

AMTL 100 (CALCULUS)  
Re-Midterm Exam

Date: 10/12/2024

Total Marks: 30

Time: 2 hours

1. Find the following limits

(a)  $\lim_{x \rightarrow 0^+} x^{\sin x}$  [3]

(b)  $\lim_{x \rightarrow 0^+} (\csc x - \cot x)$  [2]

2. Find the intervals on which the following function is increasing and those on which it is decreasing. Also, find all local and global extrema of the function. [6]

$$f(x) = x^2 + \frac{1}{x^2} \quad \text{for } x \neq 0$$

3. Consider the sequence given by

$$a_1 = 2, \quad a_{n+1} = \frac{1}{2} \left( a_n + \frac{3}{a_n} \right) \quad \text{for } n \geq 1$$

(a) Prove that the sequence is bounded from below. [2]

(b) Prove that the sequence is decreasing. [2]

(c) Conclude that the sequence converges and find its limit. [3]

4. Determine whether the following series converge or diverge.

(a)  $\sum_{n=1}^{\infty} \frac{1 + n \ln n}{n^2 + 3}$  [3]

(b)  $\sum_{n=1}^{\infty} \ln \left( 1 + \frac{1}{n} \right)$  [3]

(c)  $\sum_{n=1}^{\infty} \frac{e^n}{e^n + n}$  [3]

5. Use Taylor's theorem to estimate the error when  $e^x$  is approximated by  $1 + x + \frac{x^2}{2}$  for  $|x| \leq 0.1$ . [3]