electric field is then applied perpendicular to the beam by the two parallel deflection plates on either side. This causes the beam to experience a force in the vertical direction, deflecting the point where it falls on the fluorescent screen. This can be observed by the glow of the point on the screen where the beam makes contact. The force exerted by the electric field in the vertical direction is given by $F_e = eE$.

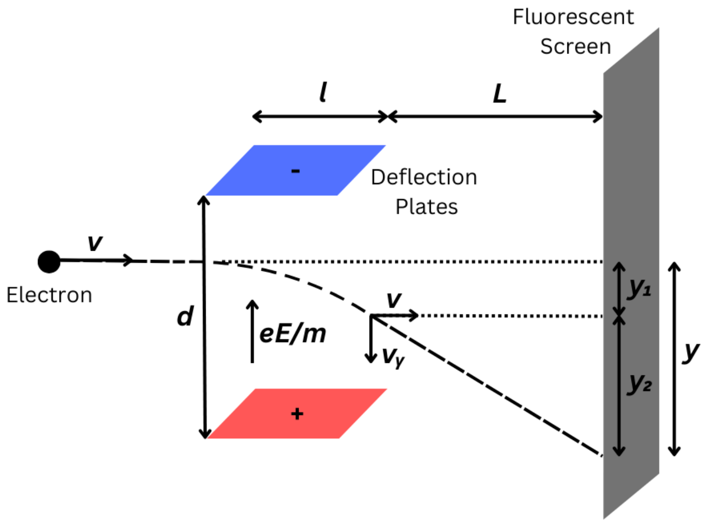

Figure 2: Schematic diagram of an electron being deflected by an electric field in a CRT
Source: made on www.canva.com

Let the length of the deflection plates be l and the distance between them be d (fig 2). Assuming classical mechanics, the electron is accelerated by a constant acceleration eE/m in the vertical direction while its horizontal velocity remains unchanged. The time spent by the particle between the plates is $t_1 = l/v$. Using Newton's laws of motion, the vertical distance travelled by the particle while being accelerated is:

$$s = ut + \frac{1}{2}at^2$$

$$\implies y_1 = \frac{1}{2}at_1^2 = \frac{1}{2}\frac{eE}{m}\frac{l^2}{v^2}$$

Once the electron leaves the space between the deflection plates, it has acquired a vertical velocity of $v_y = at_1 = eEl/mv$ which can be shown from $v = u + at$. This continues to deflect the beam vertically as the particle travels the distance between the screen and deflection plates L. The time spent travelling this path is $t_2 = L/v$, so the additional deflection of the electron beam is:

$$y_2 = v_y t_2 = \frac{eEl}{mv}\frac{L}{v}$$

Hence, the total vertical displacement of the electron being after being deflected by an electric field E is:

$$y = y_1 + y_2 = \frac{1}{2}\frac{eEl^2}{mv^2} + \frac{eElL}{mv^2}$$

$$\implies y = \frac{eEl}{mv^2}\left(\frac{l}{2} + L\right)$$

We assume that $L >> l/2$,

$$\implies y = \frac{eElL}{mv^2} \tag{6}$$

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

A pair of bar magnets are used to create a horizontal magnetic field that is also perpendicular to the electron beam, which causes it to deflect in the opposite direction due to the Lorentz force $F_b = evB$. The strength of the magnetic field can be tuned such that the magnetic force exactly cancels out the electric force, eliminating the deflection of the beam.

$$F_e = F_b$$
$$\implies eE = evB$$
$$\implies v = \frac{E}{B} \tag{7}$$

Substituting for v in equation 6 and rearranging to get the e/m ratio:

$$\frac{e}{m} = \frac{yE}{B^2 lL}$$

If we know the distance between the parallel deflection plates d and assume they produce a uniform electric field E by applying a potential V between them, we can say $E = V/d$. Putting this into the above equation,

$$\frac{e}{m} = \frac{Vy}{B^2 lLd} \tag{8}$$

In order to measure B, we use a compass placed between the bar magnets. If the bar magnets are oriented along the east-west direction, the magnetic field they produce will be perpendicular to the horizontal component of the Earth's magnetic field B_H (fig 3). Assuming the compass magnet has a magnetic moment of m, it will experience a torque of $\tau_H = mB_H 2l\cos\theta$ due to Earth's magnetic field and a torque of $\tau_B = mB2l\sin\theta$ due to the external magnetic field. At equilibrium, these torques will cancel out:

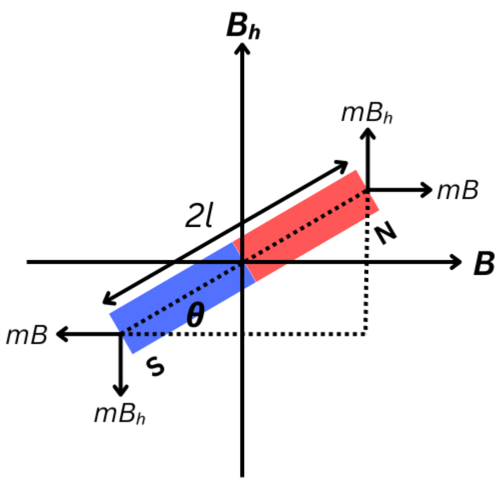

Figure 3: Compass suspended in an external magnetic field
perpendicular to the horizontal component of Earth's magnetic field
Source: Made on www.canva.com

$$\tau_2 = \tau_1$$
$$\implies mB2l\sin\theta = mB_H 2l\cos\theta$$
$$\implies B = B_H \tan\theta \tag{9}$$

Hence, by measuring the deflection of the compass θ, we can find the magnetic field produced by the bar magnets since B_H is a known constant value.

Ayaan Dutt
ayaan.dutt_ug25@ashoka.edu.in

Experimental Setup

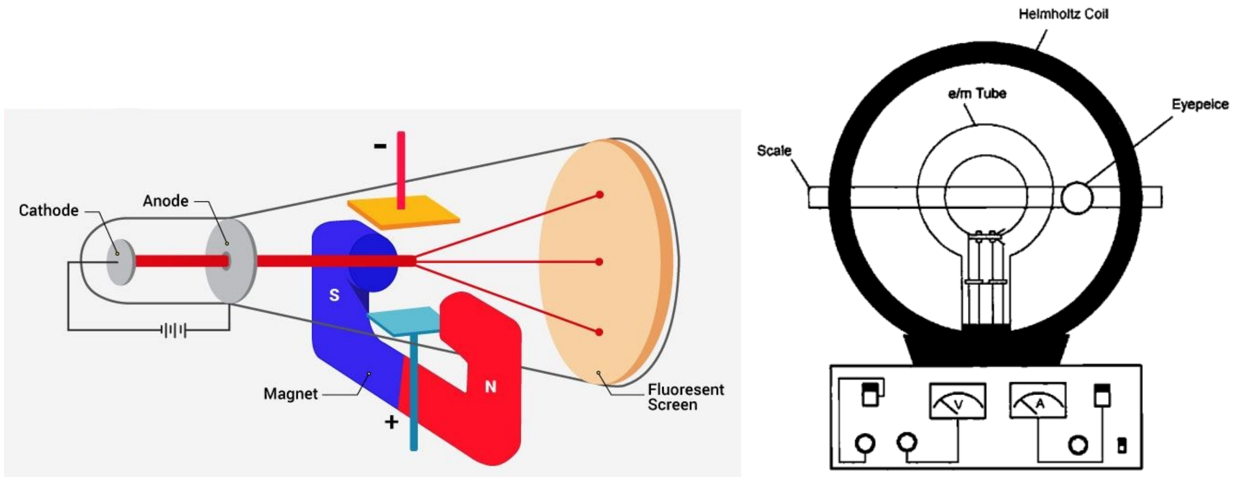

Figure 4: Experimental setup fot determining e/m ratio by Thompson's Method using a cathode ray tube (left) and a Helmholtz coil apparatus (right)
Source: (left) Charge to Mass Ratio, Byju's, (right) e/m Lab Handout, IIT-Roorkee

Instruments and materials used:

- Holmarc e/m Apparatus (Model: HO-ED-EM-01)
- Nvis 6103 Cathode Ray Tube (CRT)
- Deflection Magnetometer
- Compass
- Two Bar Magnets
- Metal Stand
- Power Supply with Variable Voltage

Least count of e/m apparatus accelerating voltage = 1 V
Least count of e/m apparatus magnetising current = 0.01 A
Least count of e/m apparatus eyepiece scale = 1 mm
Least count of CRT deflection voltage = 0.1 V
Least count of deflection magnetometer = 1 deg
Least count of metal stand scale = 1 mm
Least count of CRT screen = 1 mm

Specifications:

Number of turns in Helmholtz coils (N) = 140
Radius of Helmholtz coils (a) = 13.5 cm
Distance between deflection plates (d) = 1.4 cm
Length of deflection plates (l) = 3.15 cm
Distance between screen and plates (L) = 12 cm
Strength of Earth's magnetic field $(B_H) = 48410.8 \times 10^{-9}$ T

Procedure

Helmholtz Coil Apparatus

1. Make sure all the knobs are at their minimum before turning on the instrument. Turn the accelerating voltage to 200V and allow the filament to heat up for 5-10 minutes until a fine electron beam visibly emerges from the electron gun.

2. Keep the deflection voltage toggle switch is off and the current toggle switch is either on clockwise (CW) or counter-clockwise (CCW).

3. Apply 1.0 A of magnetising current and observe the direction in which electron beam bends. If the beam bends downwards, flip the direction of the current with the toggle switch. Adjust the current until the electron beam traces a complete circle.

4. Rotate the e/m tube such that the electron beam returns to the electron gun instead of spiralling in one direction.

5. Adjust the accelerating voltage and observe the radius of the diameter of the ring change. At regular intervals of voltage, measure the inner and outer diameter of the ring using the eyepiece and scale. Take their average to find the diameter d corresponding to each V. The slope of V vs d^2 was used to find the e/m ratio using equation 5.

CRT Apparatus

1. Using a compass, the metal stand was oriented along the east-west direction and cathode ray tube was placed in the gap with the screen facing north. The CRT was places such that the region of the body that contained the deflection plates coincided with the axis of the metal stand.

2. The CRT was attached to the power supply and the deflection voltage toggle switch was turned off. Using the focus, intensity and x-deflection control knobs, the fluorescent spot on the screen of the CRT was focused into a fine, sharp point at the center of the screen.

3. Turning the polarity of the deflection voltage to (+) and turning up the deflection voltage, the spot was displaced by 0.5 cm upwards on the screen. The corresponding deflection voltage V_1 was noted.

4. By placing a pair of bar magnets with opposite poles pointed towards each other on either side of the CRT on the metal stand, a uniform magnetic field was induced between the deflection plates. By adjusting the position of the magnets (keeping them equidistant from the CRT), the strength of the magnetic field was varied until the position of the beam returned to the origin.

5. The position of the magnets was noted as r_1 and r_2. The magnets were removed and the last two steps were repeated for negative $(-)$ polarity. The deflection voltage for negative polarity was recorded as V_2 and the position of the bar magnets was r_3 and r_4.

6. The CRT was removed and the magnets were placed at r_1 and r_2 with the appropriate polarity. The deflection magnetometer replaced the CRT in the middle of the stand the net deflection of the needle was recorded in degrees (θ_1 and θ_2). Similarly, the deflection angles θ_3 and θ_4 were recorded for r_3 and r_4.

7. Steps 3 to 6 were repeated for a deflection of 0.7, 1.0, 1.2, 1.5, 1.7 and 2.0 cm on the screen. Each set of values was used to find a value of the e/m ratio using equation 8.

Observations

Helmholtz Coil Apparatus

Least count of Helmholtz coil apparatus accelerating voltage = 1 V
Least count of Helmholtz coil apparatus magnetising current = 0.01 A
Least count of Helmholtz coil apparatus eyepiece scale = 1 mm

The Helmholtz coil apparatus only required taking readings for the inner and outer diameter of the electron beam ring for different accelerating voltages. The magnetising current was set at 1.0 A throughout the experiment to simplify calculations. The following ring diameters are the average of the inner and outer diameters of the electron beam rings:

$$D_i = \frac{ID_i + OD_i}{2}$$

Accelerating Voltage (V) [V]	Diameter 1 (D_1) [m]	Diameter 2 (D_2) [m]	Diameter 3 (D_3) [m]
140	0.0725	0.0725	0.0725
160	0.0790	0.0770	0.0810
180	0.0885	0.0860	0.0850
200	0.0915	0.0925	0.0930
220	0.0975	0.0970	0.0955
240	0.1010	0.1015	0.0995
260	0.1060	0.1050	0.1020

Table 1: The average electron beam ring diameters calculated from the mean of the inner and outer diameters for different accelerating voltages

CRT Apparatus

Least count of CRT deflection voltage = 0.1 V
Least count of deflection magnetometer = 1 deg
Least count of metal stand scale = 1 mm
Least count of CRT screen = 1 mm

The CRT method involves finding the deflection voltage (V), bar magnet positions (r) and deflection angles (θ) for each displacement (y) in both the positive and negative y-direction. During the experiment, we ensured that the magnets were always equidistant from the center, so $r_1 = r_2 = r_a$ and $r_3 = r_4 = r_b$. Since the needle of the deflection magnetometer is a straight line, the deflection $\theta_1 = \theta_2 = \theta_a$ and $\theta_3 = \theta_4 = \theta_b$. The average deflection voltage is $V = (V_1 + V_2)/2$ and the average angle of deviation is $\theta = (\theta_a + \theta_b)/2$.

Using equation 9, we have also tabulated the strength of the magnetic field corresponding to each deflection angle.

$$B = B_H \tan\theta$$

y [cm]	V [V]	r_a [cm]	r_b [cm]	θ_a [deg]	θ_b [deg]	θ [rad]	B [T]
0.5	140	9.50	9.50	50.0	36.0	0.75	4.51e-05
0.7	160	7.00	7.00	62.0	50.0	0.98	7.18e-05
1.0	180	4.50	4.50	75.0	63.0	1.20	1.26e-04
1.2	200	3.50	3.40	80.0	67.0	1.28	1.63e-04
1.5	220	2.20	2.10	84.0	72.0	1.36	2.28e-04
1.7	240	1.40	1.50	87.0	75.0	1.41	3.06e-04
2.0	260	0.600	0.700	89.0	77.0	1.45	3.94e-04

Table 2: The magnetic field B (calculated from the magnetometer deflection θ) required to cancel out the deflection y caused by voltage V

Analysis and Error Analysis

Helmholtz Coil Apparatus

The diameter of the electron beam ring was calculated as the average of the three data sets tabulated in table 1. The error in D was taken to be the standard deviation corresponding to each mean value.

$$D = \frac{1}{3}\sum_{i=1}^{3} D_i$$

$$\Delta D = \sqrt{\sum_{i=1}^{3}\frac{(D_i - D)^2}{3}}$$

Plotting the square of these values against the accelerating voltage V:

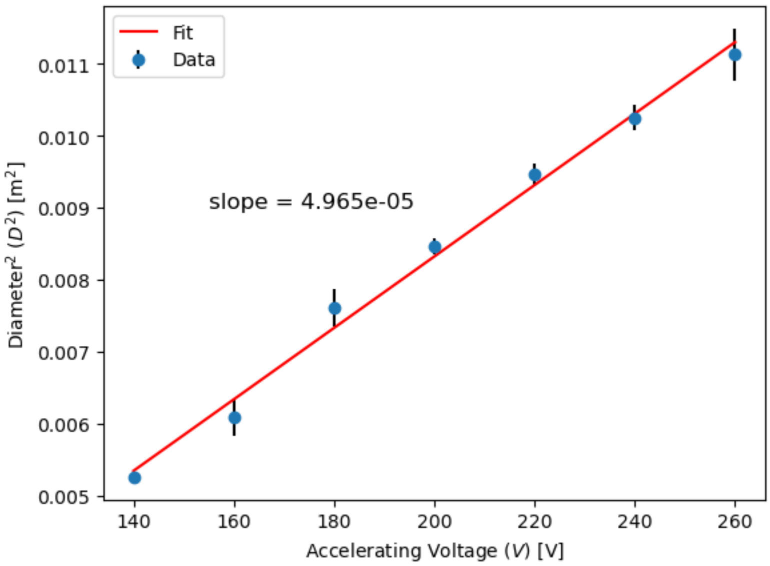

Figure 5: The square of the diameter varies linearly with respect to the accelerating voltage

$$\frac{e}{m} = (9.2 \times 10^6)\frac{V}{I^2 D^2}$$

This linear relationship is in accordance with what we expect from equation 5 since $I = 1$A was held constant throughout this experiment. The slope gives us a value of $V/d^2 = 4.965 \times 10^{-5}\ m^2/V$. Putting this into the above equation, we find the value of the e/m ratio:

$$\frac{e}{m} = 1.853 \times 10^{11}\ C/kg$$

The error in e/m can be found by quadrature:

$$\Delta\left(\frac{e}{m}\right) = \frac{e}{m}\sqrt{\left(\frac{\Delta slope}{slope}\right)^2 + \left(2\frac{\Delta I}{I}\right)^2}$$

The error in the slope was found from the regression fit using the diagonal elements of the covariance matrix to be $\Delta slope = 0.2 \times 10^{-5}\ m^2/V$. The error $\Delta I = 0.01$ A is simply the least count error of the instrument. Calculating these values, we find:

$$\Delta\left(\frac{e}{m}\right) = 0.08 \times 10^{11}\ C/kg$$

Hence, the experimentally determined value of e/m is:

$$\frac{e}{m} = (1.85 \pm 0.08) \times 10^{11}\ C/kg$$

This value has a percentage error of 5.18% compared to the known value of an electron's charge-to-mass ratio, which is within the acceptable margin of error.

CRT Apparatus

The deflection angles were averaged and put into equation 9 to determine the magnetic field corresponding to a certain separation between the bar magnets. This value of B has been tabulated in table 2. The constants in equation 8 are listed below:

$$\frac{e}{m} = \frac{Vy}{B^2 lLd}$$

Distance between deflection plates (d) = 1.4 cm
Length of deflection plates (l) = 3.15 cm
Distance between screen and plates (L) = 12 cm

This gives us a series of e/m values, one for each value of deflection.

y [cm]	V [V]	B [T]	e/m [C/kg]	Error [C/kg]	% Error [%]
0.5	140	4.51e-05	3.11e+11	0.677e+11	77.5
0.7	160	7.18e-05	2.49e+11	0.456e+11	42.3
1.0	180	1.26e-04	1.63e+11	0.402e+11	6.99
1.2	200	1.63e-04	1.41e+11	0.488e+11	19.7
1.5	220	2.28e-04	1.11e+11	0.664e+11	36.5
1.7	240	3.06e-04	8.05e+10	0.813e+11	54.0
2.0	260	3.94e-04	6.58e+10	1.07e+11	62.4

Table 3: The e/m values and their percentage errors determined from the CRT Thompson method

The error in e/m can once again be calculated by finding the quadrature error. Note that l, L and d are considered to be known constants:

$$\Delta\left(\frac{e}{m}\right) = \frac{e}{m}\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta y}{y}\right)^2 + \left(2\frac{\Delta B}{B}\right)^2}$$

The error in V and y are simply the least count errors associated with their measurements. However, B has not been measured directly. Instead, it if found from the deflection in the magnetometer θ which has its own error which propagates into B. The propagation of error in B is given by:

$$\frac{\Delta B}{B} = \frac{\Delta \tan\theta}{\tan\theta} = \frac{\partial \tan\theta}{\partial\theta}\frac{\Delta\theta}{\theta} = \sec^2\theta\frac{\Delta\theta}{\theta}$$

Substituting this back into the above equation, we get:

$$\Delta\left(\frac{e}{m}\right) = \frac{e}{m}\sqrt{\left(\frac{\Delta V}{V}\right)^2 + \left(\frac{\Delta y}{y}\right)^2 + \left(2\sec^2\theta\frac{\Delta\theta}{\theta}\right)^2}$$

Plugging in the appropriate values, we can determine the series of errors given in table 3. The best value of e/m we found was:

$$\frac{e}{m} = (1.6 \pm 0.4) \times 10^{11}\ C/kg$$

The known value of $e/m = 1.758 \times 10^{11}\ C/kg$ lies within the quadrature error of most of the experimentally determined results and has a percentage error of only 6.99% for the best value.

Results

The aim of this experiment was to determine the value of the charge-to-mass ratio (e/m) of an electron using Thompson's method. Two different experimental setups were used - the Helmholtz coil apparatus and the cathode ray tube (CRT) apparatus. Both instruments use the influence of known electric and magnetic fields to observe changes in a beam of electrons and determine their charge-to-mass ratio.

From the Helmholtz coil apparatus, we determined the following value of e/m:

$$\frac{e}{m} = (1.85 \pm 0.08) \times 10^{11}\ C/kg$$

This value has a percentage error of only 5.18% compared to the known value.

The CRT apparatus gave us a series of e/m values corresponding to different deflection voltages. The best value we determined was:

$$\frac{e}{m} = (1.6 \pm 0.4) \times 10^{11}\ C/kg$$

This value has a percentage error of 6.99% compared to the known value.

Discussion

The Helmholtz coil apparatus is prone to significant error due to parallax and the diffused nature of the electron beam. The primary measurement in this experiment is the diameter of the electron beam ring which is found by aligning an eyepiece with cross-hairs at the edge of the circle and measuring its horizontal position. However, the vertical position of the eyepiece has to be estimated since there is no way to determine the exact center of the loop. It is not possible to verify if the eyepiece is aligned along the diameter. Furthermore, the electron beam forms an indistinct edge, requiring human judgement (and therefore human error) while determining the edge of the ring. This human error can be minimized by taking multiple sets of data, but since there is no external reference, any systematic errors cannot be removed. These are additional sources of error in D that have not been accounted for in our analysis.

The velocity of the electrons also play a critical role in determining the value of e/m. The hole in the anode results in a non-uniform electric field that accelerates the electrons to a velocity that is slightly lower than the theoretically expected value. This applies to both the Helmholtz and CRT apparatus. Additionally, in the Helmholtz apparatus, collisions with helium atoms within the e/m tube may further reduce the electrons velocity. Since the equation for e/m is inversely proportional to D^2, and D is directly proportional to the velocity v, the experimental values for e/m is likely to be substantially impacted by these two factors.

In the CRT apparatus, the position of the deflected point on the fluorescent screen is subject to parallax error. This was resolved by using a reference point on the body of the CRT to orient one's line of sight before taking a measurement. In our case, we aligned the top of the y-axis with a small screw which was on the central axis of the CRT before taking any measurements.

During the experiment we noticed a critical piece of information was not emphasised in the experiment - the position of the CRT with respect to the metal stand has a significant impact on the results of the experiment. This is because the magnetic field of the bar magnets must act in the same region as the length of the deflection plates inside of the CRT. If the magnetic field acts before or after the electric field, it may still be possible to reverse the deflection on the screen, but the path travelled by the electron beam will be far more complicated and cannot be explained by our given theory. For instance, if the electron beam does not enter the electric field horizontally, its vertical velocity will altered when exiting the field since the initial velocity had a vertical component. This will in turn have an effect on the amount of deflection observed and therefore the strength of magnetic field required to correct it. Hence, it is advised that the position of the deflection plates be externally marked on the CRT for future iterations of the experiment to allow for easy alignment with the magnetic field.

References

[1] e/m by Thomson Method, Ashoka University Lab Handout, https://drive.google.com/file/d/10ZiKb305Bkc24dZoyzj0fYk0f2f-Y35W/view

[2] Instruction Manual and Experiment Guide for the PASCO scientific Model SE-9638, e/m Apparatus, PASCO Scientific, https://www.nhn.ou.edu/~johnson/Education/Juniorlab/e_Over_m/Pasco9638.pdf

[3] Charge to Mass Ratio of Electrons, BYJU'S, https://byjus.com/chemistry/charge-to-mass-ratio/

[4] The Discovery of the Electron: The J.J. Thompson Experiment, Embry-Riddle Aeronautical University, https://physicsx.erau.edu/Courses/CoursesS2024/PS315/Charge-to-Mass/Classic_eoverm.pdf

[5] Electron Charge-to-Mass Ratio, Physik-Institut, University of Zurich, https://www.physik.uzh.ch/~matthias/espace-assistant/manuals/en/anleitung_etom_e.pdf

[6] Determining e/m by Thompson Method, IIT-Roorkee Lab Handout, https://iitr.ac.in/Academics/static/Department/Physics/PHC-114/2._ebym_thomson.pdf

[7] Nvis 6103: e/m by Thompson Method, Nvis Technologies Pvt. Ltd., https://www.nvistech.com/wp-content/uploads/2023/12/Nvis6103.pdf