

**Tutorial Sheet 8: Second Order ODEs**

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- (1) Find the solutions of the following initial value problems:
- (a)  $y'' - 2y' - 3y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$
  - (b)  $y'' + 10y = 0$ ,  $y(0) = \pi$ ,  $y'(0) = \pi^2$
- (2) Find all solutions of the following differential equations:
- (a)  $4y'' - y = e^x$
  - (b)  $y'' + 4y = \cos x$
  - (c)  $y'' + 9y = \sin 3x$
- (3) Verify that  $y_1 = \frac{\cos x}{x}$  is a solution to the differential equation  $xy'' + 2y' + xy = 0$ , and then use reduction of order method to find a second linearly independent solution.
- (4) Use the method of undetermined coefficients to find a particular solution of each of the following equations:
- (a)  $y'' + 4y = \cos x$
  - (b)  $y'' - y' - 2y = x^2 + \cos x$
  - (c)  $y'' + 9y = x^2 e^{3x}$
- (5) Solve the following initial value problems:
- (a)  $x^2 y'' - 2xy' + 2y = 0$ ,  $y(1) = 1.5$ ,  $y'(1) = 1$
  - (b)  $x^2 y'' + 3xy' + y = 0$ ,  $y(1) = 3$ ,  $y'(1) = -4$
  - (c)  $x^2 y'' - 3xy' + 4y = 0$ ,  $y(1) = 0$ ,  $y'(1) = 3$
- (6) Use the variation of parameters method to solve the following equations:
- (a)  $y'' + 9y = \csc 3x$
  - (b)  $x^2 y'' - 2xy' + 2y = x^3 \sin x$
  - (c)  $y'' - 4y' + 4y = \frac{6e^{2x}}{x^4}$