

INDIAN INSTITUTE OF TECHNOLOGY DELHI - ABU DHABI  
**AMTL100: CALCULUS**  
**Tutorial Sheet 2**

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(1) Find the following limits

$$(a) \lim_{x \rightarrow \infty} (\sqrt{x+9} - \sqrt{x+4}) \quad (b) \lim_{x \rightarrow \infty} (\sqrt{x^2+3x} - \sqrt{x^2-2x}) \quad (c) \lim_{x \rightarrow \infty} \frac{x + \sin x + 2\sqrt{x}}{x + \sin x}$$

(2) Explain why the equation  $\cos x = x$  has at least one solution.

(3) Show that the equation  $x^3 - 15x + 1 = 0$  has three solutions in the interval  $[-4, 4]$ .

(4) If  $f(x) = x^3 - 8x + 10$ , show that there are values  $c$  for which  $f(c)$  equals

(a)  $\pi$ ; (b)  $-\sqrt{3}$ ; (c) 5,000,000.

(5) If the product function  $h(x) = f(x) \cdot g(x)$  is continuous at  $x = 0$ , must  $f(x)$  and  $g(x)$  be continuous at  $x = 0$ ? Give reasons for your answer.

(6) Give an example of functions  $f$  and  $g$ , both continuous at  $x = 0$ , for which the composite  $f \circ g$  is discontinuous at  $x = 0$ .

(7) Suppose that  $f : [0, 1] \rightarrow [0, 1]$  be a continuous function. Show that there must exist a number  $c$  in  $[0, 1]$  such that  $f(c) = c$ .

(8) Find the horizontal asymptotes of  $f(x) = \frac{\sqrt{x^2+4}}{x}$ .

(9) Find the vertical asymptotes of  $f(x) = \frac{x^2+x-6}{x^2+2x-8}$ .