AMTL 101 (Linear Algebra and Differential Equations)
Major Exam

Date: 22/05/2025 Total Marks: 40 Time: 2 hours

1. Consider the IVP: $y' = x + y,\ y(0) = 0.$ [5 = 2 + 3]
   (a) Find the solution of the given IVP.
   (b) Using the Picard's approximation, find a formula for the n-th iterate $y_n(x)$ for the above IVP. Also, show that $y_n(x)$ converges to the solution obtained in (a).

2. Find the general solution of the following ODE using the power series method. [5]
$$y'' - 2xy' - 2y = 0.$$

3. Solve the following initial value problem using Laplace transform method: [5]
$$y'' - 2y' = \delta(t-1), \quad t > 0$$
$$y(0) = 1, \quad y'(0) = 0.$$

4. Find the Laplace transform of the function [4]
$$f(t) = \begin{cases} t, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ 3 - t, & 2 < t < 3 \\ 0, & t > 3. \end{cases}$$

5. Find the inverse Laplace transforms of the following functions. [4 = 2 + 2]
   (a) $F(s) = \dfrac{1}{s^2 - 2s - 3}$
   (b) $G(s) = \tan^{-1}\left(\dfrac{3}{s}\right)$

6. Let $X(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$, $A = \begin{pmatrix} -4 & 12 \\ -3 & 8 \end{pmatrix}$ and $g(t) = e^{2t}\begin{pmatrix} 6t \\ 1-t \end{pmatrix}$. [6 = 3 + 3]
   (a) Find two linearly independent solutions for the homogeneous system $X' = AX$.
   (b) Use variation of parameters formula, to find a particular solution for the non-homogeneous system $X' = AX + g(t)$.

7. Consider the following initial value problem: [6 = 2 + 2 + 2]
$$y' = \frac{10}{3}xy^{2/5}, \quad y(x_0) = y_0.$$
   (a) Show that for any x_0, y_0 in $\mathbb{R}$ the above IVP has at least one solution.
   (b) For $y_0 \neq 0$ and $x_0 \in \mathbb{R}$, show that the above IVP has a unique solution.
   (c) For $x_0 = 0$ and $y_0 = 0$, show that the above IVP has more than one solution.

8. Prove that there is no onto linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^5$. Give an example of an onto linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^3$. [5 = 3 + 2]

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