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Zeeman Effect

Lab Report 1

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Aim

- Observe the Zeeman Effect for a mercury vapour lamp
- Calibrate the magnetic field
- Find the spacing of the Fabry-Perot etalon
- Determine the value of the Bohr Magneton (μ_0)

Theoretical Background

When a strong magnetic field is applied to a mercury vapour lamp, certain transitions are excited in the atoms which are observed as the Zeeman effect. The energy splitting can be measured by filtering all the wavelengths except 546.1nm light and passing it through a Fabry-Perot Etalon to observing the interference pattern produced. Energy splitting in the 6^3P_2 to 7^3S_1 transition leads to the formation of 3 lines - a central π-line which is polarised parallel to the direction of the magnetic field and two σ-lines which are polarised perpendicular to the magnetic field. Using a polarisation filter it is possible to omit the central line and observe the splitting of the spectral lines.

The Fabry-Perot Etalon consists of two parallel flat glass plates coated on the inner surface with a partially transmitting metallic layer. Let the separation between the surfaces be t. An incoming ray at angle θ will undergo multiple reflections, producing several parallel rays with a path difference of $\delta = 2t\cos\theta$ (fig 1). For constructive interference to occur, the path difference must be:

$$n\lambda = 2t\cos\theta \tag{1}$$

From this we find that $n = (2t\cos\theta)/\lambda$ and the central fringe will form at $n_0 = 2t/\lambda$. Note that the central fringe need not be a complete maxima or minima and therefore n_0 is not necessarily an integer. Putting n_0 back into equation 1, we can find θ_n:

$$\theta_n = \sqrt{\frac{2(n_0 - n)}{n_0}} \tag{2}$$

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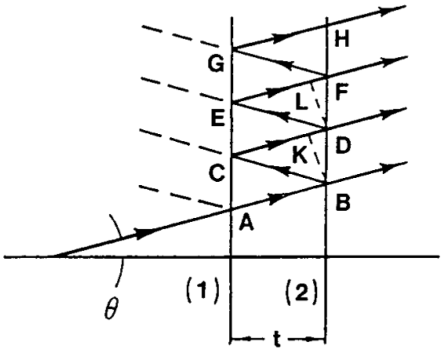

Figure 1: A schematic diagram of a ray passing through a Fabry-Perot Etalon
Source: Zeeman Effect, PHYWE series of publications

Using a lens of focal length f to focus the parallel rays (fig 2), we can bring the maxima into focus as a ring. The radius of this ring will be $r_n = f \tan\theta_n \approx f\theta_n$ for small values of θ_n. Using equation 2 to substitute θ_n, we get:

$$r_n^2 = \frac{2f^2}{n_0}(n_0 - n) \tag{3}$$

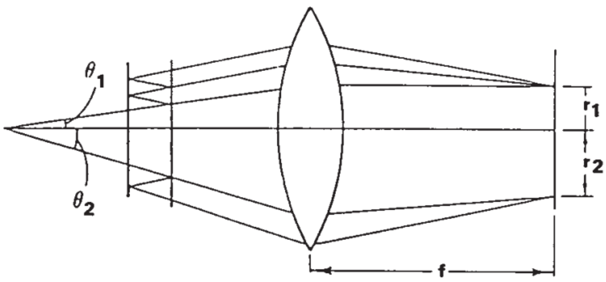

Figure 2: A lens focuses parallel beams from the Etalon into a ring on the focal plane
Source: Zeeman Effect, PHYWE series of publications

If n_1 is the order of the interference of the first maxima, let $n_1 - n_0 = \epsilon$ where ϵ is some number between 0 and 1 (since n_0 may not be an integer). Thus, the pth ring will form at $n_p = n_0 - \epsilon - (p-1)$, or $n_0 - n_p = p - 1 + \epsilon$. Substituting this into equation 3 and taking the difference of two consecutive radii, we find:

$$r_{n+1}^2 - r_n^2 = \frac{2f^2}{n_0} \tag{4}$$

Since $n_0 = 2t/\lambda$, the Etalon spacing must be:

$$t = \frac{f^2\lambda}{r_{n+1}^2 - r_n^2} \tag{5}$$

If a magnetic field is applied to the sample, the spectral line splits into two more components. Let us assume that the central ring is blocked out by the polariser so we only observe two rings for each order. Let the wavelengths of these components be λ_a and λ_b with corresponding fractional orders ϵ_a and ϵ_b:

$$\epsilon_a = \frac{2t}{\lambda_a} - n_{1,a} = 2tk_a - n_{1,a}$$
$$\epsilon_b = \frac{2t}{\lambda_b} - n_{1,b} = 2tk_b - n_{1,b}$$

Since the wavelengths are very close, we can assume $n_{1,a} = n_{1,b}$. Hence, we can show that the difference in the wave numbers of the two components is:

$$\Delta k = k_a - k_b = \frac{\epsilon_a - \epsilon_b}{2t} \tag{6}$$

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From equation 4 and the fact that $n_0 - n_p = p - 1 - \epsilon$, we can show that:

$$\epsilon_a = \frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - p$$

$$\epsilon_b = \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2} - p$$

Putting these into equation 6, we get the difference in wave number as:

$$\Delta k = \frac{1}{2t}\left(\frac{r_{p+1,a}^2}{r_{p+1,a}^2 - r_{p,a}^2} - \frac{r_{p+1,b}^2}{r_{p+1,b}^2 - r_{p,b}^2}\right) \tag{7}$$

Let the difference between the squares of consecutive radii be denoted as Δ, so:

$$\Delta_a^{p+1,p} = r_{p+1,a}^2 - r_{p,a}^2 = \frac{2f^2}{n_{0,a}}$$

$$\Delta_b^{p+1,p} = r_{p+1,b}^2 - r_{p,b}^2 = \frac{2f^2}{n_{0,b}}$$

This difference is nearly equal so we may assume that $\Delta_a^{p+1,p} = \Delta_b^{p+1,p}$ regardless of the order p. We may take the average of these values to experimentally determine Δ. Similarly, let the difference of the radii of the two components of the same order be $\delta_{a,b}^{p+1,p} = r_{p+1,a}^2 - r_{p+1,b}^2$. This can also be considered to be the same for all p and their average can be taken to determine δ. This leaves us with the formula:

$$\Delta k = \frac{1}{2t}\frac{\delta}{\Delta} \tag{8}$$

Note that this equation is independent of the actual length of the radii or amplification of the interference plot since it only depends on the ratio δ/Δ.

As the transition from 6^3P_2 to 7^3S_1 occurs, there is a shift in the frequencies of the various components when a magnetic field is applied (fig 3). The frequencies of the π-lines are:

$$\nu_0 - \frac{\mu_0 B}{2h},\ \nu_0,\ \nu_0 + \frac{\mu_0 B}{2h}$$

Where μ_0 is Bohr's Magneton, h is Planck's constant and B is the strength of the magnetic field. Hence, the shift in frequency due to Zeeman splitting is:

$$\Delta\nu = \mu_0 B/2h \tag{9}$$

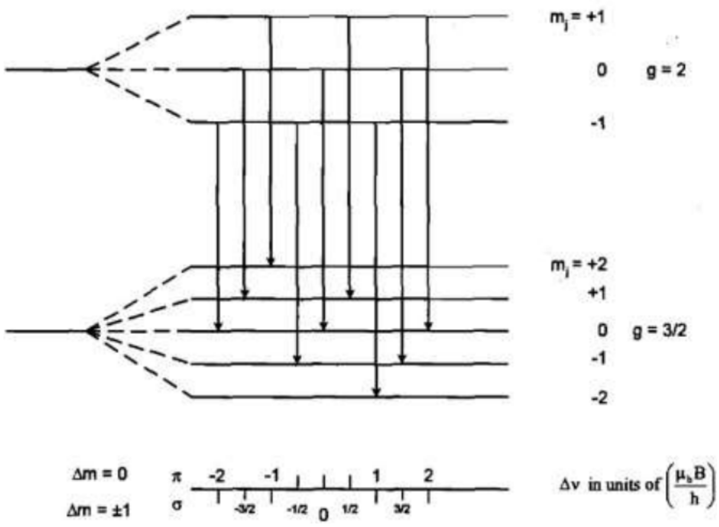

Figure 3: Energy transition diagram for 6^3P_2 to 7^3S_1 in a mercury vapour lamp
Source: Zeeman Effect Experiment, IIT Roorkee Lab Handout

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Hence, the shift in wave number is equal to $\Delta k = \Delta\nu/c = \mu_0 B/2hc$. Subsituting this into equation 8, we find:

$$\frac{\mu_0 B}{2hc} = \frac{1}{2t}\frac{\delta}{\Delta}$$

Therefore, Bohr's Magneton is given by:

$$\mu_0 = \frac{hc}{Bt}\frac{\delta}{\Delta} \tag{10}$$

Experimental Setup

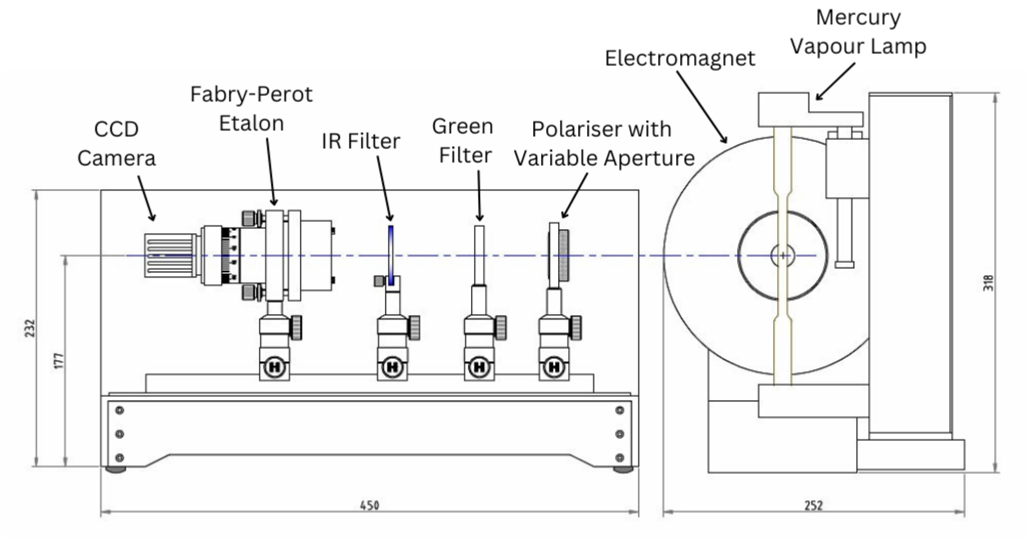

Figure 4: Experimental Setup for Zeeman Effect
Source: Zeeman Effect Apparatus, Holmarc Website

Components of apparatus:

- Rail-based platform
- IR filter
- Green filter
- Polariser with variable aperture
- Fabry-Perot etalon
- Zoom lens assembly
- CCD camera
- Electromagnet with variable power supply
- Digital Gauss meter
- Hall probe
- Laser
- Mercury lamp
- Laptop with Holmarc camera application
- Gloves

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Resolution of camera = 320×240 pixels
Focal length of etalon lens (f) = 65 mm
Least count of electromagnet power supply = 0.01A
Least count of gauss meter = 0.0001 T

Procedure

Calibration of the Electromagnet

1. Attach the hall probe to the Gauss meter and mount it in the air gap between the pole pieces of the electromagnet such that the plane of the probe is perpendicular to the field.
2. Turn on the Gauss meter and note down the residual magnetic field detected while the electromagnet power supply is off.
3. Turn on the electromagnet power supply and change the current from 0A to 2A and note down the Gauss meter reading at regular intervals. Plot the current I vs the magnetic field B to determine the calibration formula. Use the zero error as the intercept of the linear fit.

Zeeman Effect

1. Place the rail-based platform such that its axis is perpendicular to the magnetic field of the electromagnet. Mount a laser behind the electromagnet and align its beam along the axis of the platform and parallel to its surface.
2. Adding the components one by one to the platform and ensuring that the laser beam is perpendicular to each component's surface, mount the green filter, IR filter and polariser with variable aperture in the given order. The perpendicularity of the components can be verified by adjusting their angle until the beam is reflected back into the source.
3. After the polariser, mount the Fabry-Perot etalon with the zoom lens assembly facing the screen. Adjust the mirrors until the beam is focussed into a single point on the screen. Do NOT attach the camera to the etalon while the laser is on as the beam may damage the camera's sensor.
4. Replace the laser with a mercury vapour lamp ensuring that the discharge tube is placed in the air gap between the pole pieces of the electromagnet, perpendicular to the field. Be gentle with the discharge tube as it is very delicate. Once it is in position, switch on the lamp but keep the electromagnet off.
5. Connect the camera to the laptop with the Holmarc camera application and attach the camera to the etalon. Upon running the software, it should be possible to view the image formed by the filtered light passing through the etalon.
6. Adjust the screws on the etalon until a clear circular interference pattern comes into view. If there is any asymmetry of light intensity, move the components along the rail and fine-tune their angle until the image is symmetric, with distinct well-defined fringes.
7. Use the contrast and exposure sliders in the camera application along with the variable aperture of the polariser to find the best balance between the brightness and sharpness of the fringes.
8. Switch on the electromagnet (ensuring that the power supply is initially at 0A) and slowly increase the magnetic field until the fringes begin to split into three distinct lines corresponding to each order.
9. Adjust the angle of the polariser until the central fringe disappears (this happens because the central line is polarized perdendicular to the split spectral lines). It should now be possible to observe a single central fringe (at 0A) splitting into 2 distinct fringes that gradually move apart as the magnetic field intensity is increased. Do not run the electromagnet for more than 2 minutes to prevent permanent residual magnetism from building up in the coils.

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10. Take pictures of the interference pattern at regular intervals of current supplied to the electromagnet. In Image-J software, create a stack of these images and draw a radial line passing through the center of the pattern. Using the "plot profile" tool, examine the line intensity profile of each image and use the peaks to measure the radii of each fringe.

11. Using equations 5 and 10, determine the spacing of the Fabry-Perot etalon and estimate a value for Bohr's Magneton. Use the result from the calibration to find B from I.

Observations

Calibration of the Electromagnet

Least count of electromagnet power supply = 0.01 A
Least count of gauss meter = 0.0001 T

We noted that when the electromagnet was switched off, the hall probe detected a residual magnetic field of -0.079 T from the electromagnet. This was later used to determine the intercept of the calibration curve.

We measured the magnetic field at intervals of 0.1A in a range from 0-2A. Five sets of data were taken and their mean and standard deviation was used to plot a graph. The mean values of magnetic field strength (in Tesla) are tabulated below (table 1).

Current I [A]	Magnetic Field B [T]
0.0	-0.0454
0.1	-0.0210
0.2	0.0038
0.3	0.0314
0.4	0.0580
0.5	0.0854
0.6	0.1166
0.7	0.1444
0.8	0.1746
0.9	0.2024
1.0	0.2308
1.1	0.2614
1.2	0.2896
1.3	0.3196
1.4	0.3498
1.5	0.3790
1.6	0.4086
1.7	0.4368
1.8	0.4652
1.9	0.4934
2.0	0.5220

Table 1: Magnetic field produced by the electromagnet corresponding to increasing input currents

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Zeeman Effect

Images of the interference pattern were captured on the Holmarc camera application. The resolution of camera was set to 320×240. The pixel to length conversion formula given in the handout was 1 pixel = 2.8 microns for a resolution of 1280×1024. Assuming a linear conversion, we estimated the conversion formula for our resolution to be:

$$1 \text{ pixel} = \frac{\sqrt{1280 \times 1024}}{\sqrt{320 \times 240}} \times 2.8 = 11.56 \text{ microns} \tag{11}$$

Images were taken at intervals of 0.1 A of the electromagnet power suppy and analysed in the Image-J software using the "plot profile" tool. Each set of images was imported as a stack and a radial line passing through the center of the interference pattern was drawn. The line was chosen based on which region of the image displayed the clearest splitting of fringes. A line intensity profile was plotted for each image (fig 5).

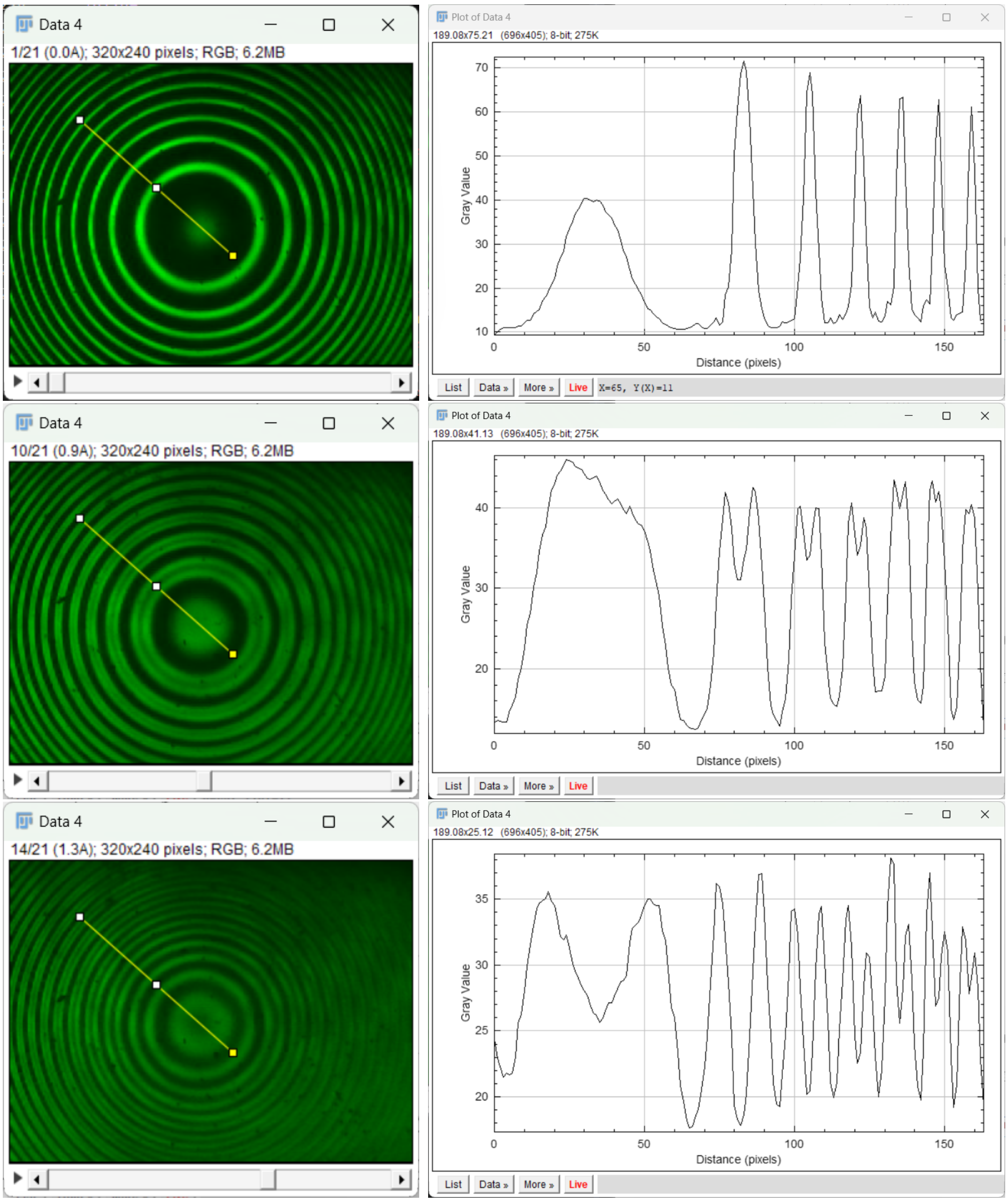

Figure 5: The line intensity profile for the interference patterns when $I = 0.0, 0.9, 1.3$ A. The fringes of the interference pattern begin to split as the magnetic field is increased which can be measured by the positions of the intensity peaks on the right

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The position of the peaks was determined using the "argrelextrema" function from the "scipy" package in Python which finds the local maxima for any dataset. The values were found (and manually verified) in pixels before being converted to meters using equation 11. The radii of the various finges were found by taking the necessary differences to determine δ and Δ, as well as the etalon spacing t.

Analysis and Error Analysis

Calibration of the Electromagnet

Since the zero error of the electromagnet (i.e. the residual magnetism) was found to be -0.079 T, this was fixed as the intercept for the linear fit of the data in table 1. Hence, an equation of the form $y = ax - 0.079$ was fitted to the data. The data showed a strong linear trend with minimal error.

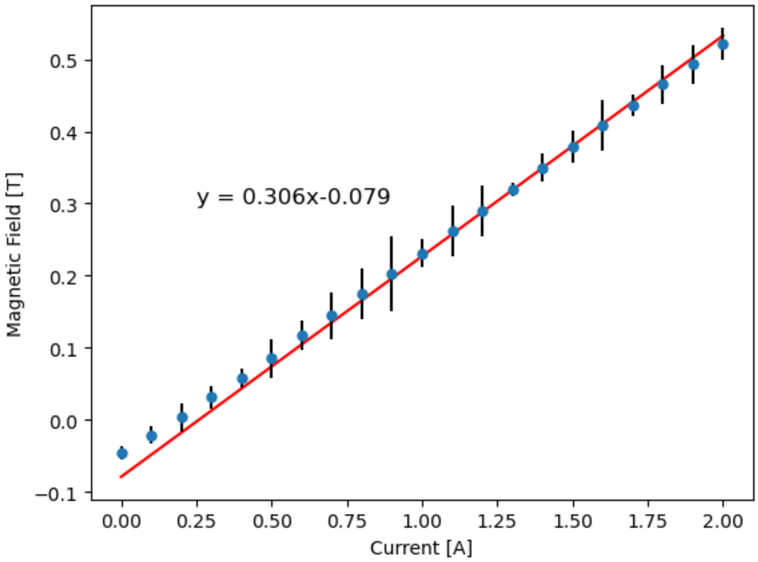

Figure 6: The slope and intercept of the graph describes how the magnetic field varies with current in the given electromagnet.
Note: The errorbars are magnified by ×20 to display the variance in the data.

The graph is plotted for the mean values of the magnetic field from 5 sets of data and the errorbars are equal to the standard deviation corresponding to each value of I. In fig 6 the errorbars have been magnified by ×20 to display the variance of the data. The calibration equation is therefore:

$$B = 0.306I - 0.079 \tag{12}$$

Zeeman Effect

The spacing of the Fabry-Perot etalon is given by equation 5, seen again below:

$$t = \frac{f^2\lambda}{r_{n+1}^2 - r_n^2}$$

The value of f was given to be 65 mm according to the Holmarc handout and the wavelength of light filtered by the green filter is 546.1 nm. The radii in this equation are measured when $B = 0$.

Since t is the same regardless of which pairs of radii we take, we chose 13 consecutive radii for each data set, which gave us 12 values of t. We had three data sets, so we were able to compute 36 total values of t, which were plotted as a histogram to determine the mean value (fig 7).

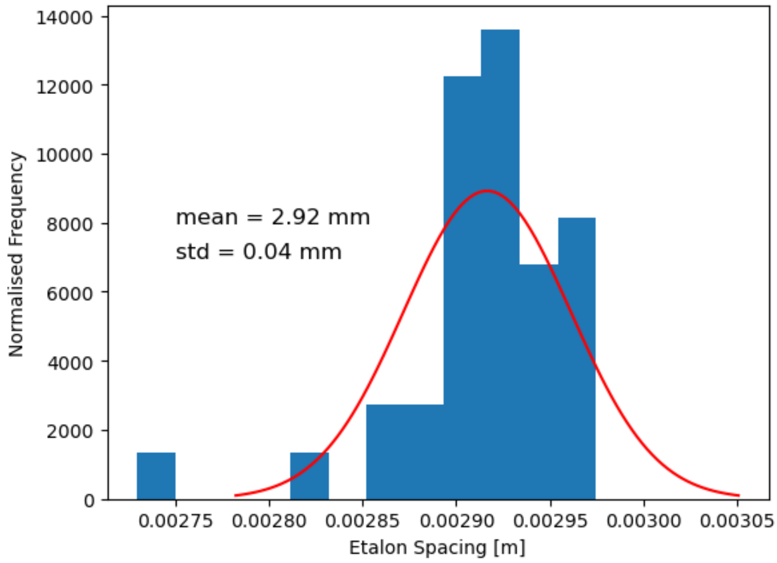

Figure 7: Assuming a normal distribution, we calculated the mean and standard deviation of the etalon spacing values

Hence, assuming the error is equal to one standard deviation, we find:

$$t = 2.92 \pm 0.04 \text{ mm}$$

We also calculated the values of δ and Δ using the equations $\delta_{a,b}^{p+1,p} = r_{p+1,a}^2 - r_{p+1,b}^2$ and $\Delta^{p+1,p} = r_{p+1}^2 - r_p^2$ for every pair of radii. Since we showed that both values of delta are independent of p, we averaged the values corresponding to each pattern. Hence, we were left with a mean value of δ/Δ corresponding to each value of current applied to the electromagnet. Using the calibration equation (equation 12), we converted the input current to magnetic field B, and plotted it against δ/Δ. The standard deviation of δ/Δ from the three data sets was used as the error.

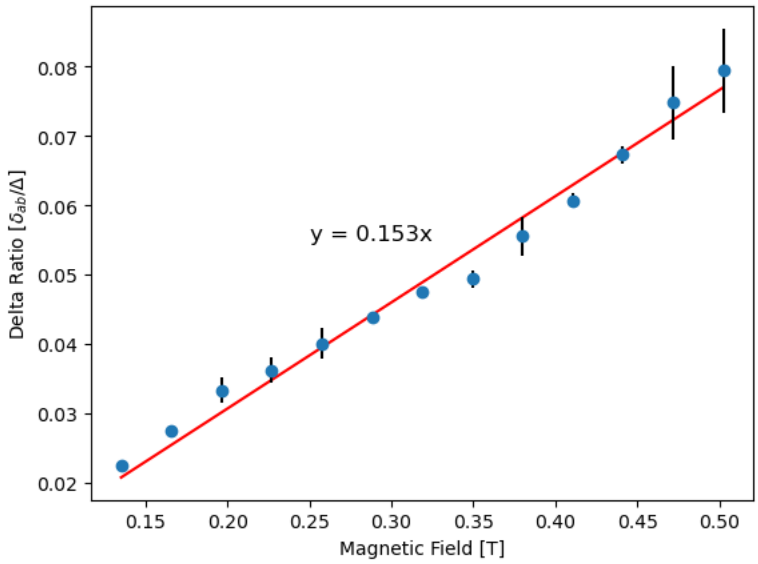

Figure 8: The delta ratio varies linearly with the magnetic field and we expect it to be zero when $B = 0$ since no splitting will be observed. Hence, the intercept was set as 0.

Using equation 10, we can calculate the value of Bohr's Magneton from the slope of this graph. Using $h = 6.626 \times 10^{-34}$ Js and $c = 3 \times 10^8$ m/s, we calculated the value of Bohr's Magneton to be 10.4×10^{-24} J/T. The standard deviation of the error in the slope (found from the scipy curvefit

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function) gave us an error of 0.1×10^{-24} J/T. Hence, the experimentally detemined value of Bohr's Magneton is:

$$\mu_0 = (10.4 \pm 0.1) \times 10^{-24} \mathrm{J/T}$$

The known value of Bohr's Magneton is 9.27×10^{-24} J/T, which means our value has a percentage error of 12.2%.

Results

The electromagnet used produces a magnetic field that linearly changes with the current supplied to it. The calibration equation is:

$$B = 0.306I - 0.079$$

The Fabry-Perot etalon was experimentally found to have a spacing of 2.92 ± 0.04 mm based on the spacing of the fringes in the interference pattern observed through it.

As a magnetic field was applied to the mercury discharge tube, the fringes in the interference pattern split into two more fringes which is known as the Zeeman effect. The slight shifts in wavelength due to the different energies of the spectral lines forms two new interference patterns that are superimposed on each other. By measuring the difference between the fringes, we were able to determine the value of Bohr's Magneton.

$$\mu_0 = (10.4 \pm 0.1) \times 10^{-24} \mathrm{J/T}$$

This value has a 12.2% error and therefore lies within a reasonable margin of error of the known value.

Discussion

We assumed a linear conversion for pixels to microns however this assumption requires greater discussion to determine the true conversion formula. Resolution describes a 2-dimensional space whereas we are finding a 1-dimensional length scale from it. Hence, the length per pixel may scale non-linearly with resolution. This conversion is critical for finding the etalon spacing since the actual spacing of the fringes is used in the calculation. However, the Bohr's Magneton calculation is independent of this scaling (assuming t is known) since the ratio δ/Δ is dimensionless.

We also used the known value of $f = 65$ mm based on the focal length of the etalon given in the apparatus description, however we also adjusted the focus on the camera itself. This would alter the focal plane on which the image is formed, thereby changing the distance between the etalon and "screen". However, this adjustment could not be accounted for in our analysis.

References

[1] Zeeman Effect, PHYWE series of publications, `https://www.nikhef.nl/~h73/kn1c/praktikum/phywe/LEP/Experim/5_1_10.pdf`

[2] Zeeman Effect Experiment, SES Instruments Pvt. Ltd., IIT Roorkee Lab Handout, `https://iitr.ac.in/Academics/static/Department/Physics/CMP%20laboratory/Zeeman_effect.pdf`

[3] Zeeman Effect Handout, Physics Lab 4 Handout, Ashoka University

[4] Zeeman Effect Apparatus (Model HO-ED-S-04A), Holmarc Opto-Mechatronics Ltd., `https://www.holmarc.com/zeeman_effect_apparatus.php`