Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 1: Linear Algebra

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(1) Suppose A and B are square matrices of the same size. Prove or disprove the following statements.
   (a) $\det(cA) = c\det(A)$ for any $c \in R$.
   (b) $\det(A + B) = \det(A) + \det(B)$.
   (c) $\text{trace}(cA) = c\,\text{trace}(A)$ for any $c \in R$.
   (d) $\text{trace}(A + B) = \text{trace}(A) + \text{trace}(B)$.
   (e) $\text{trace}(AB) = \text{trace}(BA)$.

(2) Suppose we have a system of two linear equations in three unknowns x, y, z:

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

Can this system have a unique solution? Interpret geometrically the following statements: (a) the system has no solutions. (b) the system has infinitely many solutions.

(3) Solve the following system of equations (for finding x, y in $\mathbb{C}$). Think of more than one way of solving and decide which method is better.

$$(1 - i)x + (1 + i)y = 2 + 3i$$

$$(1 + i)x + (1 - i)y = 3 - i$$

(4) Prove that for any system of linear equations $AX = b$ with real coefficients there are only three possibilities: (a) no solution (b) unique solution (c) infinitely many solutions.