

Tutorial Sheet 10: Picard's Iteration and Existence-Uniqueness Theorem

(1) Find the n -th Picard's iterate y_n for the following IVP: $y' = x^2 + y$, $y(0) = 0$.

(2) Apply the Picard's iteration method to $y' = 2y^2$, $y(0) = 1$.

(3) Consider the IVP:

$$(x^2 - 1)y' = 4y, \quad y(x_0) = y_0.$$

(a) Find the values of (x_0, y_0) for which a unique solution is guaranteed by the existence-uniqueness theorem.

(b) Show that if $(x_0, y_0) = (1, 0)$, then the IVP has infinitely many solutions.

(4) Find all the initial conditions, such that corresponding IVP, with the ODE

$$(x^2 - 4x)y' = (2x - 4)y$$

has no solution, a unique solution and more than one solution, respectively.