Indian Institute of Technology Delhi - Abu Dhabi
AMTL100: CALCULUS
Practice problem Sheet 2

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(1) Determine the radius of convergence of the series $\sum a_n x^n$, where a_n is given by:
(a) $1/n^n$, (c) $n^n/n!$, (b) $n^\alpha/n!$, (c) $(n!)^2/(2n)!$, (d) $(\ln n)^{-1}$, $\quad n \geq 2$, (f) $n^{-\sqrt{n}}$.

(2) Discuss the convergence or the divergence of the series with nth term:
(a) $2^n e^{-n}$, (b) $n^n e^{-n}$, (c) $e^{-\ln n}$, (d) $(\ln n)e^{-\sqrt{n}}$, (e) $n!e^{-n}$, (f) $n!e^{-n^2}$.

(3) Discuss the convergence or the divergence of the series whose nth term is:
(a) $\frac{n!}{3\cdot 5\cdot 7\cdots(2n+1)}$, (b) $\frac{(n!)^2}{(2n)!}$, (c) $\frac{2\cdot 4\cdots(2n)}{3\cdot 5\cdots(2n+1)}$, (d) $\frac{2\cdot 4\cdots(2n)}{5\cdot 7\cdots(2n+3)}$.

(4) Use power series operations to find the Taylor series at $x = 0$ for the following functions:
(a) xe^x, (b) $x^2 \sin x$, (c) $\frac{x^2}{2} - 1 + \cos x$, (d) $\sin x - x + \frac{x^3}{3!}$, (e) $x \cos \pi x$, (f) $x^2 \cos(x^2)$, (g) $\cos^2 x$ (Hint: $\cos^2 x = (1 + \cos 2x)/2$.), (h) $\sin^2 x$, (i) $\frac{x^2}{1-2x}$, (j) $x \ln(1+2x)$, (k) $\frac{1}{(1-x)^2}$, (l) $\frac{2}{(1-x)^3}$, (m) $x \tan^{-1} x^2$, (n) $\sin x \cdot \cos x$, (o) $e^x + \frac{1}{1+x}$, (p) $\cos x - \sin x$, (q) $\frac{x}{3} \ln(1 + x^2)$, (r) $\ln(1+x) - \ln(1-x)$.

(5) Find the first four nonzero terms in the Maclaurin series for the functions:
(a) $e^x \sin x$, (b) $\frac{\ln(1+x)}{1-x}$, (c) $(\tan^{-1} x)^2$, (d) $\cos^2 x \cdot \sin x$, (e) $e^{\sin x}$, (f) $\sin(\tan^{-1} x)$.

(6) Write down the first four terms in the binomial series for the given function:
(a)$(1 + 3x)^{-6}$, $\quad$ (b)$\sqrt[3]{8 - 2x}$.