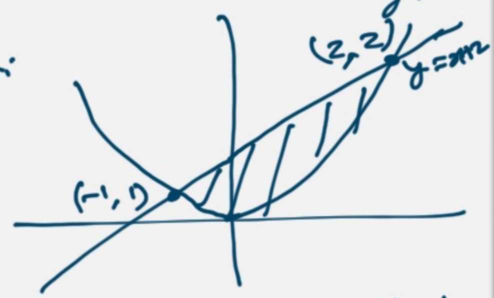
Tutorial 11

1. (a) $I = \iint_R x^2 \, dA$; R is the region bounded by $y = x^2$ & $y = x+2$



Soln: Pts. of intersection:

$x^2 = x+2$

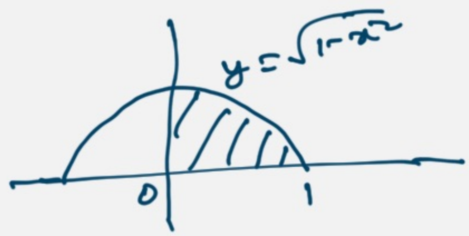
$\Rightarrow \quad x = -1$ or $x = 2$

$(-1,1)$ & $(2,2)$

$$I = \int_{-1}^{2} \int_{x^2}^{x+2} x^2 \, dy \, dx = \int_{-1}^{2} x^2 (x+2-x^2) \, dx$$

1. (b) $\iint_R (x^2+y^2) \, dA$;

$R : \{ (x,y) : 0 \le y \le \sqrt{1-x^2}, \ 0 \le x \le 1 \}$

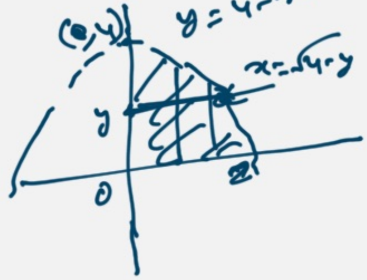


Soln: Polar coords.

$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{1} r^2 \cdot r \, dr \, d\theta$$

$$= \frac{\pi}{2} \times \frac{1}{4} = \frac{\pi}{8} .$$

2.(c) $\displaystyle\int_0^2\int_0^{4-x^2} \frac{x e^{2y}}{4-y}\,dy\,dx$

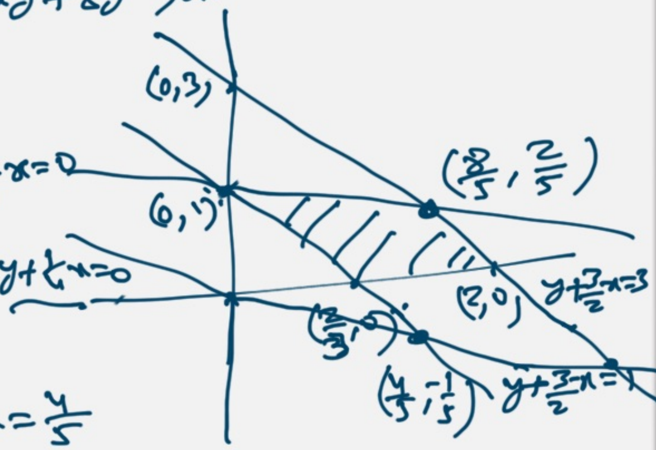


$\displaystyle = \int_0^4 \left(\int_0^{\sqrt{4-y}} x\,dx\right)\frac{e^{2y}}{4-y}\,dy$

$\displaystyle = \int_0^4 \frac{1}{2}\cancel{(4-y)}\cdot\frac{e^{2y}}{\cancel{(4-y)}}\,dy$

$\displaystyle = \frac{1}{4}e^{2y}\Big|_0^4 = \frac{1}{4}\left(e^8-1\right)$

(4) $\displaystyle\iint_R \left(3x^2+14xy+8y^2\right)dA$



$y+\frac{3}{2}x = 1$; $y+\frac{1}{4}x = 0$

$-\frac{1}{4}x = 1-\frac{3}{2}x$

$\Rightarrow \left(\frac{3}{2}-\frac{1}{4}\right)x = 1$

$\Rightarrow \frac{5}{4}x = 1 \Rightarrow x = \frac{4}{5}$

$y = -\frac{1}{5}$

$y+\frac{3}{2}x = 3$; $y+\frac{1}{4}x = 1$ $\Rightarrow 3-\frac{3}{2}x = 1-\frac{1}{4}x$

$\Rightarrow \frac{5}{4}x = 2 \Rightarrow x = \frac{8}{5}$

$y = \frac{3}{5}$

$3x^2 + 14xy + 8y^2 = (3x+2y)(x+4y)$

Put $u = 3x+2y$; $v = x+4y$

$J = \frac{\partial(x,y)}{\partial(u,v)} \Rightarrow \frac{1}{J} = \frac{\partial(u,v)}{\partial(x,y)}$

$= \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$

$= 10$

$\Rightarrow J = \frac{1}{10}$

$\iint_R (3x+2y)(x+4y)\,dA$

$= \int_{v=0}^{4} \int_{u=2}^{6} uv \cdot \frac{1}{10}\,du\,dv$

(5) (a) $$\iiint_D z^2(x^2+y^2)\,dV$$

$$D = \{\, x^2+y^2 \le 1 \,,\ -1 \le z \le 1 \,\}$$

Cylindrical coords:

$$dV = r\,dr\,d\theta\,dz$$

$$\int_{z=-1}^{1}\int_{\theta=0}^{2\pi}\int_{r=0}^{1} z^2 \cdot r^2 \cdot r\,dr\,d\theta\,dz$$

$$= 2\pi \times \frac{2}{3} \times \frac{1}{4} = \frac{\pi}{3}.$$