probability and statistics

- ## $Sure, let's present the solution in a more textual format:$*

$A) Sample space$\\(S\\)$:$

$The sample space for flipping a balanced coin three times independently consists of all possible outcomes. Since each,$

\\($S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$\\)

$B) Assigning values of$ \\(X\\) $to each sample point:$

Let \\(X\\) $represent the difference between the number of heads and the number of tails. For each outcome, we calcul$

- \\($X(HHH) = 3 - 0 = 3$\\)
- \\($X(HHT) = 2 - 1 = 1$\\)
- \\($X(HTH) = 2 - 1 = 1$\\)
- \\($X(HTT) = 1 - 2 = -1$\\)
- \\($X(THH) = 1 - 2 = -1$\\)
- \\($X(THT) = 1 - 2 = -1$\\)
- \\($X(TTH) = 0 - 3 = -3$\\)
- \\($X(TTT) = 0 - 3 = -3$\\)

$C) Probability distribution function of$ \\(X\\)$:$

$Wefindtheprobabilityforeachvalueof$ \(X \) :

- \($P(X=-3)=\frac{2}{8}=\frac{1}{4}$ \)

- \($P(X=-1)=\frac{3}{8}$ \)

- \($P(X=1)=\frac{2}{8}=\frac{1}{4}$ \)

- \($P(X=3)=\frac{1}{8}$ \)

$D)Probability$ \($P(X\leq 1)$ \) :

\[$P(X\leq 1)=P(X=-3)+P(X=-1)+P(X=1)=\frac{1}{4}+\frac{3}{8}+\frac{1}{4}=\frac{5}{8}$ \]

$E)Probability$ \($P(X<1)$ \) :

\[$P(X<1)=P(X=-3)+P(X=-1)=\frac{1}{4}+\frac{3}{8}=\frac{5}{8}$ \]

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`$$F) Mean \( \mu - E(X) \):$`$

\[E(X) = (-3) \times \frac{1}{4} + (-1) \times \frac{3}{8} + (1) \times \frac{1}{4} + (3) \times \frac{1}{8} = -\frac{1}{8}]

\G) Variance (\sigma^2 - \text{Var}(X)):

\[\text{Var}(X) = \frac{15}{4} - \left(-\frac{1}{8}\right)^2 = \frac{15}{4} - \frac{1}{64} = \frac{239}{64}]

\So, the variance of (X) is (\frac{239}{64}).$$`

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