

AMTL 100 (CALCULUS)  
Major Exam

Date: 16/12/2024

Total Marks: 40

Time: 2 hours

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1. Use Lagrange multiplier method to find the maximum value of  $xy$  subject to the constraint  $x + y = 4$ . [5]

2. Find the local minima, local maxima and saddle points of the function: [5]

$$f(x, y) = x^3 - y^3 - 2xy + 6$$

3. Find the limit of  $f$  as  $(x, y) \rightarrow (0, 0)$  or show that the limit does not exist. [5]

$$f(x, y) = \sin\left(\frac{x^3 - y^3}{x^2 + y^2}\right)$$

4. Find the area of the surface generated by revolving the following curve about the  $x$ -axis. [5]

$$y = x^3, \quad 0 \leq x \leq 1$$

5. Using appropriate transformations evaluate the following double integral. [5]

$$\int_0^{2/3} \left( \int_y^{2-2y} (x + 2y)e^{(y-x)} dx \right) dy$$

6. Evaluate the following volume integrals [5]

$$\iiint_D e^{(x^2+y^2+z^2)^{3/2}} dV,$$

where  $D = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 \leq 1\}$ .

7. Determine whether the following series converges or diverges. [5]

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$$

8. Find the radius and interval of convergence of the following power series. [5]

$$\sum_{n=1}^{\infty} \left( \frac{n}{n+2} \right) x^n$$