Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 3: Linear Algebra

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(1) Find the inverse of the following matrices (if they exist) by performing elementary row operations.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(2) Let V be a vector space over a field $\mathbb{F}$ and $\mathbf{0}$ denote the zero vector. Prove the following:

(a) $c.\mathbf{0} = \mathbf{0}$ for any $c \in \mathbb{F}$.

(b) $0.v = \mathbf{0}$ for any $v \in V$.

(c) $(-1).v = -v$ for any $v \in V$.

(3) Show that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ (with the usual addition and multiplication) is a field.

(4) Let $V = \{(a_1, a_2) : a_1, a_2 \in \mathbb{C}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{C}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) \text{ and } c.(a_1, a_2) = (ca_1, ca_2).$$

Is V a vector space over $\mathbb{C}$ under these operations?