Indian Institute of Technology Delhi - Abu Dhabi

**AMTL100: CALCULUS**

**Tutorial Sheet 8**

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(1) Use Riemann integral to prove that:

(a) $\lim_{n\to\infty}(\frac{1}{n+1} + \frac{1}{n+2} + \ldots \frac{1}{2n}) = \ln 2$

(b) $\lim_{n\to\infty} \frac{1}{n}(\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \ldots \sin\frac{n\pi}{n}) = \frac{2}{\pi}$

(2) Discuss the convergence/divergence of the following improper integrals of the first kind:

(a) $\int_0^\infty e^{-x}\cos x \ dx$

(b) $\int_1^\infty \frac{dx}{x^2(e^x+1)}$

(c) $\int_1^\infty \frac{x+1}{x^{3/2}} \ dx$

(d) $\int_0^\infty \frac{dx}{x^2+\sqrt{x}}$

(3) Discuss the convergence/divergence of the following improper integrals of the second kind:

(a) $\int_1^2 \frac{\sqrt{x}}{\ln x} \ dx$

(b) $\int_0^1 \frac{\sin(x^2)}{\sqrt{x}} \ dx$

(c) $\int_1^{\pi/2} \frac{\tan x}{x^{3/2}} \ dx$

(d) $\int_0^3 \frac{\ln x}{\sqrt{|2-x|}} \ dx$

(4) Using Beta and Gamma functions, evaluate the following:

(a) $\int_0^\infty e^{-x^2} \ dx$

(b) $\int_0^{\pi/2} \sqrt{tanx} \ dx$

(c) $\int_0^1 x^m(\ln(1/x))^n \ dx$

(d) $\int_0^{\pi/2} \sin^4\theta\cos^6\theta \ d\theta$