

Method of undetermined coefficients

This method is applicable when the homogeneous part is of constant coefficient linear ODE and the nonhomogeneous part is one of the following functions:

- exponential
- polynomial
- sine or cosine
- sum or products of above functions

Example: $y'' - 2y' + y = e^t$

$$y_h(t) = c_1 e^t + c_2 t e^t$$

(because char. equa. $m^2 - 2m + 1 = 0$)

$$\therefore e^t, t e^t$$

$$y_p(t) = A t^2 e^t$$

$$y_p'(t) = A(t^2 e^t + 2t e^t)$$

$$y_p''(t) = A(t^2 e^t + 4t e^t + 2e^t)$$

$$\therefore A t^2 [(4t^2 + 4t + 2) - 2(t^2 + t)] = e^t$$

$$\Rightarrow 2A = 1 \Rightarrow A = \frac{1}{2}$$

$$\therefore y_p(t) = \frac{1}{2} t^2 e^t$$

$$\text{Example: } y'' - y = t^2$$

$$y_h(t) = c_1 e^t + c_2 t e^t$$

What should be $y_p(t)$?

$$y_p(t) = A t^2 + Bt + C$$

(polynomial of the same degree as $r(t) = t^2$)

$$y_p'(t) = 2At + B$$

$$y_p''(t) = 2A$$

$$\therefore 2A - (At^2 + Bt + C) = t^2$$

$$\Rightarrow (2A - C) - Bt - (A+1)t^2 = 0$$

$$\Rightarrow 2A - C = 0, B = 0, A + 1 = 0$$

$$\Rightarrow A = -1, B = 0, C = -2$$

$$\therefore y_p(t) = -t^2 - 2$$

$$\therefore y = c_1 e^t + c_2 t e^t - t^2 - 2$$

Modification Rule for polynomial.

If $y = \text{constant}$ is a solution of the homogeneous part but $y = t^k$ is not a solution, then we multiply

the particular by t

i.e. if $r(t) = a_0 + a_1 t + \dots + a_n t^n$

then $y_p(t) = t(b_0 + b_1 t + \dots + b_n t^n)$

and then determine b_0, b_1, \dots, b_n .

Example: $y'' - y = \sin(t)$

$$y_h(t) = c_1 e^t + c_2 t e^t$$

$$\text{We take } y_p(t) = A \sin t + B \cos t$$

$$\text{Then } y_p' = A \cos t - B \sin t$$

$$y_p'' = -A \sin t - B \cos t$$

$$\therefore -A \sin t - B \cos t = \sin t$$

$$\Rightarrow -2A \sin t - 2B \cos t = 0$$

$$\Rightarrow -2A = 1 \text{ and } -2B = 0$$

$$\Rightarrow A = \frac{1}{2}, B = 0$$

$$\therefore y_p = -\frac{1}{2} \sin t$$

Example: $y'' + y = \sin t$

$$y_h(t) = c_1 \sin t + c_2 \cos t$$

$$y_p(t) = t(A \sin t + B \cos t)$$

$$y_p'(t) = t(A \cos t - B \sin t) + (A \sin t + B \cos t)$$

$$y_p''(t) = t(A \sin t - B \cos t) + A \cos t - B \sin t$$

$$\therefore t(-A \sin t - B \cos t) + 2(A \sin t - B \cos t) = \sin t$$

$$\Rightarrow A = 0, -2B = 1 \Rightarrow B = \frac{1}{2}$$

$$\therefore y_p(t) = -\frac{1}{2} t \cos t$$

Summary: To solve a nonhomogeneous ODE with constant coefficients homogeneous part we first solve the homogeneous ODE and then apply the following rule to find a particular solution:

find a particular solution:

$r(t) \quad | \quad y_p(t)$

(i) $B e^{rt}$

if $A e^{rt}$ is not a solution of the homogeneous part

if $A t e^{rt}$ is not a solution of the homogeneous part

if $A t^2 e^{rt}$ is not a solution of the homogeneous part

if $A t^3 e^{rt}$ is not a solution of the homogeneous part

if $A t^4 e^{rt}$ is not a solution of the homogeneous part

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