Indian Institute of Technology Delhi - Abu Dhabi

AMTL100: CALCULUS

Tutorial Sheet 3

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(1) Show that the following functions have exactly one zero in the given interval.
   (i) $f(x) = x^4 + 3x + 1,\ [-2, -1]$
   (ii) $f(x) = x^3 + \frac{4}{x^2} + 7,\ (-\infty, 0)$
   (iii) $f(\theta) = 2\theta - \cos^2\theta + \sqrt{2},\ (-\infty, \infty)$
   (iv) $f(\theta) = \tan\theta - \cot\theta - \theta,\ (0, \pi/2)$

(2) Find the local and global extrema (if any) of the following functions.
   (i) $f(x) = \sin x - \cos x,\ 0 \le x \le 2\pi$
   (ii) $f(x) = x^3 - 3x^2,\ -\infty < x \le 3$
   (iii) $f(x) = \frac{x-2}{x^2-1},\ 0 \le x < 1$
   (iv) $f(x) = \sec^2 x - 2\tan x,\ -\pi/2 < x < \pi/2$

(3) Suppose that $f'(x) \le 1$ for $1 \le x \le 4$. Show that $f(4) - f(1) \le 3$.

(4) Prove that $f(x) = x - \ln x$ is increasing in the interval $(1, \infty)$ and use it to show that $\ln x < x$ if $x > 1$.

(5) Find the absolute maximum value of $f(x) = x^2 \ln(1/x)$ and say where it is assumed.

(6) Suppose that the first derivative of $y = f(x)$ is

$$y' = 6(x + 1)(x - 2)^2.$$

At what points, if any, does the graph of f have a local maximum, local minimum, or point of inflection?

(7) Find the Taylor polynomials of orders 1, 2, and 3 of the function f around a.
   (i) $f(x) = e^{2x},\ a = 0$
   (ii) $f(x) = \ln x,\ a = 1$
   (iii) $f(x) = \tan x,\ a = \pi/4$
   (iv) $f(x) = \sqrt{1 - x},\ a = 0$

(8) Estimate the error in the approximation of $\sinh x = x + x^3/3!$ when $|x| < 0.5$.