AMTL 101 (Linear Algebra and Differential Equations)
Midterm Exam

Date: 27/02/2025 | Total Marks: 30 | Time: 90 mins

1. Write down all possible 3×3 real RRE matrices of rank 2. [4]

2. Find a condition on a, b, c, d so that [4]

$$\{(1,1,1,1),(1,0,1,0),(1,1,0,1),(a,b,c,d)\}$$

is a linearly dependent set in $\mathbb{R}^4$.

3. Consider the system of linear equations: [6]

$$\begin{aligned} x_1 + x_2 + 3x_3 + 2x_4 &= 4 \\ x_1 + 2x_2 + 4x_3 + 3x_4 &= 5 \\ x_1 + 3x_2 + 2x_3 + ax_4 &= 4 \\ x_1 + 2x_2 + x_3 &= b \end{aligned}$$

Find all possible real numbers a and b such that the system has
(a) no solutions, (b) a unique solution or (c) infinitely many solutions. Also, find all the solutions when the system has infinitely many solutions.

4. Let W_1 and W_2 be two subspaces of $\mathbb{R}^4$ defined by [2+2+2=6]

$$W_1 = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w=0, y+z=0\},$$
$$W_2 = \{(x,y,z,w) \in \mathbb{R}^4 : x+y+z=0, y+z+w=0\}.$$

   (a) Find a basis for $W_1 \cap W_2$.
   (b) Find the dimension of $W_1 + W_2$.
   (c) Find a basis for $W_1 + W_2$.

5. Prove or disprove the following statements. [2+2+2=6]

   (a) For any $A, B \in M_{n\times n}(\mathbb{R})$, $W = \{X \in M_{n\times n}(\mathbb{R}) : AXB = BXA\}$ is a subspace of $M_{n\times n}(\mathbb{R})$.
   (b) Let W_1, W_2, W_3 be subspaces of a vector space V such that $\dim(W_1) = \dim(W_2) = \dim(W_3) = 1$ and $W_i \cap W_j = \{0\}$ for $i \neq j$. Then $\dim(W_1 + W_2 + W_3) = 3$.
   (c) If $\{u, v, w\}$ is linearly independent, then $\{u - 2v, 4v - 2w, w - u\}$ is linearly independent.

6. Prove or disprove that the following is a linear transformation. [2+2=4]

   (a) $T : \mathbb{C} \to \mathbb{C}$ given by $T(z) = \bar{z}$, where $\bar{z}$ denotes the complex conjugate of z and $\mathbb{C}$ is considered as a vector space over $\mathbb{C}$.
   (b) $T : \mathbb{R}[x] \to \mathbb{R}[x]$ given by $T(p(x)) = x^2 p(x) + p(1)$.

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