Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 2: Linear Algebra

(1) Which of the following matrices are row reduced echelon matrix. Give a reason when the matrix is not row reduced echelon.

$$\begin{pmatrix} 1 & 0 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 4 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

(2) Find row reduced echelon matrix row equivalent to each of the matrices in the previous question.

(3) Solve the following system of homogeneous linear equations by reducing the coefficient matrix into row reduced echelon form:
   (a) $x - 2y - 3z = 0, 2x + y + 3z = 0, 3x - 4y - 2z = 0.$
   (b) $2x_1 + 4x_2 - 5x_3 + 3x_4 = 0, 3x_1 + 6x_2 - 7x_3 + 4x_4 = 0, 5x_1 + 10x_2 - 11x_3 + 6x_4 = 0.$
   (c) $x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0, 2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0, 3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0.$

(4) Solve the following systems of linear equations by reducing the augmented matrix to the row reduced echelon form:
   (a) $x_1 - x_2 + 2x_3 = 1, 2x_1 + 2x_3 = 1, x_1 - 3x_2 + 4x_3 = 2.$
   (b) $x_1 + 7x_2 + x_3 = 4, x_1 - 2x_2 + x_3 = 0, -4x_1 + 5x_2 + 9x_3 = -9.$
   (c) $x_2 + 5x_3 = -4, x_1 + 4x_2 + 3x_3 = -2, 2x_1 + 7x_2 + x_3 = -1.$
   (d) $-2x_1 - 3x_2 + 4x_3 = 5, x_2 - x_3 = 4, x_1 + 3x_2 - x_3 = 2.$

(5) Write down all possible values of $a, b, c, x, y \in \mathbb{R}$ for which the following matrix is a row-reduced echelon matrix.

$$\begin{pmatrix} a & b & 0 & 0 \\ 0 & c & 1 & 1 \\ 0 & 0 & x & y \end{pmatrix}$$

(6) Consider the following system of linear equations:

$$\begin{aligned} x + 2y - z &= 2 \\ 2x + y + 3z &= 1 \\ 3x + 4y + 5z &= d \\ -x + 2y + z &= 2 \end{aligned}$$

Which of the following statements is/are true?
   (a) The system has a solution for $d = 2$.
   (b) The system has a solution for $d = 4$.
   (c) The system has infinitely many solutions for $d = 10$.
   (d) The system has a unique solution for some $d > 0$.