Indian Institute of Technology Delhi - Abu Dhabi
AMTL100: CALCULUS
Tutorial Sheet 5

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(1) Find the sum of the series or show that it diverges.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$

(b) $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n}\right)$

(c) $\sum_{n=1}^{\infty} \left(1 - \frac{7}{4^n}\right)$

(d) $\sum_{n=0}^{\infty} \cos\left(\frac{n\pi}{2}\right)$

(2) Determine if each series converges or diverges. If a series converges, find its sum.

(a) $\sum_{n=1}^{\infty} \left(\ln\sqrt{n+1} - \ln\sqrt{n}\right)$

(b) $\sum_{n=1}^{\infty} \left(\sqrt{n+4} - \sqrt{n+3}\right)$

(c) $\sum_{n=1}^{\infty} \frac{4}{(4n-3)(4n+1)}$

(d) $\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)}\right)$

(e) $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$

(3) Determine if each series converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^{\frac{5}{2}}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n3^n}$

(c) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

(d) $\sum_{n=1}^{\infty} \sin \frac{1}{n}$

(e) $\sum_{n=2}^{\infty} \frac{1}{n!}$

(f) $\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$

(g) $\sum_{n=1}^{\infty} \frac{1}{n \sqrt[n]{n}}$

(h) $\sum_{n=1}^{\infty} \frac{1+\cos n}{n^2}$

(4) Suppose that $a_n > 0$ and $\lim_{n\to\infty} n^2 a_n = 0$. Prove that $\sum a_n$ converges.

(5) Prove that if $\sum a_n$ is a convergent series of non-negative terms, then $\sum a_n^2$ converges.