

INDIAN INSTITUTE OF TECHNOLOGY DELHI - ABU DHABI
AMTL100: CALCULUS
Tutorial Sheet 2

- (1) Find the following limits
- (a) $\lim_{x \rightarrow \infty} (\sqrt{x+9} - \sqrt{x+4})$ (b) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x})$ (c) $\lim_{x \rightarrow \infty} \frac{x + \sin x + 2\sqrt{x}}{x + \sin x}$
- (2) Explain why the equation $\cos x = x$ has at least one solution.
- (3) Show that the equation $x^3 - 15x + 1 = 0$ has three solutions in the interval $[-4, 4]$.
- (4) If $f(x) = x^3 - 8x + 10$, show that there are values c for which $f(c)$ equals
(a) π ; (b) $-\sqrt{3}$; (c) 5,000,000.
- (5) If the product function $h(x) = f(x) \cdot g(x)$ is continuous at $x = 0$, must $f(x)$ and $g(x)$ be continuous at $x = 0$? Give reasons for your answer.
- (6) Give an example of functions f and g , both continuous at $x = 0$, for which the composite $f \circ g$ is discontinuous at $x = 0$.
- (7) Suppose that $f : [0, 1] \rightarrow [0, 1]$ be a continuous function. Show that there must exist a number c in $[0, 1]$ such that $f(c) = c$.
- (8) Find the horizontal asymptotes of $f(x) = \frac{\sqrt{x^2+4}}{x}$.
- (9) Find the vertical asymptotes of $f(x) = \frac{x^2+x-6}{x^2+2x-8}$.