Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 4: Linear Algebra

---

(1) Find the largest possible number of linearly independent vectors in $\mathbb{R}^4$ among $v_1 = (1,-1,0,0)$, $v_2 = (1,0,-1,0)$, $v_3 = (1,0,0,-1)$, $v_4 = (0,1,-1,0)$, $v_5 = (0,1,0,-1)$, $v_6 = (0,0,1,-1)$.

(2) Let V be a vector space over $\mathbb{R}$. Show that $\{u, v, w\}$ is linearly independent if and only if $\{u+v, v+w, w+u\}$ is linearly independent.

(3) Show that $S = \{(1+i, 1-i), (1-i, 1+i), (2, i), (3, 2i)\}$ is linearly independent in the vector space $\mathbb{C}^2$ over $\mathbb{R}$.

(4) Let V be the vector space of all real polynomials over the field $\mathbb{R}$. Is $W = \{p(x) \in V : p(1) = 0\}$ a subspace of V?

(5) In each case show directly that $W_1 + W_2 = V$ and find $\dim(W_1 \cap W_2)$. Also, verify the formula $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.

(a) $V = \mathbb{R}^2$, W_1 and W_2 are distinct lines through the origin.

(b) $V = \mathbb{R}^3$, W_1 is the xy-plane and W_2 is the yz-plane.

(c) $V = M_{n\times n}(\mathbb{R})$, $W_1 = \{A \in V : A \text{ is upper triangular}\}$, $W_2 = \{A \in V : A \text{ is lower triangular}\}$.

(d) $V = M_{n\times n}(\mathbb{R})$, $W_1 = \{A \in V : A^t = A\}$, $W_2 = \{A \in V : A^t = -A\}$.