1. Use the definition to find the derivative of each of the following functions:
   (a) $f(x) := x^3$ for $x \in \mathbb{R}$,
   (b) $g(x) := 1/x$ for $x \in \mathbb{R}, x \neq 0$
   (c) $h(x) := \sqrt{x}$ for $x > 0$
   (d) $k(x) := 1/\sqrt{x}$ for $x > 0$

2. Show that $f(x) := x^{1/3}, x \in \mathbb{R}$, is not differentiable at $x = 0$.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := x^2$ for x rational, $f(x) := 0$ for x irrational. Show that f is differentiable at $x = 0$, and find $f'(0)$.

4. Differentiate and simplify:
   (a) $f(x) := \frac{x}{1+x^2}$
   (b) $g(x) := \sqrt{5 - 2x + x^2}$
   (c) $h(x) := \left(\sin x^k\right)^m$ for $m, k \in \mathbb{N}$
   (d) $k(x) := \tan\left(x^2\right)$ for $|x| < \sqrt{\pi/2}$

5. Let $n \in \mathbb{N}$ and let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := x^n$ for $x \geq 0$ and $f(x) := 0$ for $x < 0$. For which values of n is f' continuous at 0 ? For which walues of n is f' differentiable at 0 ?

6. Deternine where each of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is differentiable and find the derivative:
   (a) $f(x) := |x| + |x + 1|$
   (b) $g(x) := 2x + |x|$
   (c) $h(x) := x|x|$
   (d) $k(x) := |\sin x|$

7. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) := x^2 \sin t/x^2$) for $x \neq 0$, and $g(0) := 0$. Show that g is differentiable for all $x \in \mathbb{R}$. Also show that the derivative g' is not bounded on the interval $[-1, 1]$.

8. If $r > 0$ is a rational number, let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := x^r \sin(1/x)$ for $x \neq 0$, and $f(0) := 0$. Decermine those values of r for which $f'(0)$ exists.

9. Given that the function $h(x) := x^3 + 2x + 1$ for $x \in \mathbb{R}$ has an inverse h^{-1} on $\mathbb{R}$, find the value of $\left(h^{-1}\right)'(y)$ at the points corresponding to $x = 0, 1, -1$.

10. For each of the following functions on $\mathbb{R}$ to $\mathbb{R}$, find points of relative extrema, the intervals on which the function is increasing, and chose on which it is decreasing:
    (a) $f(x) := x^2 - 3x + 5$
    (b) $g(x) := 3x - 4x^2$
    (c) $h(x) := x^3 - 3x - 4$
    (d) $k(x) := x^4 + 2x^2 - 4$

11. Find the points of relative extrema, the intervals on which the following functions are increasing, and those on which they are decreasing:

(a) $f(x) := x + 1/x$ for $x \neq 0$

(b) $g(x) := x/(x^2 + 1)$ for $x \in \mathbb{R}$

(c) $h(x) := \sqrt{x} - 2\sqrt{x+2}$ for $x > 0$

(d) $k(x) := 2x + 1/x^2$ for $x \neq 0$

12. Find the points of relative extrema of the following functions on the specified domain:

(a) $f(x) := |x^2 - 1|$ for $-4 \leq x \leq 4$

(b) $g(x) := 1 - (x-1)^{2/3}$ for $0 \leq x \leq 2$

(c) $h(x) := x\,|x^2 - 12|$ for $-2 \leq x \leq 3$

(d) $k(x) := x(x-8)^{1/3}$ for $0 \leq x \leq 9$

13. Let $a_1, a_2, \cdots, a_n$ be real numbers and let f be defined on $\mathbb{R}$ by

$$f(x) := \sum_{i=1}^{n} (a_i - x)^2 \qquad \text{for} \qquad x \in \mathbb{R}.$$

Find the unique point of relative minimum for f.

14. Let $a > b > 0$ and let $n \in \mathbb{N}$ satisfy $n \geq 2$. Prove that $a^{1/n} - b^{1/n} < (a-b)^{1/n}$. [Hint: Show that $f(x) := x^{1/n} - (x-1)^{1/n}$ is decreasing for $x \geq 1$, and evaluate f at I and a/b.]

15. Use the Mean Value Theorem to prove that $|\sin x - \sin y| \leq |x - y|$ for all x, y in $\mathbb{R}$.

16. Use the Mean Value Theorem to prove that $(x-1)/x < \ln x < x - 1$ for $x > 1$. [Hint: Use the fact that $D \ln x = 1/x$ for $x > 0$.]

17. Let $f : [a, b] \to \mathbb{R}$ be continuous on $[a, b]$ and differentiable in (a, b). Show that if $\lim_{x \to a} f'(x) = A$, then $f'(a)$ exists and equals A. [Hint: Use the definition of $f'(a)$ and the Mean Value Theorem.]

18. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) := 2x^4 + x^4 \sin(1/x)$ for $x \neq 0$ and $f(0) := 0$. Show that f has an absolute minimum at $x = 0$, but that its derivative has both positive and negative values in every neigtborhood of 0.

19. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) := x + 2x^2 \sin(1/x)$ for $x \neq 0$ and $g(0) := 0$. Show that $g'(0) = 1$, but in every neighborhood of 0 the derivative $g'(x)$ takes on both positive and negative values. Thus g is not monotonic in any neighborhood of 0.

20. Evaluate the following limits, where the domain of the quolient is as indicated.

(a) $\lim_{x \to 0+} \frac{\ln(x+1)}{\sin x}$ $(0, \pi/2)$

(b) $\lim_{x \to 0+} \frac{\tan x}{x}$ $(0, \pi/2)$

(c) $\lim_{x \to 0+} \frac{\ln \cos x}{x}$ $(0, \pi/2)$

(d) $\lim_{x \to 0+} \frac{\tan x - x}{x^3}$ $(0, \pi/2)$

21. Evaluate the following limits:

(a) $\lim_{x \to 0} \frac{\operatorname{Arctan} x}{x}$ $(-\infty, \infty)$

(b) $\lim_{x \to 0} \frac{1}{x(\ln x)^2}$ $(0, 1)$

(c) $\lim_{x \to 0+} x^3 \ln x$ $(0, \infty)$

(d) $\lim_{s \to \infty} \frac{x^3}{e^x}$ $(0, \infty)$

22. Evaluate the following limits:

(a) $\lim_{x \to \infty} \frac{\ln x}{x^2}$ $(0, \infty)$

(b) $\lim_{t\to\infty} \frac{\ln x}{\sqrt{x}}$ $(0, \infty)$

(c) $\lim_{x\to 0} x \ln \sin x$ $(0, \pi)$

(d) $\lim_{x\to\infty} \frac{x+\ln x}{x \ln x}$ $(0, \infty)$

23. Evaluate the following linuts:

(a) $\lim_{x\to 0+} x^{2x}$ $(0, \infty)$

(b) $\lim_{x\to 0}(1 + 3/x)^x$ $(0, \infty)$

(c) $\lim_{x\to\infty}(1 + 3/x)^x$ $(0, \infty)$

(d) $\lim_{x\to +\infty} \left(\frac{1}{x} - \frac{1}{\text{Arctan}\, x} \right)$ $(0, \infty)$

24. Evaluate the following limits:

(a) $\lim_{x\to\infty} x^{1/x}$ $(0, \infty)$

(b) $\lim_{x\to 0+}(\sin x)^x$ $(0, \pi)$

(c) $\lim_{x\to 0+} x^{\sin x}$ $(0, \infty)$

(d) $\lim_{x\to\pi/2-}(\sec x - \tan x)$ $(0, \pi/2)$

25. Try to use L'Hospital's Rule to find the limit of $\frac{\tan x}{\sec x}$ as $x \to (\pi/2)^-$. Then evaluate directly by changing to sines and cosines.