Indian Institute of Technology Delhi - Abu Dhabi

AMTL100: CALCULUS

Tutorial Sheet 11

(1) Evaluate the following double integrals.
   (a) $\iint_R x^2\ dA$, where R is the region bounded by $y = x^2$, $y = x + 2$.
   (b) $\iint_R (x^2 + y^2)\ dA$, where $R : 0 \leq y \leq \sqrt{1 - x^2}, 0 \leq x \leq 1$.
   (c) $\iint_R (a^2 - x^2 - y^2)\ dA$, where R is the region $x^2 + y^2 \leq a^2$.

(2) Evaluate the following double integrals by changing the order of integration, if needed.
   (a) $\int_0^3 \int_{-y}^{y} (x^2 + y^2) dxdy$
   (b) $\int_0^{\pi} \int_x^{\pi} \frac{\sin y}{y} dydx$
   (c) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx$

(3) Find the volume of the following.
   (a) Region under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x$, $x = 0$, and $x + y = 2$ in the xy-plane.
   (b) Region bounded in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.

(4) Use the given transformations to transform the following integrals and evaluate them.
   (a) $u = 3x + 2y$, $v = x + 4y$ and $I = \iint_R (3x^2 + 14xy + 8y^2) dA$, where R is the region in the first quadrant bounded by the lines $y + \frac{3}{2}x = 1$, $y + \frac{3}{2}x = 3$, $y + \frac{1}{4}x = 0$,and $y + \frac{1}{4}x = 1$.
   (b) $u = x + 2y$, $v = x - y$ and $I = \int_0^{2/3} \int_y^{2-2y} (x + 2y) e^{(y-x)} dA$

(5) Evaluate the following volume integrals.
   (a) $\iiint_D (z^2x^2 + z^2y^2) dV$, where $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, -1 \leq z \leq 1\}$
   (b) $\iiint_D xyz dV$, where $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 \leq 1, 0 \leq z \leq x^2 + y^2\}$
   (c) $\iiint_D e^{(x^2+y^2+z^2)^{3/2}} dV$, where $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$