Indian Institute of Technology Delhi - Abu Dhabi

AMTL100: CALCULUS

Tutorial Sheet 10

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(1) Determine whether the limit as $(x, y) \to (0, 0)$ exists for the following functions.

(a)

$$f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2-y^2}, & x^2 \neq y^2 \\ 0, & x^2 = y^2 \end{cases}$$

(b)

$$f(x,y) = \begin{cases} x \sin\left(\frac{1}{y}\right) + y \sin\left(\frac{1}{x}\right), & xy \neq 0 \\ 0, & xy = 0 \end{cases}$$

(c)

$$f(x,y) = \begin{cases} \frac{\sin(xy)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(2) Examine the continuity of the following functions.

(a)

$$f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6}, & (x,y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases}$$

(b)

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & \text{otherwise} \end{cases}$$

(3) Find local minima and local maxima points of the function $f(x, y) = xye^{-(x^2+y^2)}$.

(4) Find the maximum and minimum values of $f(x, y) = 3x+4y$ subject to the constraint $x^2 + 4xy + 5y^2 = 10$.

(5) Use Lagrange multiplier method to find the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ which are nearest and farthest from the straight line $3x + y - 9 = 0$.

(6) A farmer wishes to build a rectangular storage bin, without a top, with a volume of 500 cubic meters. Find the dimensions of the bin that will minimize the amount of material needed in its construction.