ACM 11

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Problem 1: Mathematica as a Calculator

Question 1

In[1]:=
```
valNum = 354 224 848 179 261 915 075;
IntegerQ[Sqrt[(5 * (valNum^2)) + 4]] || IntegerQ[Sqrt[(5 * (valNum^2)) - 4]]
```

Out[2]=
```
True
```

This evaluates to True, thus indicating that the number 354,224,848,179,261,915,075 is in the fibonacci sequence.

Question 2

In[3]:=
```
valNum2 = 2 305 843 008 138 952 128;
valNum2 == (DivisorSigma[1, valNum2]) / 2
```

Out[4]=
```
False
```

This evaluates to False, thus indicating that the number 2,305,843,008,138,952,128 is not a perfect number (i.e. it is not equivalent to half the sum of all of its positive divisors).

Question 3

In[5]:=
```
NextPrime[Prime[PrimePi[10^9]]]
```

Out[5]=
```
1 000 000 007
```

This shows that the smallest prime number above 1 billion is 1,000,000,007.

Question 4

In[6]:=
```
SetPrecision[ZetaZero[20], 100]
```

Out[6]=
```
0.5000000000000000000000000000000000000000000000000000000000000000000000000000000\
  0000000000000000000 +
 77.14484006887480537268266485630463701579603244923446104176523145315113916425371\
   508940828869469973776 i
```

Evaluating this gives the 20-th zero of the Riemann zeta function.

Question 5

In[7]:=
```
Sum[((3^k) - 1) / (4^k) * Zeta[k + 1], {k, 1, Infinity}]
```

Out[7]= π

This computes the given sum to be Pi.

Problem 2: High-Dimensional Balls and Monte Carlo

Part 1

In[8]:=
```
Clear[n, R]
V[n_, R_] = ((Pi^(n / 2)) * (R^n)) / Gamma[(n / 2) + 1]
S[n_, R_] = ((n + 1) * (Pi^((n + 1) / 2)) * (R^n)) / Gamma[(((n + 1) / 2) + 1)]
D[V[n, R], R] == S[(n - 1), R]
```

Out[9]= $$\frac{\pi^{n/2} R^n}{\text{Gamma}\left[1+\frac{n}{2}\right]}$$

Out[10]= $$\frac{(1+n)\,\pi^{\frac{1+n}{2}} R^n}{\text{Gamma}\left[1+\frac{1+n}{2}\right]}$$

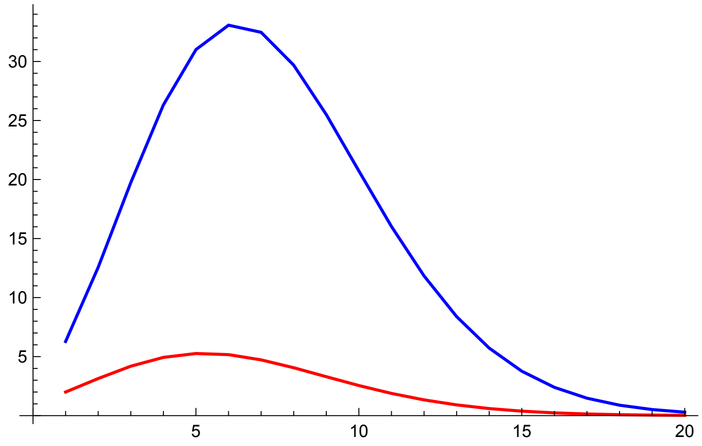
Out[11]= True

This creates the V(n, R) and S(n, R) functions and then confirms the given identity, S(n-1, R) = derivative of V(n, R) with respect to R.

Part 2

In[12]:=
```
dataV = Table[{n, V[n, 1]}, {n, 1, 20}];
dataS = Table[{n, S[n, 1]}, {n, 1, 20}];
ListLinePlot[{dataV, dataS}, PlotStyle → {Red, Blue}]
```

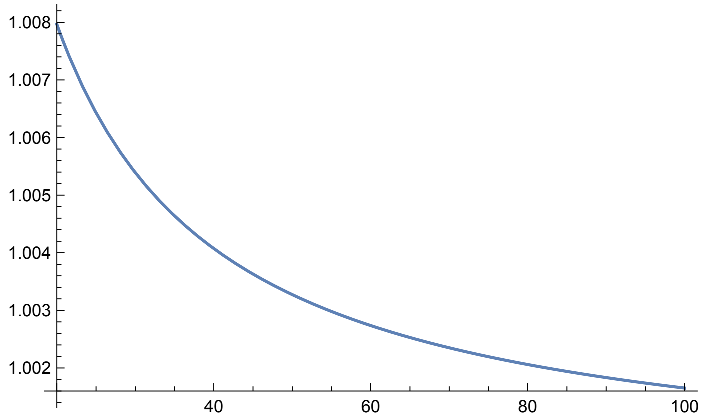
Out[14]=



Part 3

In[15]:=
```
approxS[n_, R_] := ((n + 1) * (Pi ^ ((n + 1) / 2)) * (R ^ n)) /
   (Sqrt[2 * Pi] * Exp[(- (n + 1)) / 2] * (((n + 1) / 2) ^ ((n + 2) / 2)));
dataRatio = Plot[approxS[n, 1] / S[n, 1], {n, 20, 100}]
```

Out[16]=



The limit of this ratio as n approaches infinity does not depend on R. This is because the only thing different between approxS and the S function is the denominator (the Gamma part), which does not depend on R. Thus, the R^n term in the numerator of both functions gets eliminated.

Part 4

In[17]:=
```
n = 10;
m = 10 ^ 6;
points = RandomReal[{-1, 1}, {m, n}];
valPoints = Map[Norm, points];
numInRange = Count[valPoints, _? (# ≤ 1 &)];
currentVolume = N[((2 ^ n) * numInRange) / 10 ^ 6];
piApprox = (Gamma[(n / 2) + 1] * currentVolume) ^ (2 / n)
relativeError = Abs[Pi - piApprox]
```

Out[23]= 3.13389

Out[24]= 0.00770617

This approximates Pi using a sample size of 10^6 in a 10 dimensional space. This reports the approximation (~3.13363) and the relative error (0.00796099).

Problem 3: Mandelbrot Set

Part 1

In[25]:=
```
mandelbrot[cx_, cy_, nmax_] :=
 Module[{n = 0, zn = 0},
  While[(n < nmax),
   zn = (zn^2) + (cx + I * cy);
   If[Norm[zn] ≤ 2, n = n + 1, Break[]]];
  n
 ]
mandelbrot[1, 0, 100]
```

Out[26]= 2

This creates the mandelbrot function and confirms that it works using an easily computable c value (it confirms that c = 1 gives m =2).

Part 2

In[27]:=
```
Timing[mandelbrot[Sqrt[5] / 100, 0, 1000]]
mandelbrotC = Compile[{cx, cy, nmax}, mandelbrot[cx, cy, nmax]]
```

Out[27]= {173.991, 1000}

Out[28]= CompiledFunction[Argument count: 3 Argument types: {_Real, _Real, _Real}]

In[29]:=
```
Timing[mandelbrotC[Sqrt[5] / 100, 0, 1000]]
```

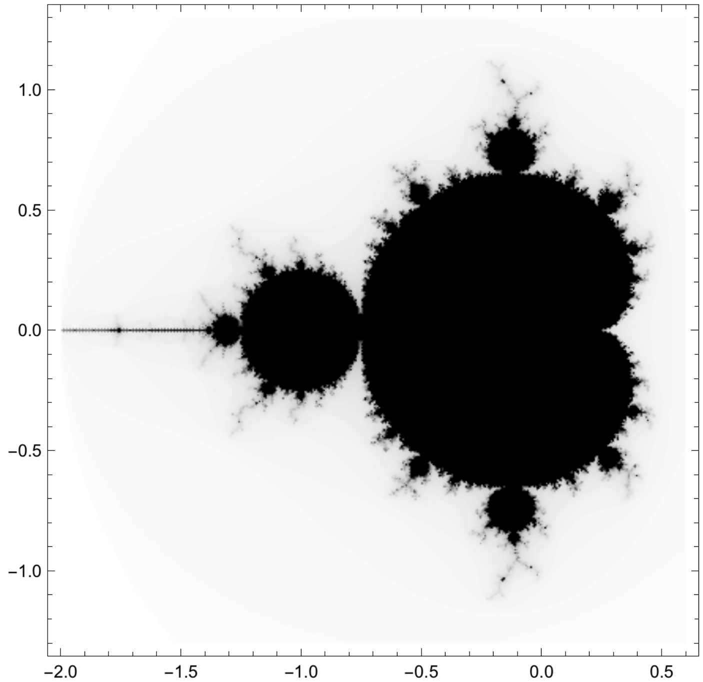
Out[29]= {0.004639, 1000.}

This runs the function on a more complicated c value, with a large nmax. I then created a compiled version of the mandelbrot and showed that the compiled version ran faster. To compare times, I used the Timing function. This showed that the uncompiled version took 0.49704 seconds while the comiled version took 0.004748 seconds.

Part 3

In[30]:=
```
DensityPlot[-mandelbrotC[x, y, 100], {x, -2, 0.6},
 {y, -1.3, 1.3}, ColorFunction → GrayLevel, PlotPoints → 100]
```

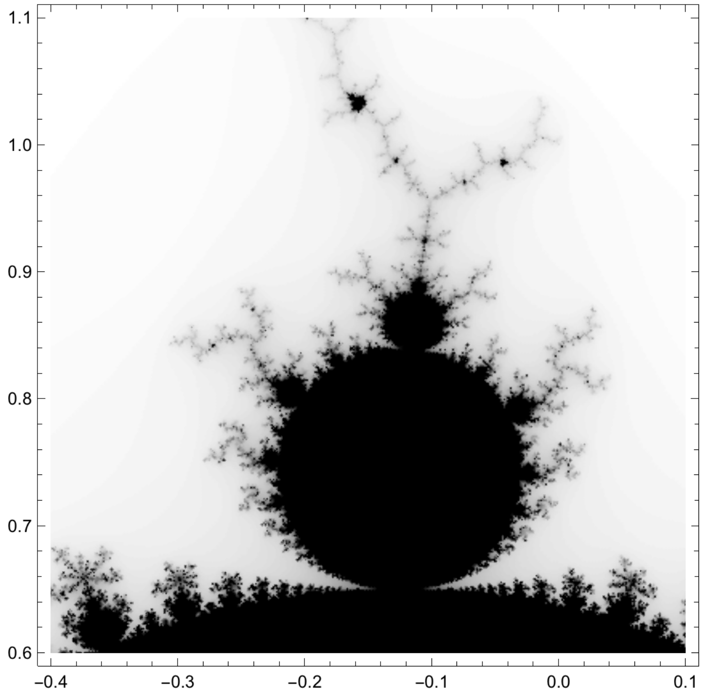
Out[30]=



Part 4

In[31]:=
```
DensityPlot[-mandelbrotC[x, y, 100], {x, -0.4, 0.1},
 {y, 0.6, 1.1}, ColorFunction → GrayLevel, PlotPoints → 100]
```

Out[31]=



Part 5

In[33]:=
```
DensityPlot[-mandelbrotC[x, y, 1000], {x, -0.75, -0.747}, {y, 0.06, 0.063},
  ColorFunction → GrayLevel, PlotPoints → 300, ImageSize → Large]
```

Out[32]=