Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 5: Linear Algebra

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(1) Let V be the vector space of all real polynomials and let T be the derivative operator on V. Find the nullspace and the range space of T.

(2) Find the rank and nullity of the following linear transformations. Also write a basis for the range space in each case.

(a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y - z, x - y + z, x + y + z)$.

(b) Assume that $0 \le m \le n$ and $T : \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(x_1, x_2, \ldots, x_n) = (x_1, x_2, \ldots, x_m)$.

(3) Suppose V is a finite dimensional vector space and there is a linear operator $T : V \to V$ whose nullspace and range space are the same. What can you say about the dimension of V?

(4) Give an example of a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ satisfying $\ker(T) = \text{range}(T)$.

(5) Write the matrix representations of the linear operators with respect to the ordered basis $\mathcal{B}$.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x + y, x)$, $\mathcal{B} = \{(1, 1), (1, -1)$.

(b) $T : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ defined by $T\left(\begin{bmatrix} x & y \\ z & w \end{bmatrix}\right) = \begin{bmatrix} x + w & z \\ z + w & x \end{bmatrix}$,

$\mathcal{B} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.