Hall Effect

Lab Report 6

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Abstract

In this experiment we use a Hall probe and electromagnet to determine the relationship between Hall voltage and probe current, magnetic field and temperature. This setup demonstrates how current behaves in a semiconductor in the presence of a perpendicular magnetic field. The probe current and magnetic field are varied by changing the current supplied to the Hall probe and electromagnet, respectively. The Hall voltage also implicitly depends on temperature, which is investigated by supplying a heater current to a filament that raises the temperature of the p-type Germanium semiconductor.

Introduction

André-Marie Ampère made preliminary observations that led to the discovery of the Hall effect in the 1820s. However, it was only after the field of electromagnetism was formalized by James Clerk Maxwell that the Hall effect could be explained via the Lorentz force. In 1879, Edwin Hall observed that when an electrical current passes through a conductor placed in a magnetic field, a potential proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and to the magnetic field. This effect is known as the Hall effect, and it is the basis of many practical applications and devices such as magnetic field measurements, and position and motion detectors. In this experiment, we aim to:

- Calibrate the electromagnet and determine the calibration curve that relates the current supplied to the magnetic field.
- Find Hall Voltage as a function of magnetic field at constant probe current to calculate:
  1. Charge carrier density
  2. Charge carrier mobility
  3. Hall coefficient
- Measure Hall voltage as a function of probe current at a constant magnetic field.
- Determine Hall Voltage as a function of conductor temperature.

Theoretical Background

In the classical Hall effect experiment, we consider a metal conductor with electrons as the only available charge carrier available. When a potential V_x is applied across the metal strip, a current I_x is induced in the positive x-direction, giving the electrons a drift velocity of $\vec{v_d} = -v_x\hat{x}$ (figure 1). The drift velocity is the average velocity of the charge carriers over the volume of the conductor; each charge carrier (i.e. electron) may move in a seemingly random way within the conductor, but under the influence of applied fields there will be a net transport of carriers along the length of the conductor.

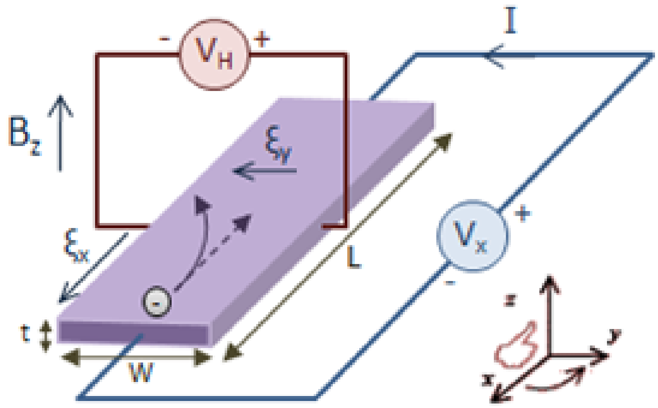

Figure 1: Schematic diagram of the Hall Effect experiment
Source: Hall effect, Wikipedia

The current I_x flowing through the conductor is equal to the current density J_x times the cross sectional area A:

$$\vec{I_x} = \vec{J_x}A = -nev_xwt\ \hat{x}$$
$$\implies v_xw = -\frac{I_x}{nte} \tag{1}$$

Where n is the charge density in the conductor, e is the charge of an electron, w is the width of the conductor and t is the thickness of the conductor. If the conductor is placed in a magnetic field $\vec{B} = B_z\hat{z}$, the electrons will experience a Lorentz force $\vec{F_B} = q\vec{v_d} \times \vec{B} = -ev_xB_z\hat{y}$. This will cause the electrons to drift towards the margins of the conductor, leading to a build up of negative charge on one side and a corresponding positive charge on the other. This charge distribution will induce its own electric field $\vec{E} = -E_y\hat{y}$ which will in turn exert an opposite force $\vec{F_E} = -e\vec{E} = eE_y\hat{y}$ on the electrons. At steady state, the magnetic and electric forces will cancel out:

$$\vec{F_B} + \vec{F_E} = 0$$
$$\implies ev_xB_z = eE_y$$
$$\implies E_y = v_xB_z \tag{2}$$

In this experiment, we measure the potential difference V_H generated across the conductor due to the interaction between the electric and magnetic fields, known as the Hall voltage. This potential difference is produced by the induced electric field:

$$V_H = -\int_0^w E_y dy = -E_yw \tag{3}$$

Substituting for E_y and v_x from equations 1 and 2 in equation 3, we find:

$$V_H = -(v_xw)B_z$$
$$\implies V_H = \frac{I_xB_z}{nte} \tag{4}$$

Assuming that the perpendicularity of the current and magnetic field is fixed, we can simply say:

$$V_H = \frac{IB}{nte} \tag{5}$$

Where I is the current in the probe, B is the external magnetic field, n is the density of charge carriers, e is the charge per electron and t is the thickness of the conductor. The above equation also holds true for a p-type Germanium semiconductor such as the one used in this experiment. This is because the only difference between the two systems is the charge of the primary charge carrier (positive for the p-type semiconductor and negative for the metal), but due to symmetry, the behaviour of the system is maintained.

A useful experimentally measurable material property known as the Hall coefficient is given by:

$$R_H = \frac{E_y}{J_x B_z} = \frac{(V_H/w)}{(I_x/wt)\, B_z} = \frac{V_H t}{I_x B_z} = \frac{1}{ne}$$

Hence,

$$R_H = \frac{1}{ne} \tag{6}$$

Where n is the density of charge carriers (in the case of the p-type semiconductor, these are the positively charged holes).

Charge mobility μ is another important quantity that is related to the drift velocity of the charge carriers:

$$\mu = \frac{v_d}{E_x} = \frac{v_d}{\rho J_x} = \frac{1}{\rho n e}$$

We get the above from the vector form of Ohm's law ($\vec{E} = \rho\vec{J}$), where ρ is the electrical resistivity of the semiconductor. Putting R_H into the above equation, we get:

$$\mu = \frac{R_H}{\rho} \tag{7}$$

Putting charge mobility (equation 7) into our expression for Hall voltage (equation 5), we find:

$$V_H = \frac{IBR_H}{t} = \frac{IB\mu\rho}{t} \tag{8}$$

There are two types of scattering mechanisms that influence the mobility of electrons and holes: lattice scattering and impurity scattering. In metals we know that lattice vibrations cause the mobility to decrease with increasing temperature. However, the mobility of the carriers in a semiconductor is also influenced by the presence of charged impurities. Impurity scattering is caused by crystal defects such as ionized impurities. At lower temperatures, carriers move slower, so there is more time for them to interact with charged impurities. As a result, as the temperature decreases, impurity scattering increases, and the mobility decreases. The total mobility then is the sum of the lattice-scattering mobility and the impurity-scattering mobility. For a Germanium semiconductor, the approximate temperature dependence of mobility due to impurity scattering is $\mu \propto T^{3/2}$ whereas the temperature dependence of mobility due to lattice scattering is $\mu \propto T^{-3/2}$ (which dominates at higher temperatures). Hence, we can assume that the impurity-scattering effects will be negligible at higher temperatures.

$$V_H \propto \mu \propto \frac{1}{T^{3/2}} \tag{9}$$

Experimental Setup

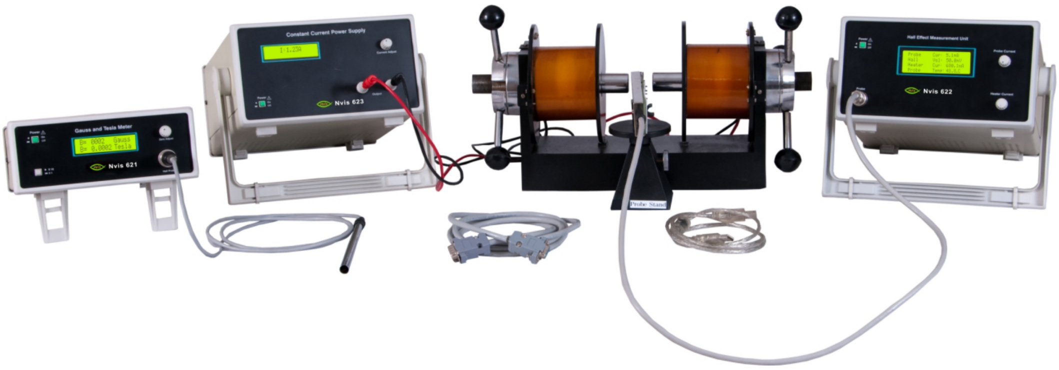

Figure 2: Nvis 6101 Hall Effect experimental setup
Source: Hall Effect Setup, NVIS Technologies Pvt. Ltd.

We used the Nvis 6101 Hall Effect setup for this experiment. The setup consists of the following instruments:

- Gauss and Tesla Meter Nvis 621
- InAs Gauss probe
- Hall Effect Measurement Unit Nvis 622
- Electromagnet with Current Power Supply Nvis 623
- Hall Probe with Oven

Least count of Hall voltage (V_H) = 0.1 mV
Least count of probe current (I) = 0.1 mA
Least count of electromagnet current (I_B) = 0.01 A
Least count of Gauss meter (B) = 10 Gauss = 0.001 T
Least count of temperature (T) = 0.1 °C

The Hall probe has four contacts and the imperfect arrangements of these contacts causes a Hall voltage reading to be displayed even if the probe is kept outside of the electromagnet. This is known as the zero field potential (V_0). Hence, this zero error value must be subtracted from all values of Hall voltage readings.

Note: The oven mechanism in the Hall probe appeared to be internally damaged during the experiment and any results derived from its use should be subject to scrutiny.

Procedure

Calibration

1. Mount the InAs probe between the poles of the electromagnet such that its surface is parallel to the faces of the poles.
2. Connect the electromagnet to the power supply and the InAs probe to the Gauss meter. Switch on both instruments and ensure that the current supply is initially zero.
3. At zero current, note down the magnetic field detected by the Gauss meter. This is the residual permanent magnetism which must be added to the final readings to adjust for zero error.

4. Gradually increase the current supply and note down the magnetic field at regular intervals. Be sure to take note of the sign of the field as it may cross zero at some point due to the zero error.

5. Plot the current input against the magnetic field (after adjusting for zero error) and fit a curve to the data. The function of this curve will act as the calibration equation for the following parts.

Magnetic Field

1. Replace the InAs probe with the Hall probe connected to the Hall effect measurement unit, such that its surface of the probe is perpendicular to the magnetic field generated by the electromagnet.

2. Fix the probe current at a constant value (we used $I = 5.0$mA) and note down the zero field potential V_0 by observing the hall voltage while the probe is outside the electromagnet.

3. With the probe between the poles of the electromagnet, gradually increase the current supplied to the magnet and note down the hall voltage at regular intervals of magnetic field. Call these readings V_1.

4. Flip the probe to the other side of the magnet so that the direction of the magnetic field is effectively reversed with respect to the current. Repeat the previous step to find a second set of Hall voltage values V_2.

5. Determine the average Hall voltage produced by the magnetic field by averaging the deviation of the hall voltage from the zero field potential for both orientations. This gives us an average hall voltage:
$$V_H = \frac{|V_1 - V_0| + |V_2 - V_0|}{2}$$

6. Plot the magnetic field (calculated from the calibration curve from the previous part) against the average Hall voltage. Use the slope to determine the charge carrier density, Hall coefficient and charge carrier mobility.

Probe Current

1. Repeat the same steps as the previous part, except this time keep the magnetic field constant (we used $I_B = 2.00$A) and vary the probe current to see its effect on Hall voltage.

2. For each value of probe current, note down V_1, V_2 and V_0.

3. Calculate the average Hall voltage as in the previous part and plot it against the probe current. Verify the relationship between V_H and I. You can also use the slope to determine the charge carrier density, Hall coefficient and charge carrier mobility.

Temperature

1. Repeat the same steps as the previous part, except this time keep both the magnetic field constant (we used $I_B = 2.85$A) and the probe current constant (we used $I = 5.0$mA). In this part, use the heater current knob on the measurement unit to raise the temperature of the probe and observe how Hall voltage changes.

2. For each temperature, note down V_1, V_2 and V_0.

3. Calculate the average Hall voltage as in the previous part and plot it against the temperature. Verify the relationship between V_H and T. You can also use the slope to determine the charge carrier density, Hall coefficient and charge carrier mobility.

Observations and Analysis

Calibration

In order to calibrate the electromagnet, we first use the InAs Gauss probe connected to a Gauss meter to determine the relationship between the current supplied to the electromagnet and the subsequent magnetic field produced. We measured the magnetic field for zero current to be -0.97T. We plotted the magnetic field against the current input values to determine the calibration curve:

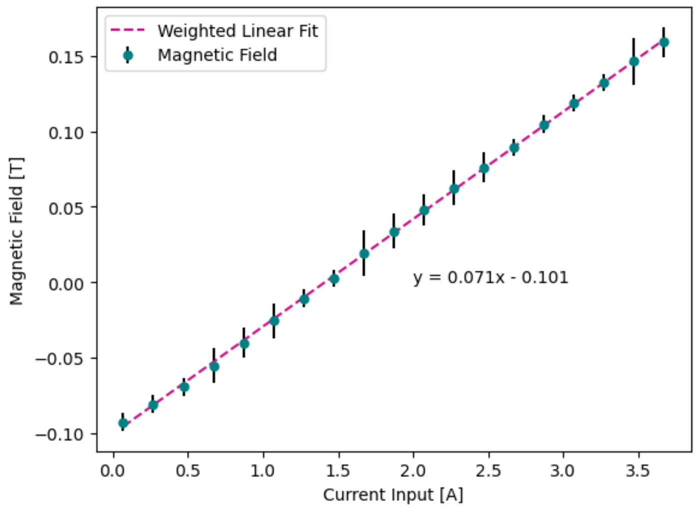

Figure 3: The magnetic field produced by the electromagnet increases linearly with respect to the current input

The graph is plotted for the mean values of the magnetic field from 3 sets of data and the errorbars are equal to the standard deviation corresponding to each value of I. The function from the linear fit of the above data is:

$$B = 0.071 I_B - 0.101 \tag{10}$$

Magnetic Field

Using the calibration equation (equation 10), we converted the current supply I_B to magnetic field B. Keeping the probe current I constant, we measured the change in the Hall voltage with respect to the magnetic field. Plotting the mean V_H against B and using the standard deviation of 3 data sets as error bars, we find the relationship to be linear:

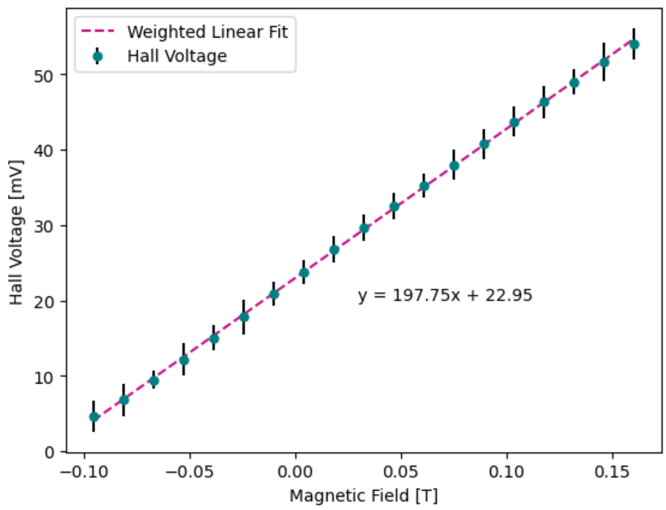

Figure 4: The Hall voltage varies linearly with respect to the magnetic field, as theoretically expected

Using the polyfit function from the `numpy` package, we determined the slope of the graph to be $a = 197.7 \pm 0.9$ mV/T$= 0.1977 \pm 0.0009$ V/T. The error in slope was determined from the diagonal elements of the covariance matrix of the weighted linear fit. From equation 5, we know:

$$\frac{V_H}{B} = \frac{IR_H}{t}$$

Hence, we can find the value of the Hall coefficient R_H, where $I = 5$mA, $t = 0.5$mm, and $V_H/B =$slope:

$$R_H = \left(\frac{V_H}{B}\right)\frac{t}{I} = (1.977 \pm 0.009) \times 10^{-2} \text{ m}^3\text{C}^{-1}$$

From equation 6, the charge density of the semiconductor (assuming $e = 1.6 \times 10^{-19}$C) is:

$$n = \frac{1}{R_H e} = (3.16 \pm 0.07) \times 10^{20} \text{ m}^{-3}$$

Finally, the charge carrier mobility from equation 7, taking $\rho = 0.1\Omega$m is:

$$\mu = \frac{R_H}{\rho} = (0.1977 \pm 0.0009) \text{ m}^2\Omega^{-1}\text{C}^{-1}$$

Probe Current

Keeping the magnetic field constant by supplying a constant current supply to the electromagnet, we were able to determine the relationship between the Hall voltage and probe current. We plotted the mean Hall voltage against the probe current and used the standard deviation of 3 data sets to determine the errorbars.

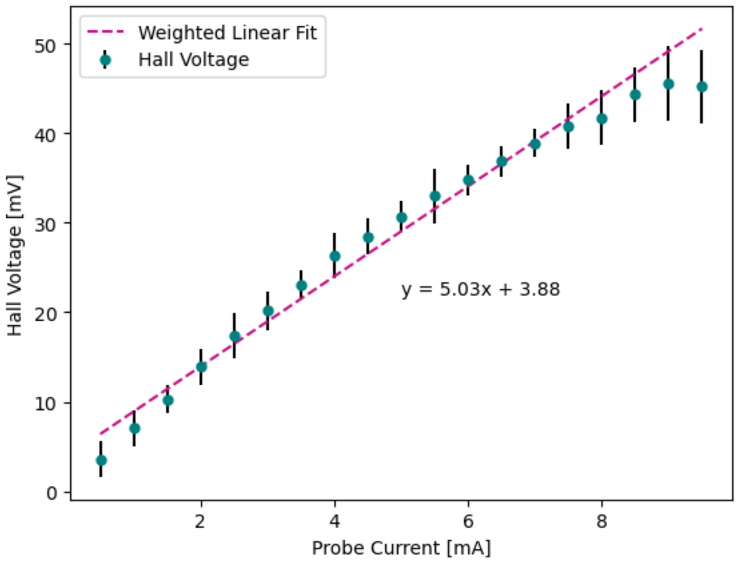

Figure 5: The trend of the data appears to have some slight non-linearity towards the end of the data set. It is possible that this may be due to magnetic hysteresis of the electromagnetic coils from extended use, causing a change in the magnetic field during the experiment.

Fitting a weighted linear trend to the data, we find the slope to be $a = (5.0 \pm 0.2)$V/A. The error in slope was determined from the diagonal elements of the covariance matrix of the weighted linear fit. From equation 5, we know:

$$\frac{V_H}{I} = \frac{BR_H}{t}$$

From the calibration curve, $I_B = 2.00$A corresponds to a magnetic field of $B = 0.04$T. Hence, we can find the value of the Hall coefficient R_H, where $B = 0.04$T, $t = 0.5$mm, and $V_H/I =$slope:

$$R_H = \left(\frac{V_H}{I}\right)\frac{t}{B} = (6.0 \pm 0.2) \times 10^{-2} \text{ m}^3\text{C}^{-1}$$

From equation 6, the charge density of the semiconductor (assuming $e = 1.6 \times 10^{-19}$C) is:

$$n = \frac{1}{R_H e} = (1.1 \pm 0.3) \times 10^{20} \text{ m}^{-3}$$

Finally, the charge carrier mobility from equation 7, taking $\rho = 0.1\Omega$m is:

$$\mu = \frac{R_H}{\rho} = (0.60 \pm 0.02) \text{ m}^2\Omega^{-1}\text{C}^{-1}$$

Temperature

Finally, we plotted the Hall voltage against the temperature by using the heater current to change the temperature of the probe. The magnetic field and probe current were both held constant at $B = 0.1$T and $I = 5.0$mA, respectively. Three data sets were collected and the mean values were plotted while the standard deviation was used to plot the errorbars.

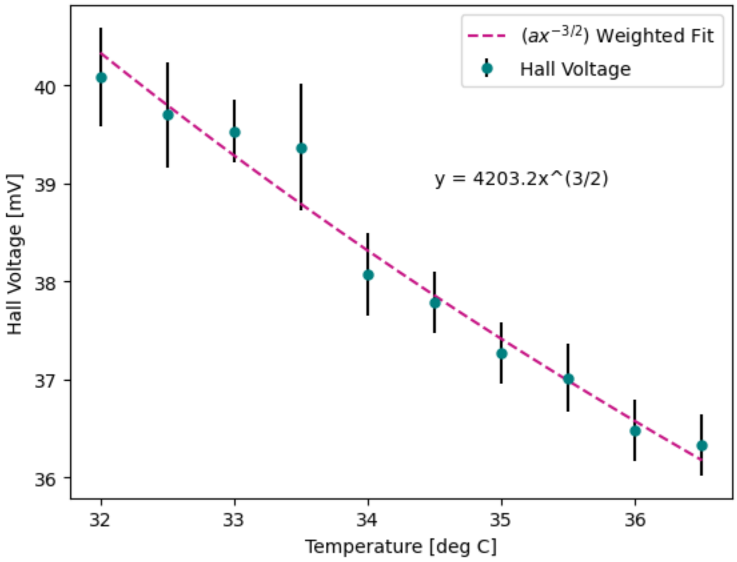

Figure 6: The Hall voltage appears to decrease as the temperature increases, suggesting that lattice-scattering dominates at these temperatures since the charge carrier mobility of the material is decreasing as the temperature rises.

Theoretically, we expect the Hall voltage to fall off to the $-3/2$ power of T since $V_H \propto 1/T^{(3/2)}$. We fit a function of this form to the data and found that the data appears to approximate this relationship. However, it should be noted that the heater current malfunctioned during the experiment and the data from this part may not be accurate.

Error Analysis and Discussion

The Hall voltage in all the parts is calculated by measuring the deviation of the voltage from the zero field potential using both directions of magnetic field and finding their average. This is done to approach a more accurate value of V_H that is independent of the orientation of the probe with respect to the magnetic field.

$$V_H = \frac{|V_1 - V_0| + |V_2 - V_0|}{2}$$

Since each value in the above equation contains its own error, the error propagates additively to give:

$$\Delta V_H = \frac{1}{2}(\Delta V_1 + \Delta V_2 + 2\Delta V_0)$$

This was used to determine the errorbars plotted in figures 4, 5 and 6. The error in the Hall coefficient is given by quadrature. For the magnetic field part, the slope error was given by $\Delta a_1 = 0.9\text{mV/T}$ and the current error is $\Delta I = 0.1\text{mA}$. The thickness of the semiconductor was taken from the handout and is assumed to be a constant:

$$\Delta R_H = R_H \sqrt{\left(\frac{\Delta a_1}{a_1}\right)^2 + \left(\frac{\Delta I}{I}\right)^2} = 0.009 \times 10^{-2}\ \text{m}^3/\text{C}$$

The error in both the density of charge carriers n and charge carrier mobility μ originates entirely from R_H since they are related to R_H by a multiplicative constant. So,

$$\Delta n = n \left(\frac{\Delta R_H}{R_H}\right) = 0.07 \times 10^{20}\ \text{m}^{-3}$$

And,

$$\Delta \mu = \mu \left(\frac{\Delta R_H}{R_H}\right) = 0.0009\ \text{m}^2\Omega^{-1}\text{C}^{-1}$$

Similarly, for the probe current part, the slope error is $\Delta a_2 = 0.2\text{V/A}$ and $\Delta B = B(\Delta m/m) + \Delta C = 0.0008\text{T}$ where m is the calibration slope and c is the calibration intercept. Their respective errors were calculated from the covariance matrix generated from the weighted linear polyfit from the `numpy` package:

$$\Delta R_H = R_H \sqrt{\left(\frac{\Delta a_2}{a_2}\right)^2 + \left(\frac{\Delta B}{B}\right)^2} = 0.2 \times 10^{-2}\ \text{m}^3/\text{C}$$

The error in both the density of charge carriers n and charge carrier mobility μ originates entirely from R_H since they are related to R_H by a multiplicative constant. So,

$$\Delta n = n \left(\frac{\Delta R_H}{R_H}\right) = 0.3 \times 10^{20}\ \text{m}^{-3}$$

And,

$$\Delta \mu = \mu \left(\frac{\Delta R_H}{R_H}\right) = 0.02\ \text{m}^2\Omega^{-1}\text{C}^{-1}$$

Additional sources of error may be:

1. Magnetic hysteresis occurs when an external magnetic field causes the atomic dipoles in metals such as iron to align themselves with it, thereby magnetizing the material. Even when the field is removed, part of the alignment is retained, leading to permanent magnetism. This permanent residual magnetism may contribute to the magnetic field experienced by the charge carriers, altering the expected behaviour under a given current supply. Prolonged use of the electromagnet increases the probability of hysteresis and should therefore be avoided. We also measured the residual magnetic field at zero current supply to remove it from the calibrated data.

2. In our calculations, we make the assumption that the majority charge carriers (i.e. holes) are the sole charge carriers in the material. However, in semi-conductors, both positive holes and negative electrons behave as charge carriers and a more nuanced understanding of the Hall coefficient would lead us to the formula:

$$R_H = \frac{\sigma_h^2 R_h + \sigma_e^2 R_e}{(\sigma_h + \sigma_e)^2}$$

Where σ_h and σ_e are the conductivity corresponding to the holes and electrons, and R_h and R_e are the Hall coefficients for the holes and electrons (corresponding to equation 6), respectively. We exempt this equation from our analysis since the contribution from each charge carrier cannot be ascertained based on the information we collect in this experiment. Hence, we simply assume that the contribution from the electrons is negligible ($\sigma_e \to 0$). However, this approximation may not be accurate for the regime in which we operate the instruments and may be leading to significant deviations from expected results.

3. The heater current apparatus malfunctioned during the experiment and led to significant fluctuations in the temperature (as evidenced by the large standard deviation in the data) and erratic changes in the device display. As a result, it should be noted that the data collected in this part may not be precise and may contain errors.

4. The angle and the position of the probe with respect to the poles of the electromagnet has a significant impact on the hall voltage detected. This is because only the component of the magnetic field that is perpendicular to the surface of the probe will act on the charge carriers in the expected manner. Hence, it must be ensured that the centre of the probe (in the middle of the 4 contacts) is placed between the poles of the electromagnet and the hall voltage should be maximised when the magnetic field is entirely perpendicular to the probe (since the entire field will act on the charge carriers).

Results

We experimentally verified that the Hall voltage V_H is directly proportional to the probe current I and magnetic field B, in accordance with the derived expression:

$$V_H = \frac{IB}{nte}$$

Where n is the charge carrier density, e is the charge of the electron and t is the thickness of the semiconductor element. From the above expression we calculated the Hall coefficient $R_H = 1/ne$, charge carrier mobility $\mu = R_H/\rho$ and charge carrier density n.

	R_H [10^{-2} m^3/C]	n [10^{20} m^{-3}]	μ [$m^2\Omega^{-1}C^{-1}$]
Magnetic Field	1.977 ± 0.009	3.16 ± 0.07	0.1977 ± 0.0009
Probe Current	6.0 ± 0.2	1.1 ± 0.3	0.60 ± 0.02

Table 1: The experimentally determined values of the Hall coefficient, charge carrier density and charge carrier mobility

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