MODAL ANALYSIS OF COMBUSTION INSTABILITIES

Establishing Spatio-temporal correlation for better understanding of Instabilities

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Project for ME278: Practical Introduction to Data Analysis

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Introduction:

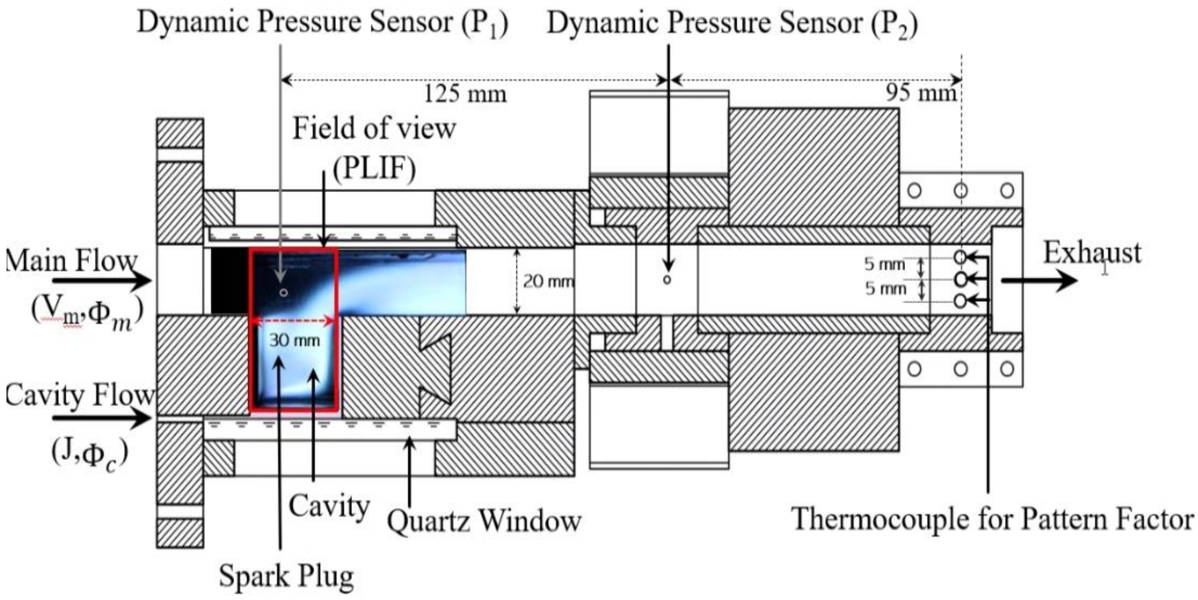
This project is about analysing instabilities in combustion of hydrocarbon when some amount of hydrogen is added to it. With the world moving towards carbon neutral environment if it is possible to design combustors combusting hydrogen it will be a great step forward towards that.

Now here we are trying to simulate based on experimental observations the effect of hydrogen in combustion, one of the main positives for using hydrogen is its high calorific value (141.8 MJ/Kg.), whereas methane has calorific value of (55.5 MJ/Kg)* and the fact that combustion of hydrogen is spatially more uniform it forms a more spread out flame at the exit which aids gas turbine blades to have more uniform temperature distribution which in turn causes lesser thermal shock compared to methane.

But the main problem if combustion of hydrogen is the instability of flame in time. Hence this project focusses on analysing the same. For analysing this pressure at certain regions has been calculated and using La Vision IRO we have extracted 1000 snapshots in time of flame forming inside at a sampling frequency of 5000 Hz.

Now we have done the spatio-temporal decomposition. POD and DMD are used to extract the complex flow dynamics inside the combustor. Both are based on SVD algorithm, for reducing high dimensional data. These techniques seek to take advantage of this fact in order to produce low-rank dynamical system capable of accurately modelling the full spatio-temporal evolution of the complex system without prior knowledge of the governing equations in the flow. In POD the modes are arranged in descending order of energy.

Experimental Setup:



- The setup has a main duct in which the main flow is occurring. A cavity is present inside the combustor where the primary combustion takes place. The air fuel mixture is mixture is injected from the small channel below the cavity and also by the main flow.
- There are two pressure sensors present inside the combustor, one above the cavity and one downstream in the main flow channel.
- The chemiluminiscence(near infrared) images are captured using a high speed camera.

Experimental Data and Applied Methods:

- We have collected data from three different experiments having percentage hydrogen as follows
    1. 0%
    2. 10%
    3. 50%
- The data we have obtained are as follows,
    - Dynamic pressure (p1, p2) obtained from sensors
    - Image obtained by La-Vision Highspeed IR0
- The methods applied are ,
    - Time series analysis (FFT, Wavelet transform) for pressure data and heat release evolution in time.
    - Modal decomposition (POD, DMD) on image data
    - Recurrence plot for all time series to observe oscillation.
    - Time delay embedding to capture limit cycle oscillations.

Pressure Data

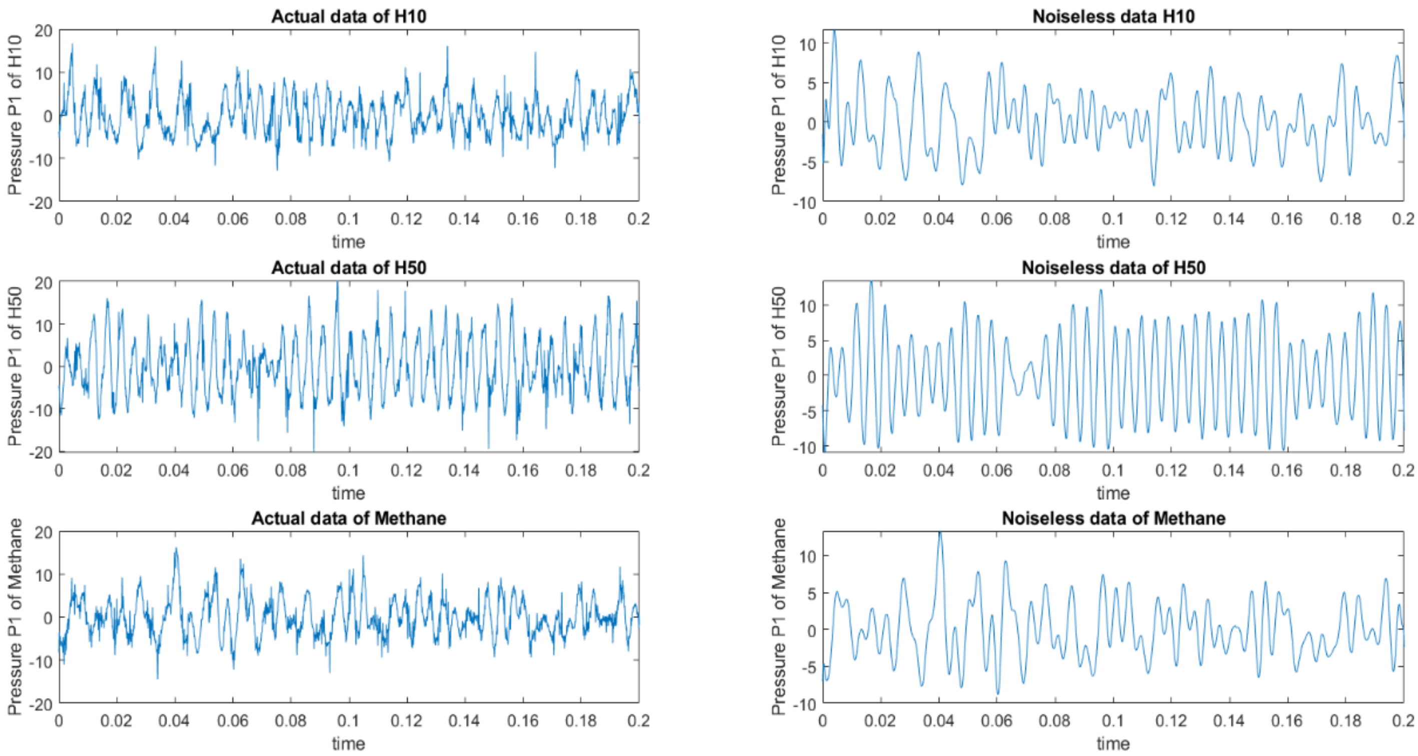
Pressure data is collected from two probes P1 and P2. From P1 and P2 pressure data we plot the time series FFT spectrum and wavelet transform. Data is taken at a sampling frequency of 10000 points per second. We perform the following functions on the mean removed dataset

- The time series data for h10, h50 and methane are plotted. Savitzsky-golay filter is applied to see the noiseless plot.
- We then plot the fft spectrum of the pressure data and note its peak.
- We perform spectrogram analysis to get a visual representation of the spectrum of frequencies of a signal as it varies with time.
- Then we do wavelet transform of the dataset.

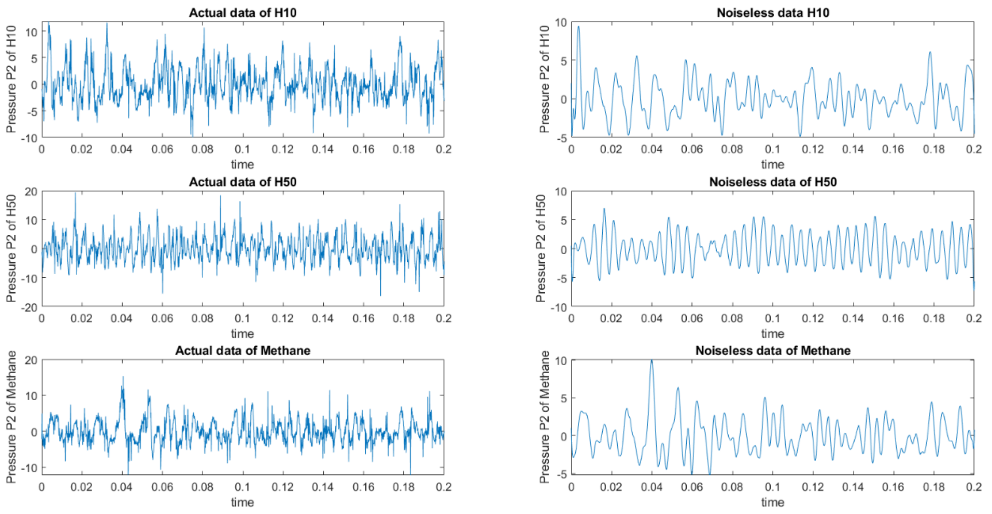
Dataset

	Equivalent ratio	P1 mean (kPa)	P2 mean (kPa)
Methane	0	-5.5594	-7.6603
H10	0.1	-5.4154	-7.5710
H50	0.5	-5.4975	-7.3094

Time series analysis (P1 pressure data)



P2 Pressure Data

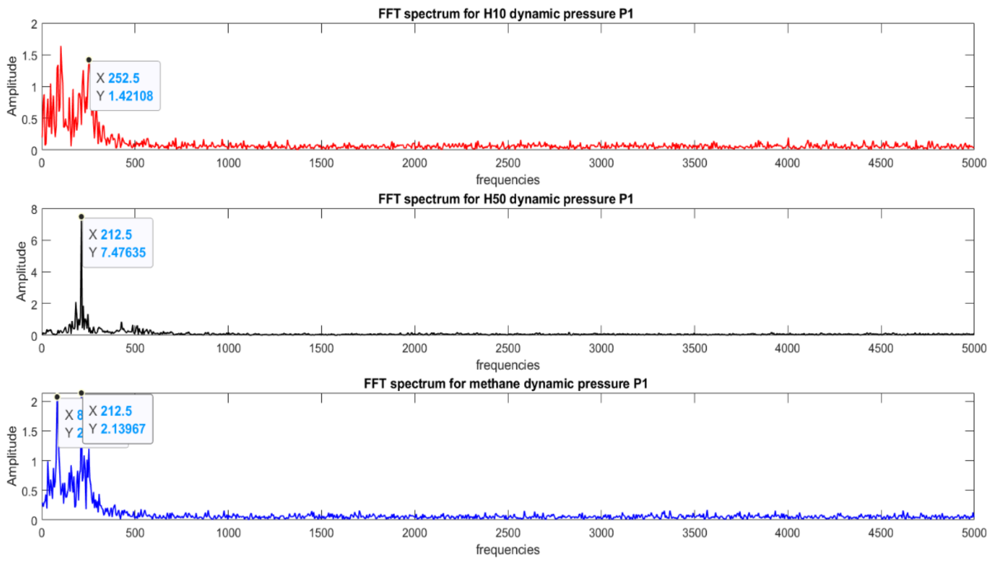


We have applied savitzsky-golay filter with window width 15 and order 21. As we can see the h50 pressure data is associated with more amplitude. This will also be evident from the fft spectrum.

We also that P2 pressure shows more oscillation which is the exit of the combustor.

FFT Spectrum

P1 Pressure Data

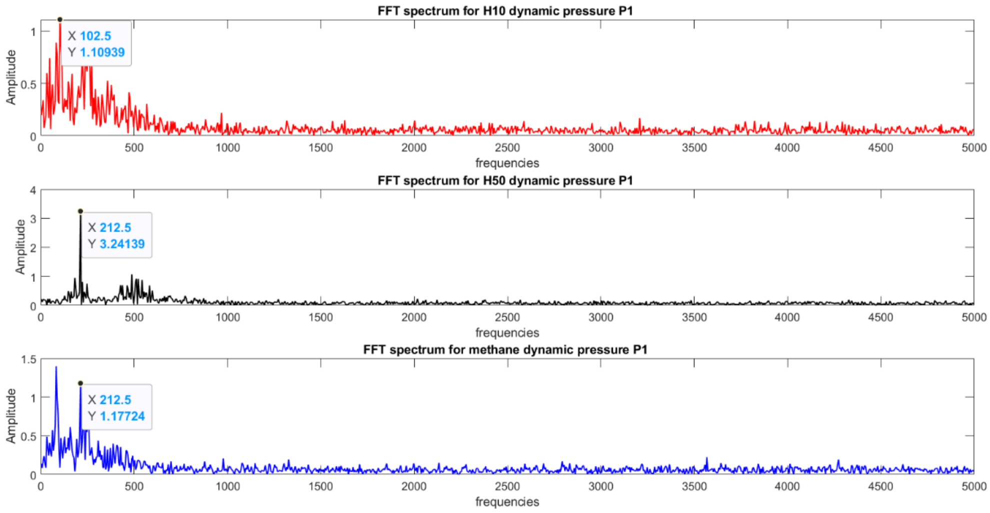


We perform FFT to obtain the more dominant frequencies. As we can see there is more oscillation in FFT spectrum of methane and h10 as compared to h50. H50 and methane both shows frequency peaks at 212.5 Hz and H10 shows peak at 252

Hz. In the FFT spectrum the number of peaks whose amplitude greater than 0.5 is noted as

peak_h10	62
peak_h50	83
peak_meth...	64

P2 Pressure Data



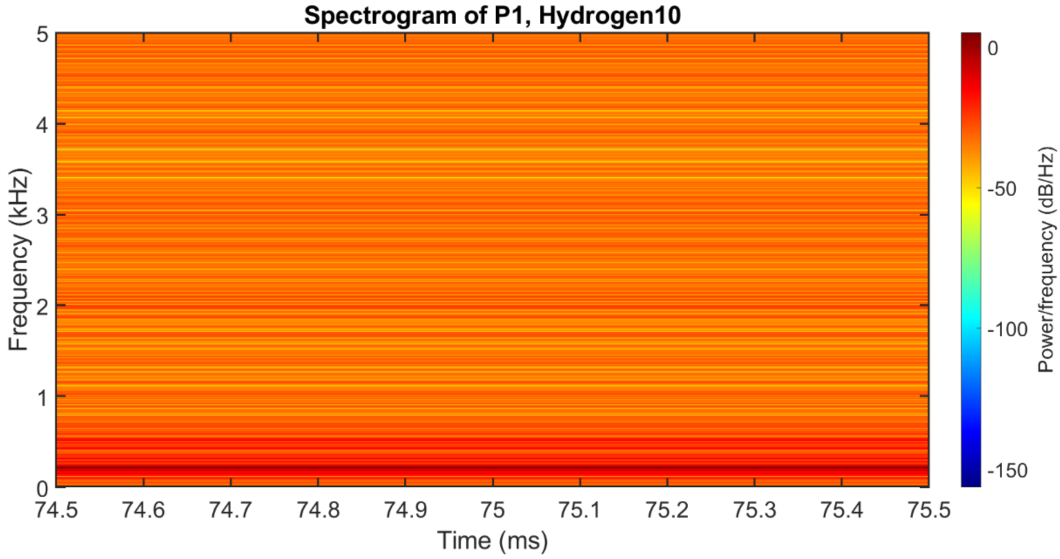
Here the peaks for H10 comes at 102.5 Hz and 257.5Hz. The peak for H50 and Methane remains at 212.5 Hz. In the FFT spectrum the number of peaks whose amplitude greater than 0.5 is noted as

peak_h10_...	78
peak_h50_...	94
peak_meth...	70

This gives the indication of higher oscillations for higher hydrogen content.

Now that we have all information about frequency, we need to obtain info having temporal resolution. For this we have done spectrogram for p1 region for hydrogen ratio 0.5.

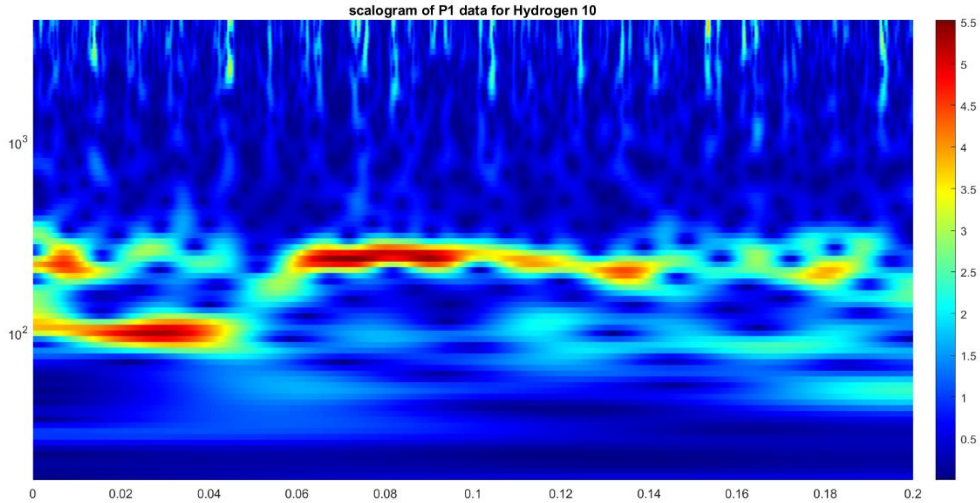
Spectrogram Analysis: -

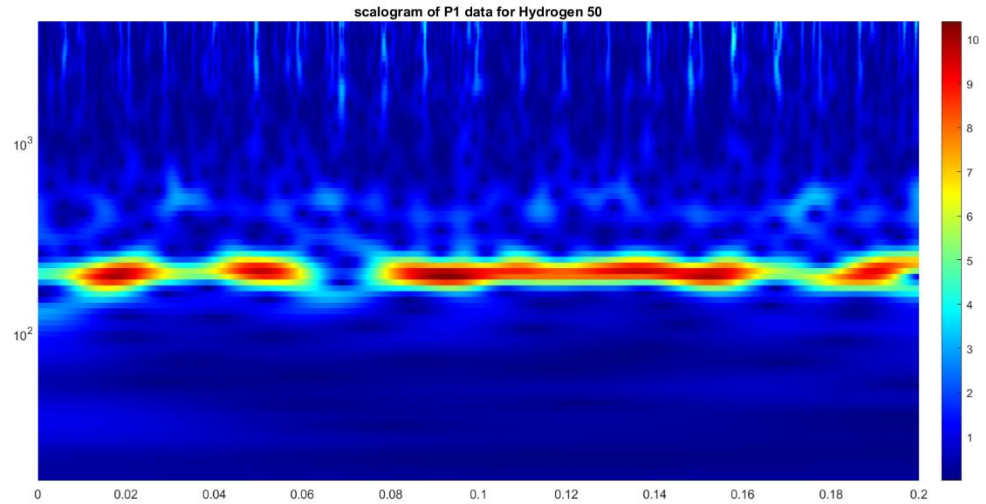


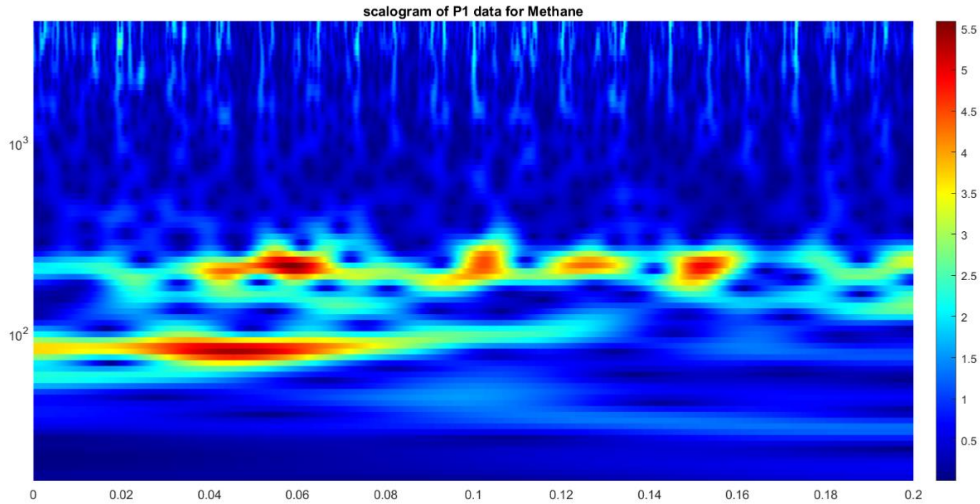
The spectrogram does not give us good information about temporal evolution of the frequencies. Thus, we have next computed Wavelet transform for the to get a better a time resolution for the higher frequencies. We have used Morse Wavelet.

Wavelet Transform: -

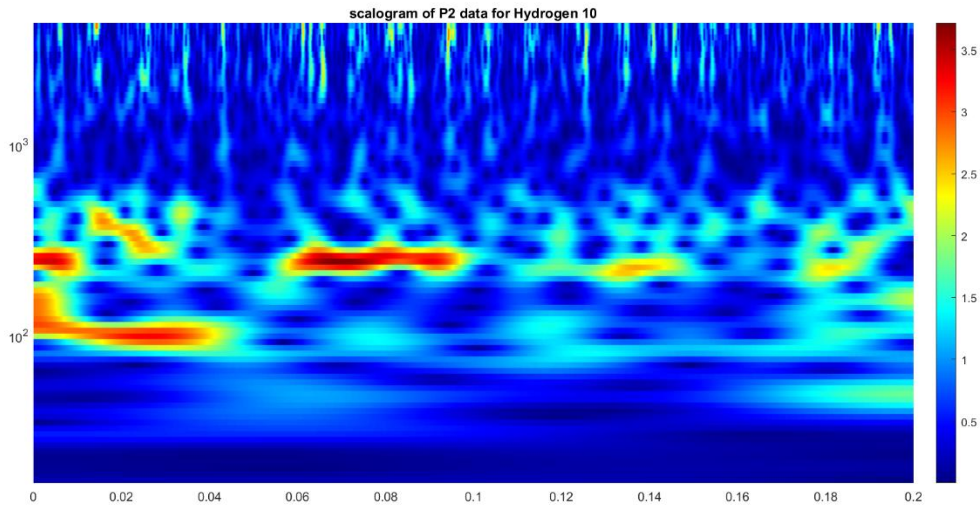
Pressure Data P1:

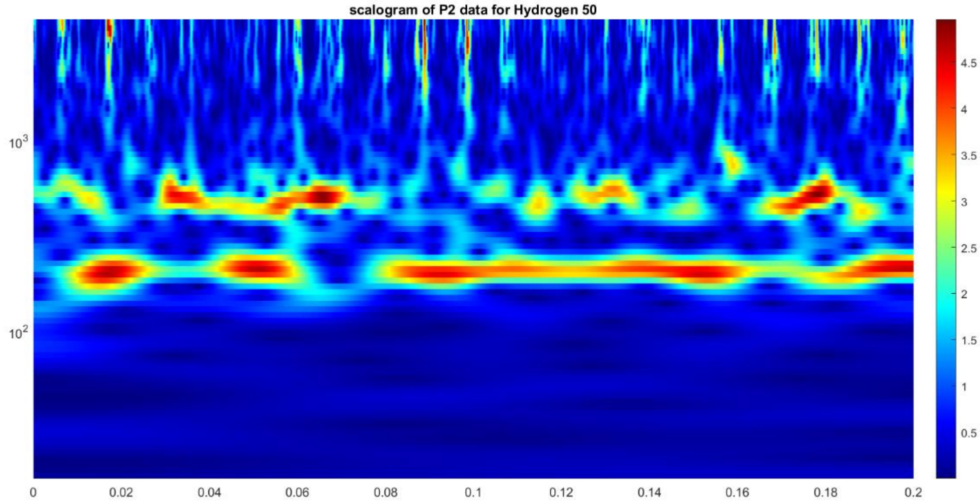


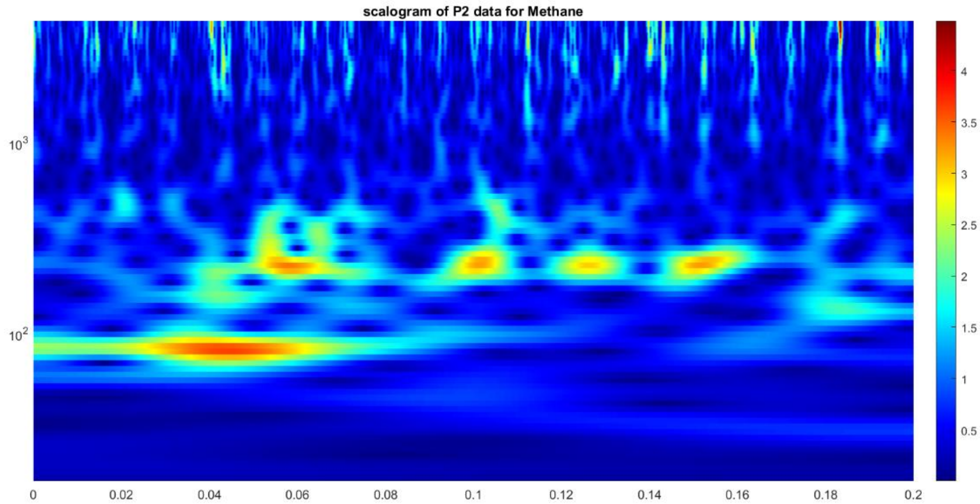




Pressure Data P2:



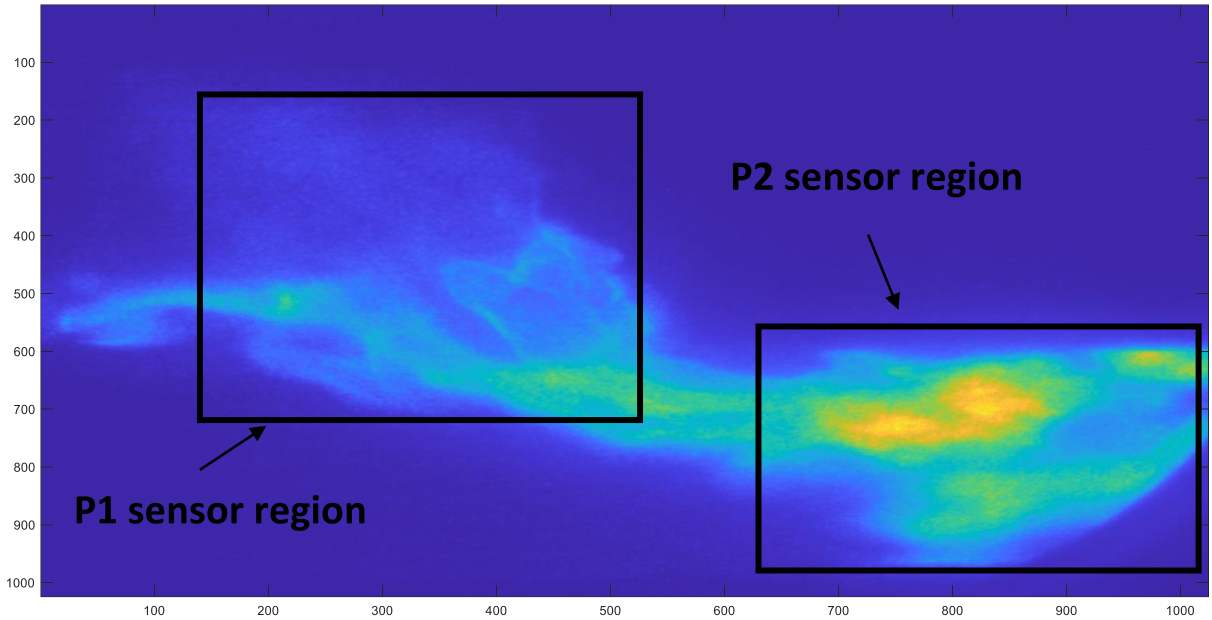




In the P2 dataset we can see H50 shows more oscillation. This means adding more hydrogen the oscillations grow up and for more hydrogen different higher frequencies appear and persists also for longer time. Dominant frequencies appear around 200 Hz and pressure data shows more instabilities near combustor exit in the P2 region.

Time series analysis of heat release rate:

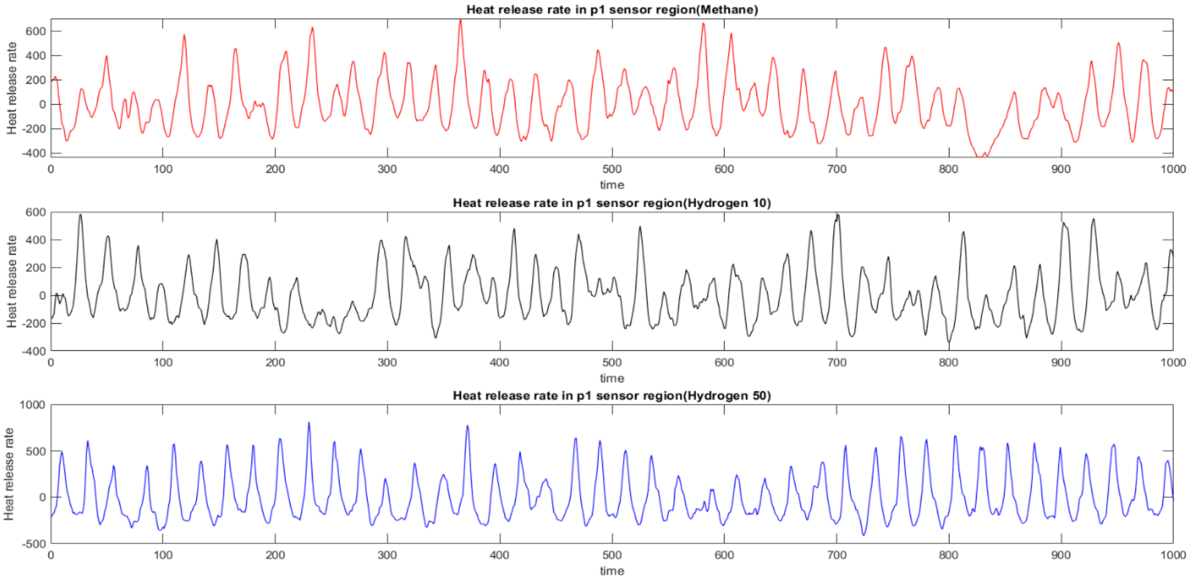
- From the pictures obtained from experiment are further analysed for time evolution (FFT, Wavelet Transform).
- The images are sampled at a frequency of 5000 HZ. Hence, we have data for 0.2 seconds which is 1000 snapshots.
- To obtain the time series both in region where p1 sensor and p2 sensor are present, we have taken a box in snapshots and calculated mean value for 1000 such snapshots and hence time obtained time series.
- Here from a snapshot region are specified for p1 and p2 sensors.



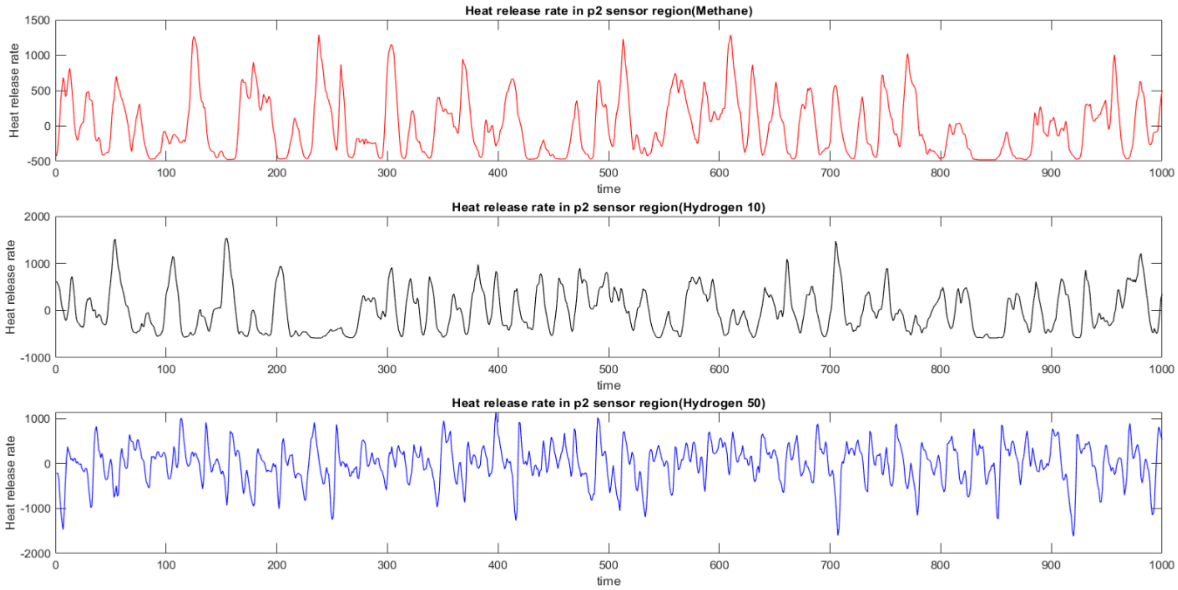
Equivalence ratio(H2/CH4)	Mean	
	P1 sensor region	P2 Sensor region
0	647.1	481.38
0.1	630.3	585.57
0.5	807.18	186.05

- Now the time series obtained are plotted. The mean values of all three series are given in table,

- Now plotting the time series of p1 sensor region for all the models,

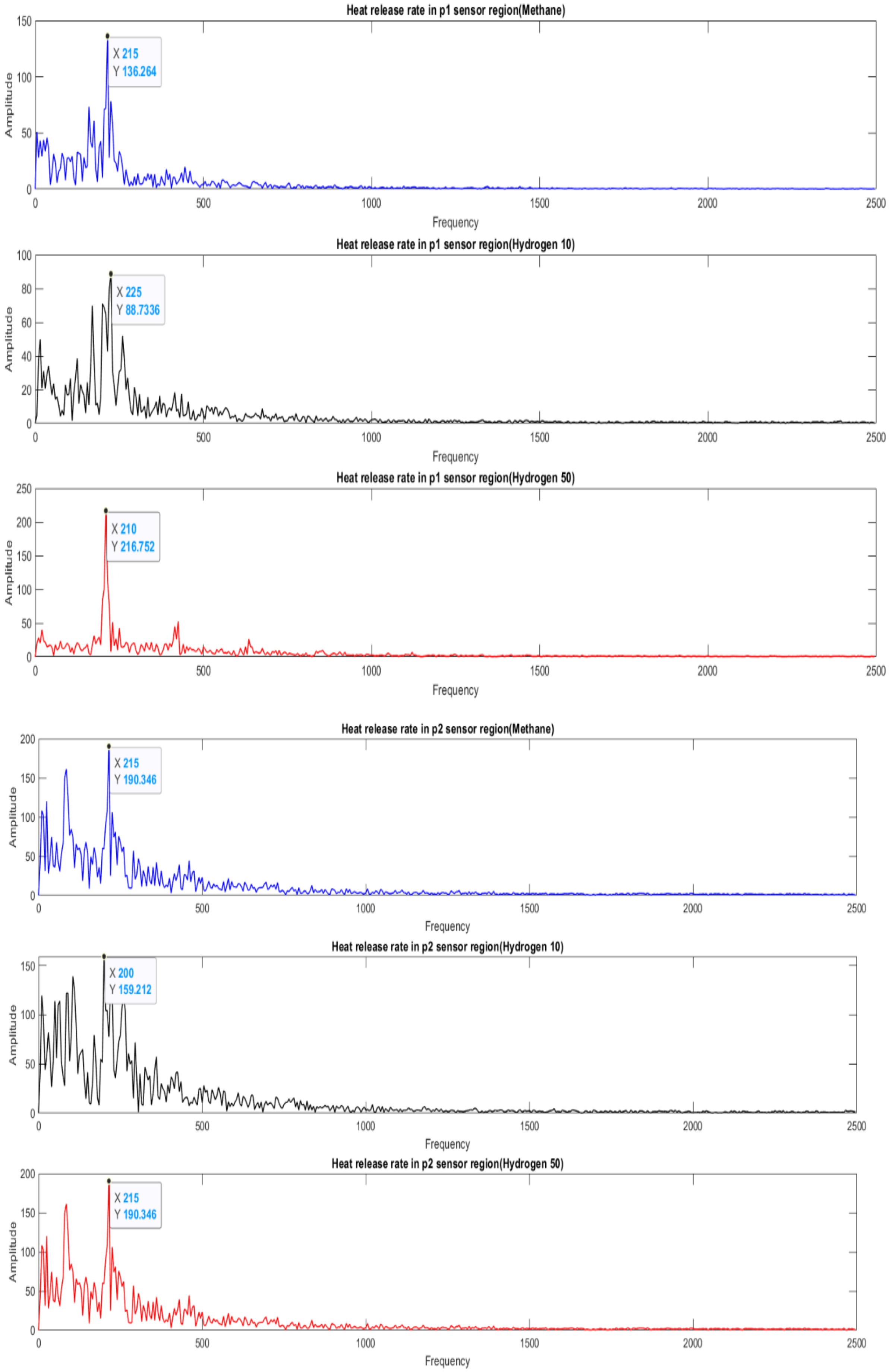


- P2 sensor region is also plotted,



- Now here mean has been subtracted which would be beneficial while doing FFT as we are more intended to visualize the oscillations of the phenomenon.

- Fourier transform is performed to obtain dominant frequencies,



- As it can be observed oscillations are more in the p2 sensor region which is basically the exit of combustor.
- FFT s does not really give us a good insight as the data is not long enough and there are lot of frequencies involved, but qualitative differences are what we can observe as FFT contains the most resolution in frequencies.
- To have a clearer idea about the oscillations we have calculated no of frequencies having an amplitude more than 30 units in p2 sensor region.

Workspace

Name	Value
a0	114
a10	130
a50	172

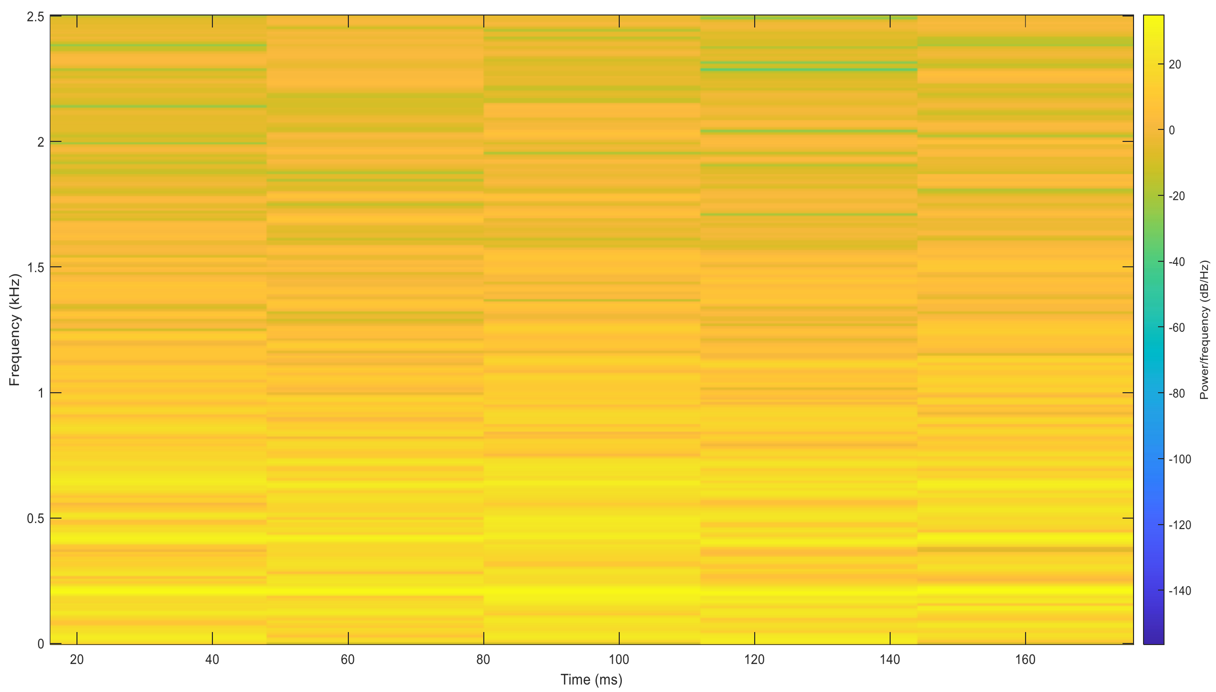
a0 – Methane

a10 – Hydrogen 10

a50 – Hydrogen 50

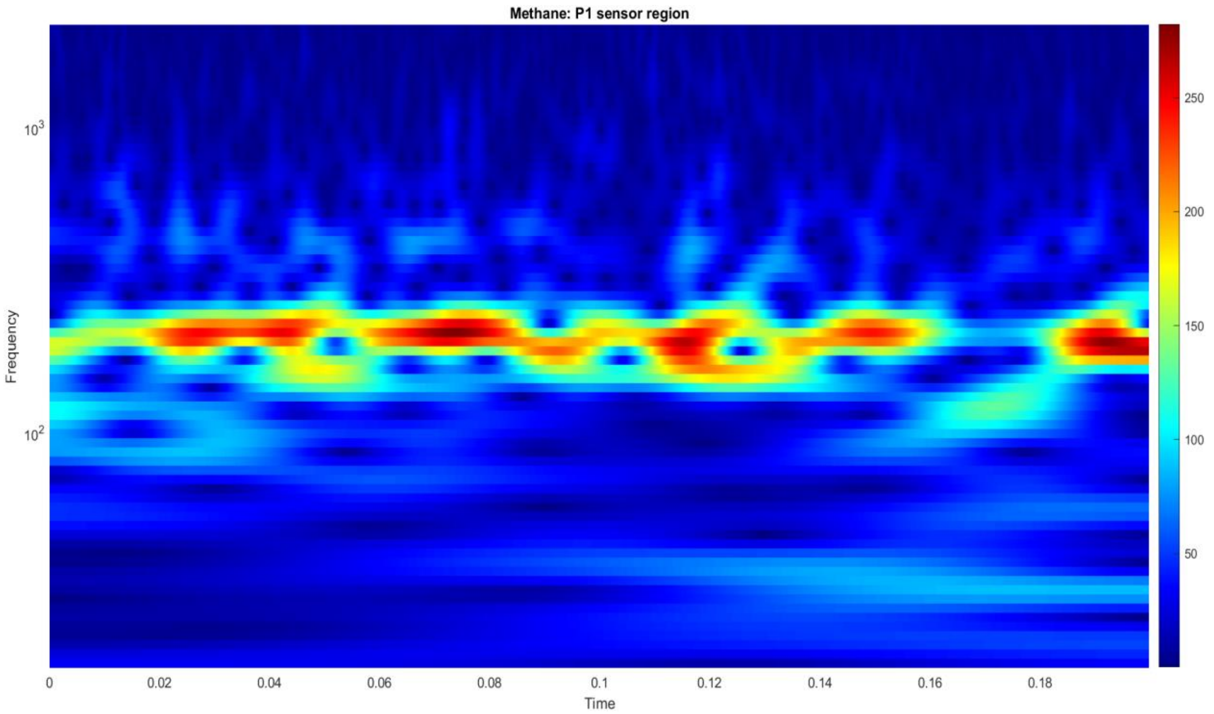
- Which also gives the indication of higher oscillations for higher hydrogen content.
- Now that we have all information about frequency, we need to obtain info having temporal resolution. For this we have done spectrogram for p2 region for hydrogen ratio 0.5.

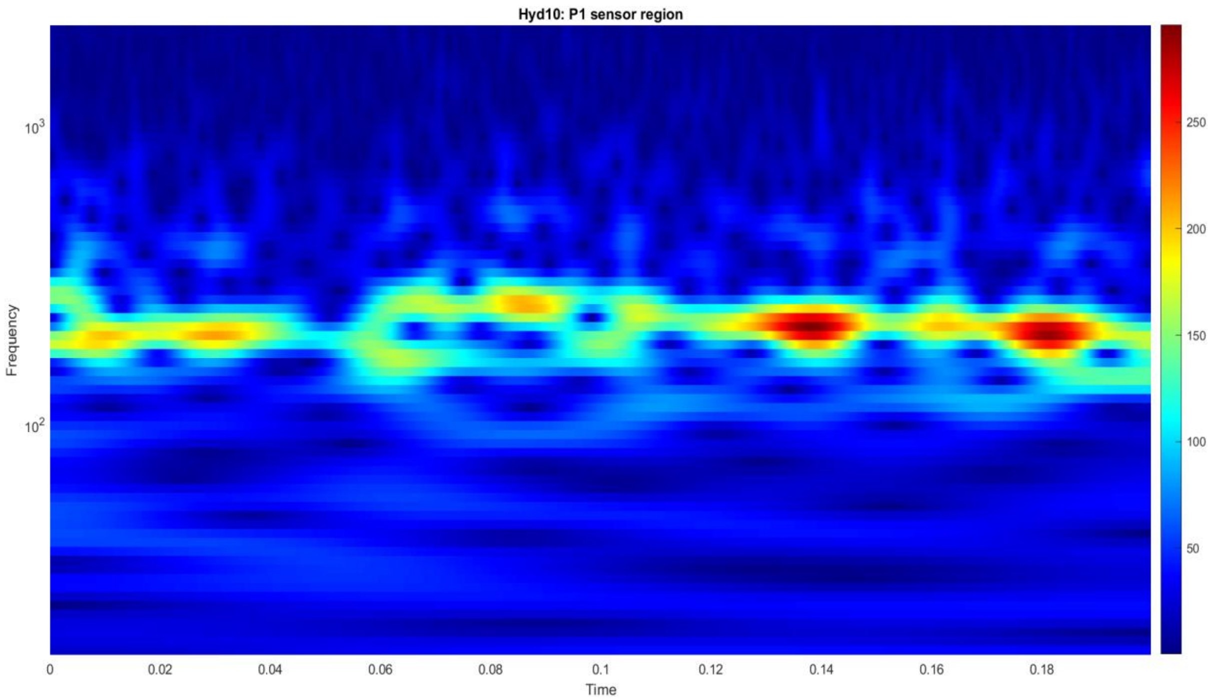
Spectrogram:

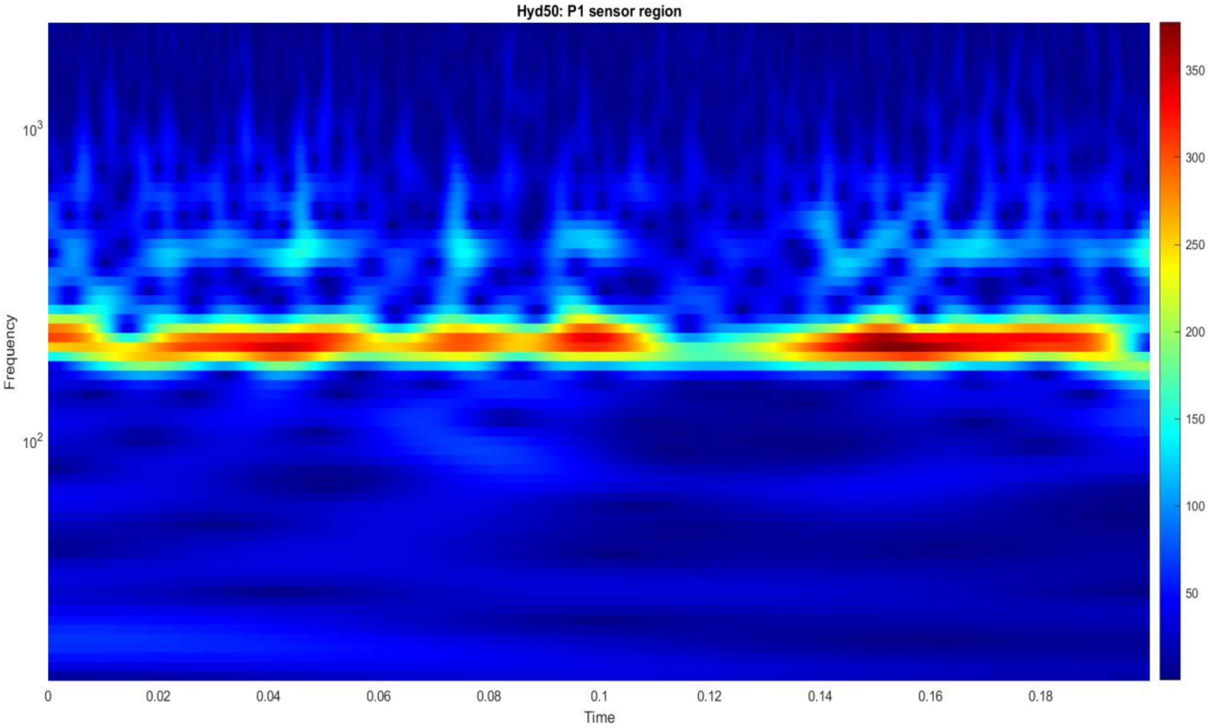


- This is evident that spectrogram does not give better idea in temporal axes. Hence, we have opted for wavelet transform (using Morse wavelet) which increases temporal resolution for higher frequencies which gives better result.

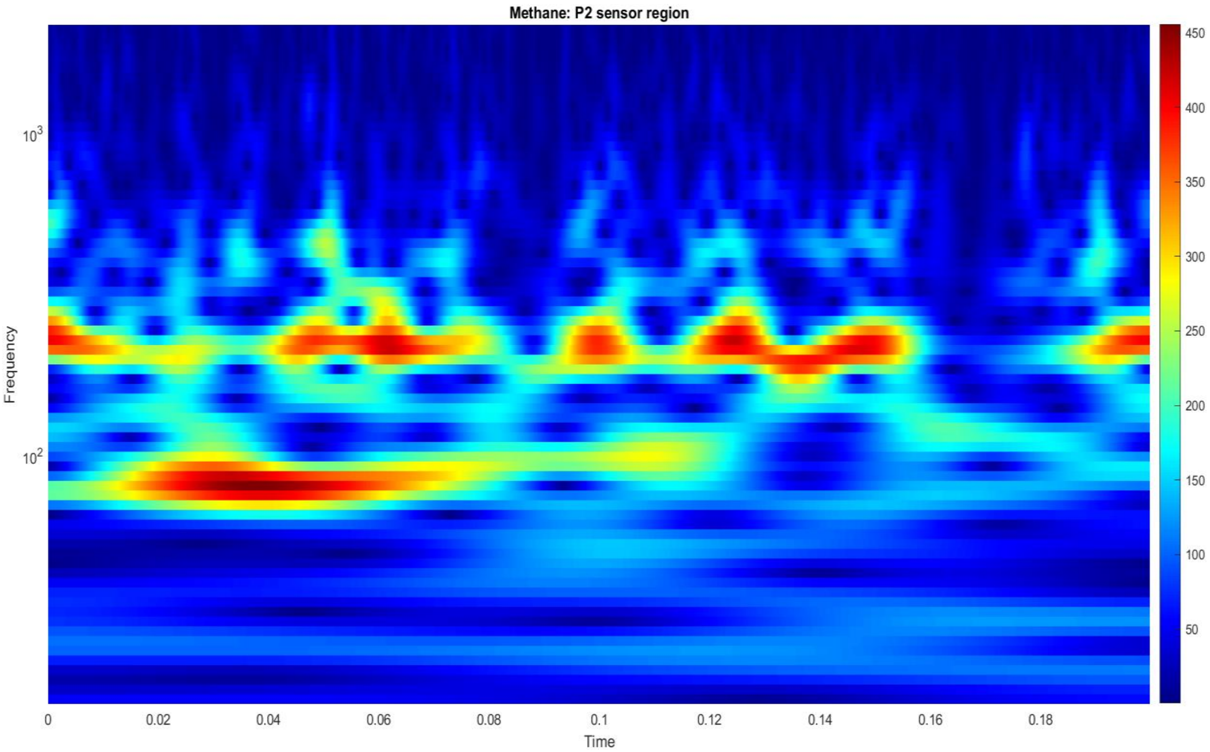
Wavelet transform:



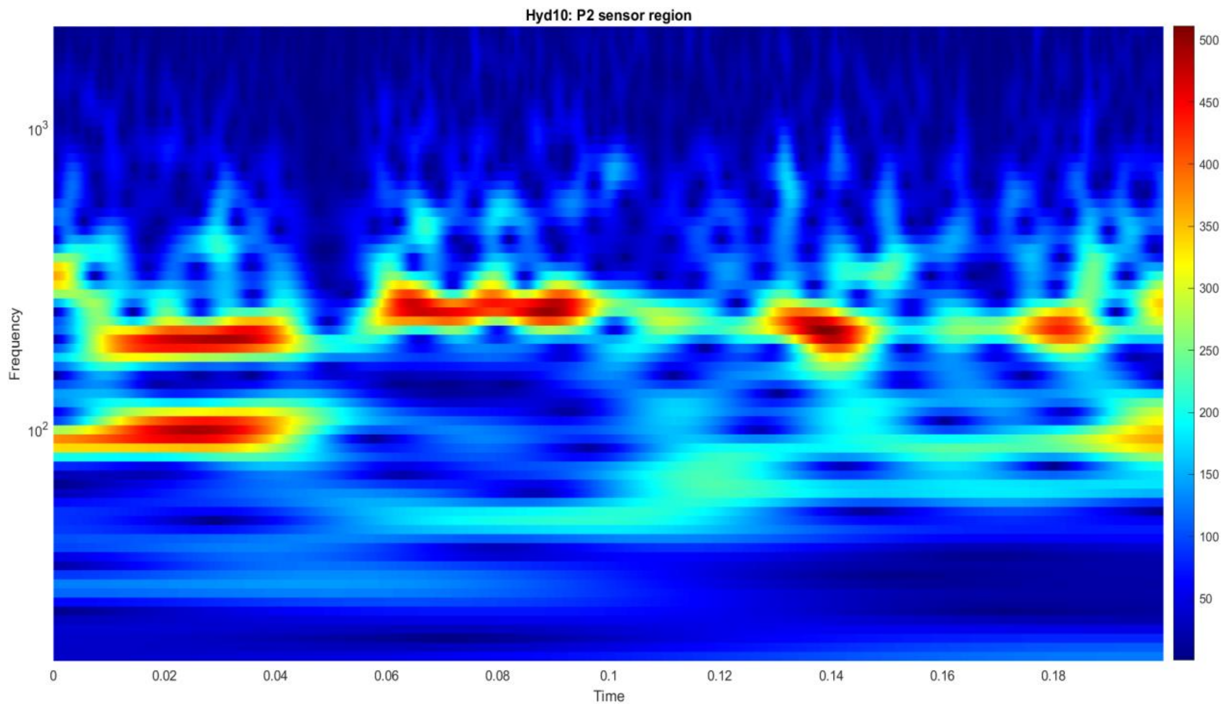


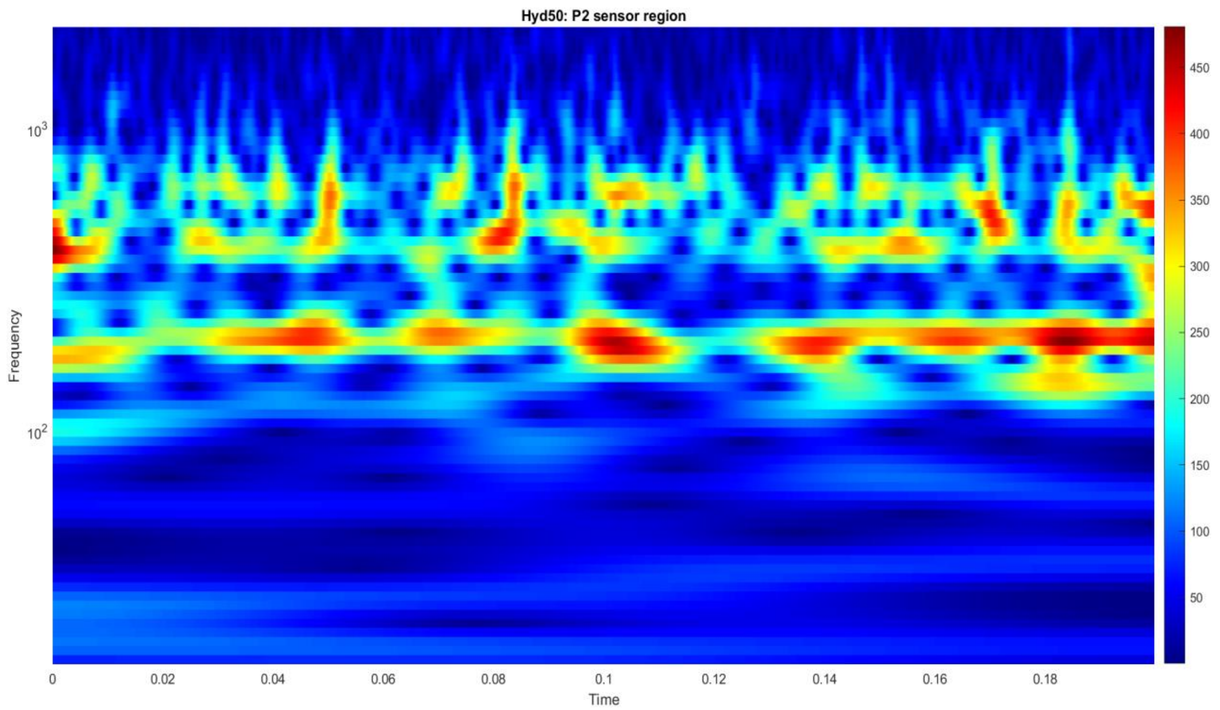


Hyd50: P1 sensor region
Frequency
Time
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
10^3
10^2
350
300
250
200
150
100
50



Methane: P2 sensor region
Frequency
Time
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
10^3
10^2
450
400
350
300
250
200
150
100
50





- Now it is clear that with adding more hydrogen the oscillations grow up and for more hydrogen different higher frequencies appear and persists also for longer time. As we see the dominant frequencies are around 200

Hz for, but appearance of higher frequencies is more prominent in 50% hydrogen mixture. And, at the exit of combustor the instabilities are far more than that in p1 region.

Spacio-Temporal Decomposition

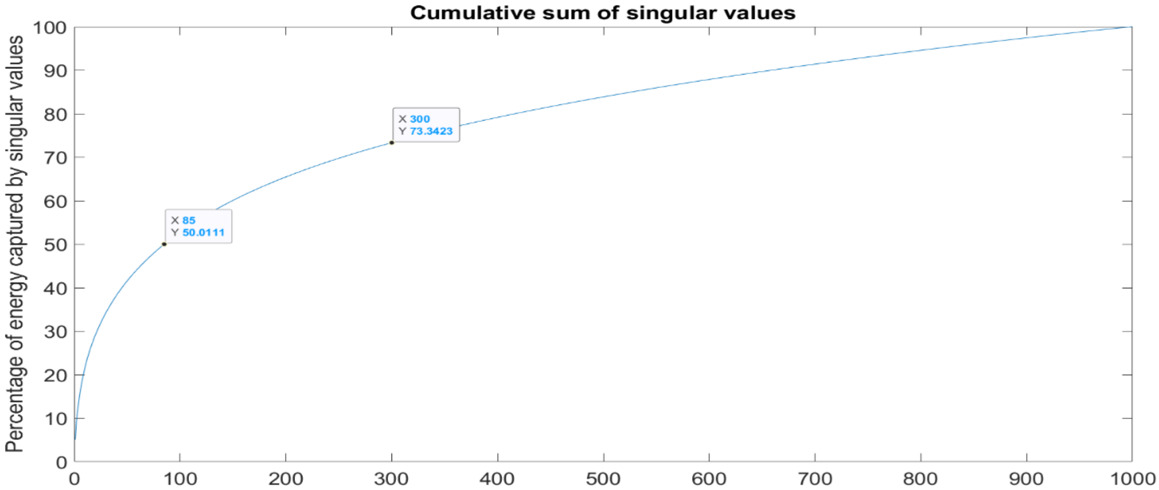
- *Proper Orthogonal Decomposition*:

  To find spatial and temporal coherence in the data we have POD of three datasets given to us.

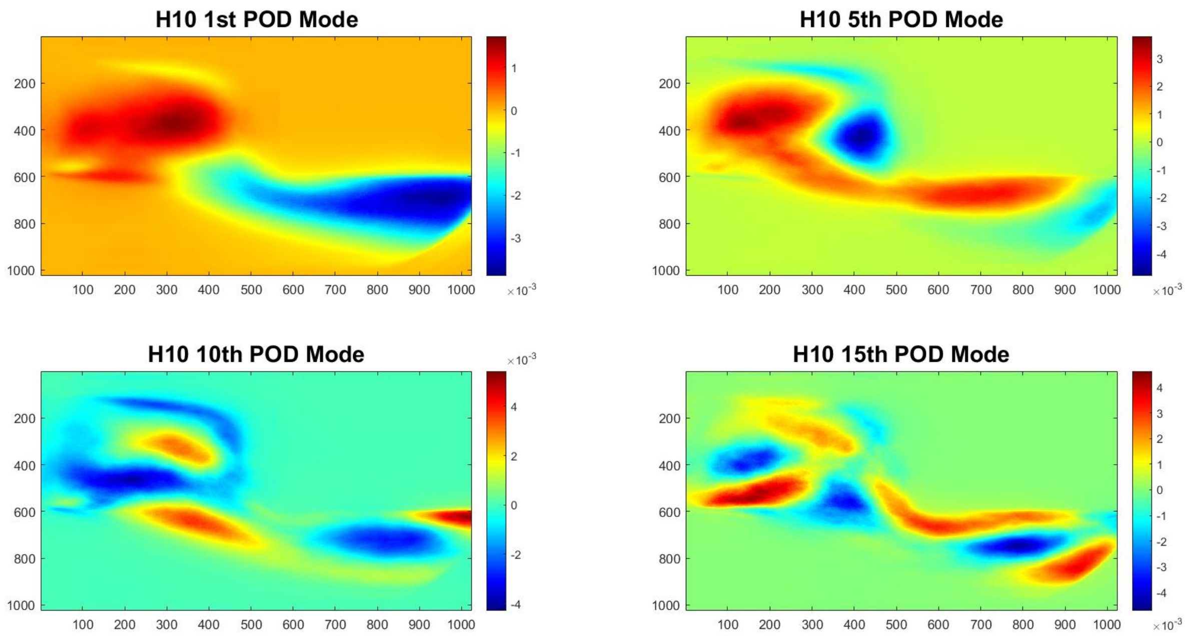
10% Hydrogen:

For the 10% Hydrogen data, we have 1000 snapshots 1024-by-1024 images of combustion. So, first we vectorize the data and created the 1024^2-by-1000 i.e. 1048576-by-1000 snapshot matrix.

The POD modes and time dynamics are calculated by doing SVD of the snapshot matrix. The Singular value distribution is shown below,

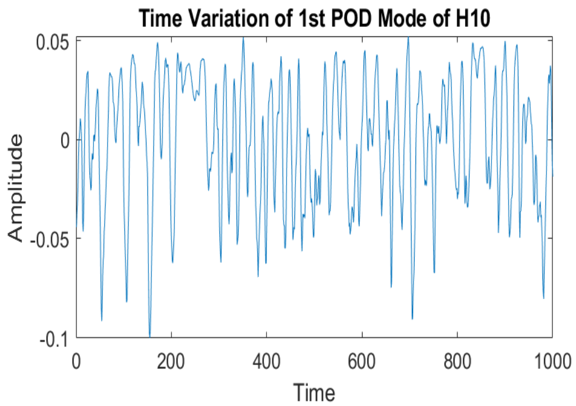


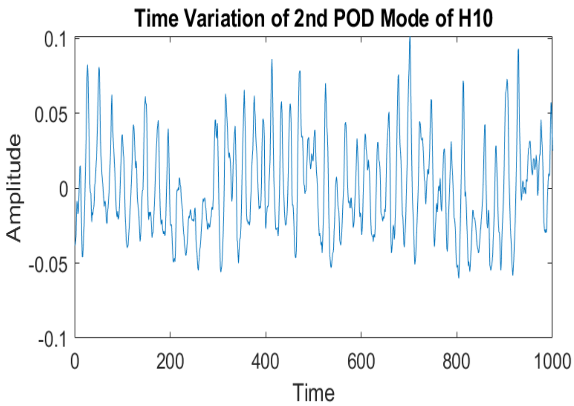
We can see that, up to 85th mode the singular values capture over 50% of the covariance of the data. The POD modes are shown below,

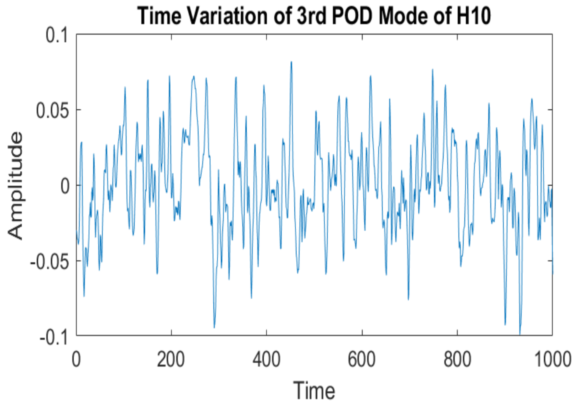


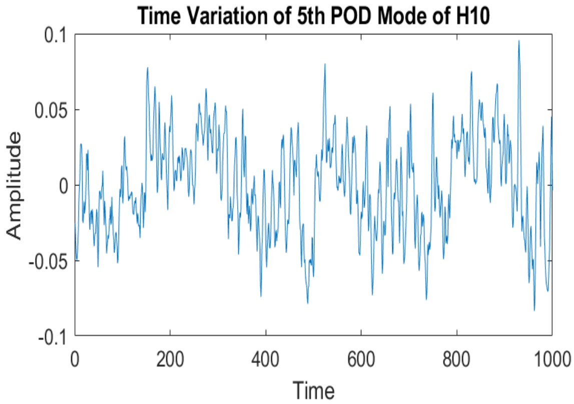
We can see the 1st mode captures the most corelation, since the flow is most corelated in space over time in those two regions where the combustion happens and blows off. In the subsequent modes, we can see smaller regions which are less coherent in space over time.

Now, for the time dynamics of the modes we have calculated the snapshot matrix by multiplying corresponding column of U with the corresponding row of V^T for a particular mode. Time dynamics of the modes are shown below,

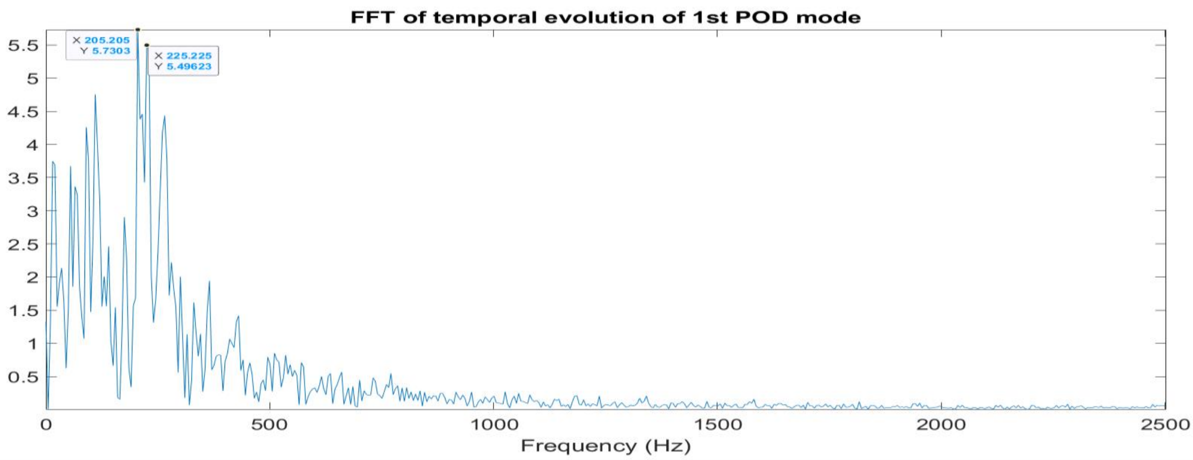


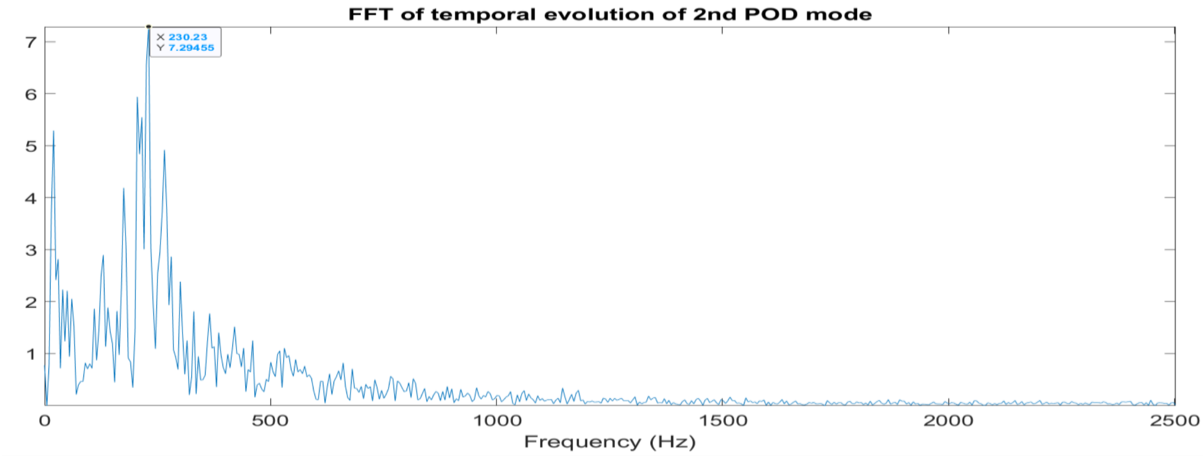


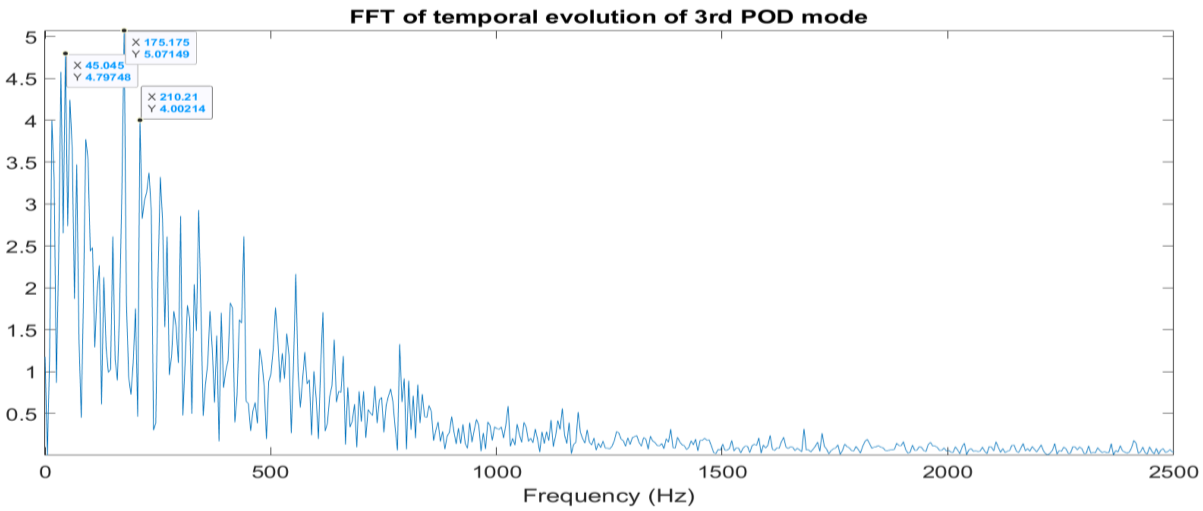


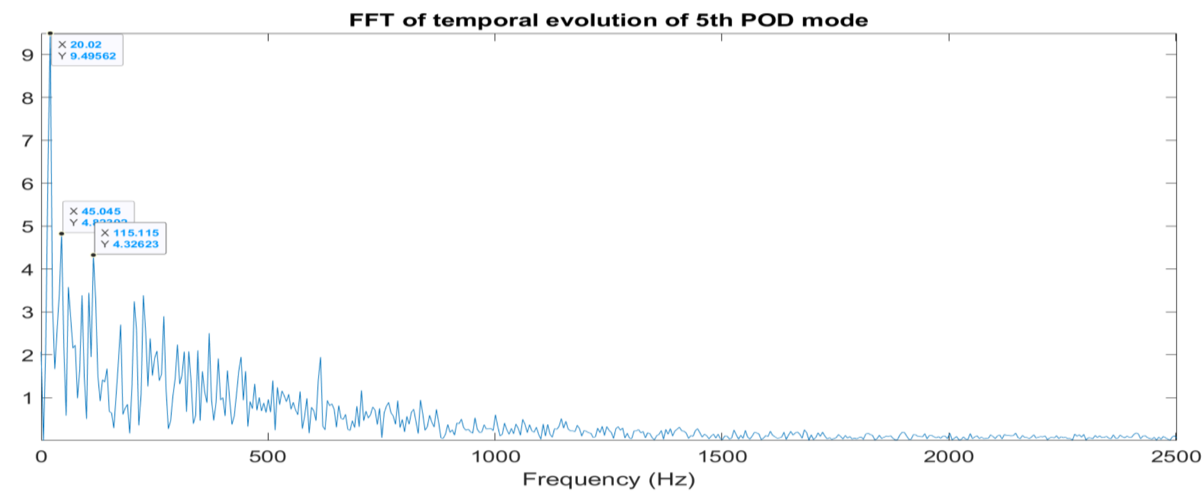


To see in which frequencies they oscillate, we have also done FFT of the time dynamics of the modes and compare how these matches with Pressure data and time-averaged data of snapshot matrices.









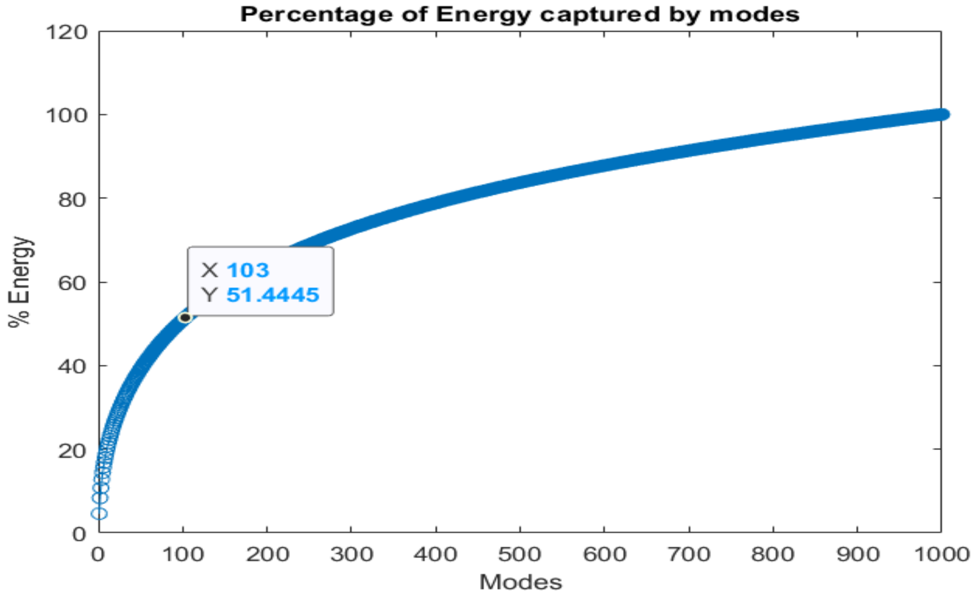
We can see the from 1st mode the dominant frequency almost matches with the frequency from Pressure data which tells about how 1st mode captures the time oscillations in the actual data.

We have reconstructed the spatial-temporal images by considering only first 85 POD modes and see how it matches with real spatial-temporal images.

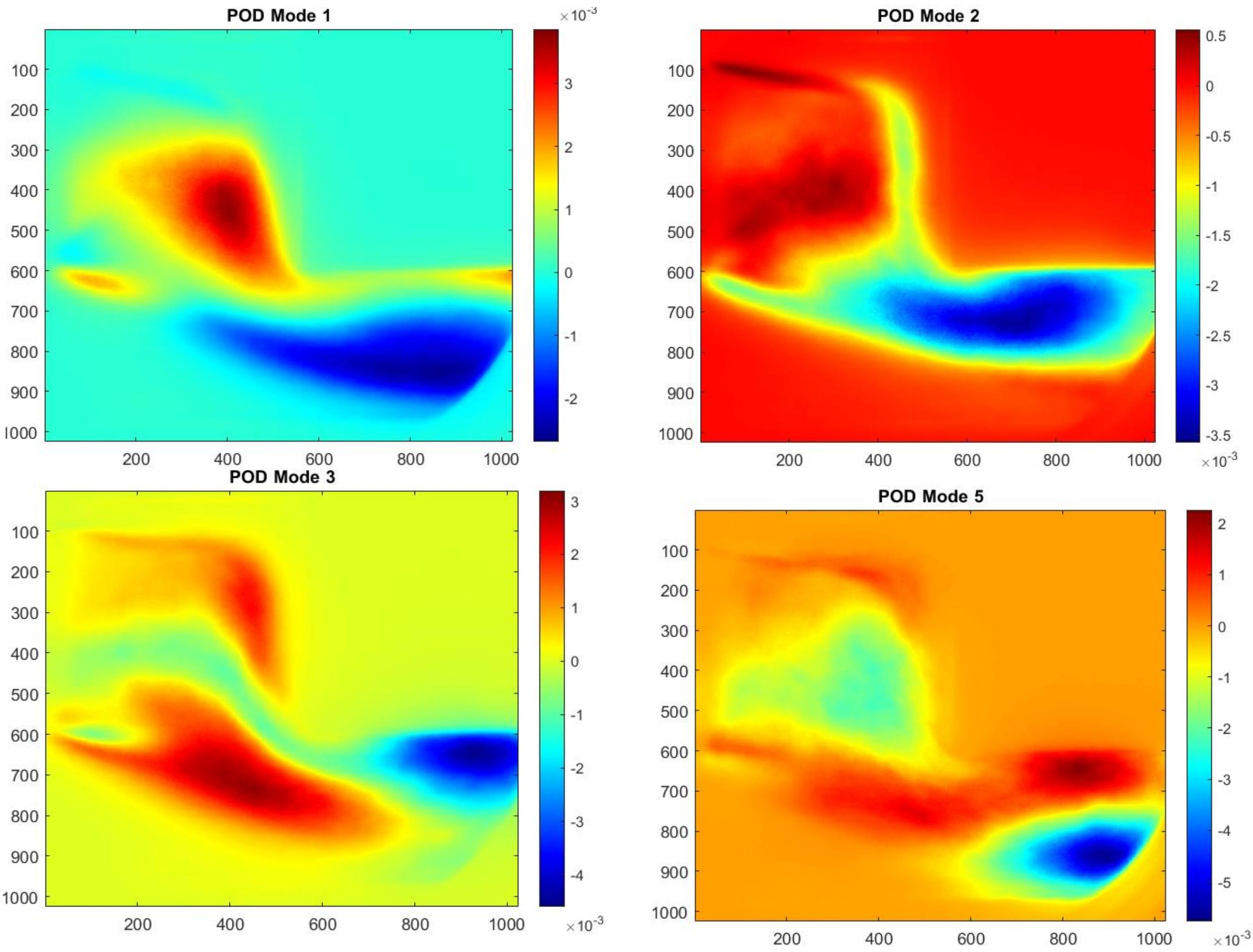
Click **here** to see the corresponding reconstructed gif.

50% Hydrogen:

Now, we have also done POD for 50% Hydrogen dataset. The Singular value distribution for Hydrogen 50 data is shown below,

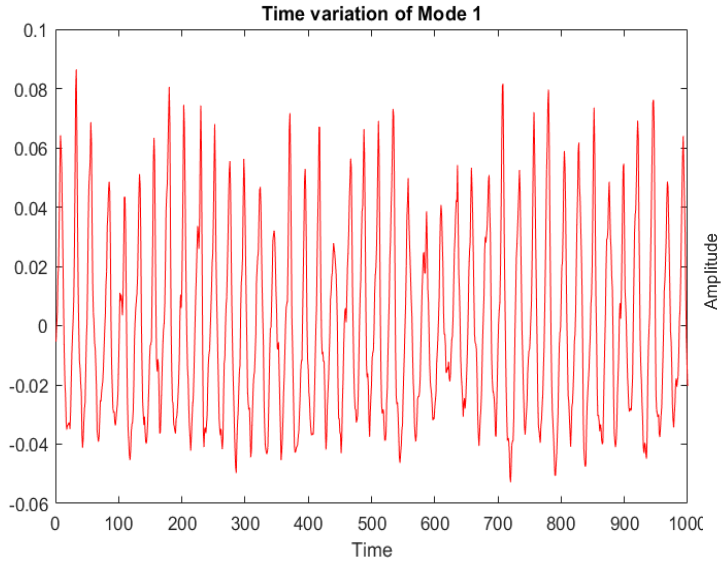


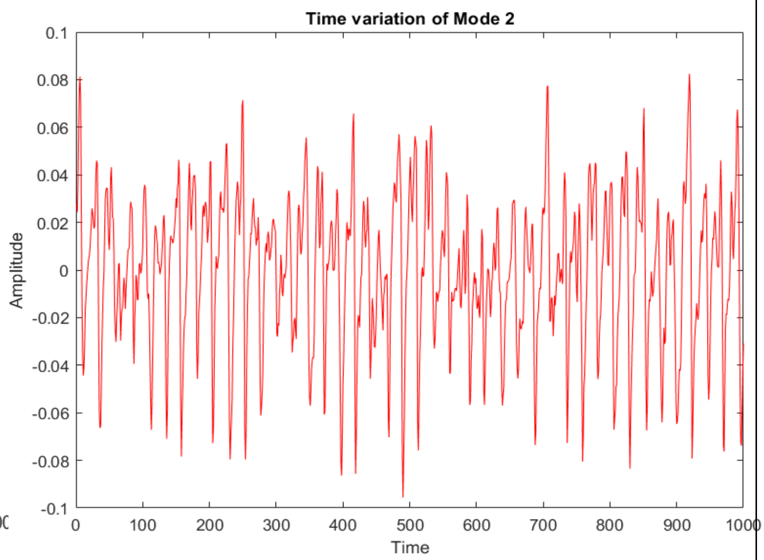
Here we can see that, up to almost 100th mode the singular values capture over 50% of the covariance of the data. The POD modes are shown below,

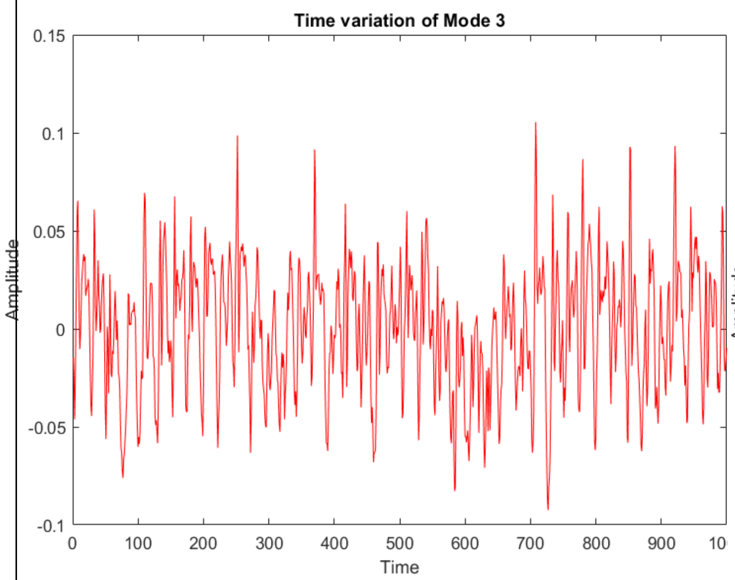


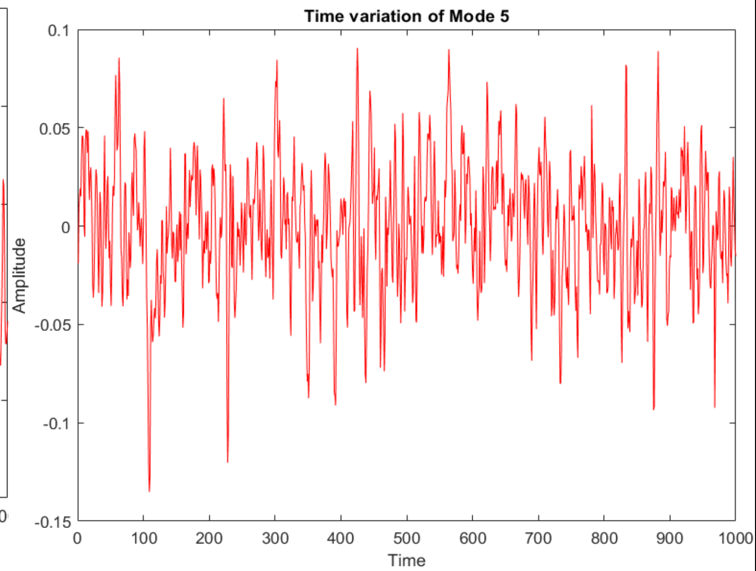
Here also we can see the 1st mode captures the most corelation, the flow is most corelated in space over time in those two regions where the combustion happens and blows off. In the subsequent modes, we can see smaller regions which are less coherent in space over time.

Now, for the time dynamics of the modes we have calculated the snapshot matrix by multiplying corresponding column of U with the corresponding row of V^T for a particular mode. Since, gif can't be played in pdf, so we have plotted the 1st column of matrix V which are shown below,

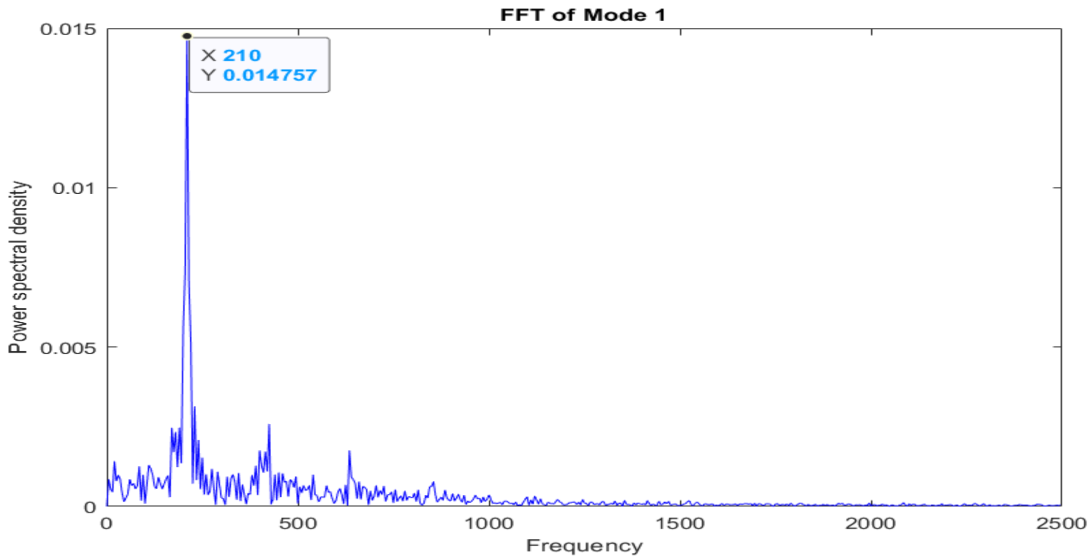


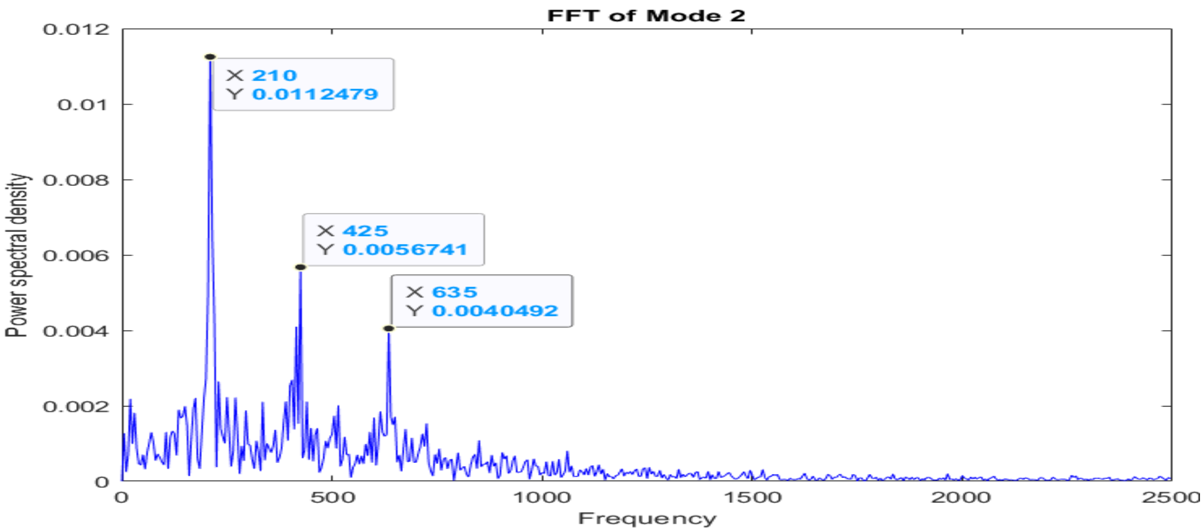


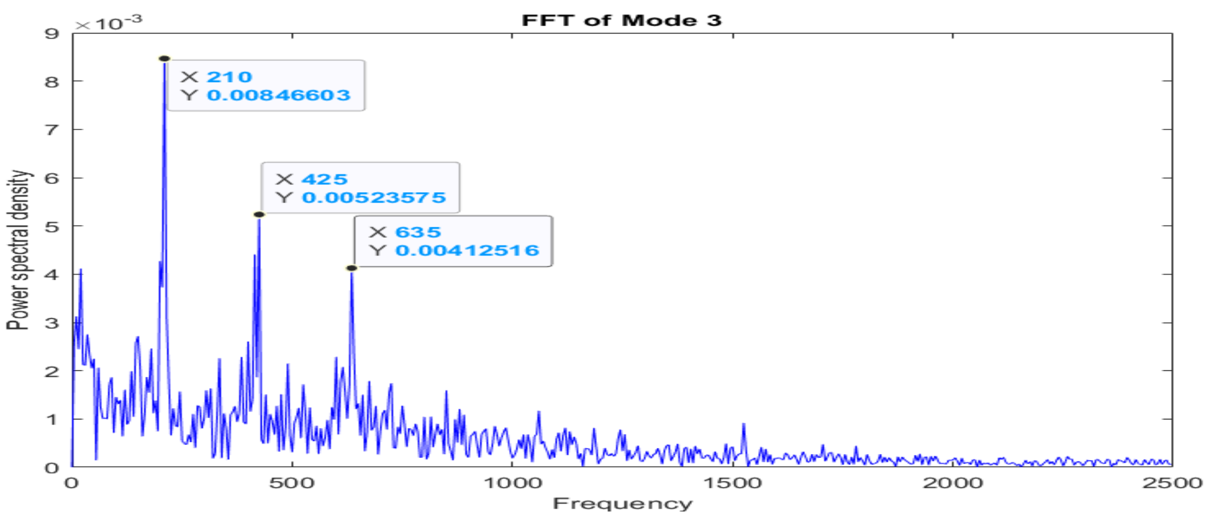


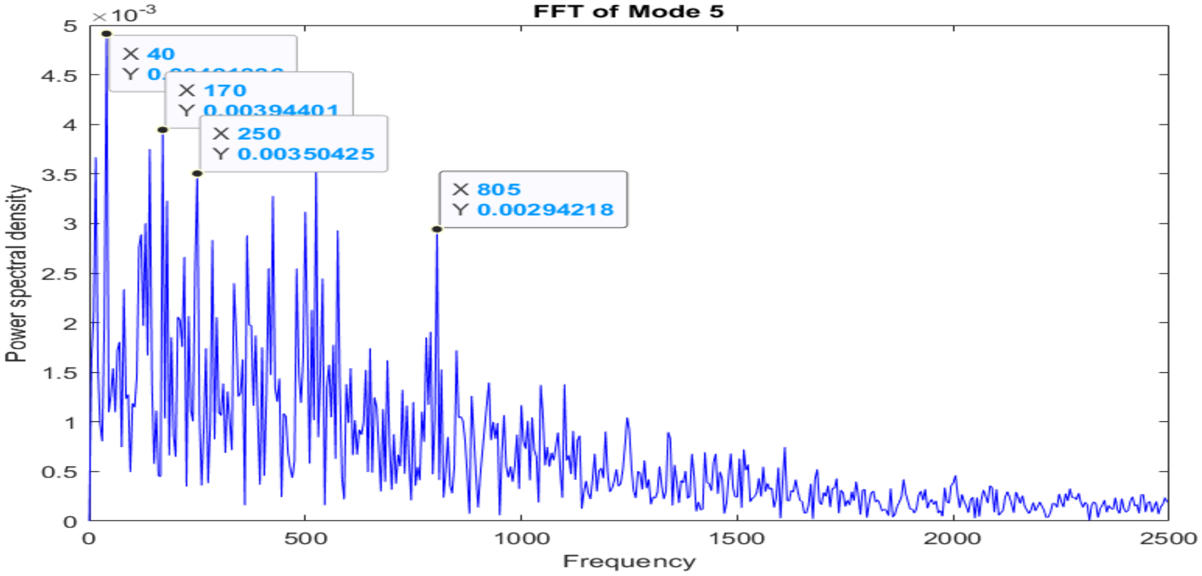


Same as before we have done FFT of the time dynamics of the modes and compare how these matches with Pressure data and time-averaged data of snapshot matrices,







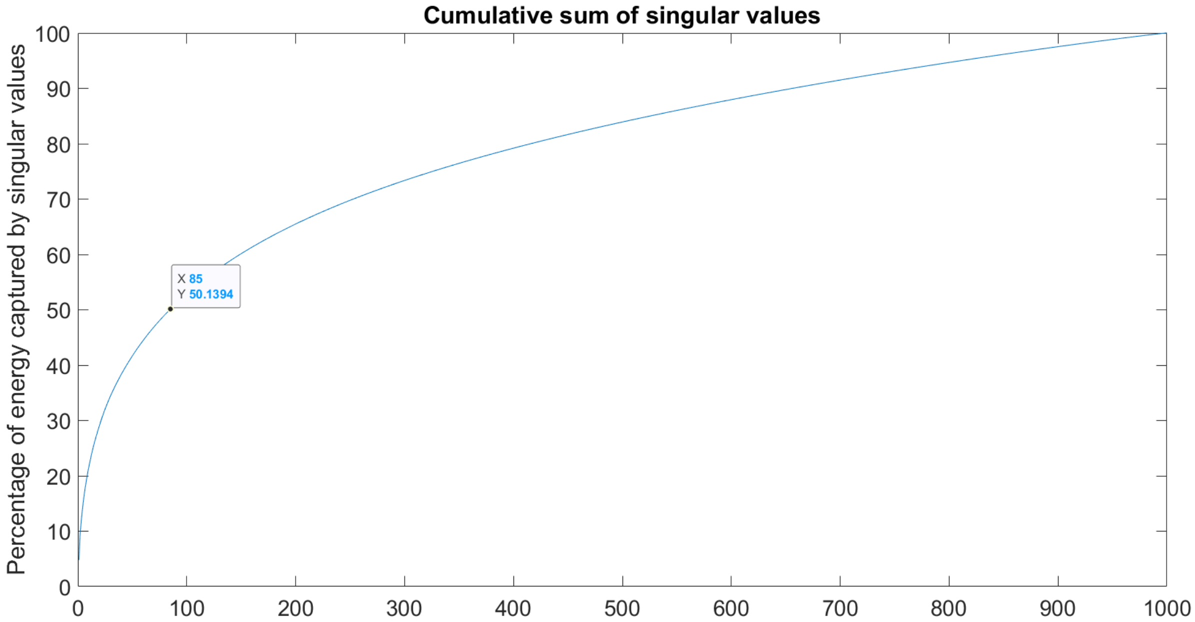


We can see here also the from 1st mode the dominant frequency almost matches with the frequency from Pressure data which tells about how 1st mode captures the time oscillations in the actual data. In further modes we observe that there are more than 1 dominant frequencies showing chaotic oscillations, which are not important.

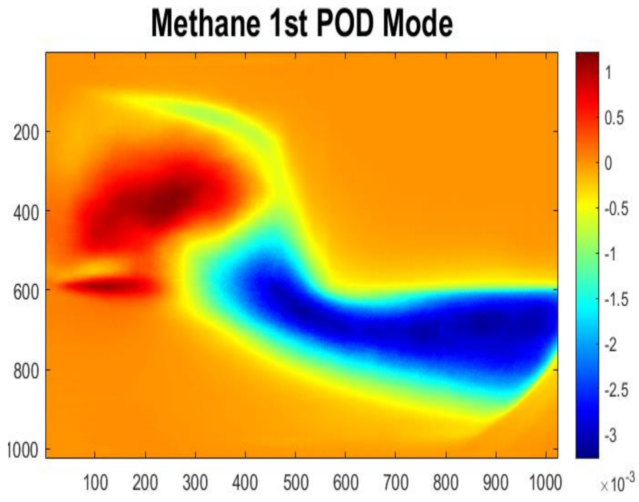
We have reconstructed the spatial-temporal images by considering only first 100 POD modes and see how it matches with real spatial-temporal images. Click **here** to see the corresponding reconstructed gif.

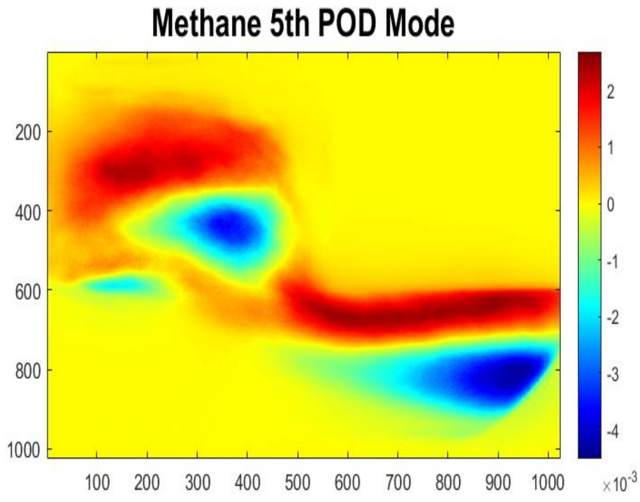
Methane:

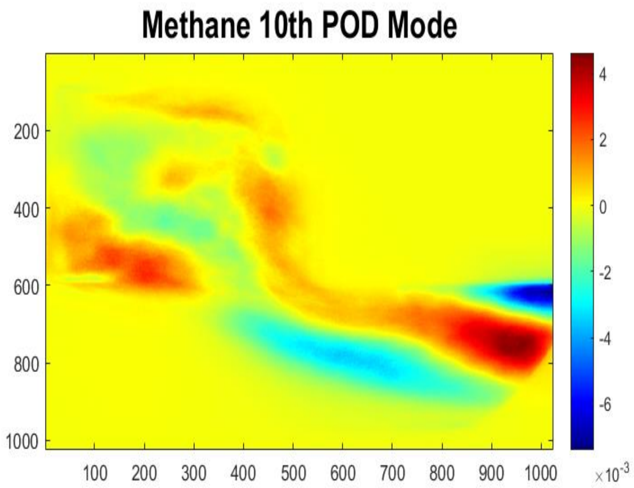
Since now, we have done analysis of fuel with some Hydrogen content in it. Now, we also have data without any hydrogen content, only methane in the fuel. Here also, we have done POD of the vectorized snapshot matrix. The cumulative Singular value distribution for Methane data is shown below,

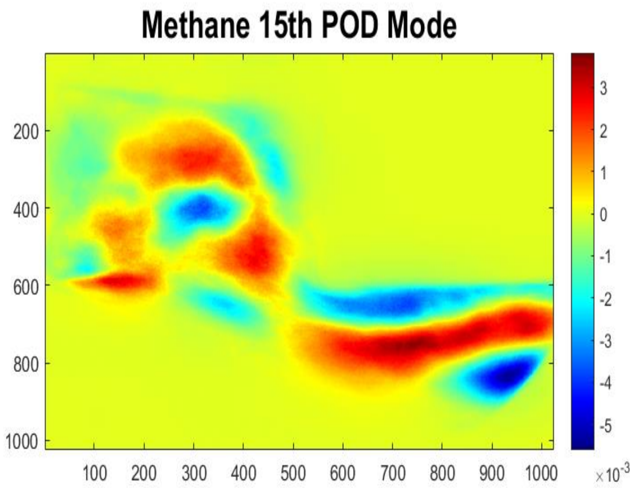


Here also we can see that, up to 85th mode the singular values capture over 50% of the covariance of the data. The POD modes are shown below,



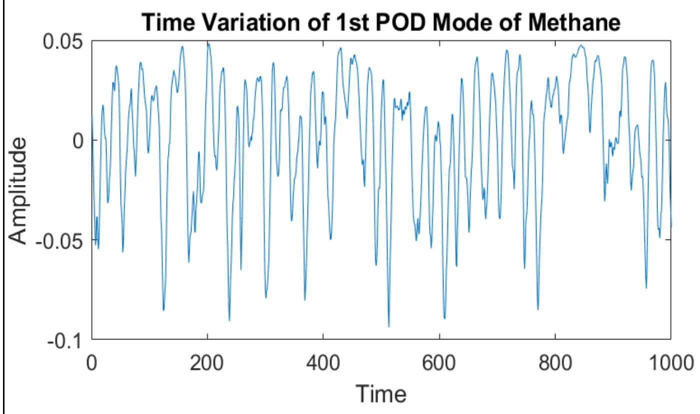


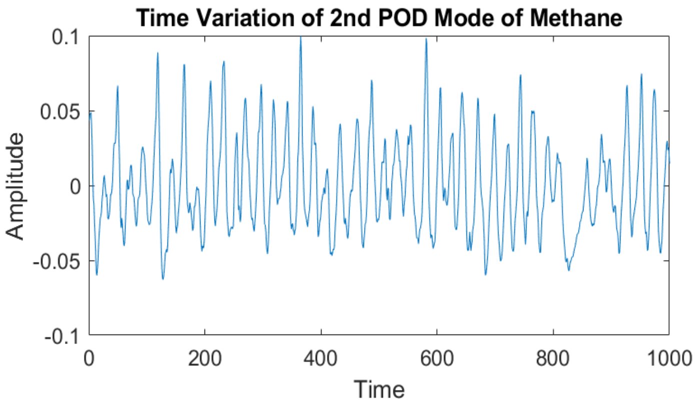


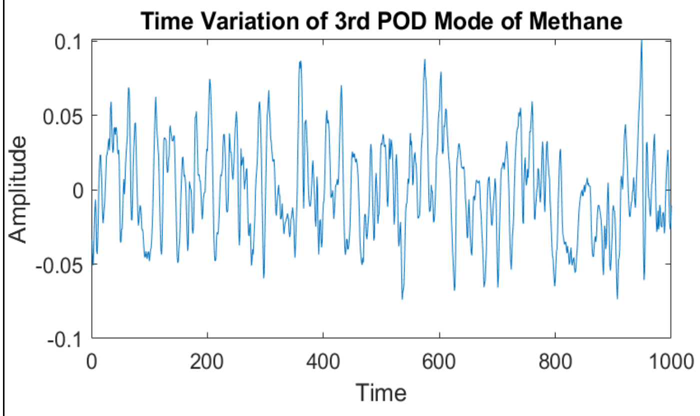


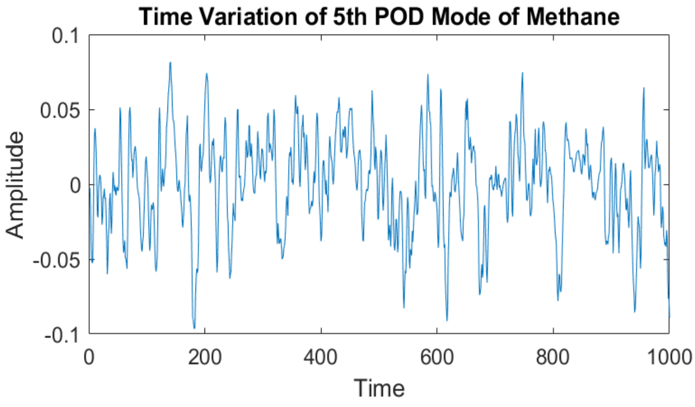
Here also we can see the 1st mode captures the most corelation. In the subsequent modes, we can see smaller regions which are less coherent in space over time.

Now, for the time dynamics of the modes we have calculated the snapshot matrix by. Since, gif can’t be played in pdf, so we have plotted the 1st column of matrix V as time vaiation. Time variations for first 5 modes are shown below.



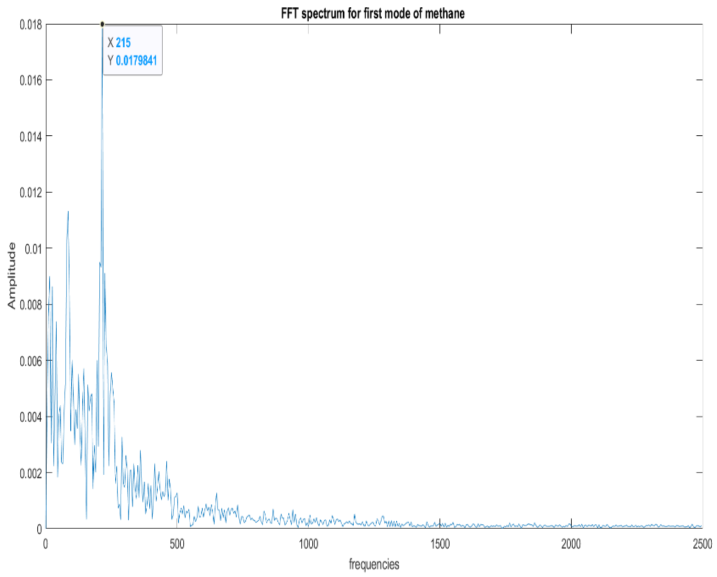


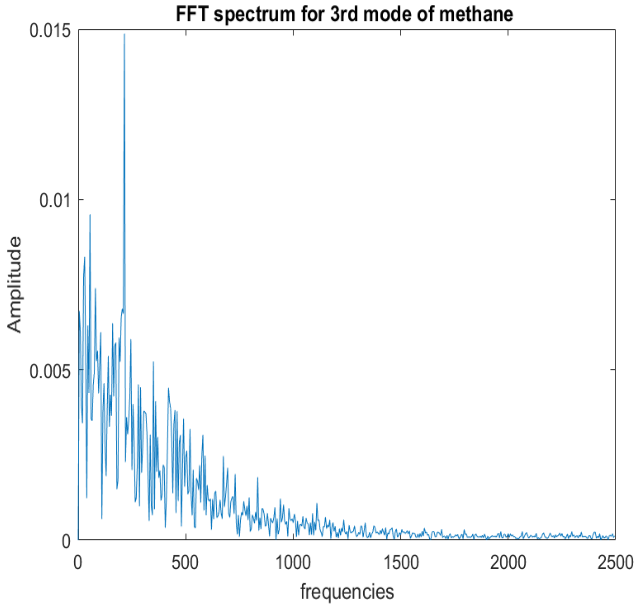


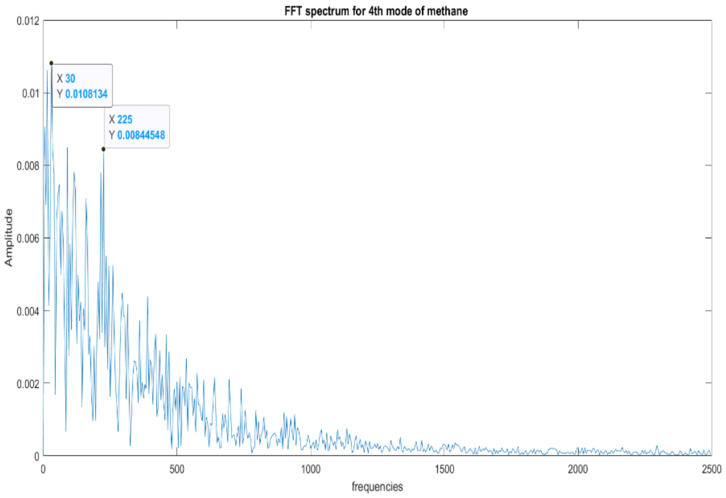


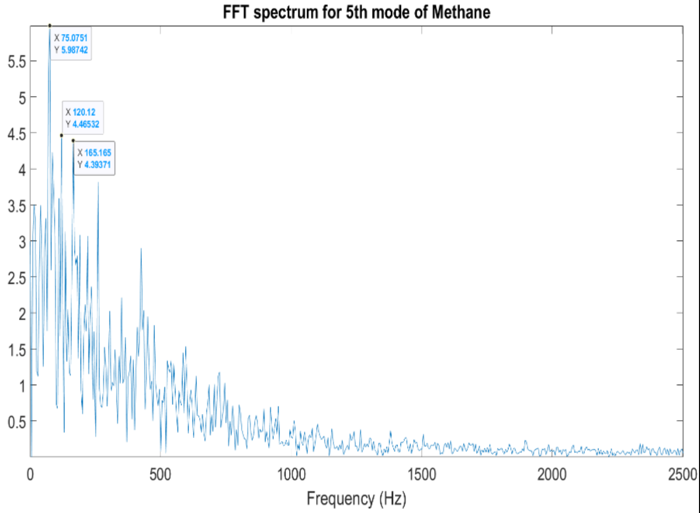
We see that 1st mode captures the oscillation quite good. In further modes abrupt time oscillations are seen.

Same as before we have done FFT of the time dynamics of the modes and compare how these matches with Pressure data and time-averaged data of snapshot matrices.









We have reconstructed the spatial-temporal images by considering only first 85 POD modes and see how it matches with real spatial-temporal images. Click **here** to see the corresponding reconstructed gif.

Dynamic mode decomposition of Hydrogen (50%) :

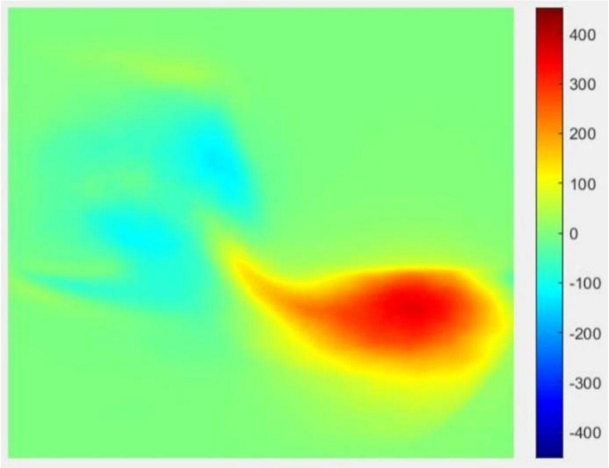


Fig. 1 DMD mode 1

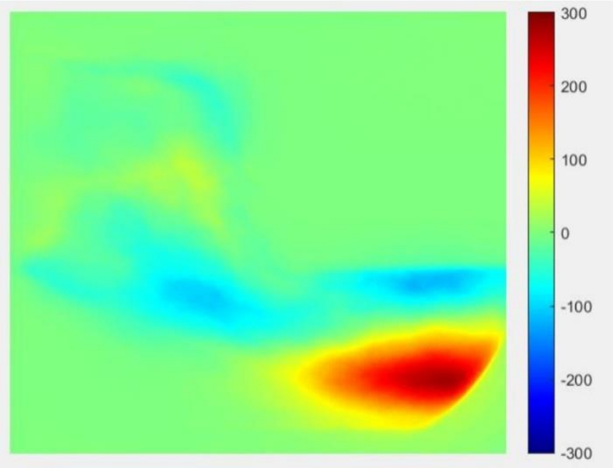


Fig. 2 DMD mode 3 (real)

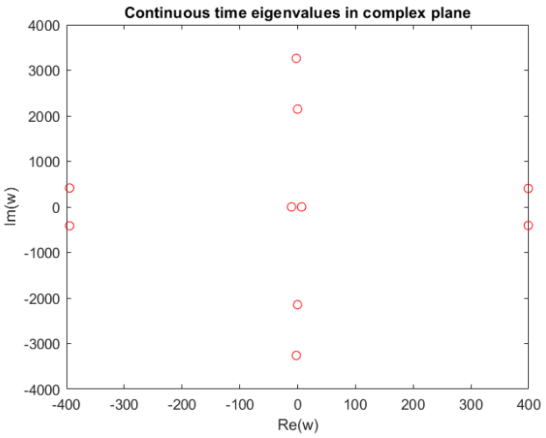


Fig. 3

To analysis the flow dynamics apart from the oscillatory behavior observed in POD modes the Dynamic mode decomposition was done for the chemiluminescence images (Frame rate: 5000 Hz and 1000 frames for time 0.2 secs). The best fit linear operator A was computed by projecting it into the rank 10 POD subspace of the snapshot matrix. Initial computation from DMD modes showed decay and oscillatory behavior of the mode, which can be due to the noise in the images. For removing noise, the forward-backward DMD was done to find the best fit operator as follows:

$$X_2 = A_{for}X_1,\ X_1 = BX_2,$$

$$B^{-1} = A_{back}$$

$$A = \sqrt{A_{for}A_{back}}$$

The modes with continuous eigenvalues having very small real part (those lying along the y-axis of Fig.1) were plotted, for observing the oscillatory flow behaviors. Fig.1 and Fig.2 shows DMD modes 1 and 3 and the corresponding temporal variation are given by the Fig.4 & 5. The DMD modes shows a convective flow structure in the combustor cavity and duct. The first few modes showed larger spatially coherent structures. The first mode also shows a slow decay with a higher frequency of oscillations and mode 3 shows a complete periodic oscillation.

Other modes either die out quickly and ramp up in time without oscillations, hence excluded from the analysis.

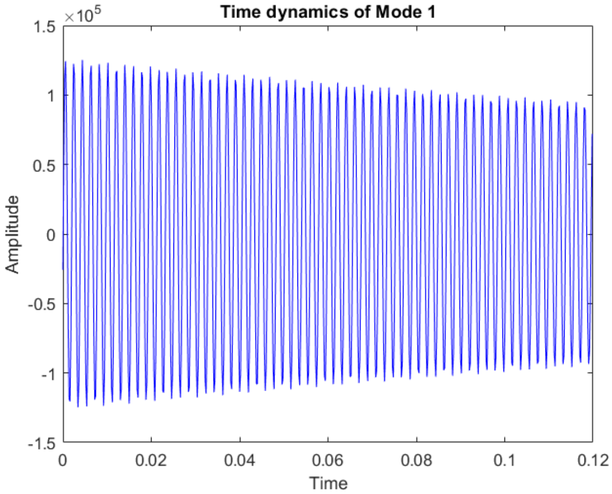


Fig. 4

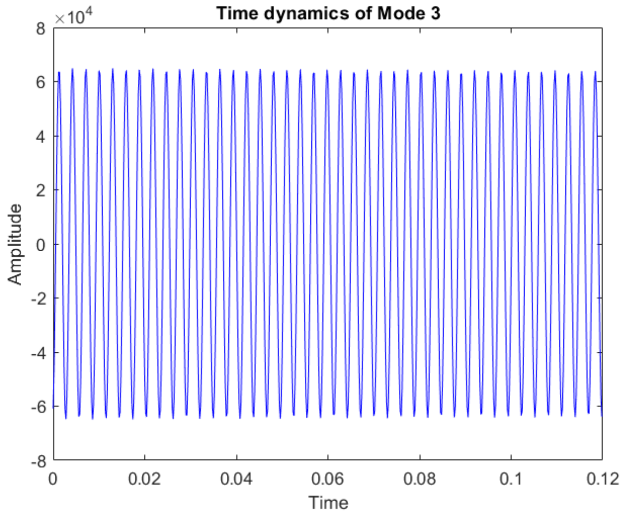


Fig. 5

Phase Space Reconstruction and Recurrence plot:

Phase space reconstruction was done using the dynamic pressure time series data to understand the dynamics of the combustion and verify the results obtained from previous analysis of Proper Orthogonal Decomposition and Dynamic mode Decomposition. For this Time delay embedding method has been used as explained in the following section.

In order to extract information about latent variables involved in the process, we have used the pressure data available with us from experiment. First, delayed versions of sample signal were created by deciding a delay time $\tau = n\Delta t$, (where n = 1,2,3………N-1), from this a Hankel matrix was constructed with the delayed vectors as follows:

$$D = [x_i \; ; x_{i+\frac{\tau}{\Delta t}} ; \dots \dots \dots \dots \dots ; \; x_{i+\frac{(d-1)\tau}{\Delta t}}]$$

In our work we have considered d = 2000 and time delay τ = 0.0001 sec, n = 1.

Singular value decomposition of the Hankel matrix was done to get the dominant singular values as shown in Fig.6. We, choose the first three columns of the right singular matrix (V) with the highest three singular values for reducing the dimension of the system from d to 3. The three latent variables V(: ,1), V(: ,2) and V(: ,3) are then plotted in the 3D phase space for dynamics pressures 1 and 2 for Methane, Hydrogen (10%) and Hydrogen (50%) as shown in Fig.7 (a)-(f).

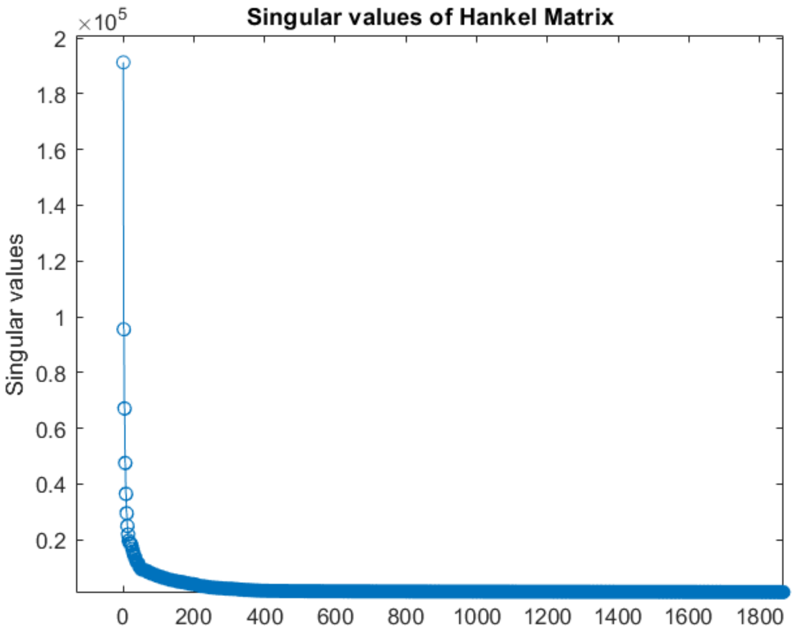


Fig. 6 Singular values of Hankel matrix

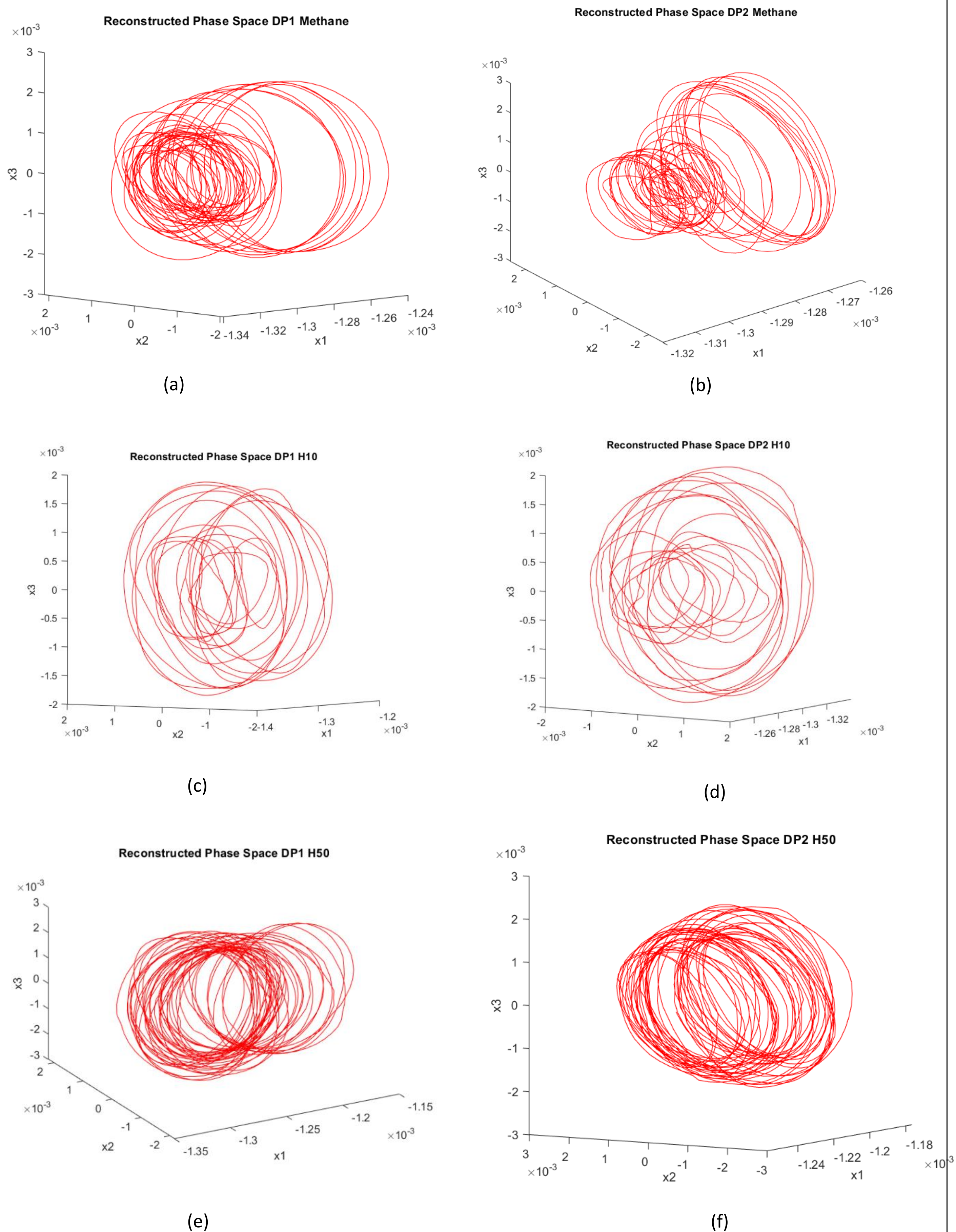
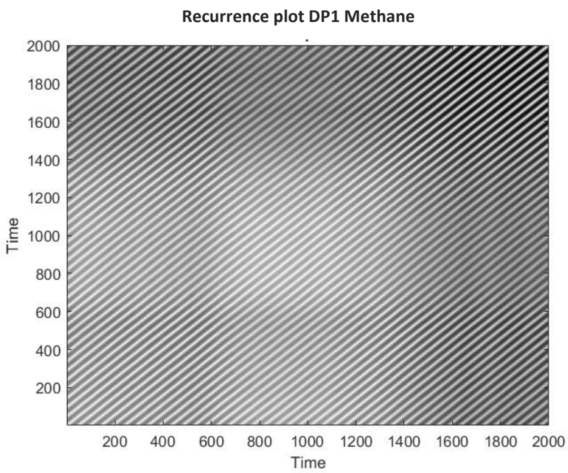


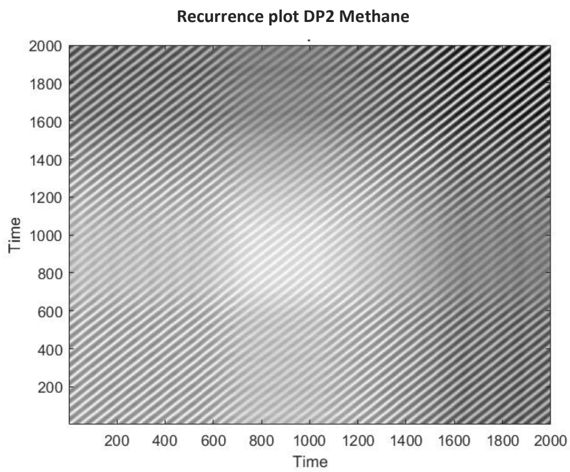
Fig. 7 Phase space reconstruction using latent variables

From the above 3D phase plots the quasiperiodic nature of system can be inferred from the cyclic trajectories, phase trajectories seem to move aorund inside a finite cylindrical space passing close to each other. The dimension of Hankel matrix was chosen to be very high (i.e. 2000) to remove false neighbours in the phase space, it was chosen by observation from various values of d. Change in the diameter of the cyclic path suggests variation in the frequency of oscillations. Increase in amount of hydrogen in the fuel makes the phase portrait more dense suggesting the shift from quasiperiodic to more periodic oscillations.

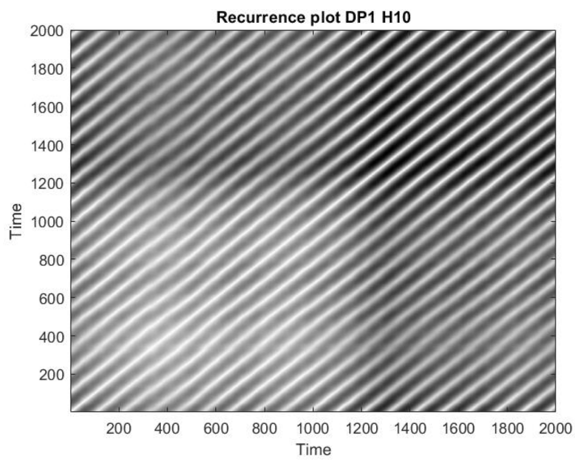
To get more idea about the the nature of oscillations, recurence plots were used to visualize the dynamics as shown below in Fig.8 (a) – (f).



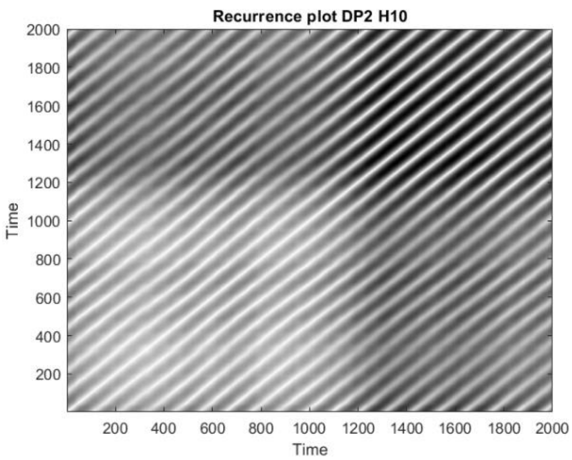
(a)



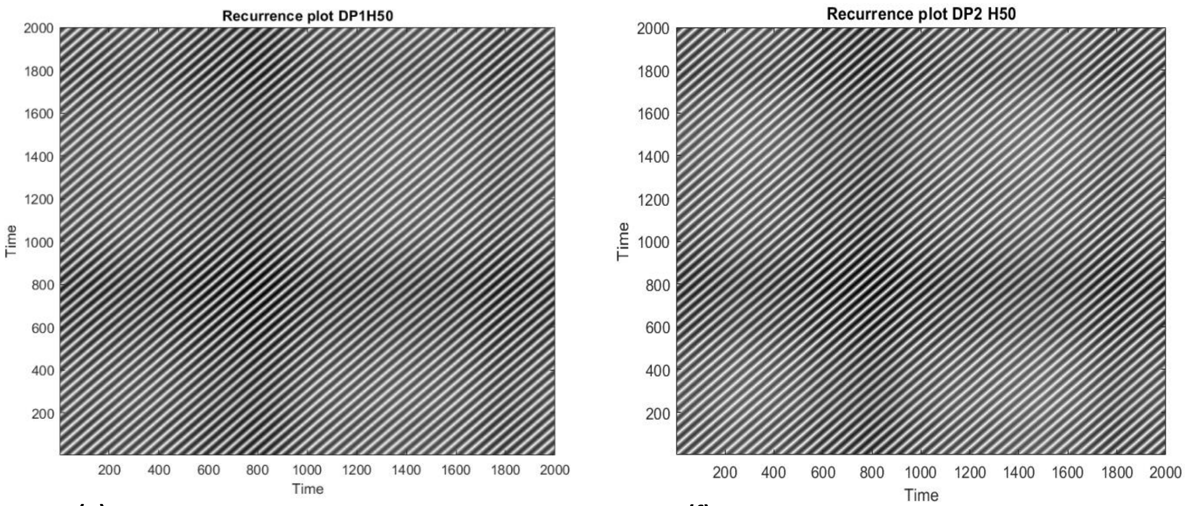
(b)



(c)



(d)



(e) (f)

<u>Fig.8 Recurrence plots</u>

The above plots are computed for time:0-0.2 secs. Sampling frequency : 10,000 Hz.

The oscillatory nature of phase trajectories is clearly evident from the recurrence plots. We observe clear diagonal lines in lighter shade than the background suggesting that the trajectories come very close to their previously traversed paths but not exactly following same path, which is the characteristics of quasiperiodic orbits. As hydrogen is added the phase trajectories seem to become dense (passing very close), which can also be inferred from the Fig. 3 (e) and (f) where there are lighter diagonal lines in darker backgorund as compared to Methane and 10% Hydrogen where there are weak oscillations represented by the light square patches. The darkar patches represent the time when the oscillations are stronger (close to limit cycle oscillations). These kind of oscillations were oberved to get activated after 0.16 secs where as limti cycle kind of oscillations persists from startr to end for 0.2 secs in case of 50% Hydrogen.

The frequency deduced from the recurrence plot also matches closely to the results obtained from FFT of pressure data and space averaged time series from the chemiluniscene images in the cavity and along the exhaust duct. Frequency for Methane : 205 Hz, Hydrogen (10%) : 90 Hz and Hydrogen (50%): 210 Hz.