**Tutorial Sheet 11: Laplace Transforms**

(1) Find the Laplace transforms of the following functions.
$$\cos^2 \omega t, \ e^t \cosh 3t, \ \sin 2t \cos 2t, \ e^{-\alpha t} \cos \beta t.$$

(2) Find the inverse Laplace transforms of the following functions.
$$\frac{5s}{s^2 - 25}, \ \frac{1 - 7s}{(s-3)(s-1)(s+2)}, \ \frac{2s^3}{s^4 - 1}, \ \frac{2}{s^2 + s + \frac{1}{2}}.$$

(3) Solve the following IVPs using Laplace transform.
  (a) $y'' - y' - 2y = 0; \ y(0) = 8, y'(0) = 7.$
  (b) $y'' + 2y' - 3y = 6e^{-2t}; \ y(0) = 2, y'(0) = -14.$

(4) Find the Laplace transforms of the following functions ($u$ is the unit step function).
$$tu(t - 1), \ e^{-2t}u(t - 3), \ 4u(t - \pi)\cos t.$$

(5) Find the inverse Laplace transforms of the following functions.
$$\frac{e^{-3s}}{s^3}, \ \frac{3(1 - e^{-\pi s})}{s^2 + 9}, \ \frac{se^{-2s}}{s^2 + \pi^2}.$$

(6) Solve the following IVPs using Laplace transform.
  (a) $y'' + 6y' + 8y = e^{-3t} - e^{-5t}; \ y(0) = 0, y'(0) = 0.$
  (b) $y'' + 3y' + 2y = 4t$ if $0 < t < 1$ and $8$ if $t > 1$; $y(0) = 0, y'(0) = 0.$
  (c) $y'' + 4y' + 5y = \delta(t - 1); \ y(0) = 0, y'(0) = 3.$

(7) Find inverse Laplace transforms of the following functions.
$$\ln\left(\frac{s + 2}{s + 3}\right), \ \cot^{-1}\frac{s}{\pi}.$$

(8) Compute convolution of the following functions.
$$1 * \sin t, \ \cos 2t * \sin 2t, \ u(t - 1) * t^2, \ u(t - 3) * e^{2t}.$$

(9) Solve the following integral equations using Laplace transform.
  (a) $y(t) = 2t - 4\int_0^t y(\tau)(t - \tau)d\tau$
  (b) $y(t) = 1 - \sinh t + \int_0^t (1 + \tau)y(t - \tau)d\tau$