Indian Institute of Technology Delhi - Abu Dhabi

AMTL101

Tutorial Sheet 6: Linear Algebra

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(1) Find the eigenvalues of A, B and C:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{pmatrix}, \; B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}, \; C = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

(2) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $a + b = c + d$. Show that $(1, 1)$ is an eigenvector of A and find the eigenvalues.

(3) Show that the following matrix A is diagoanlizable and find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{2}{5} & \frac{9}{5} & \frac{2}{5} \\ -\frac{1}{5} & \frac{2}{5} & \frac{6}{5} \end{pmatrix}.$$

(4) Use diagonalizability to compute A^k for any natural number k:

$$A = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}.$$

Also, compute the same using the Cayley-Hamilton theorem.

(5) Show that the eigenvalues of A and A^T are the same. Show by an example that the eigenvectors of A and A^T need not be the same.