# What is Fuzzy Logic?

# *Always be on Focus → Master Slide*

**CS-464**

- **Evolutionary Computation**
  - W4-5: Genetic Algorithm
  - W6: Particle Swarm
- **Fuzzy Logic**
  - W2: Introduction to Fuzzy Logic
  - W3: Fuzzy Logic Control
- **Machine Learning**
  - W8: Introduction to ML
  - W9: Clustering Technique
  - W10: Linear Regression
  - W11: Logistic Regression
  - W13: Support Vector Machine
  - W14: Neural Network

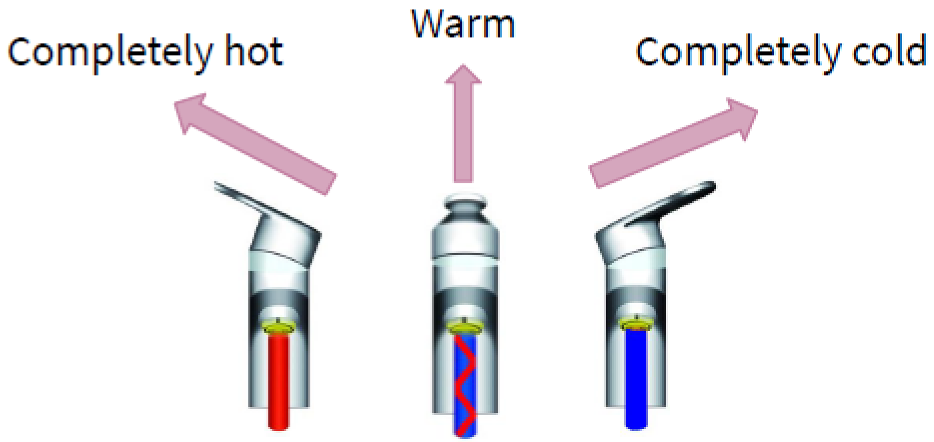
# Motivation

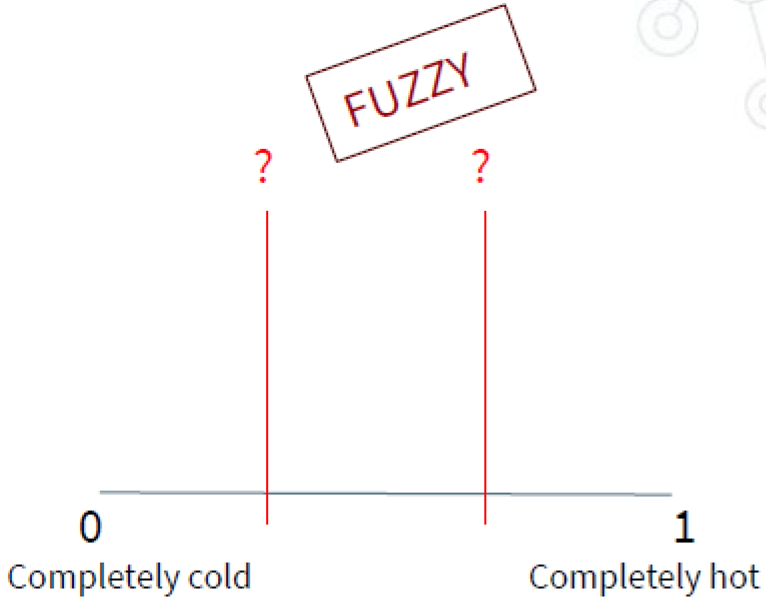
In classical logic, conclusions can only be either TRUE or FALSE (1 or 0)

Example:
"The test is difficult" (if not difficult, it's easy)
"The test is easy" (if not easy, it's difficult)

However in real-life, conclusions may be PARTIAL truths:

"The test is normal"
"The test is rather difficult"
"The test is somewhat easy"
...



# Fuzzy Logic

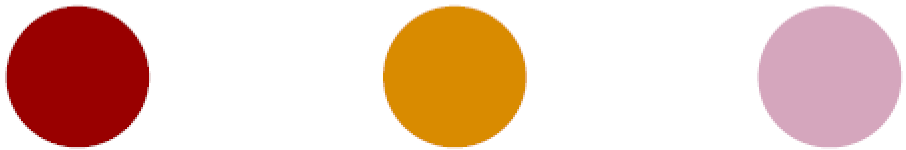
Fuzzy logic is a form of logic in which the truth values of variables can be any real number between 0 and 1.

In contrast with Boolean logic:
variables can only be either 0 or 1.

Example:
- "Do you find that movie interesting?"
- "A little" (not good, not bad) → fuzzy

Inspired by the way human processes information with imprecise or partial knowledge

Red or Brown? Orange or Yellow? Pink or Purple?



# Linguistic variables

In mathematics: variables usually take
numerical values.

In Fuzzy Logic: non-numerical variables
are often used to represent facts and rules
→ LINGUISTIC VARIABLES

Example of "temperature" variable:
In maths (°C): -10, -5, 0, 7, 9, 15, 20, 22, 30 ...
In Fuzzy Logic: cold, warm, hot … (linguistic  values)


10° C!!!
Is it "cold", "warm", or "hot"?

It depends, but personally:
40% cold, 60% warm
(I'm living in France)

Cold, Warm, and Hot → FUZZY SETS

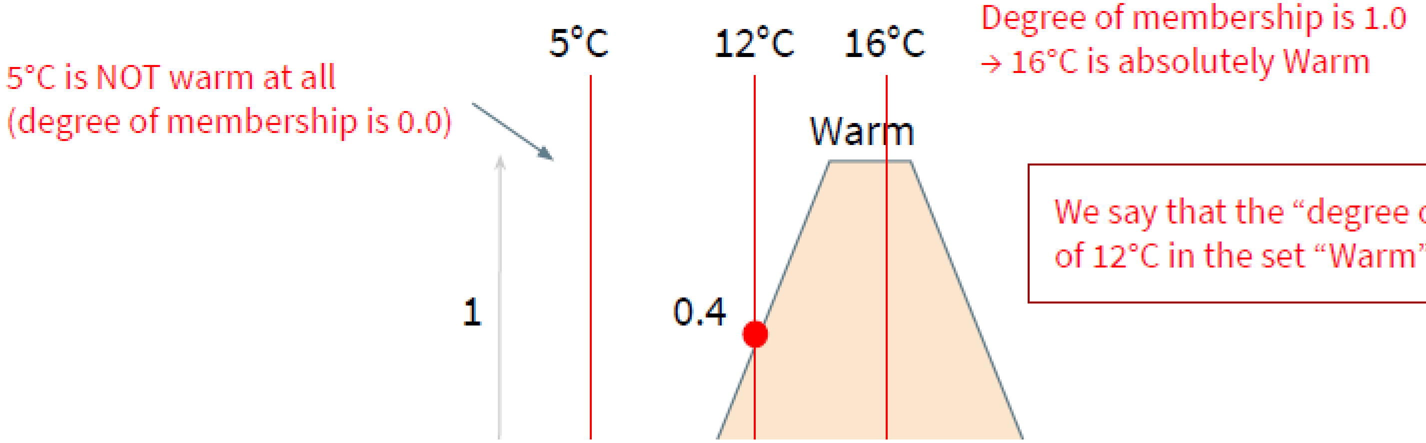
# Fuzzy set

In classical logic, given a set *A*, for any element *x*, then either: $x \in A$ or $x \notin A$
(*x* is either a member or not a member of *A*)

**Example:**
France is a member of EU (degree of membership = 1)
USA is not a member of EU (degree of membership = 0)

In Fuzzy logic, an element is allowed to be **partially** a member of a set, i.e. the degree of membership is not restricted to {0, 1} but any value in [0, 1]

**Example:**
The set "Warm" is described by the following trapezoid, then for a temperature of 12°C:




We say that the "degree of membership" of 12°C in the set "Warm" is **0.4**

# Fuzzy set

Given an element space *U*, a Fuzzy set *A* is represented by a function

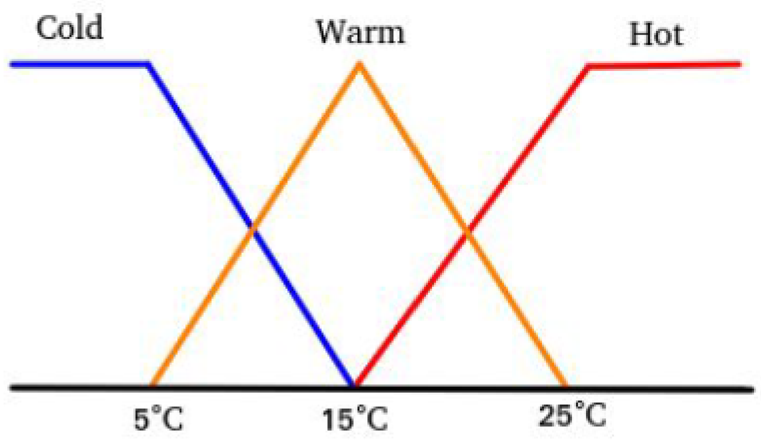
$$m : U \to [0, 1]$$

Then, for any $x \in U$ :

$$\begin{cases} \text{if} \quad m(x) = 0 \ \Rightarrow \ \text{x does not belong to A} \\ \text{if} \quad m(x) = 1 \ \Rightarrow \ \text{x fully belongs to A} \\ \text{if} \quad 0 < m(x) < 1 \ \Rightarrow \ \text{x partially belongs to A} \end{cases}$$

*U* is also called the **universe of discourse**.
*m* is also called the **membership function** of the fuzzy set *A*.
*m(x)* = degree of membership of *x* in *A*.

Usually, *m* is often represented by a **triangle** or a **trapezoid** (and sometimes sigmoid curves).
For example the fuzzy sets: Cold, Warm, and Hot can be represented by the following shapes:



# Fuzzification

## Fuzzy control system

Fuzzy control system is a control system based on Fuzzy logic

Usually contains three stages:

- Input (***Fuzzification***)
- Processing (***INFERENCE***)
- Output (***Defuzzification***)

**FUZZIFICATION**

Mapping numerical inputs into fuzzy sets in order to calculate the degrees of membership.
**E.g**: given a temperature as an input, after fuzzification we have: 0.0 Cold, 0.4 Warm, 0.6 Hot

**INFERENCE**

Using "IF … THEN …" rules to infer possible outputs, given result from fuzzification stage.
E.g: IF temperature IS Hot THEN fan_speed IS High
IF temperature IS Warm THEN fan_speed IS Medium

**DEFUZZIFICATION**

Grouping possible outputs from inference stage then compute the final output which is a numerical value.

# Use case

Fuzzy control can be applied in ABS - Anti-lock Braking System in automotives.

ABS is used to prevent the wheels from locking up during braking a vehicle, thereby maintaining tractive contact with the road surface.

There are several factors as the inputs for an ABS but we just take wheels' **speed** and brake's **temperature** into our example.

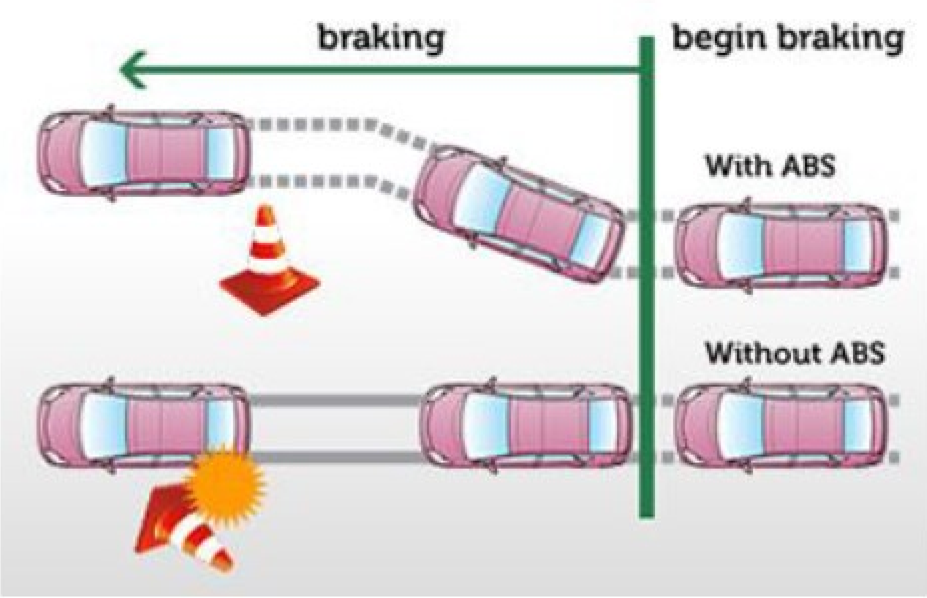
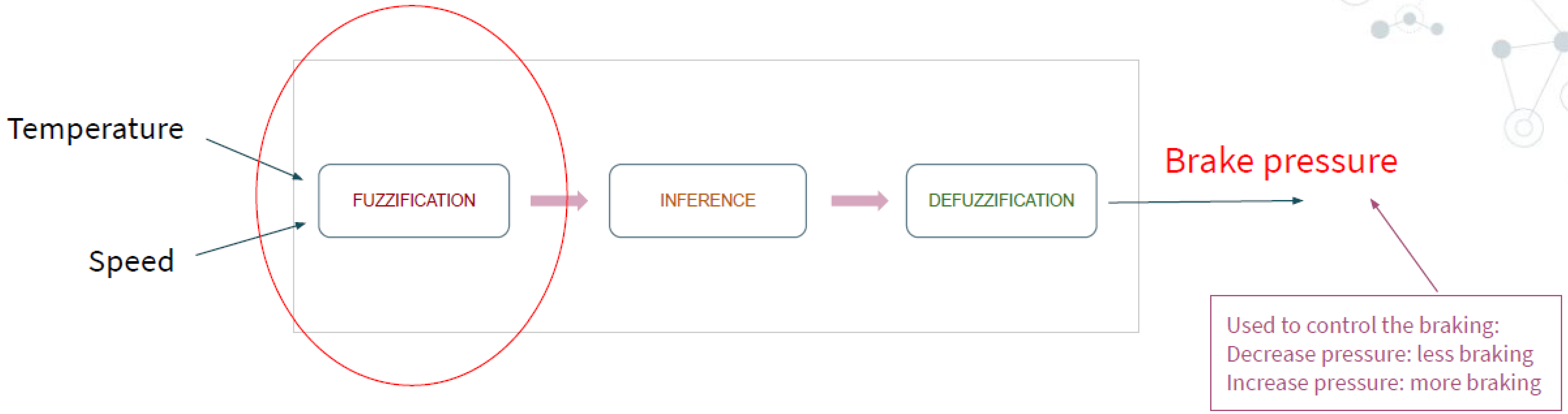



*Image: toyota-malaysia.com*

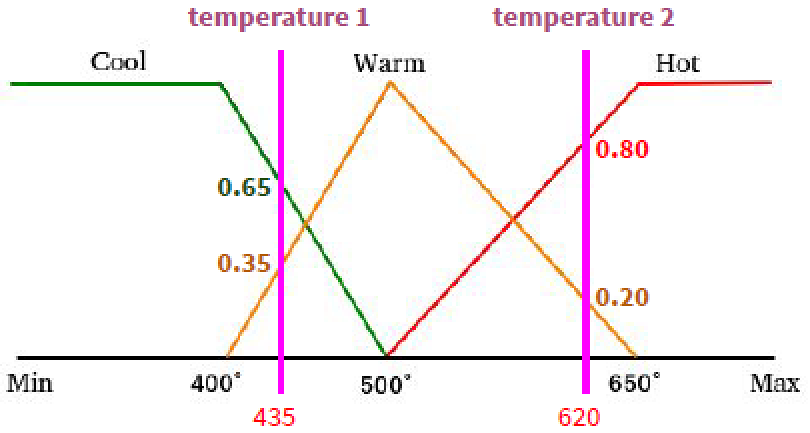
# Use case


Temperature
Speed
FUZZIFICATION
INFERENCE
DEFUZZIFICATION
Brake pressure
Used to control the braking:
Decrease pressure: less braking
Increase pressure: more braking

# Fuzzification

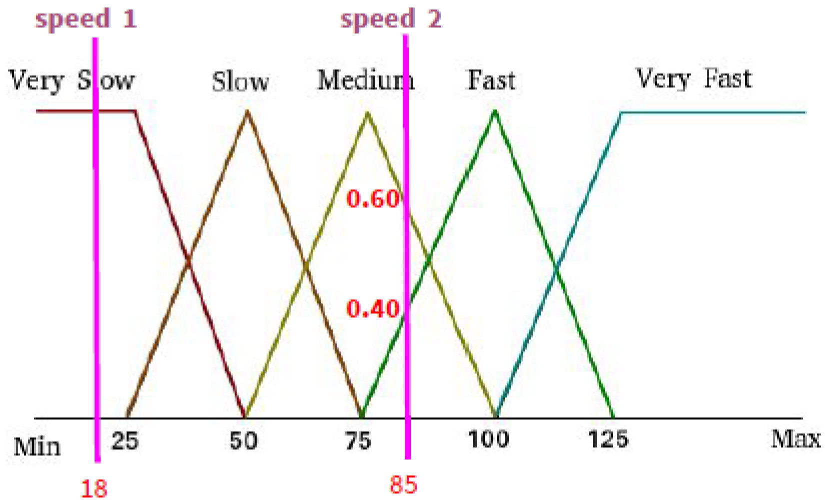
## Temperature

We define 3 Fuzzy sets for linguistic values: Cool, Warm, and Hot, represented by these membership functions:




## Speed

We define 5 Fuzzy sets for linguistic values: Very Slow, Slow, Medium, Fast, Very Fast, represented by these membership functions:




temperature 1 → 0.65 **Cool**, 0.35 **Warm**, 0.00 Hot
temperature 2 → 0.00 **Cool**, 0.20 **Warm**, 0.80 **Hot**

speed 1 → 1.00 Very Slow
speed 2 → 0.60 Medium, 0.40 Fast

NEXT: INFERENCE RULES

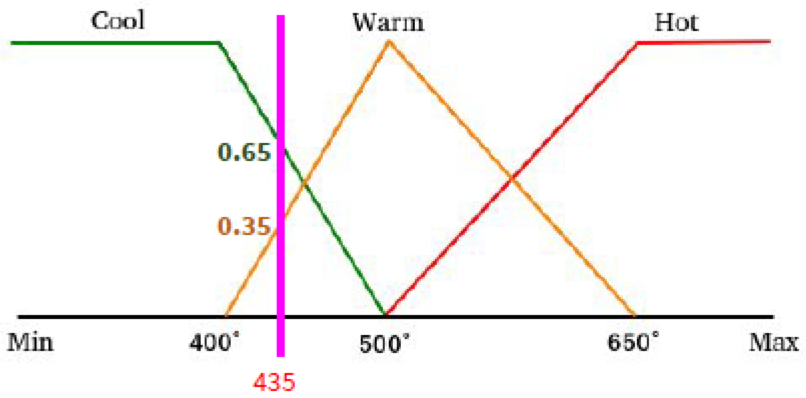
Fuzzification: *converting numerical input values into truth values (degrees of membership)*

# Inference Rules

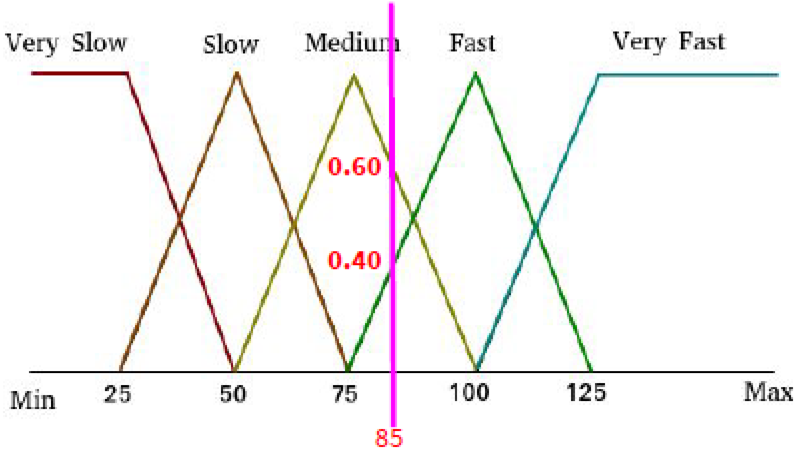
## Fuzzification

Converting each numerical input value into its truth values (degrees of membership)

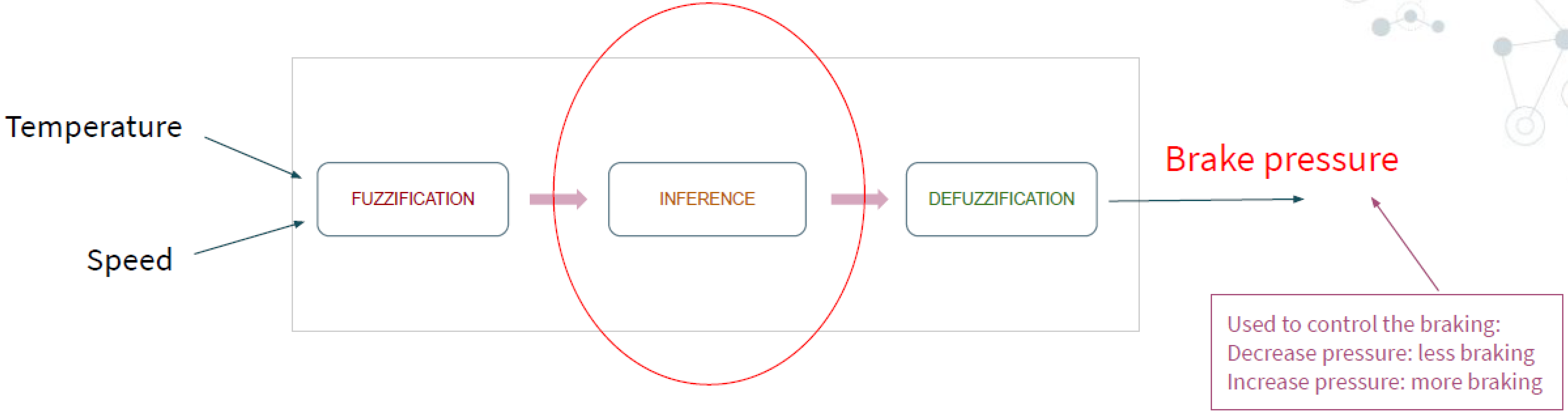
Brake temperature




Wheel speed



# Inference rules



# Inference rules

In this stage, we use a set of **IF-THEN** rules to map truth values (fuzzification results) into desired output values

Examples:

**IF** temperature **IS** Warm **AND** speed **IS** Slow **THEN** brake_pressure **IS** Decreased

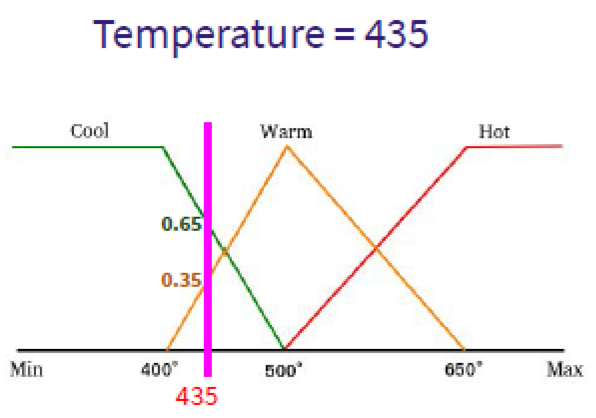
**IF** temperature **IS** **NOT** Hot **AND** speed **IS** Medium **THEN** brake_pressure **IS** Unchanged

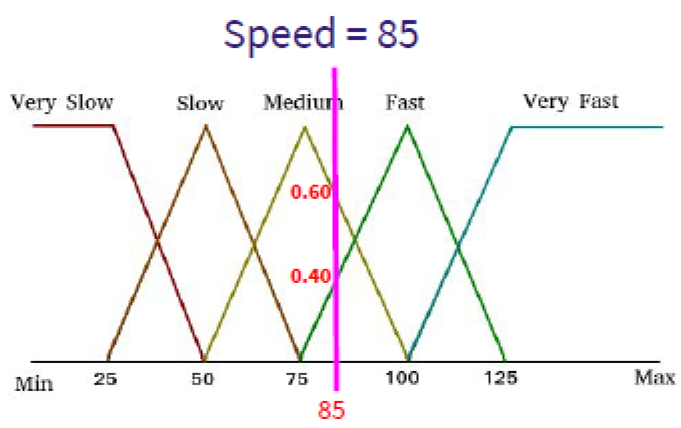
**IF** temperature **IS** Cool **AND** (speed **IS** Fast **OR** speed **IS** Very Fast) **THEN** brake_pressure **IS** Increased

...

These values are then combined to get a compromised output value (numerical)

# Inference rules




**IF** temperature **IS** Warm **AND** speed **IS** Slow **THEN** brake_pressure **IS** Decreased

0.35 0.00 → 0.00

**IF** temperature **IS NOT** Hot **AND** speed **IS** Medium **THEN** brake_pressure **IS** Unchanged

1.00 0.60 → 0.60

**IF** temperature **IS** Cool **AND** (speed **IS** Fast **OR** speed **IS** Very Fast) **THEN** brake_pressure **IS** Increased

0.65 0.40 0.00 → 0.40

0.40

**AND(x, y) = MIN(x, y)**

**NOT(x) = 1 - x**

**OR(x, y) = MAX(x, y)**

**HOW TO COMBINE THESE TRUTH VALUES** (of brake_pressure) **TO GET THE FINAL NUMERICAL OUTPUT?**

# Defuzzification

## Inference rules

INPUTS: temperature=435, speed=85

**IF** temperature **IS** Warm **AND** speed **IS** Slow **THEN** brake_pressure **IS** Decreased

0.35 0.00 → 0.00

**IF** temperature **IS NOT** Hot **AND** speed **IS** Medium **THEN** brake_pressure **IS** Unchanged

1.00 0.60 → 0.60

**IF** temperature **IS** Cool **AND** (speed **IS** Fast **OR** speed **IS** Very Fast) **THEN** brake_pressure **IS** Increased

0.65 0.40 0.00 → 0.40

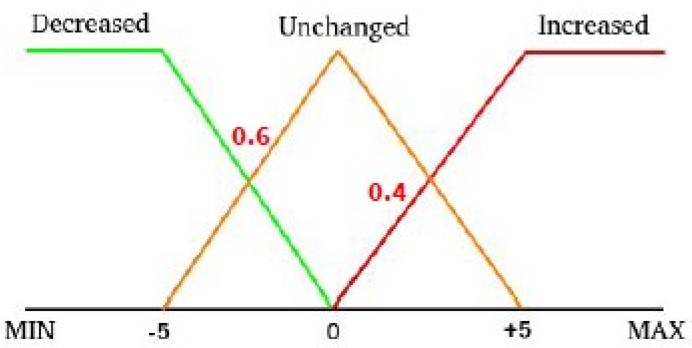
Use **DEFUZZIFICATION** to combine these outputs

# Defuzzification

From inference stage, we have:
brake_pressure: **0.6** Unchanged, **0.4** Increased

*→ Objective: finding a compromised numerical value from these outputs of inference stage*

*Membership functions for brake_pressure*




1 Plot the membership functions for the output

# Defuzzification

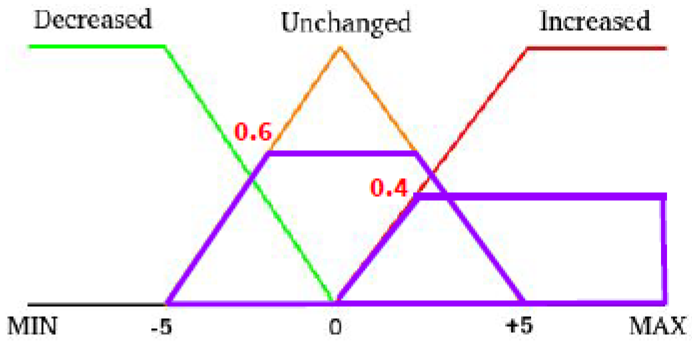
From inference stage, we have:
brake_pressure: **0.6** Unchanged, **0.4** Increased

*→ Objective: finding a compromised numerical value from these outputs of inference stage*

1. Plot the membership functions for the output

2. For each truth value (degree of membership), cut the membership function at that value

*Membership functions for brake_pressure*



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From inference stage, we have:
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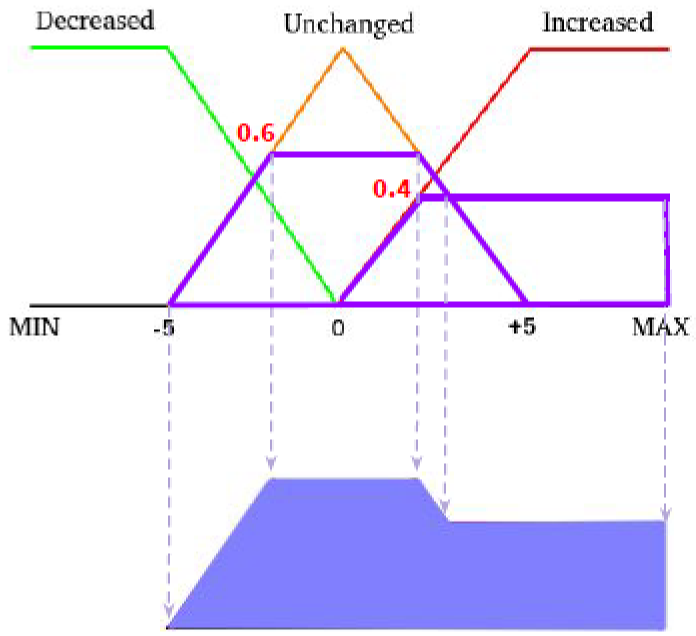
*→ Objective: finding a compromised numerical value from these outputs of inference stage*

1. Plot the membership functions for the output
2. For each truth value (degree of membership), cut the membership function at that value
3. Combine the resulting curves to get an area using the OR operator

*Membership functions for brake_pressure*



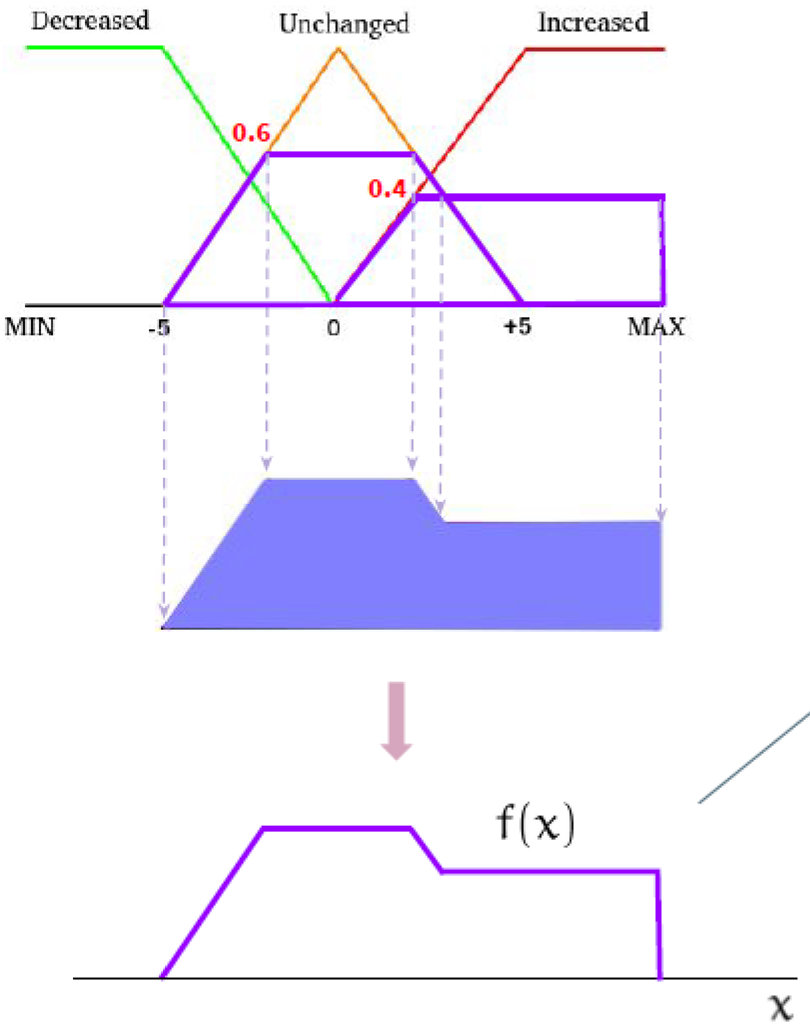
# Defuzzification

From inference stage, we have:
brake_pressure: **0.6** Unchanged, **0.4** Increased

*→ Objective: finding a compromised numerical value from these outputs of inference stage*

1. Plot the membership functions for the output
2. For each truth value (degree of membership), cut the membership function at that value
3. Combine the resulting curves to get an area using the OR operator
4. Find the center-of-gravity (COG) of the area under the combined curve, then this value is the final output (numerical)

*Membership functions for brake_pressure*




$\int f(x)\,dx$ : the area bounded by the curve of f(x)

$$x^* = \frac{\int x f(x)\,dx}{\int f(x)\,dx}$$

is the center-of-gravity, i.e. the final output

# Defuzzification

From inference stage, we have:
brake_pressure: **0.6** Unchanged, **0.4** Increased

*→ Objective: finding a compromised numerical value from these outputs of inference stage*

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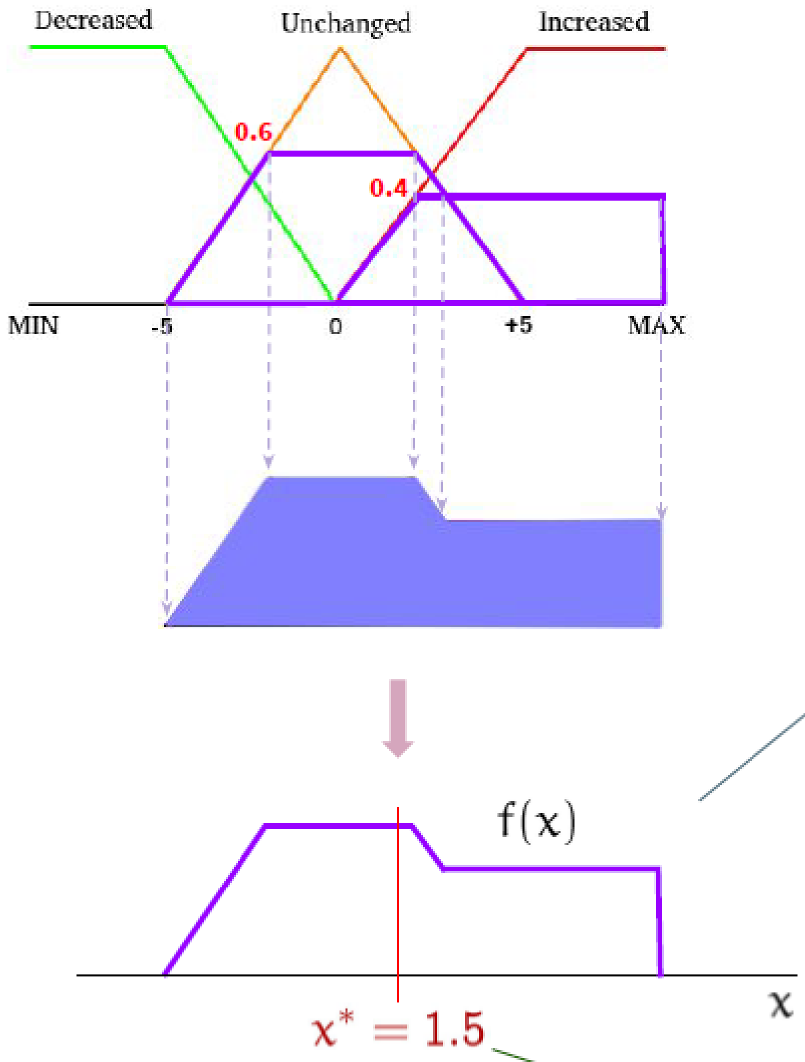
*Membership functions for brake_pressure*




Center-of-gravity DEFUZZIFICATION

$\int f(x)\,dx$ : the area bounded by the curve of f(x)

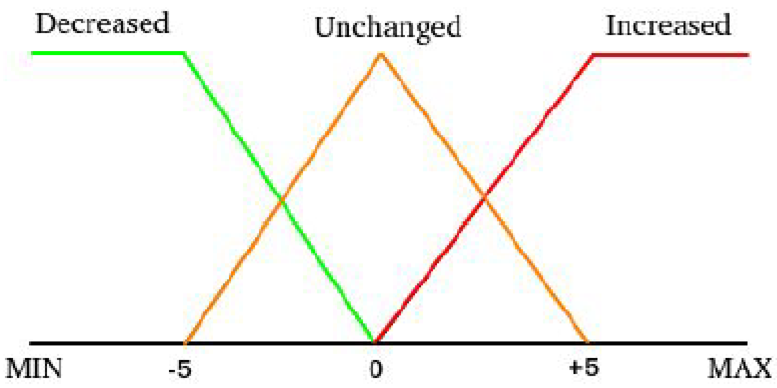
$$x^* = \frac{\int x f(x)\,dx}{\int f(x)\,dx}$$

is the center-of-gravity, i.e. the final output

For temperature = 435 and speed = 85, the Fuzzy system outputs brake_pressure = 1.5

# Defuzzification

Defuzzification is the process of converting truth values (of the output) from inference stage into a compromised numerical value which is the final output of the system



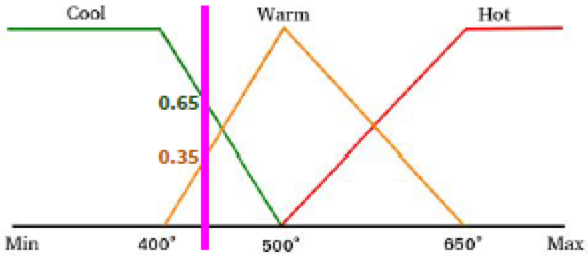

There are several defuzzification methods:

Center-of-gravity (the most common)
Center-of-sum
Weighted average
First-of-maxima
Last-of-maxima
Mean-of-maxima
...

# Overview of Fuzzy control system

**FUZZIFICATION**

Mapping each numerical input variable into its membership functions to compute truth values

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**INFERENCE**

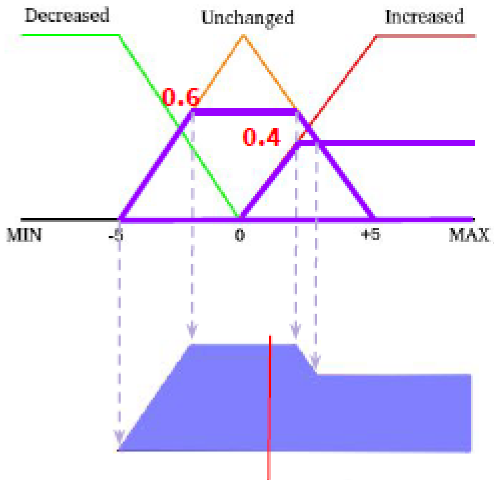
Given the truth values of input variables, using "IF … THEN …" rules to infer possible outputs

**IF** temperature **IS** Warm **AND** speed **IS** Slow **THEN** brake_pressure **IS** Decreased
**IF** temperature **IS NOT** Hot **AND** speed **IS** Medium **THEN** brake_pressure **IS** Unchanged
**…**

↓

**DEFUZZIFICATION**

Grouping possible outputs to compute a compromised numerical value, which is the final output of the system




$$x^* = \frac{\int x f(x)\,dx}{\int f(x)\,dx}$$