Quiz 1 Solutions
AMTL101

1. The augmented matrix for the given system of linear equations is:

$$[A:b] = \begin{pmatrix} 2 & 3 & 1 & 2 & 8 \\ 3 & 2 & 2 & 1 & 8 \\ 0 & 5 & -1 & 4 & 8 \end{pmatrix}$$

$$\xrightarrow[R_1 \to \frac{R_1}{2}]{} \begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} & 1 & 4 \\ 3 & 2 & 2 & 1 & 8 \\ 0 & 5 & -1 & 4 & 8 \end{pmatrix}$$

$$\xrightarrow[R_2 \to R_2 - 3R_1]{} \begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} & 1 & 4 \\ 0 & -\frac{5}{2} & \frac{1}{2} & -2 & -4 \\ 0 & 5 & -1 & 4 & 8 \end{pmatrix}$$

$$\xrightarrow[R_2 \to -\frac{2}{5}R_2]{} \begin{pmatrix} 1 & \frac{3}{2} & \frac{1}{2} & 1 & 4 \\ 0 & 1 & -\frac{1}{5} & \frac{4}{5} & \frac{8}{5} \\ 0 & 5 & -1 & 4 & 8 \end{pmatrix}$$

$$\xrightarrow[R_1 \to R_1 - \frac{3}{2}R_2]{R_3 \to R_3 - 5R_2} \begin{pmatrix} 1 & 0 & \frac{4}{5} & -\frac{1}{5} & \frac{8}{5} \\ 0 & 1 & -\frac{1}{5} & \frac{4}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The above matrix is the RRE form of $[A:b]$; free variables are x_3 and x_4. Putting $x_3 = \lambda$ and $x_4 = \mu$, we get

$$x_1 + \frac{4}{5}\lambda - \frac{1}{5}\mu = \frac{8}{5},$$
$$x_2 - \frac{1}{5}\lambda + \frac{4}{5}\mu = \frac{8}{5}.$$

$\therefore$ Solution space $(A) =$

$$\left\{ \left(\frac{8}{5} - \frac{4}{5}\lambda + \frac{1}{5}\mu,\ \frac{8}{5} + \frac{1}{5}\lambda - \frac{4}{5}\mu,\ \lambda,\ \mu \right);\ \lambda, \mu \in \mathbb{R} \right\}.$$

2.

$$[A:I] = \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ -1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow[R_2 \to R_2 + R_1]{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow[R_2 \to \frac{1}{2} R_2]{} \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow[R_3 \to R_3 + R_2]{} \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 1 \end{pmatrix}$$

$$\xrightarrow[R_3 \to 2R_3]{} \begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow[R_1 \to R_1 - 3R_3]{R_2 \to R_2 - \frac{5}{2} R_3} \begin{pmatrix} 1 & 2 & 0 & 4 & -3 & -6 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{pmatrix}$$

$$\xrightarrow[R_1 \to R_1 - 2R_2]{} \begin{pmatrix} 1 & 0 & 0 & -2 & 1 & 4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{pmatrix}$$

So, the inverse of A is $\begin{pmatrix} -2 & 1 & 4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$.

3. It is easy to see that $(1,0) \in W$ since $1^2 + 0^2 = 1 \leq 1$, but $2(1,0) = (2,0) \notin W$ because $2^2 + 0^2 = 4 > 1$. $\therefore$ W is not closed with respect to scalar multiplication hence, not a subspace of $\mathbb{R}^2$ over the field $\mathbb{R}$.