NEW TOPICS ADDED FROM ACADEMIC SESSION 2021-22 ONWARDS
SECOND SEMESTER
ENGINEERING MECHANICS [ES 114]
UNIT - I

SYSTEM OF FORCES

When several forces of different magnitude and direction act upon a body, they constitute a force system. Considering the plane in which forces are applied and depending upon the position of line of action, forces may be classified as:

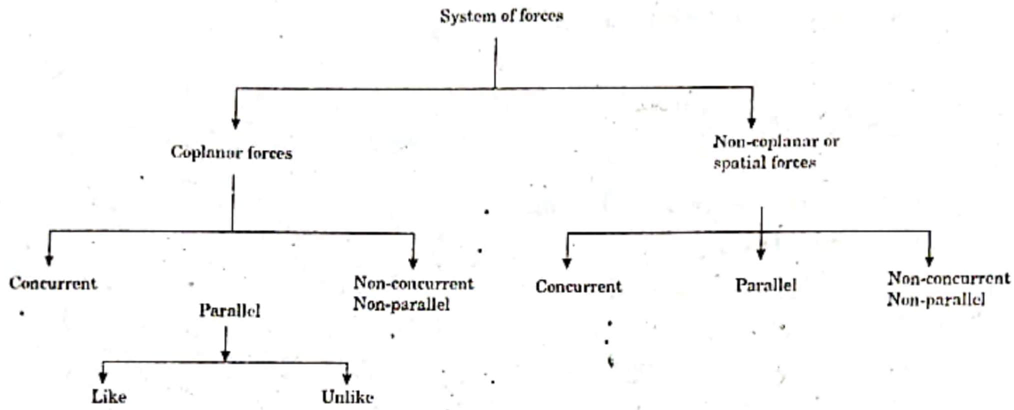


Fig. 1

Force is an external agent which tends to change the speed or direction of a system. A force is applied whenever the system needs to be accelerated or decelerated. A force on a rigid body may produce one or both of the following effects:

— change its state of rest or motion
— accelerate or retard its motion
— change its shape or size
— turn or rotate it, and
— keep it in equilibrium.

A force is a vector quantity determined completely by its

— magnitude
— point of application
— line of action, and
— direction.

RESOLUTION OF FORCES

Finding the components of a given force in two given directions is called resolution. These component forces will have the same effect on the body as the given single force. Let the given force be R and let it be required to find its components in directions making angles α and β with its line of action.

With reference to parallelogram OACB, the sides OA and OB represent the components of the given force R along OX and OY respectively. That is

$$OA = P \text{ and } OB = Q$$

Further $\angle OCA = \angle BOC$
(alternate angles)

$$= \beta$$

$$\therefore \quad \angle OAC = 180° - (\alpha + \beta)$$

Applying sine rule to ΔOAC, we get

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin\{180-(\alpha+\beta)\}}$$

AC is parallel and equal to OB which represents Q

$$\therefore \quad \frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

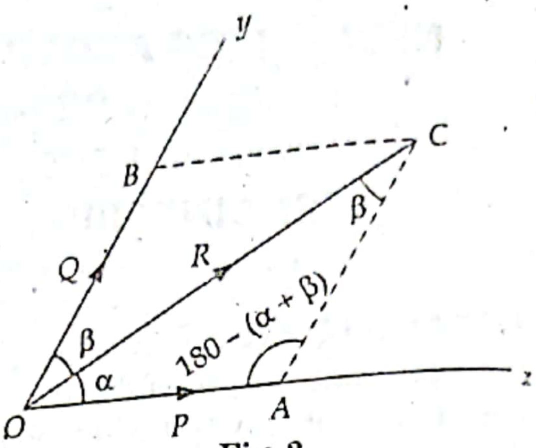


Fig. 2

Thus the resolved parts of the given force R are

$$P = R\frac{\sin\beta}{\sin(\alpha+\beta)} \text{ and } Q = R\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

When the force R is to be resolved along perpendicular directions, then OX and OY are at right angles and OACB becomes a rectangle

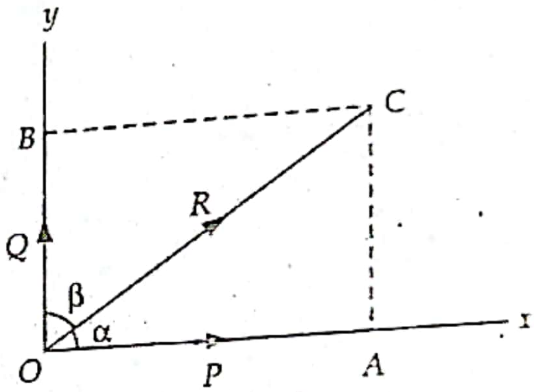


Fig. 3

$$\alpha + \beta = 90° \quad \text{or} \quad \beta = 90° - \alpha$$

$$\therefore P = R\frac{\sin\beta}{\sin(\alpha+\beta)} = R\frac{\sin(90°-\alpha)}{\sin 90°} = R\cos\alpha$$

i.e., the projection of OC (representing R) on OX is a measure of resolved component along the direction OX

$$Q = R\frac{\sin\alpha}{\sin(\alpha+\beta)} = R\frac{\sin\alpha}{\sin 90°} = R\sin\alpha$$

i.e., the projection of OC (representing R) on OY is a measure of resolved component along the direction of OY When the components P and Q are at right angles to each other, they are called the rectangular components of force R.

Q.1. Two cables which have known tensions of 40 N and 60 N are attrached to the top of a tower AB as shown in Fig. 4. What tension will be induced in the guy wire AC if the resultant of the forces exerted at the top A by the cables acts vertically downwards?

Ans. Since the resultant of the forces acts vertically downwards,

we have $\Sigma F_x = 0$ and that gives

$40 \cos 15° - 60 \cos 30° + T \cos\theta = 0$ ...(i)

In the right angled triangle ABC

$$\theta = \tan^{-1}\left(\frac{AB}{BC}\right) = \tan^{-1}\left(\frac{15}{10}\right) = 56.31°$$

Substituting $\theta = 56.31°$ in expression (i), we get

$40 \cos 15° - 60 \cos 30° + T \cos 56.31° = 0$

or $38.64 - 51.96 + 0.555\, T = 0$

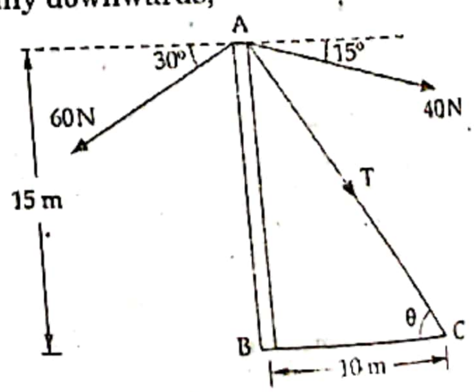


Fig. 4

$\therefore$ Tension in the guy wire $T = \dfrac{51.96 - 38.64}{0.555} = 24$ N

RESULTANT OF COPLANAR-CONCURRENT FORCES

Analytical method (Principle of resolved parts): The resultant of a number of a coplanar-concurrent forces acting on a body is worked out analytically by adopting the step-by-step procedure outlined below:

(i) Find the components of each force in the system in two mutually perpendicular X and Y directions.

(ii) Make algebraic addition of components in each direction to get two components ΣF_x and ΣF_y.

(iii) Obtain the resultant both in magnitude and direction by combining the two components forces ΣF_x and ΣF_y which are mutually perpendicular.

$$\text{Resultant, } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

and its inclination θ to X-axis is given by

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

This analytical method is based on theorem of resolved parts which states that the algebraic sum of the resolved parts of two forces in a given direction is equal to the resolved part of their resultant in the same direction".

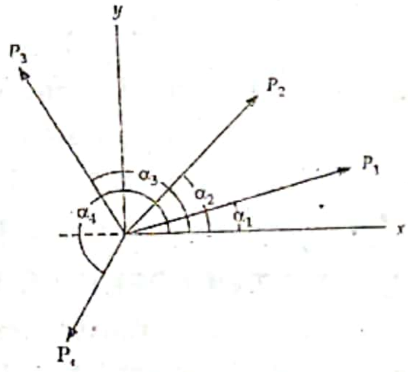


Fig. 5

With reference to Fig. 5 P_1, P_2, P_3 and P_4 are the concurrent forces meeting at point O and making angles α_1, α_2, α_3 and α_4 with OX.

Resolving along x-axis and y-axis, we get

$$\Sigma F_x = P_1 \cos\alpha_1 + P_2 \cos\alpha_2 + P_3 \cos\alpha_3 + P_4 \cos\alpha_4$$
$$\Sigma F_y = P_1 \sin\alpha_1 + P_2 \sin\alpha_2 + P_3 \sin\alpha_3 + P_4 \sin\alpha_4$$

The usual practice for analytical solution is to examine the orientation of the given forces in different quandrants and identify their inclination :

Q.2. Determine the magnitude and direction of the resultant of the following set of forces acting on a body.

(i) 200 N inclined 30° with east towards north.

(ii) 250 N towards the north,

(iii) 300 N towards north west, and

(iv) 350 N inclined at 40° with west towards south.

What will be the equilibrium of the given force system?

Ans. The given force system has been depicted in Fig. 6

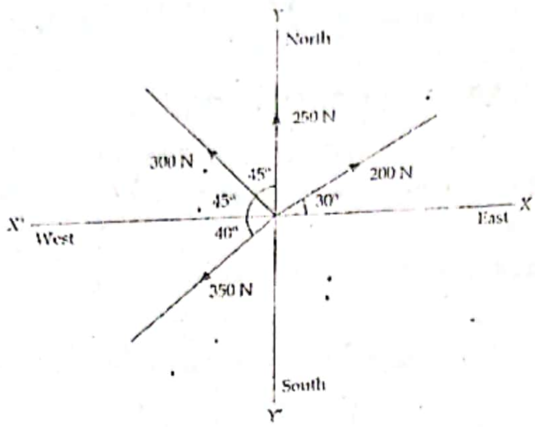


Fig. 6

Resolving all the forces along X-direction

$$\Sigma F_x = 200 \cos 30° + 250 \cos 90° + 300 \cos 135° + 350 \cos 220°$$
$$= 200 \times 0.866 + 250 \times 0 + 300 \times (-0.707) + 350 \times (-0.766)$$
$$= 173.2 + 0 - 212.1 - 268.1 = -307 \text{ N (along OX')}$$

Resolving all the forces along Y-direction

$$\Sigma F_y = 200 \sin 30° + 250 \sin 90° + 300 \sin 135° + 350 \sin 220°$$
$$= 200 \times 0.5 + 250 \times 1 + 300 \times 0.707 + 350 \times (-0.642)$$
$$= 100 + 250 + 212.1 - 224.7 = 337.4 \text{ N (along OY)}$$

RESULTANT OF COPLANAR, NON-CONCURRENT FORCE SYSTEM

Below are outlined the different steps which are followed to determine the resultant of coplanar, non-current force system in magnitude, direction and position:

1. Resolve the given forces horizontally and vertically and determine

   ΣF_x = algebraic sum of the component of all forces in x-direction, and

   ΣF_y = algebraic sum of the component of all forces in y-direction

2. Determine the magnitude of the resultant force by using the relation

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

3. Find the direction of the resultant force with respect to x-axis by using the relation

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

Depending upon the sign of ΣF_x and ΣF_y decide in which of the quadrant the resultant lies.

4. Obtain the algebraic sum of the moments of all forces about any given point say O.

5. Mark the position of resultant such that it produces the same direction of moment about point O.

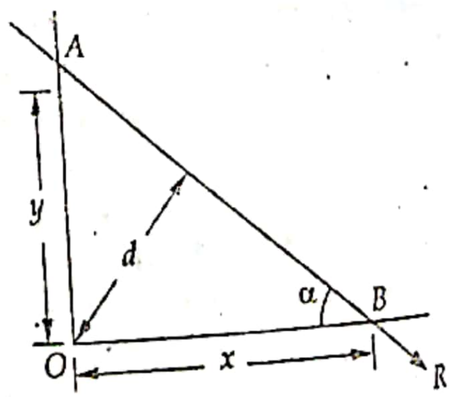


Fig. 7 (a)

6. Apply Varignon's theorem of moment to find the exact position of resultant. That is

$$\Sigma M_0 = R \times d$$

where ΣM_0 = moment of given forces about point O

d = perpendicular distance of line of action of resultant R from point O.

The x and y intercepts of the line of action of resultant can then be worked out as

$$x = \frac{d}{\sin\alpha} \quad \text{and} \quad y = \frac{d}{\cos\alpha}$$

Alternatively/ the intercepts can be determined by considering the moment of K about O as the sum of moments of its components about O (Varignon's theorem)

$$R \times d = \Sigma M_o$$

$$\Sigma F_x \times 0 + \Sigma F_y \times x = \Sigma M_o$$

$$\therefore \quad x = \frac{\Sigma M_O}{\Sigma F_y}$$

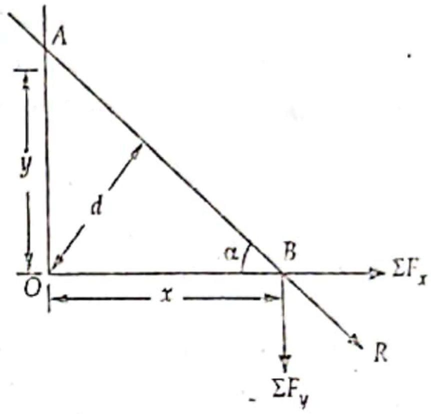


Fig. 7 (b)

Similarly by resolving the resultant into its components 4 it can be calculated that

$$y = \frac{\Sigma M_O}{\Sigma F_x}$$

Q.3. An angle bracket has been subjected to three forces and a couple as shown in Fig. 8. Determine the resultant of this system of forces. Proceed to locate the points where line of action of resultant intersects line AB and the line BC.

Ans. Resolving all the forces horizontally and vertically

$\Sigma F_x = 125 \cos 60° - 200 = -137.5$ N

$\Sigma F_y = 125 \sin 60° - 50 = 58.25$ N

Magnitude of the resultant force,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(-137.5)^2 + (58.25)^2}$$

$$= 149.33 \text{ N}$$

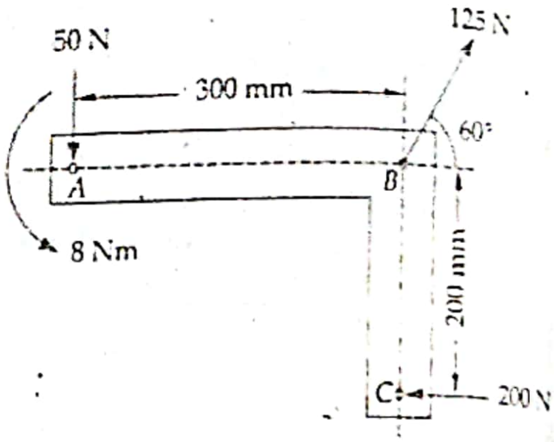


Fig. 8

If α be the angle which the resultant makes with the horizontal, then

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{58.25}{137.5} = 0.4236$$

$\therefore \alpha = 22.96°$

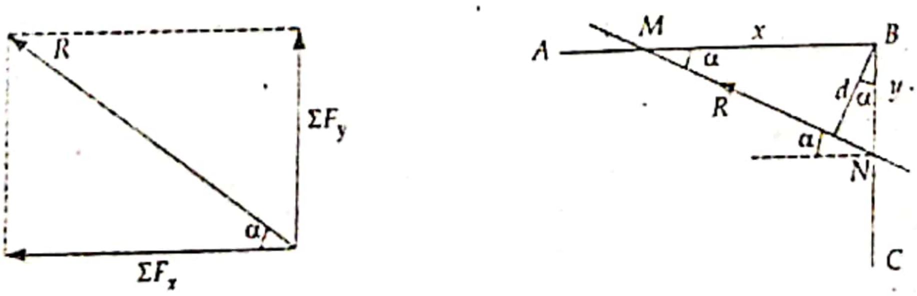


Fig. 9

(b) Taking moments of all forces about point B (clockwise moment + ve)

$$\Sigma M_b = -8 - 50 \times 0.3 + 200 \times 0.2 = 17 \text{ Nm} \quad \text{(clockwise)}$$

The resultant should lie below B and act as shown in Fig. (9) so that it can produce a clockwise (positive) moment about B,

Let d be the distance of resultant from point B. Then,

$$R \times d = \Sigma M_b$$

$$d = \frac{\Sigma M_b}{R} = \frac{17}{149.33} = 0.1138 \text{ m}$$

The intercepts on x-axis and y-axis are then worked out as

$$x = BM = \frac{d}{\sin\alpha} = \frac{0.1138}{\sin 22.96°} = 0.292 \text{ m}$$

$$y = BM = \frac{}{\cos\alpha} \frac{0.1138}{\cos 22.96°} = 0.124 \text{ m}$$

The resultant thus intersects AB at 0.292 m from B and EC at 0.124 m from B.

COUPLE

Two parallel forces equal in magnitude but opposite in direction, and separated by a finite distance are said to form a couple. If F and F′ are two such forces, then the couple is denoted by (F, F′). A body acted upn.by a couple spins round but remains at the same spot.

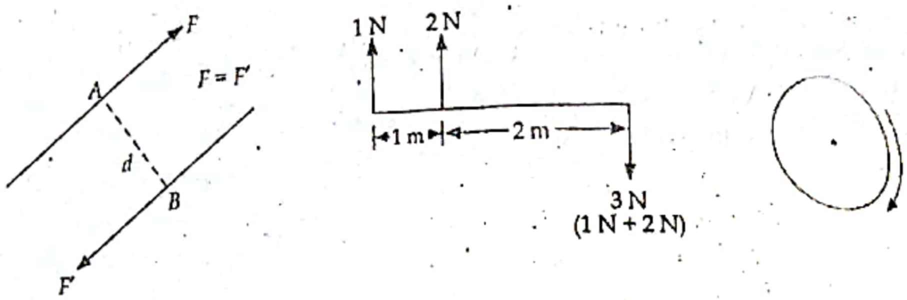


Fig. 10 Concept of a couple

With reference to Fig. 10 the moment of couple M is

$$M = F \times d$$

The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called arm of the couple.

Examples of a couple :

(i) Winding of a watch or a clock

(ii) Opening or closing a water tap

(iii) Unscrewing the cap of an ink bottle

(iv) Locking/unlocking of a lock with a key

(v) Turn of the cap of a pen

(vi) Forces applied to the handle of a screw

The salient aspects of a couple are :

(i) The algebraic sum of the vertical and horizontal components of the forces (F, F′) forming a couple is zero, i.e., the resultant of the forces (F, F′) is zero.

(ii) A zero resultant of the forces forming the couple implies that a couple cannot produce translation. The couple causes only rotation and provides a turning effect, i.e., tends to make the body turn.

(iii) The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called arm of the couple. With reference to Fig.10, the moment of couple is

$$M = F \times d$$

and

$$M = 1 \times 3 + 2 \times 2 = 7 \text{ Nm}$$

(iv) The sum of the moment of the two forces forming a couple is equal to the moment of the couple.

Consider a point A (Fig. 11) lying in the plane of the forces F and F′ that form a couple. Then the sum of the moments of the two forces about point A is

$$\begin{aligned} M &= F' d_2 - F d_1 \\ &= F d_2 - F d_1 \ (\because F' = F) \\ &= F(d_2 - d_1) = F d \\ &= \text{moment of the couple} \end{aligned}$$

The sum of the moments does not depend on the location of point A: it will have the same value and same sense.

Obviously a deduction can be made that "the algebraic sum of the moments of two forces forming a couple about any point in their plane is constant and equal to the moment of the couple".

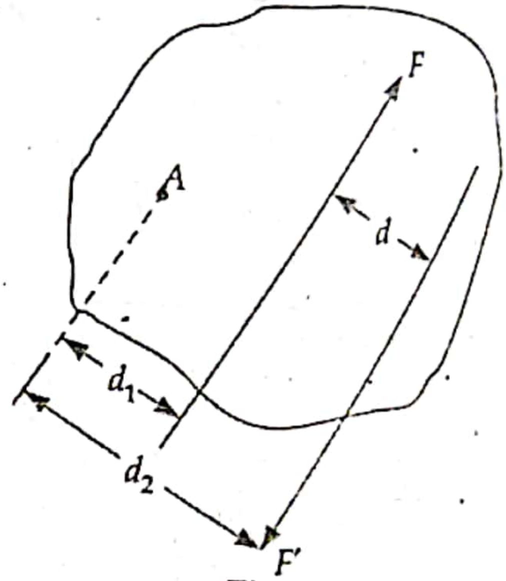


Fig. 11

(v) Two caplanar couples, whose moments are equal and opposite, balance each other.

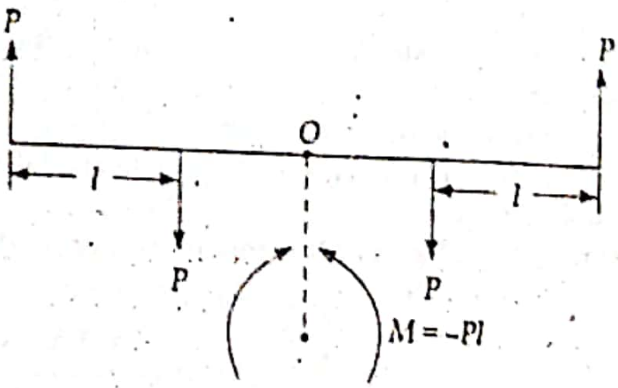


Fig. 12

With reference to Fig. 12, there act two couples whose moments are equal and opposite. These couples will balance each other and accordingly there will be no turning effect at point O.

(vi) Any two couples will be equivalent if their moments are equal, both in magnitude and direction.

Consider the system of forces depicted in Fig. 12

In Fig. 12(a): $M = P \times l = Pl$ (anticlockwise)

In Fig. 12 (b): $M = \frac{P}{2} \times 2l = Pl$ (anticlockwise)

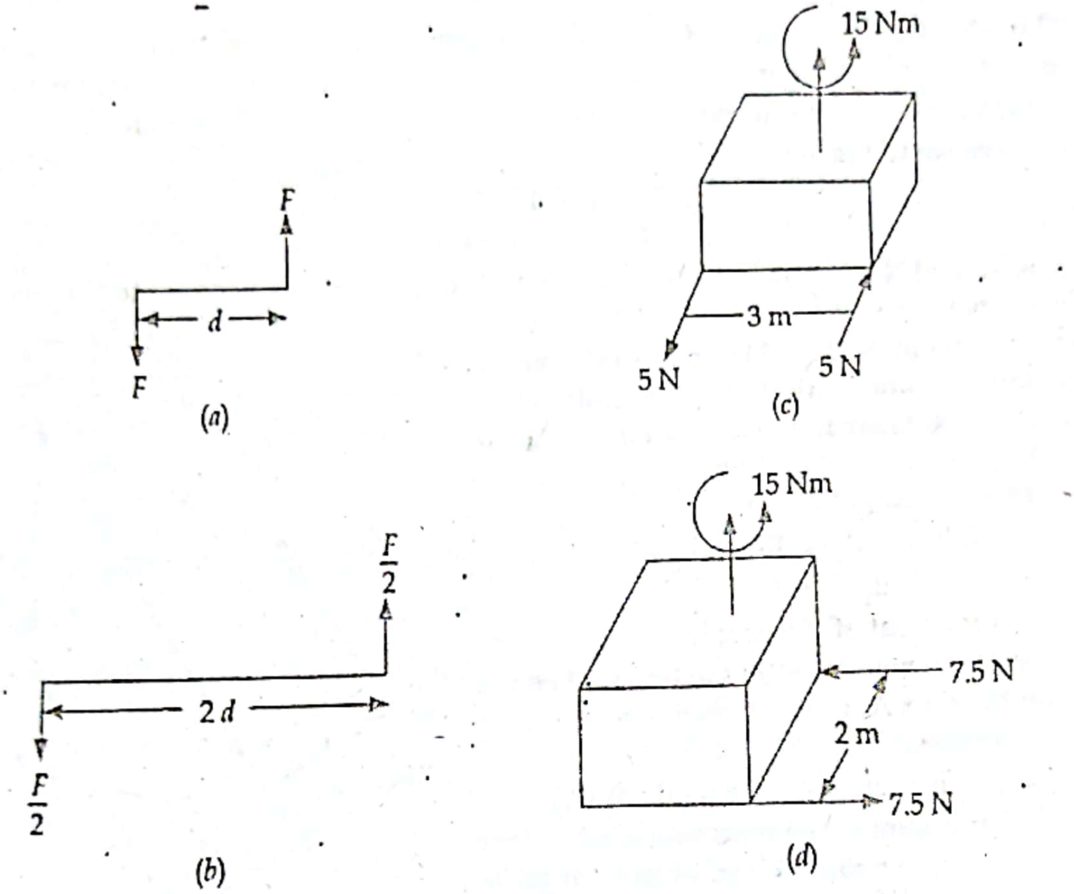


Fig. 12

Further, the two couples depicted in Fig. 12 (c) and (d) have a different force magnitude direction but they have same moment magnitude and direction. They two represent an equivalent couple.

Two equivalent couples have the same effect on the body, and it does not matter

– where the two forces form the couple

– what magnitudes and directions the forces have

The magnitude and the direction of the moment of couple are only factors which decide the equivalence of couples.

(vii) Algebraic sum of moments of a number of couples is equal to the moment of a single couple.

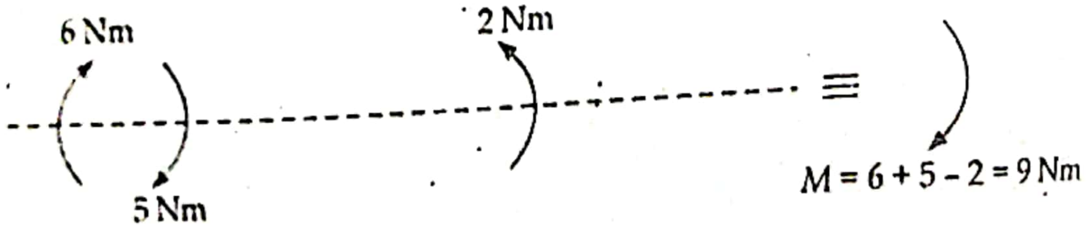


Fig. 13

(viii) A single force P and a couple M acting in the same plane on a body cannot balance other. However, they are together equivalent to a single force at a distance e = M/P from its original line of action.

Further :

- a couple cannot be balanced by a single force, but can be balanced only by a couple of opposite sense.
- any number of coplanar couples can be reduced to a single couple of moment equal to

algebraic sum of the moments of all the couples.

- the translatory effect of a couple on a body is zero.
- the effect of couple on a body remains unchanged if the couple is

(a) rotated through an angle,

(b) shifted to any other position.

(c) replaced by another pair of forces whose rotational effect is same

RESOLUTION OF FORCE SYSTEM INTO A FORCE AND A COUPLE

The following procedure is adopted for resolution of a force system into a force and a couple

(i) Determine the resultant of the given force system both in magnitude and location.

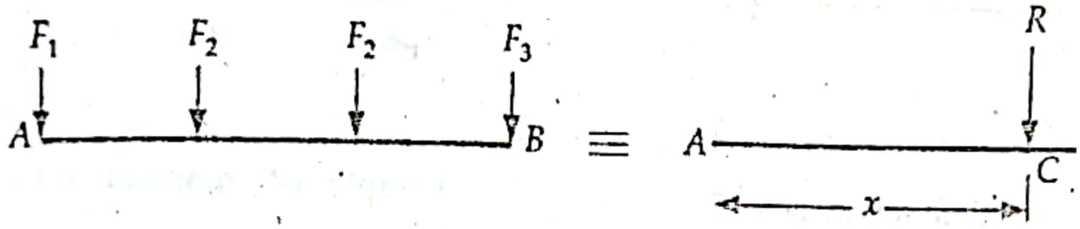


Fig. 14(a)

With reference to (Fig.14 a), R is the resultant of forces F_1, F_2, F_3 and F_4 and it acts at the distance x from a particular point A.

(ii) Introduce a force equivalent to the resultant at point A both in the upward and downward directions. This application of equal and opposite forces at point A gives zero net resultant at A and obviously there is no change in the given force system.

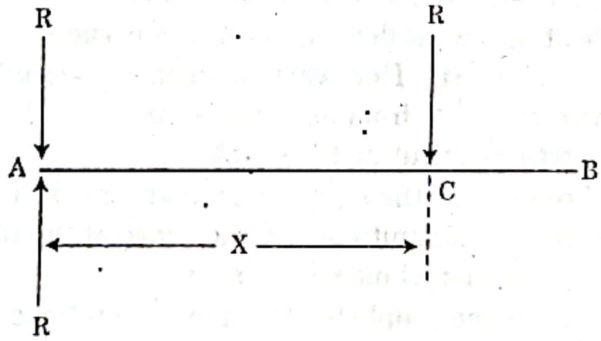


Fig. 14(b)

(iii) The downward resultant R and the upward force R applied at A constitute a couple. The moment of this couple is in anti-clockwise direction and equals the product of R and the distance x between to forces, That is $M_A = R \times x$

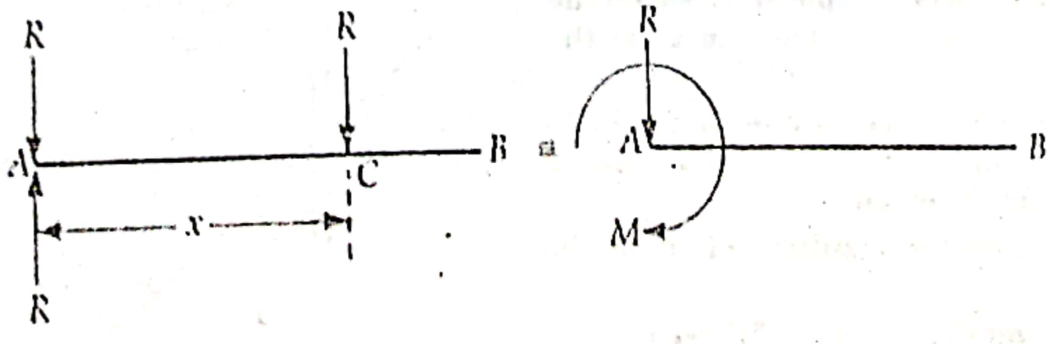


Fig. 14(c)

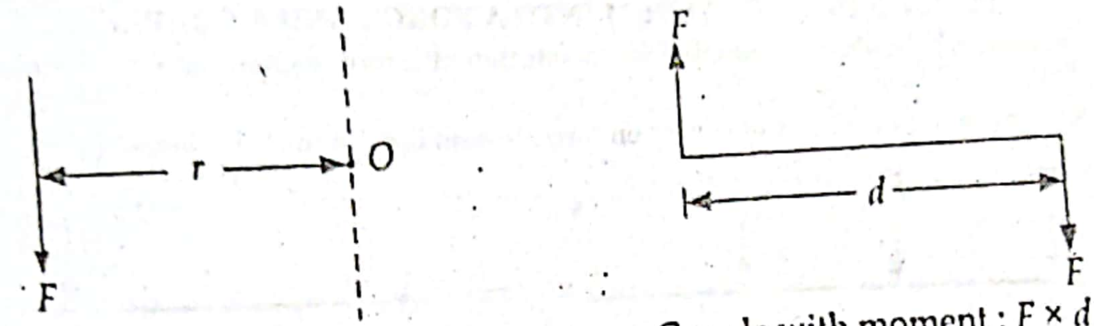
The given force system thus gets replaced with a force and a couple at point A.

Moment And couple

The differences between a moment and a couple may be stated as:

(i) Two parallel forces having the same magnitude but acting in opposite direction form a couple.

The moment of couple is the product of either of the forces and the perpendicular distance between them.



Moment about an axis : $F \times r$.

Couple with moment : $F \times d$

Fig. 15

The product of a force and the perpendicular distance of the line of action of the force from a point or axis is referred to as the moment of force about that point or axis.

(ii) Both the moment and couple have sense of direction (clockwise or anti-clockwise) and are expressed in the units Nm.

(iii) Moment of a force varies where as the moment of a couple is always constant. The moment of force about any point depends upon the perpendicular distance of that point from the line of action of force, However, the moment of couple is independent of the distance of the centre of couple from any outside point; it depends only upon the distances between the forces constituting the couple.

Further, the moment of couple has the same value, and same sense irrespective of the location of moment centre. The magnitude and the sense of the moment of force does change with change in the location of moment centre.

(iv) Both the moment of force and couple tend to cause a rotational effect on the body.

(v) Moment of force about a point can be translated into moment of force about another point and a couple.

Q.4. (a) A body is acted upon by the following two couples:

(i) a clockwise couple of 30 Nm on the *x*-axis with its centre 4 m from the origin, and

(ii) a counter clockwise couple of 50 Nm along the y-axis with its centre 2 m from the origin.

Calculate the resultant effect at the origin.

(b) Find the moment of three couples acting on a L-bar as shown in the figure.

Ans. The moment of a couple has the same value and sense irrespective of the location of moment centre.

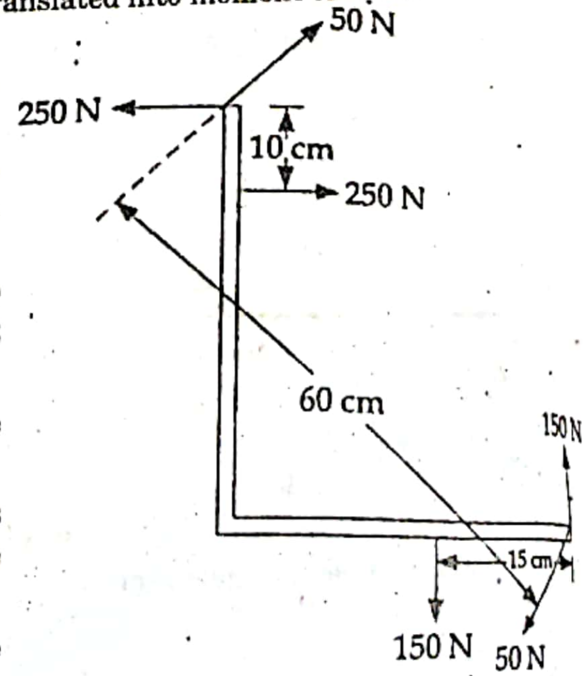


Fig. 16

∴ Resultant moment

$$= 30 - 50 = -20 \text{ Nm (anti-clockwise)}$$

(b) Resultant moment on the bar due to the three given couples is

$$= -(250 \times 0.1) + (50 \times 0.6) - (150 \times 0.15)$$
$$= -25 + 30 - 22.5$$
$$= -17.5 \text{ Nm (anticlockwise)}$$

MOMENT OF A FORCE

Moment of force about a point is defined as the turning or rotational effect of a force about that point. It is measured by the product of force and the perpendicular distance of the line of action of the force from that point.

With reference to Fig. 17

P = force acting on a body

l = perpendicular distance between the point O and line of action of force P

The moment of force P about point O

$$= P \times l$$

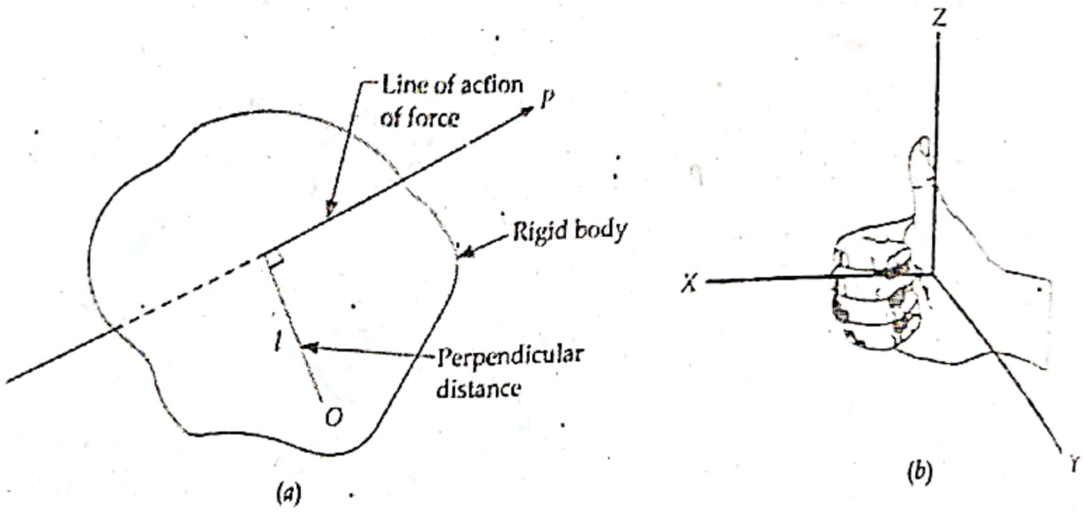


Fig. 17. Concept of moment of force and direction of a moment

The point O is called the moment centre and the distance l is the moment arm or arm of the force

- The sense of the moment is identified by applying the right hand rule (Fig, 17 b). The moment M about Y-axis is represented by the vector that points towards the direction thumb.
- The moment of force about a point is a vector which is directed perpendicular to the plane containing the moment centre and the force.
- If force is measured in newton (N) and length in metres (m), then the units of force will be Nm.
- When a force acts on a body, it causes or tends to cause a change of state of rest or of uniform rectilinear motion of the body. The action of moment tends to cause a rotation motion to the body.

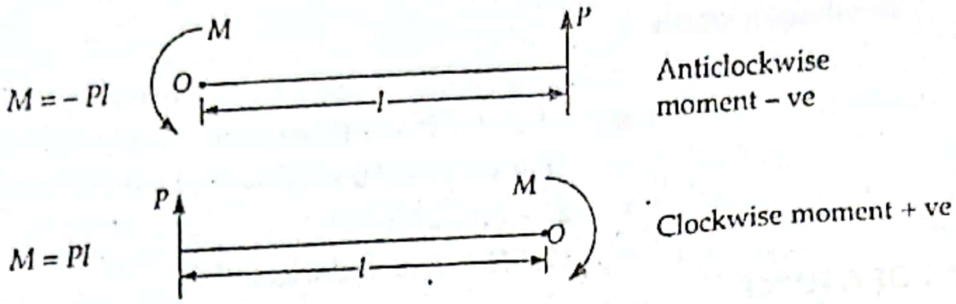


Fig. 18

• The tendency of rotation or turning of the body due to moment of force may be clockwise (in the same direction in which hands of the clock move) or anticlockwise (in a direct) opposite to the movement of hands of a clock). The corresponding moments are referred as clockwise moment and anticlockwise moment.

The general convention is to take the clockwise moments as positive and anticlockwise moments as negative (Fig. 18).

• If a number of moments act on a body (Fig. 19), then the resultant moment will be the algebraic sum of these moments, and the effect of resultant moment on the body will be governed by its turning tendency.

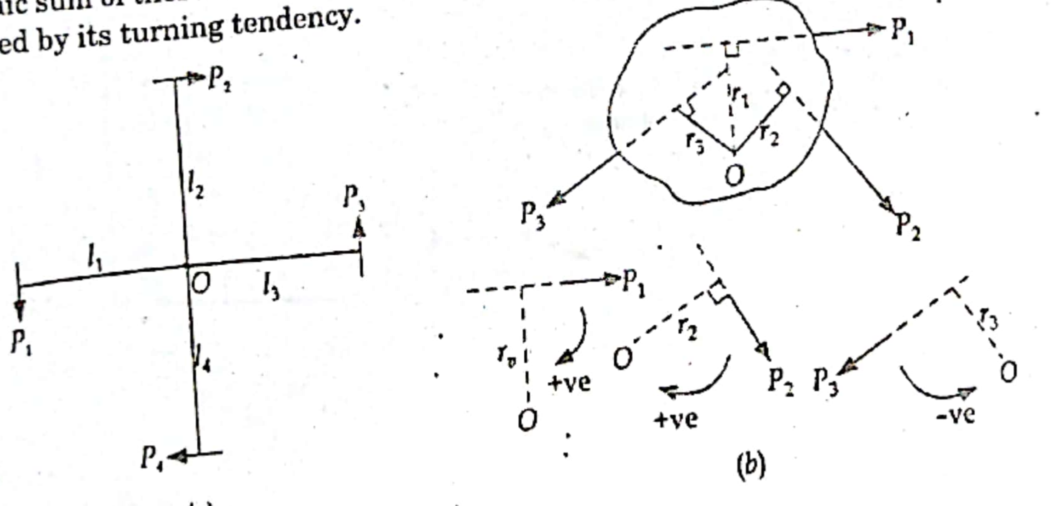


Fig. 19 Number of moments acting on a body

With reference to (Fig. 19 a), the resultant moment M about the point O is given by

$$M = -P_1 l_1 + P_2 l_2 - P_3 l_3 + P_4 l_4$$

Likewise, with reference to (Fig. 19 b), the resultant moment of forces P_1, P_2 and P_3 about point O is

$$M = r_1 P_1 + r_2 P_2 - r_3 P_3$$

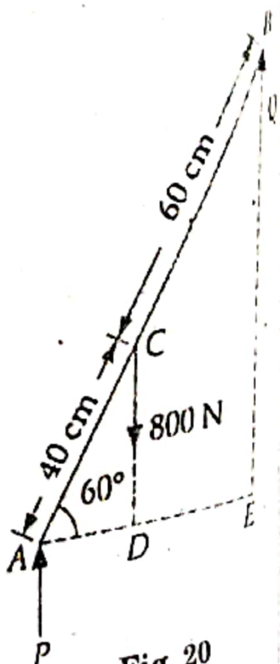


Fig. 20

Q.5. A weight of 800 N is hanging from a wooden beam which is carried by two persons as indicated in the adjoining figure. Neglecting weight of the beam, determine the load shared by each person.

Ans. Considering equilibrium of the beam,

$\Sigma F_y = 0$; $P + Q - 800 = 0$ or $P + Q = 800$

Taking moments about lower end A of the beam,

$Q \times AE - W \times AD = 0$

or $Q \times AB \cos 60° - W \times AC \cos 60° = 0$

or $Q \times (1 \times 0.5) - 800 \times (0.4 \times 0.5) = 0$

or $0.5\,Q = 160;\quad Q = \dfrac{160}{0.5} = 320$ N

and $P = 800 - Q = 800 - 320 = 480$ N

UNIT - II

BELT FRICTION

We saw that whenever a flexible member like a string, a rope, a cable or a belt passes over a smooth pulley or a cylindrical drum, as there is no frictional force developed between the two contact surfaces, the tension at both the ends of the member will be equal [refer Fig. 21(a)]. However, we know that no surface is perfectly smooth.

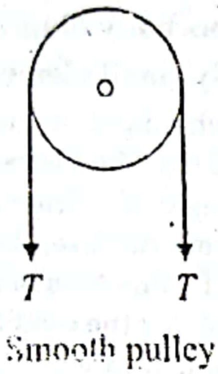


Fig. 21 (a)

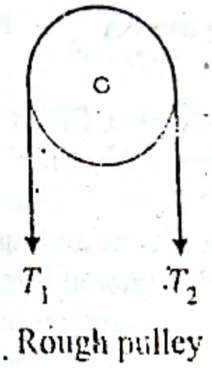


Fig. 21 (b)

Fig. 21 (c)

Hence, whenever a flexible member like a string, a rope, a cable or a belt *moves* or *tends* to move over a pulley or a cylindrical drum, frictional forces are always developed between the contact surfaces [Fig. 21(b) and (c)], which tend to oppose the relative motion. This frictional force is termed as *rope* or *belt friction*.

Unlike sliding friction, the belt friction is found not to be constant but to vary *exponentially* throughout the length of contact surface. As a result, the tension at both the ends of the rope or belt will not be equal. For impending motion of rope or belt in the *clockwise* direction, the tension on the right end will be greater than that on the left end, i.e., $T_2 > T_1$. The side of the belt with greater tension is termed as *tight side* and that with lesser tension is termed as slack side. We consider T_2 to be the tension on the tight side and T_1 to be the tension on the slack side. Hence, $T_2 > T_1$.

This belt friction is utilized in *transmitting power* as in the case of *belt drives* and power absorption as in the case of band brakes. In belt drives, both the belt and the pulley rotate by friction developed between contact surfaces. As a result, power is transmitted from one pulley to another through the belt. In the case of band brakes, the band remains fixed while drum rotates. Due to the friction developed between the band and the drum. the can be stopped tightening the band.

Relationship Between Tensions on Tight Side and Slack Side

A flat belt passing over a stationary cylindrical drum [Fig. 22 (a)]. Let the arc length AB be portion of the belt in contact with the drum subtending an angle β at the centre O of the drum. The angle β is also called *contact* or *lap angle*.

Let the belt be at the point of impending motion in the clockwise direction. Hence, we know that tension on the tight side is greater than the tension on the slack side, i.e., $T_2 > T_1$.

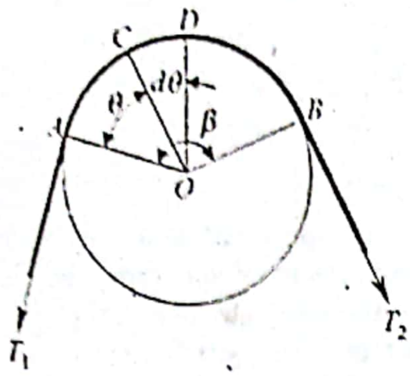


Fig. 22 (a) Flat belt passing over a stationary drum

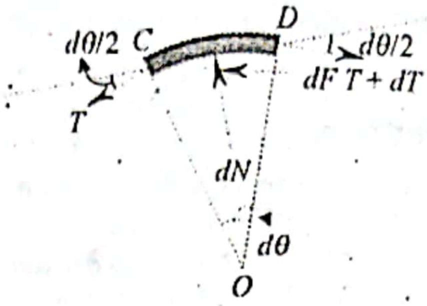


Fig 22 (b) Free- body diagram of infinitesimally small element.

Consider an infinitesimally small element CD of the belt subtending an angle dθ at the centre O the drum. Its free-body diagram is shown in Fig.22 (b). The forces acting on the element will be tension T at point C (point C is at an angle of θ from A) and tension T + dT at the point D (point D is at an angle of A + dθ from A). Also, the drum will exert a normal reaction dN and frictional force dF against the direction of motion of the belt. Since the belt is at the point of impending motion, applying the conditions of equilibrium along the normal and tangential directions to the element, we get

$\Sigma F_t = 0 \Rightarrow$

$$(T+dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - dF = 0 \qquad ...(1)$$

For an infinitesimally small element, we can assume the contact surfaces to be plane and we can apply the coulomb's laws of dry friction. Hence, $dF = \mu_s dN$. Therefore, the above equation can be written as

$$(T+dT)\cos\frac{d\theta}{2} - T\cos\frac{d\theta}{2} - \mu_s dN = 0$$

$$dT\cos\frac{d\theta}{2} - \mu_s dN = 0 \qquad ...(2)$$

$\Sigma F_n = 0 \Rightarrow$

$$dN - (T+dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} = 0$$

$$dN - (2T+dT)\sin\frac{d\theta}{2} = 0 \qquad ...(3)$$

Substituting the value of dN from the eq. (3) in the eq. (2) we get

$$dT\cos\frac{d\theta}{2} - \mu_s(2T+dT)\sin\frac{d\theta}{2} = 0 \qquad ...(4)$$

We know that as $d\theta \to 0$, $\cos\frac{d\theta}{2} \to 1$ and $\sin\frac{d\theta}{2} \to \frac{d\theta}{2}$. Hence,

$$dT - \mu_s(2T + dT)\frac{d\theta}{2} = 0 \qquad ...(5)$$

Neglecting the product of differentials $dTd\theta$ as compared to the first-order differentials,

$$dT - \mu_s T d\theta = 0$$

or

$$\frac{dT}{T} = \mu_s d\theta \qquad ...(6)$$

Upon integration between limits,

$$\int_{T_1}^{T_2} \frac{dT}{T} = \mu_s \int_0^{\beta} d\theta$$

$$\ln T_2 - \ln T_1 = \mu_s \beta$$

or
$$\ln\left(\frac{T_2}{T_1}\right) = \mu_s \beta \quad ...(7)$$

The above equation can also be written in the form

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \quad ...(8)$$

FRICTION IN JOURNAL BEARING-FRICTION CIRCLE

A journal bearing forms a turning pair as shown in Fig 23 (a). The fixed outer element of a turning pair is called a *bearing* and that portion of the inner element (i.e. shaft) which fits in the bearing is called a *journal*. The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing

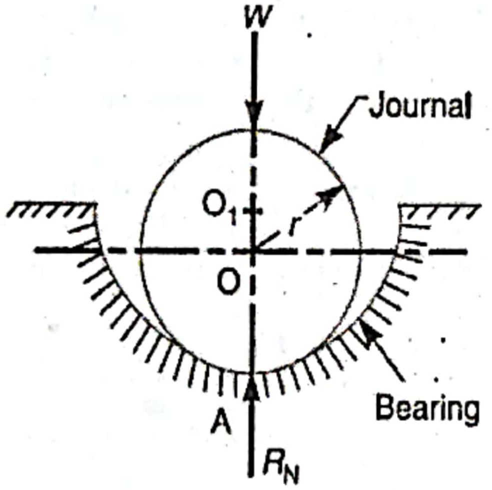


Fig. 23 (a)

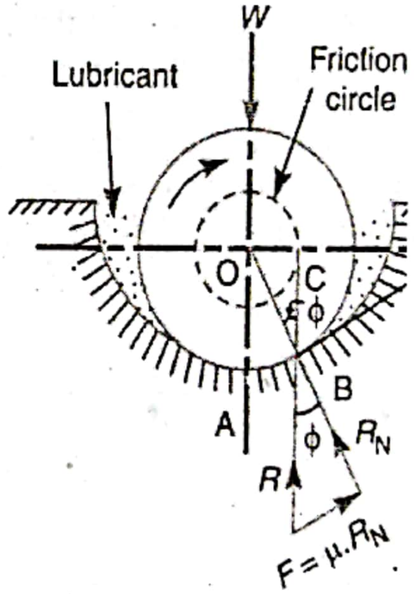


Fig. 23 (b)

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig (a). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction R_N acts vertically upwards at point A. This point A is known as **seat or point of pressure.**

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig(b). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction R does not act vertically upward, but acts at another point of pressure B.

This is due to the fact that when shaft rotates, a frictional force $F = \mu\ R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point A to point B.

In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let Ø = Angle between R (resultant of F and RN) and RN,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in metres.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$R = W$, and $T = W \times OC = W \times OB \sin\phi = W.r\sin\phi$

Since $\varnothing$ is very small, therefore substituting $\sin\phi = \tan\phi$

$T = W.r \tan\phi = \mu .W.r$..............($\mu = \tan \varnothing$)

If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$P = T\omega = T \times 2\pi N/60$ watts

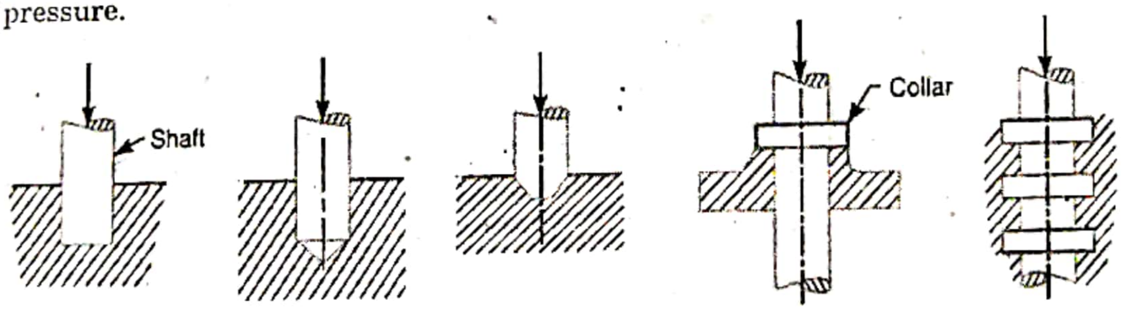
Where N = Speed of the shaft in r.p.m.

FRICTION OF PIVOT AND COLLAR BEARING

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust.

The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Fig. 24 (a) and (b) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig (d) or several collars along the length of a shaft, as shown in Fig(e) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

Fig. 24

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed that

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

FLAT PIVOT BEARING

When a vertical shaft rotates in a flat pivot bearing (known as foot step bearing), as shown in Fig, the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W = Load transmitted over the bearing surfaces,

R = Radius of bearing surface,

p = Intensity of pressure per unit area of bearing Surface between rubbing surfaces, and

μ = Coefficient of friction.

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

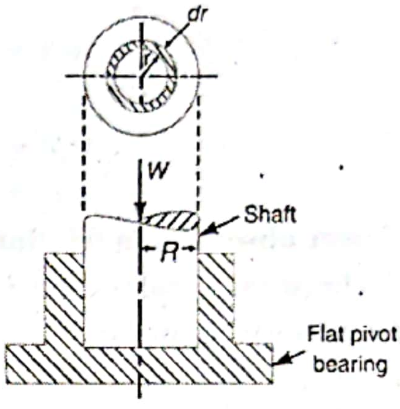


Fig. 25

Considering Uniform Pressure

When the pressure is uniformly distributed over the bearing area, then

$$P = \frac{W}{\pi R^2}$$

Consider a ring of radius *r* and thickness *dr* of the bearing area.

Area of bearing surface, A = $2\pi r.\ dr$

Load transmitted to the ring,

$\delta W = p \times A = p \times 2\pi r.\ dr$ ...(i)

Frictional resistance to sliding on the ring acting tangentially at radius r,

$Fr = \mu.\delta W = \mu\ p \times 2\pi r.\ dr = 2\pi\ \mu.p.r.dr$

Frictional torque on the ring,

$Tr = Fr \times r = 2\pi\ \mu\ p\ r.dr \times r = 2\ \pi\ \mu\ p\ r^2\ dr$ ...(ii)

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$$\therefore \text{ Total frictional torque, } T = \int_0^R 2\pi\mu\, pr^2 dr = 2\pi\mu\, p\int_0^R r^2 dr$$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi\ N/60$$

N = Speed of shaft in r.p.m.

Q.6. A conical pivot supports a load of 20 kN, the cone angle is 120° and the intensity of normal pressure is not to exceed 0.3 N/mm². The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Ans. Given : W = 20 kN = 20 × 103 N ; 2 _= 120° or _= 60° ; pn= 0.3 N/mm2 ;

N = 200 r.p.m. or _= 2_× 200/60 = 20.95 rad/s ; μ = 0.1

OUTER AND INNER RADII OF THE BEARING SURFACE.

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm.

Since the external diameter is twice the internal diameter, therefore $r_1 = 2r_2$

We know that intensity of normal pressure (p_n),

$$0.3 = \frac{W}{\pi[(r_1)^2-(r_2)^2]} = \frac{20\times10^3}{\pi[(2r_2)^2-(r_2)^2]} = \frac{2.12\times10^3}{(r_2)^2}$$

$$(r_2)^2 = 2.12 \times 10^3/0.3 = 7.07 \times 10^3 \text{ or } r_2 = 84 \text{ nm } \textbf{Ans.}$$

$$r_1 = 2r_2 = 2 \times 84 = 168 \text{ mm } \textbf{Ans.}$$

Power absorbed in friction

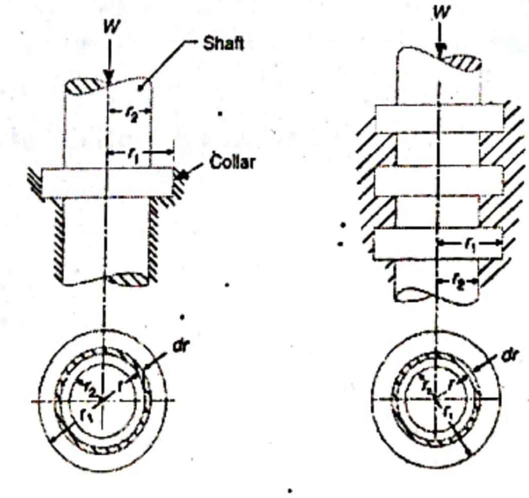
We know that total frictional torque (assuming uniform pressure),

Power absorbed in friction

P = T.ω_ = 301.76 × 20.95 = 6322 W = 6.322 kW.

FLAT COLLAR BEARING

Collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings as shown in Fig. 26 (a) and (b) respectively. The collar bearings are also known as **thrust bearings.** The friction in the collar bearings may be found below :



(a) Single collar bearing (b) Multiple collar bearing.

Fig. 26

Consider a single flat collar bearing supporting a shaft as shown in Fig (a).

Let r_1 = External radius of the collar,

r_2 = Internal radius of the collar.

Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

1. Considering Uniform Pressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2-(r_2)^2]} \quad ...(i)$$

the frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

Total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

Substituting the value of p from equation (i)

$$T = 2\pi\mu\times\frac{W}{\pi[(r_1)^2-(r_2)^2]}\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

$$= \frac{2}{3}\times\mu.W\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right]$$

Q.7. A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine 90 r.p.m., find the power absorbed in friction at the thrust block, assuming

1. uniform pressure

2. uniform wear.

Ans. Given; $n = 6$; $d_1 = 600$ mm or $r_1 = 300$ mm; $d_2 = 300$ mm or $r_2 = 150$ mm; $W = 100$ kN $= 100 \times 103$ N; $\mu = 0.12$; N = 90 r.p.m. or $= 2 \times 90/60 = 9.426$ rad/s

1. Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

$$T = \frac{2}{3}\times\mu.W\left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right]$$

$$= \frac{2}{3}\times 0.12\times 100\times 10^3\left[\frac{(300)^3-(150)^3}{(300)^2-(150)^2}\right] = 2800\times 10^3 \text{ N-mm}$$

$$= 2800 \text{ N-m}$$

Power absorbed in friction,

$$P = T_{-} = 2800 \times 9.426 = 26\,400 \text{ W} = 26.4 \text{ kW}$$

2. Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2}\times\mu.W(r_1+r_2) = \frac{1}{2}\times 0.12\times 100\times 10^3(300+15) \text{ N-mm}$$

$$= 2700 \times 10^3 \text{ N-mm} = 2700 \text{ N-m}$$

Power absorbed in friction,

$$P = T.w = 2700 \times 9.426 = 25\,450 \text{ W} = 25.45 \text{ kW}$$

FIRST TERM EXAMINATION [FEB. 2015]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time. 1 Hour **MM : 30**

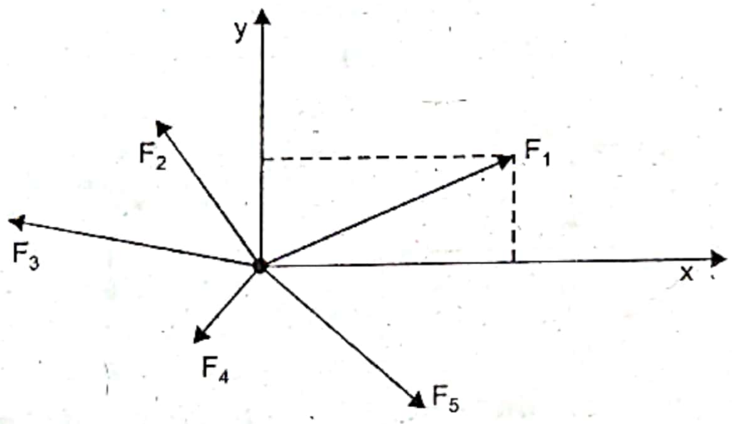
Note: *Q.No.1 is compulsory. Attempt any two more questions from the rest.*

Q.1. (*a*) Explain the term "principle of equilibrium" with conditions of equilibrium. **(2×5=10)**

Ans. If no. of forces acting on a pt. '0' (*i.e.* F_1, F_2, F_3, F_4 & F_5)

If their resultant is zero. The partical is said to be in equilibrium

$$R = \sqrt{R_x^2 + R_y^2} = 0$$

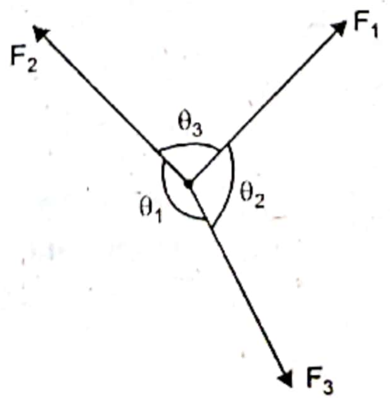


i.e., $\Sigma F_x = 0$ or $\Sigma F_y = 0$

These are equations or condition of eqn. If no. of concurent forces laying in a plane are in equilibrium.

Q.1. (*b*) State and explain the lami's theorem. **(2)**

Ans. Lami's Theoem: If three concurrent of co-planer forces are acting on a point 'O' Then the ratio of force is sine of angle of opposite two forces.

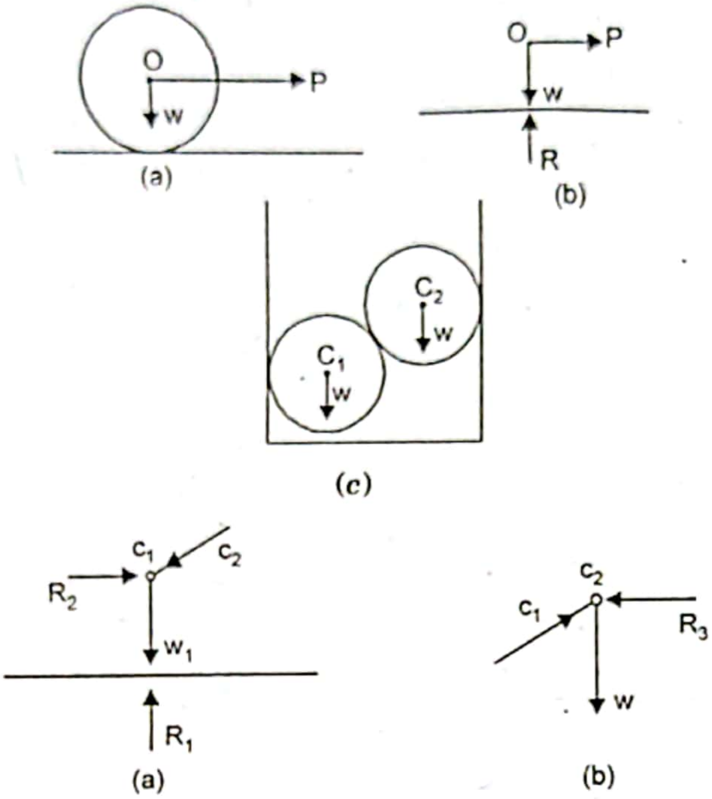


$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

Q.1. (c) Explain the term "Free body diagram" with examples.

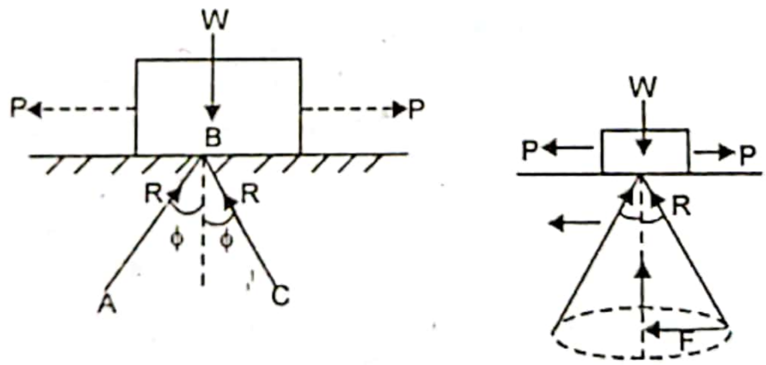
Ans. F.B.D.:→ The Free body diagram may defined as the all forces acting or a body (direct or indirect) are shown indiviually or freely.

For e.g.



Q.1. (d) Define and explain "cone of friction" with neat sketch.

Ans. Cone of friction: Conocider a block of weight W resting on horizontal surface and acting upon by force P. When we consider coplanar forces, in order fro motion not to occur in any direction, resultant R must lie within angle ABC, where ϕ is angle of friction

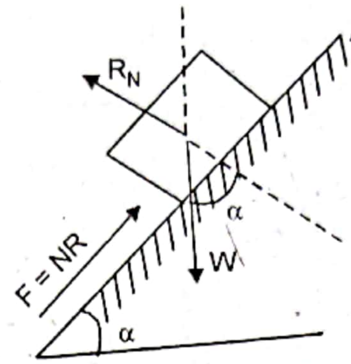


Consider force P is gradually changed through 360°. For motion not to occur, resultant reaction R must be contained with in the cone generated by revolving line AB about normal BN.

The inverted so formed with semi-central angle equal to angle of friction ϕ is called cone of friction. Now, for motion to occur resultant R will be on the surface of cone.

Q.1. (e) Explain "Angle of repose" and show that anle of repose is equal to the angle of friction.

Ans. Angle of Repose: Consider a block of weight W resting on an inclined plane OA making an angle α with the horizontal. Let angle α be increased gradually till the block is just at the point of sliding.



The block is in equlibrium under various forces.

Resolving forces along and perpendicular to plane

$$\mu R = W \sin \alpha$$
$$R = W \cos \alpha$$

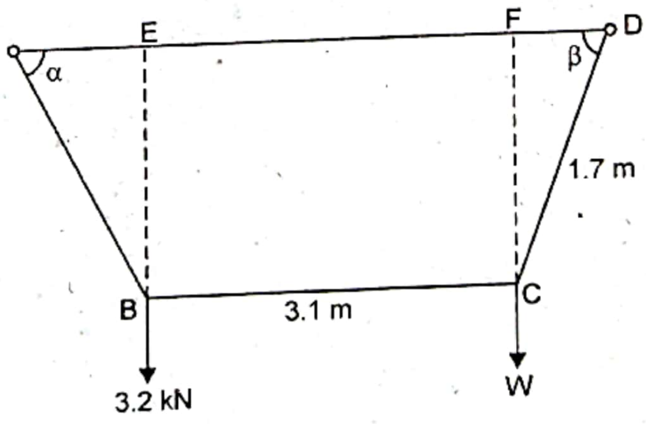
∴ $$\mu = \tan \alpha$$

In terms of angle of friction, coefficient of friction is given as $\mu = \tan \phi$

∴ $$\text{eqn. (1)} = \text{eqn. (2)}$$
$$\tan \alpha = \tan \phi$$
$$\phi = \alpha$$

The angle of inclined plane at which block resting on it is about to slide down the plane is called angle of repose.

Q.2. (*a*) Two weights are suspended from points B and C of a rope as shown in fig. (a). If distance AD is 6m, how much will be the magnitude of W to maintain the eqilibrium.



Ans. From ABE

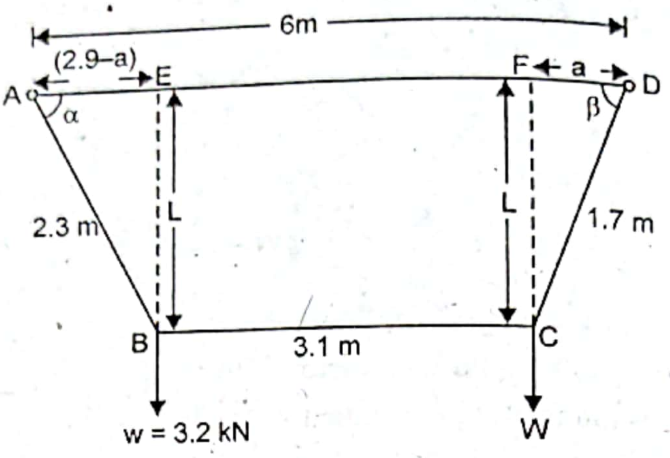
$$2.3^2 = L^2 + (2.9-a)^2 \quad ...(1)$$

From DCF

$$1.7^2 = L^2 + a^2 \quad ...(2)$$

from eqn (1) – (2)

$$2.3^2 - 1.7^2 = \cancel{L^2} + (2.9-a)^2 - \cancel{L^2} - a^2$$
$$2.4 = = 2.9^2 + \cancel{a^2} - 2 \times 2.9 \times a - \cancel{a^2}$$
$$2.4 - 8.41 = -5.8a$$
$$-6.01 = 5.8a$$
$$\boxed{a = 1.04m} = DF$$



$$\boxed{AE = 2.9 - 1.04 = 1.86m}$$

$$\cos\alpha = \frac{AE}{AB} = \frac{1.86}{2.3} = 0.8086$$

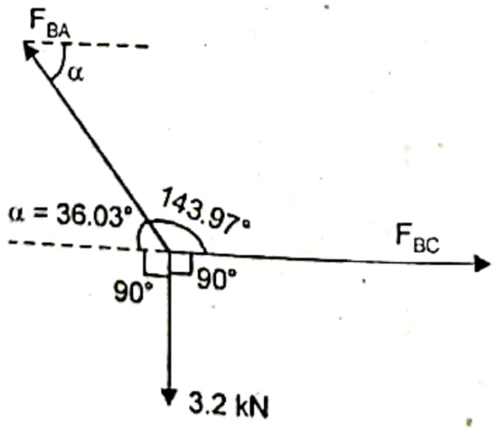
$$\alpha = \cos^{-1}(0.8086)$$

$$\boxed{a = 36.03^\circ}$$

$$\cos\beta = \frac{DF}{CD} = \frac{1.09}{1.7} = 0.612$$

$$\beta = \cos^{-1}(0.612)$$

$$\boxed{\beta = 52.26^\circ}$$



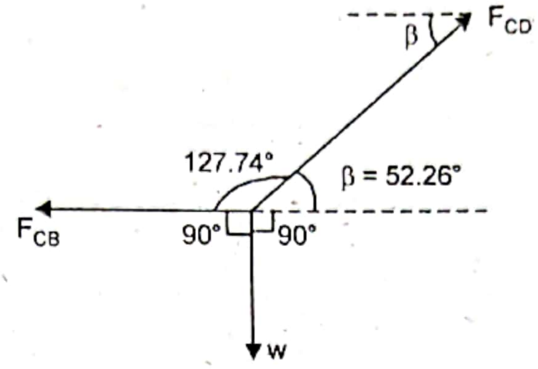
$$\frac{F_{BA}}{\sin 90^\circ} = \frac{F_{BC}}{\sin 126.03^\circ} = \frac{3.2}{\sin 143^\circ.97^\circ}$$

$$F_{BA} = \frac{3.2}{\sin 143.97^\circ} \times \sin 90^\circ$$

$$\boxed{F_{BA} = 5.44\ kN}$$

$$F_{BC} = \frac{3.2}{\sin 143.97} \times \sin 126.03$$

$$\boxed{F_{BC} = 4.39kN}$$



$$\frac{F_{CB}}{\sin(90° + 52.26°)} = \frac{W}{\sin 127.74°} = \frac{F_{CD}}{90°}$$

$$W = \frac{F_{CB}}{\sin 142.26°} \times \sin 127.74°$$

$$W = \frac{4.39}{\sin 142.26} \times \sin 127.74°$$

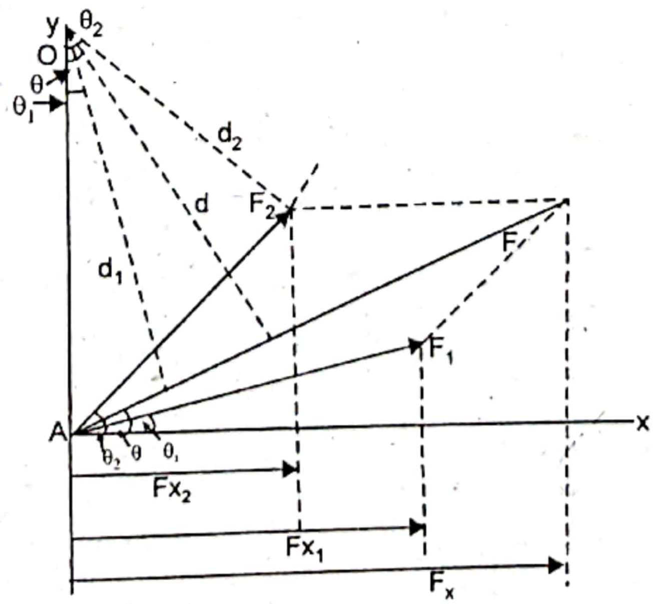
$\Rightarrow$ $\boxed{W = 5.67kN}$ Ans.

Q.2. (*b*) State and prove "Varignon's Theorem".

Ans. Varignon's Theorem: or (Principle of Moment's): Varignon's theorem states that the moment of a force any point is equal to the alzabric sum of the moments of its components about that point.

OR

Principle of moment states that the moment of the resultant of a number of force about only point is equal to the alzabric sum of the moments of all the forces.



Theorem of Varignon: "The moment of force about an axis is equal to the sum of the moment of its components about the same axis".

Moment of force F about O:–

$$Fd = F\,(OA\cos\theta) = F\cos\theta \times OA$$
$$Fd = OAFx$$

Moment of force F, about O

$$F_1d_1 = F_1\,(OA\cos\theta) = F_1\cos\theta \times OA$$
$$= Fx_1 \times OA \quad \ldots(i)$$

Moment of force F_2 about O:–

$$F_2d_2 = Fx_2 \times OA \quad \ldots(ii)$$

adding eq^n (i) and (ii)

$$F_1d_1 + F_2d_2.\ OA\,(Fx_1 + Fx_2)$$

$$\boxed{F_x = Fx_1 + Fx_2}$$

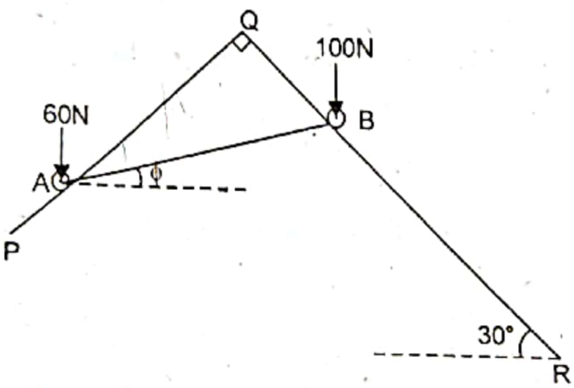
x component of resultant F = The sum of x components of forces F_1 and F_2

$$F_1d_1 \times F_2d_2 = OA \times Fx$$

$$\boxed{F_1d_1 \times F_2d_2 = F_*d}$$

So the moment of force about an axis is equal to sum of moment of its components about the same axis.

Q.3. (*a*) Two spheres weighing 60 N and f100 N are connected by flaxible string AB and rest on two mutually perpedicular planes PQ and QR as shown in fig. (b). Find the tension in the string which passes freciy through slots in smooth inclined planes.

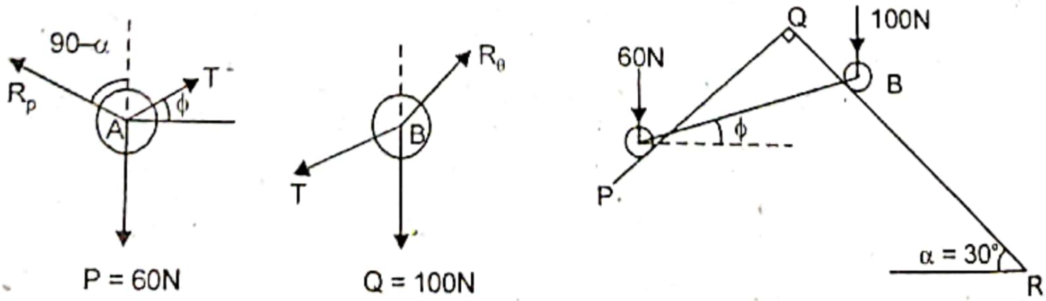


Ans. P = 60N, Q= 100N, α = 30°

Consider the eq^m. on Roller 'P'

$$\Sigma F_x = T\cos\phi - R_p\sin(90-\alpha) = 0$$

$$T\cos\phi - R_p\cos\alpha \quad \ldots(1)$$



$$\Sigma F_y = 0 \quad R_p \cos(90-\alpha) - P + T\sin\phi = 0$$

$$R_p \sin\alpha - P + T\sin\phi = 0 \quad ...(2)$$

Consider the eq^{m}. on Roller Q.

$$\Sigma F_x = RQ\sin\alpha - T\cos\phi = 0 \quad ...(3)$$

$$F_y = RQ\cos\alpha - \theta - T\sin\theta = 0 \quad ...(4)$$

From eq^{n}. 1. $$R_p = \frac{T\cos\phi}{\cos\alpha}$$

Put in equation 2

$$\frac{T\cos\phi}{\cos\alpha}\sin\alpha - P + T\sin\phi = 0$$

$$P = T(\sin\phi + \cos\phi\tan\alpha) \quad ...(5)$$

From equation 3. $$R_\theta = \frac{T\cos\alpha}{\sin\alpha}$$

Put in equation 4. $$\frac{T\cos\phi}{\sin\alpha} - \theta - T\sin\phi = 0$$

$$-Q = T(\cos\phi\cos\alpha - \sin\phi) \quad ...(6)$$

divide (5) by (6)

$$\frac{\theta}{P} = \frac{(\cos\phi\cot\alpha - \sin\phi)}{(\cos\phi\tan\alpha + \sin\phi)}$$

$$\tan\phi = \frac{P\cot\alpha - Q\tan\alpha}{P+Q}$$

$$\tan\phi = \frac{\left(60\sqrt{3} - 100\frac{1}{\sqrt{3}}\right)}{100+60} = \frac{1}{3\sqrt{3}}$$

$$\boxed{\phi = 10.9^\circ}$$

From equation (6) $$T = \frac{\theta}{\theta\cos\phi\cot\alpha - \sin\phi} = \frac{100}{\cos 10.9^\circ \cos 30^\circ - \sin 109^\circ}$$

$$\boxed{T = 66.2N}$$

Q. 3.(*b*) A ladder 8 , long weighing 200 N is resting aganist a rough vertical wall. A man of 720 N climbs the ladder. At what postion will he induce sliping? Take μ for both the contact surfaces of ladder 0.25.

Ans.

$$l = 8m$$

$$w_1 = 200 \text{ N}$$

3(
C
T
V
V

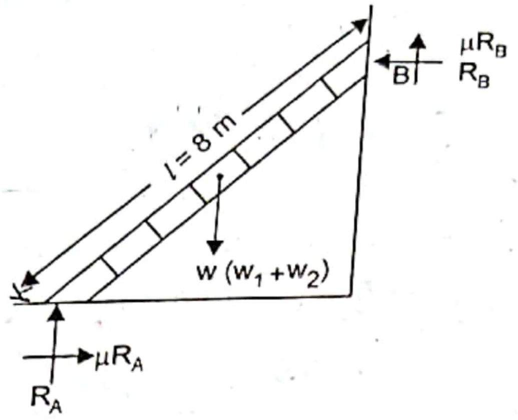
men weight $(w_2) = 720N$

$\mu = 0.25$

F.BD

$\Sigma Fx = 0$

$\mu R_A - R_B = 0 \Rightarrow 0.25\ R_A = R_B$



$\Sigma Fy = 0$

$R_A + \mu R_B - W = 0$

$R_A + 0.25\ R_A = 200 + 720$ N

$R_A = \frac{920}{1.25} = 3680N$

$R_B = 920$ N

Taking moment about A

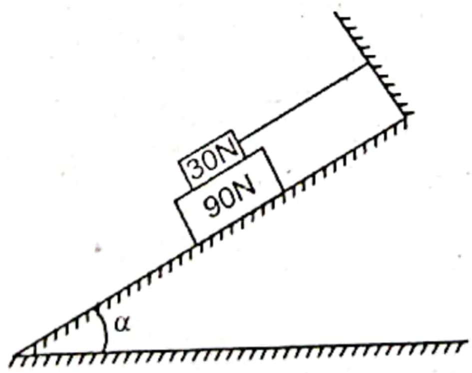
$R_B \times 8 \sin 45° + 0.25\ R_B\ (8 \text{ Cos } 45) - 920\ (4 \cos 45) = 0$

$= 5204 + 1301 - 2602$

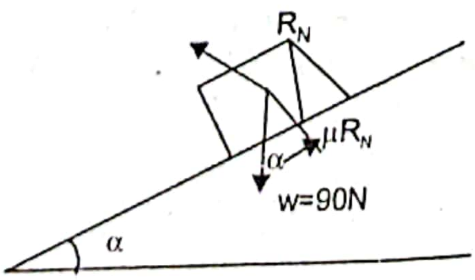
$l = 3.903\ m$

At distance 3.903 m the ladder will slip.

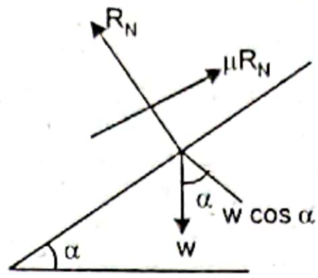
Q.4. (a) What should be the value of angle α in fig. (c), so that the motion of 90 N block impends down the plane. Take μ for all the surfaces to be 0.33.



Ans.



F.B.D



$$\Sigma Fx = 0$$

and

$$\Sigma Fy = 0$$

$$\mu R_N = w \sin \alpha \quad ...(1)$$

$$R_N = w \cos \alpha \quad ...(2)$$

$$R_N \times 0.33 = 90 \sin \alpha$$

$$\sin \alpha = \frac{0.33}{90}$$

$$R_N = 272.7 \sin \alpha$$

$$R_N = 90 \cos \alpha$$

$$90 = 272.7 \tan \alpha$$

$$\frac{90}{272.7} = \tan \alpha$$

$$\alpha = \tan^{-1} 0.33$$

$$\boxed{\alpha = 18.2^\circ}$$

Q.4. (*b*) For a flat belt drive, prove that: $T_1/T_2 = e\mu\theta$

where T_1 = tension on the tight side of the belt, T2 = tension on the slack side of the belt, μ = coefficient of friction and θ = Angle of contact.

Ans. Derive the relationship:

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}}$$

T_1 = Tension Tight side

T_2 = Tension and lack side

β = angle of contact

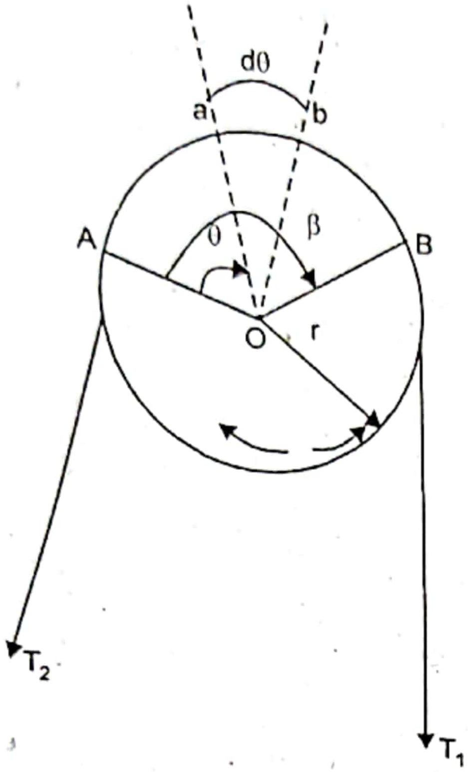
Let Tension at a = T

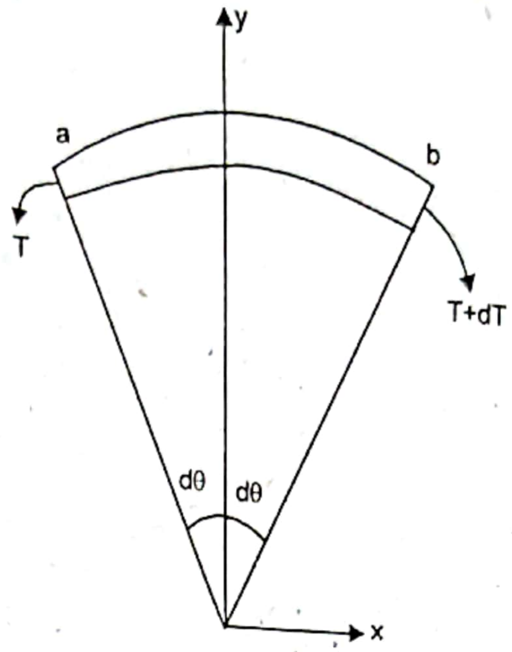
Tension at b = $T + dT$

Normal reaction = dN

Frictional force = μdN

μ = coefficient of friction





Equation of equilibrium

$$\Sigma Fx = 0$$

$$T\cos\frac{d\theta}{2} + dF - (T + dT)\cos\frac{d\theta}{2} = 0$$

$$\Sigma Fy = 0$$

$$AN = T\sin\frac{d\theta}{2} - (T + dT)\sin\frac{d\theta}{2} = 0$$

angle $d\theta$ is very small put

$$\cos\frac{d\theta}{2} \cong 1 \text{ and } \sin\frac{d\theta}{2} \cong \frac{d\theta}{2}$$

$$\frac{dQ}{2} \cong \frac{dQ}{2}$$

$$dF = \mu dN \text{ is above equation}$$

$$T + \mu dN - (T\text{-}dT) = 0$$

$$dT - \mu dN = 0$$

and $$dN - T\frac{d\theta}{2}(T + dT)\frac{d\theta}{2} = 0$$

$$dN - Td\theta = 0 \text{ after negtecting } \left(dT\frac{d\theta}{2}\right)$$ ...(2

Eliminate dN is equation (1) and (2)

$$dT = \mu T d\theta \text{ Integrating both side}$$

$$\frac{dT}{T} = \mu d\theta \text{ Integrating both side}$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\beta} \mu d\theta = \left[\log_e T\right]_{T_2}^{T_1} = \mu_{\beta}$$

$$\log \frac{T_1}{T_2} = ln \frac{T_1}{T_2} = \mu\beta$$

or $$\boxed{\frac{T_1}{T_2} = e^{\mu\beta}} \text{ also } \boxed{2.3 \log_{10} \frac{T_1}{T_2} = \mu\beta}$$

SECOND TERM EXAMINATION [APRIL 2015]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time. 1 Hour **MM : 30**

Note: *Q.No.1 is compulsory. Attempt any two more questions from the rest.*

Q.1. (a) Explain the term "statically detetminate truess" and also give methdmatical condition for rigid or perfect truss. (2×5=10)

Ans. A truss is statically determinate if the equation of static equilibrium alone are sufficient to determine the axial force's in the the member's without the need of considering their deformation.

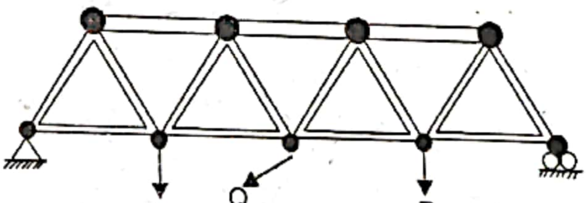
condition for perfect truss *i.e.* → $\boxed{\underbrace{m}_{\text{Member's}} = 2\underbrace{J}_{\text{Joints}} - 3}$

Q.1. (b) Explain the difference between fame, truss and machine.

Ans. Frame and Truss:

Frame: it is structure consisting or several bars or memebers pinned together and in which one or more than one of its memeber is subjected to more than two forces. They are designed to suppoert loads and stationary strucutres.

Truss: It is a system of uniform bar's or members joined together at their end's by riveting or welding and xonstructed to support loads. Every member of a truss is a two force member.



Machine: The machine are structure's designed to transmit and modify forces and contains same moving members.

Q.1.(c) State the basic assumptionsfor the perfect truss ?

Ans. Refer Q.No.1(e) First Term Exam 2018 (Page No.: 2-2018)

Q.1. (d) Explain the difference between centroid and centre of gravity.

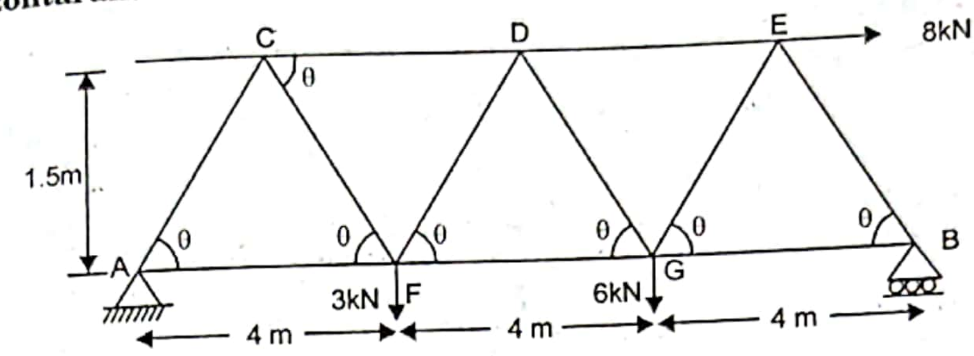
Ans. Center of gravity: C.G is that point through which the resultant of the system of parallel forces formed by the weight of all the particle's of the body passes.

Centroid: Basically centroid and C.G. are same but C.G. is for 3D body and centroid is for one dimensional body or line segments

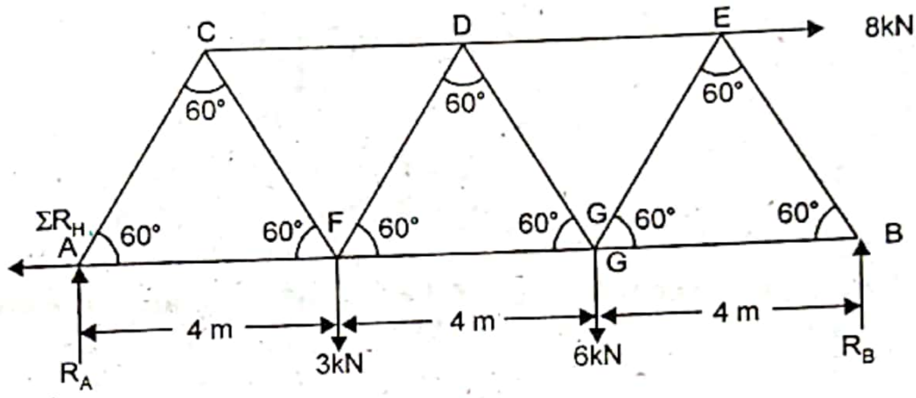
Q.1. (e) Explain that moment of area of any plane figure about a line passing through its centroid is zero.

Ans. The moment of area of a plane figure about a line passing through it's centroid is zero because the center of gravity of any plane figure is lies on it's reference line which is zero.

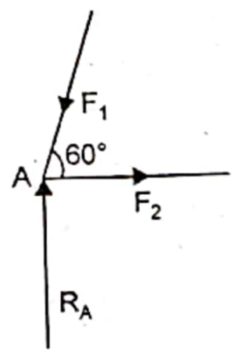
Q.2. Determine the forces in the truss shown in fig. below. which is subjected to horizontal and vertical loads. Mention the nature of forces also. **(10)**



Ans.



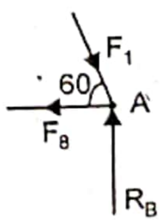
$$\Sigma F_y = 0$$
$$R_A + R_B = 9kN$$
$$\Sigma M_x = 0$$
$$R_B \times 12 = 6{\times}8 + 3{\times}4$$
$$R_B = \frac{60}{12} = 5kN$$
$$R_A = 4kN$$
$$\Sigma R_H = 8kN$$
$$R_B = \frac{60}{12} = 5kN$$



At Point A

$$\Sigma F_x = 0$$
$$F_1 = \frac{R_A}{\sin 60} = \frac{4k}{0.866} = 4.6kN \quad ...(T)$$
$$F_2 = F_1 \cos 60 = 2.3\ kN \quad ...(C)$$

At Point B

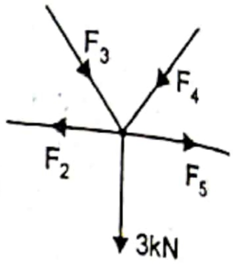


$$\Sigma F_x = 0$$
$$F_7 \cos 60 - F_8 = 0$$
$$R_B - F_7 \sin 60° = 0$$
$$F_7 = \frac{R_B}{\sin 60°} = 5.77kN(C)$$

$$F_8 = \frac{F_7}{\cos 60^\circ} = 11.5kN(T)$$

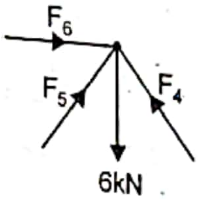
At Point F:

$$F_5 \cos 60 + F_3 \cos 60^\circ - F_3 = 0$$
$$F_3 \sin 60 - F_4 \sin 60 - 3 = 0$$



$$F_7 = 5.7\ kN\ (T)$$
$$F_3 = \frac{3 \times 5.7 \sin 60^\circ}{\sin 60^\circ} = 17.1(C)$$
$$F_4 = 4.3\ kN(C)$$

At Point E:

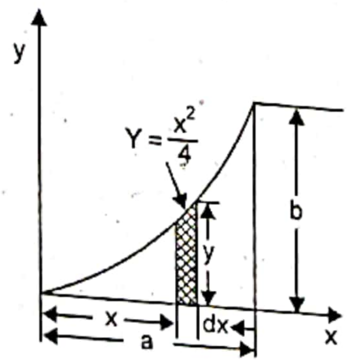


$$F_6 = F_4 \cos 60^\circ + F_5 \cos 60^\circ$$
$$F_6 = 4.3 \cos 60 + 6.2 \cos 60^\circ$$
$$F_6 = 5.25\ kN\ (T)$$

Q.3. Determine the coordinates of the C.G. of the shaded area between the parabola $Y = x^2/4$ and the straigth line Y=X.

Ans.

$$Y = \frac{x^2}{4} \qquad ...(1)$$



Equation of Parabola

$$b = Ra^2 \text{ or } R = \frac{b}{a^2}$$

$$y = \frac{b}{a^2} \times \frac{x^2}{4} \text{ or } x = \frac{a.\sqrt{y}}{\sqrt{b}}$$

By Vertical strip:

$$dA = y\,dx$$

$$n_c = \frac{\int x dA}{\int dA} = \frac{\int x.ydx}{\int ydx} = \frac{\int_o^a n\left(\frac{b}{a^2}x^2\right)dx}{\int_o^a \frac{b}{a^2}x^2.dx}$$

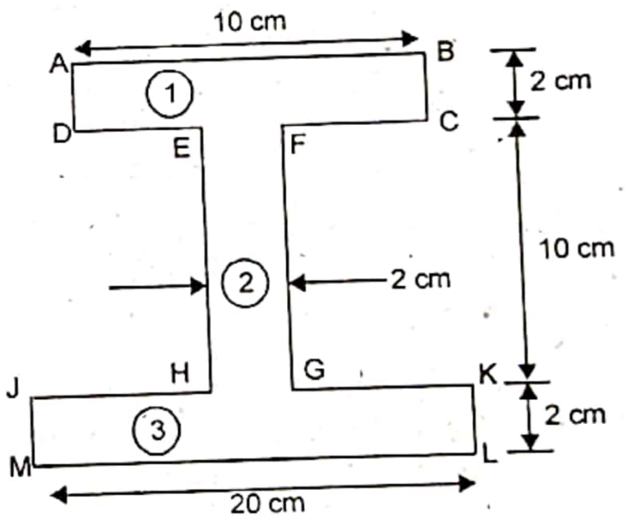
$$X_c = \frac{\left[\frac{b}{a^2}\frac{x^4}{x}\right]_0^a}{\left[\frac{b}{a^2}\frac{x^3}{3}\right]_0^a} \Rightarrow X_C = \frac{3}{4}a$$

$$Y_c = \frac{\int y\,dA}{\int dA}$$

$$Y_c = \frac{\int \frac{y}{2} y\,dx}{\int y dx} = \frac{\int_0^a \frac{1}{2} y^2 dx}{\int_0^a y dx} = \frac{\left[\frac{b^2}{2a^4}\frac{x^5}{5}\right]_0^5}{\left[\frac{b}{a^2}\frac{x^3}{3}\right]_0^a} = \frac{3}{10}b$$

$$\boxed{Y_C = \frac{3}{10}b}$$

Q.4. Find the moment of inertia of the section shown in fig. below, about the centroidal axis X-X, perpendicular to the web.



Ans.

$$Y = \frac{A_1Y_1 + A_2Y_2 + A_3Y_3}{A_1 + A_2 + A_3}$$

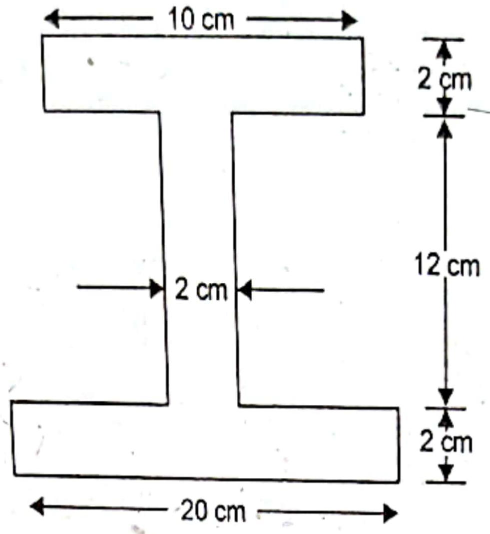
$$Y = \frac{20 + 2\times1 + 2\times12\times8 + 10\times2\times15}{20\times2 + 12\times2 + 8\times2}$$

$$= \frac{40 + 192 + 300}{40 + 24 + 16}$$

$$Y = 6.65 \text{ cm.}$$

$$I_{xx} = \Sigma I_{nn} + \Sigma A_n Y_n^2$$

$$I_{nn} = I_{11} + I_{22} + I_{33} + A_1 Y_1^2 + A_2 Y_2^2 + A_3 Y_3^2$$



Y_1; Y_2 and Y_3 are measured from C.G.

$$Y_1 = 15 - 6.65 = 8.35 \text{ cm}$$
$$Y_2 = 8 - 6.65 = 1.35 \text{ cm}$$
$$Y_3 = 6.65 - 1 = 5.65 \text{ cm}$$

$$I_{gg} = \frac{1}{12} bd^3$$

$$I_{xx} = \left[\frac{1}{12} \times 8 \times (2)^3 + \frac{1}{12} \times 2 \times 12^3 + \frac{1}{12} \times 12 \times 2^3\right]$$

$$\boxed{I_{xx} = 2212.33 cm^4}$$

$$I_{yy} = \left[\frac{1}{12} \times 2 \times 8^3 + \frac{1}{12} \times 12 \times 2^3 + \frac{1}{12} \times 12 \times 2^3\right]$$

$$\boxed{I_{yy} = 381.33\ cm^4}$$

Q.5. What is polar moment of inertia

Ans. Polar M.O.I. is defined as the M.O.I. of an area of a plane fig. with respect to an axies perpendicular to the x-y plane and passing through a pole $J_o = I_{xx} + I_{yy}$

FIRST TERM EXAMINATION [FEB. 2016]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time : 1½ Hrs. M.M. : 30

Note: *Attempt any three questions including Q no.1 which is compulsory. Select one question from each unit.*

Q.1. What is differece between Resultant and Equilibrium?

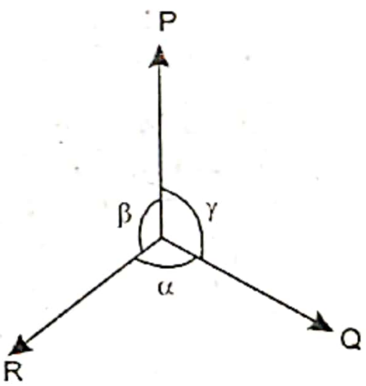
Ans.

Resultant	Equilibrium
When there are number of forces acting at a point, we can replace all forces by a single force which can produce same effect of all the forces acting together. This single force is resultant of forces	Equilibrium force is the force needed to bring system in equilibrium. Therefore it is equal but opposite to resultant of all forces.

Q.1. (b) What is essential conditions for three, non parallel, Co- planer forces to be in equilibrium? Explain with example.

Ans. The condition for three non-parallel coplanar force's which is in equilibrium condition is Lamie's Theorem which is

$$\boxed{\frac{P}{\sin\alpha} = \frac{\theta}{\sin\beta} = \frac{R}{\sin\gamma}}$$



For e.g.:

$$P = 100 \text{ N}, \theta = 50 \text{ N}$$

$$R = ? \text{ (unknown)}$$

where $\alpha = 90°, \beta = 120°, \gamma = 150°$

$$\frac{100}{\sin 90} = \frac{50}{\sin 120} = \frac{R}{\sin 150°}$$

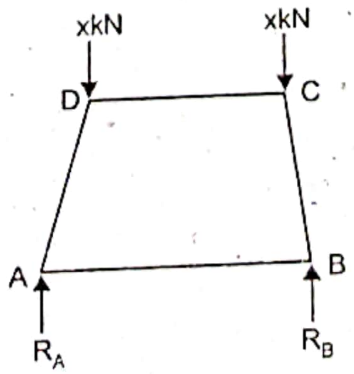
$$\boxed{R = 50 \text{ N}}$$

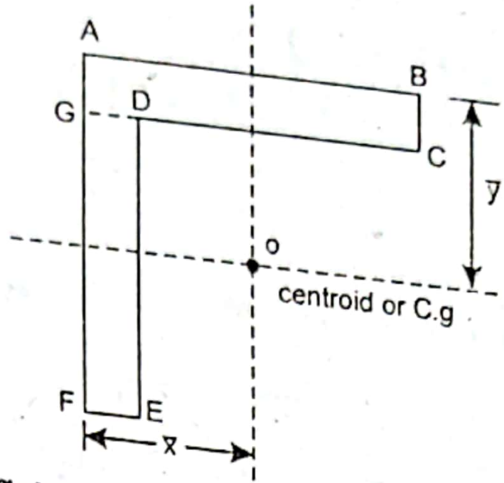
Q.1. (c) If a given area has two or more symmetry where will its centroid be located ? Explain with example.

Ans.

$$\bar{X} = \frac{a_1X_1 + a_2X_2}{a_1 + a_2}$$

$$\bar{Y} = \frac{a_1Y_1 + a_2Y_2}{Y_1 + Y_2}$$





Q.1. (d) Why is coefficient of static friction greater than coefficient of kinetic friction.

Ans. The value of static friction (μs) is always greter then the value of kinetric friction (μk) because the at this time the maximum frictional force exected at the time of impending motion i.c. the motion is just about to begin hence the maximum effect observed under the static condition that why its value is higher.

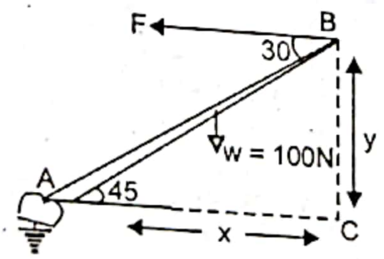
Q.1. (e) Give 2 advantages of method of section.

Ans. (a) It is used to find forces in any membrane directly.

(b) It is short and quick method.

UNIT-I

Q.2. (a) A bar AB of weight 100N is hinged at A and is pulled by a cable attached at B by Force F. Find force F and magnitude and direction of reaction A if bar is in equilibrium as shown in Fig.



Ans. Taking moment at C

$$R_A \sin 45 \times x - f \cos 15 \times y = w \times \frac{x}{2}$$

$$\tan \theta = 45$$

$$\tan 45 = \frac{y}{x}$$

$$y = x$$

$$R_A \sin 45 - F \cos 15 = \frac{100}{2}$$

$$R_A \sin 45 - F\cos 15 = 50$$

$$\Sigma f_x = 0$$

$$F \cos 15 = R \cos 45$$

$$F = R\frac{\cos 45}{\cos 15}$$

$$F = .732\text{ R}$$

$$\Sigma M_c = 0$$

$$R_A \sin 45 - F\cos 15 = 50$$

$$R_A \sin 45 - .732\, R_A \cos 15 = 50$$

$$R_A \times .707 - .732 \times .965\, R_A = 50$$

$$.707\, R_A - .706\, R_A$$

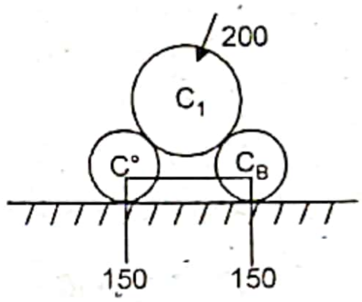
$$.001\, R_A = 50$$

$$R_A = 50\text{ KN}$$

$$F = .732 \times 50000$$

$$F = 36.6\text{ KN.}$$

Q.2. (*b*) Two identical iron spheres, each of radious 5 cm and weight 150 N are connected with string of length 16 cm and rest on horizontal smooth floor. Another sphere of radius 6 cm and weight 200 N rest over them. Determine tension in string and reaction at all contact surface.



Ans.

$$r_c = r_B = 5\text{ cm};\ l_{Bc} = 6\text{ cm}$$

$$r_A = 6\text{ cm}$$

Solving for Cylinder 1

$$\Sigma f_y = 0$$

$$R_D \sin\theta + R_\varepsilon \sin\theta = 200$$

$$2R_D \sin\theta = 200\ (R_D = R_\varepsilon)$$

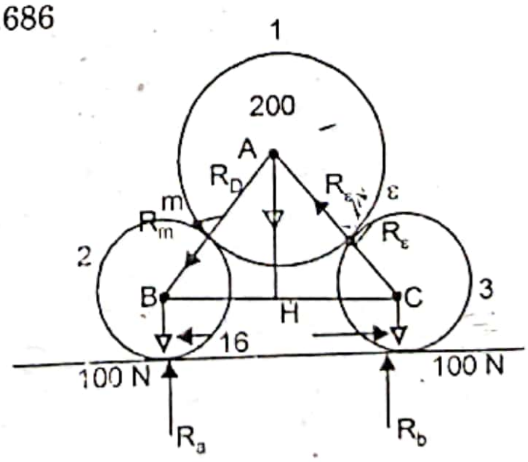
$$R_D \sin\theta = 100$$

$$\text{Sin}\,\theta = \frac{AH}{AB} = \frac{\sqrt{11^2 - 8^2}}{11} = .686$$

$$\text{Sin}\theta = .686$$

$$R_D = \frac{100}{\sin\theta} = 145.77$$

$$R_D = R_\varepsilon = 146.$$



Solving for Cyliner 2

$$\Sigma f = 0$$

$$\text{T}-R_D\cos\theta = 0$$

$$\text{T} = R_D\,\text{Cos}\theta = 146\times\frac{8}{11} = 106N$$

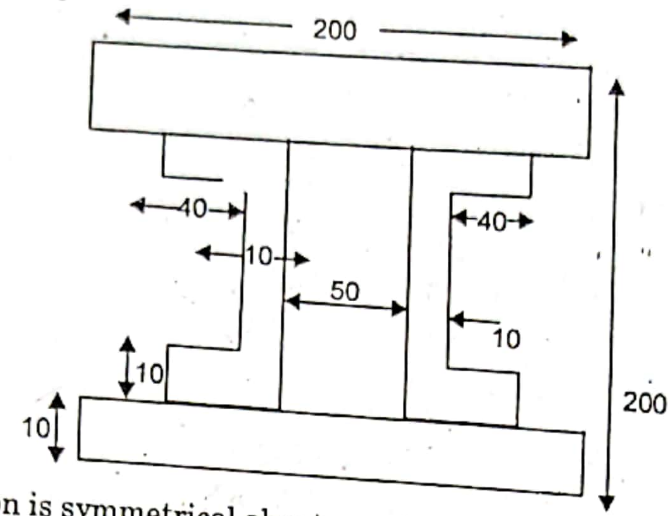
$$\Sigma f_y = 0$$

$$R_G - 100 - \text{R}_D \sin\theta = 0$$

$$R_G - 100 - 146 \times .686 = 0$$

$$R_F = R_G = 200\text{ N.}$$

Q.3. (a) Find MOI about horizontal axis passing through CG of given fig.



Ans. As section is symmetrical about xx axis

MOI of one top or bottom plate about axis through its C_G and || to $x-x$ axis.

$$I_{G1} = \frac{200 \times (10)^3}{12} = 16666 = 16.6\,\text{m}$$

Distance between C_G of plates from $x-x$ axis

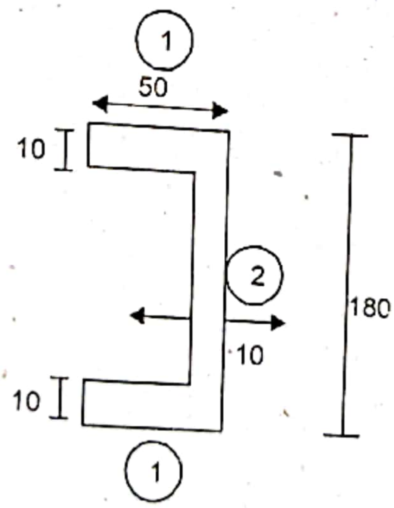
$$h_1 = 100 - 5 = 95\text{ mm}$$

M.O.I of top and bottom plate about x–x axis

$$I_{xx1} = I_{G1} + ah_1^2$$

$$2\,[7500 + 200 \times 10 \times 95^2]$$

$$= 36115000$$



M. O. I of Part (1) of one channel section about an axis through C_G and || to $x-x$ axis

$$I_{G2} = \frac{50 \times (10)^3}{12} = 4166$$

Distance of C_G of this part form x–x axis

$$h_2 = 90 - 5 = 85$$

M.O.I of part (1) about x–x

$$I_{xx2} = I_{G2} + ah_2^2$$

$$= 4\,[4166. + 50 \times 10 \times 85^2]$$

$$= 14466.664$$

Similarly M.O. I of Part 2 of channel about axis through C_G and | | to xx

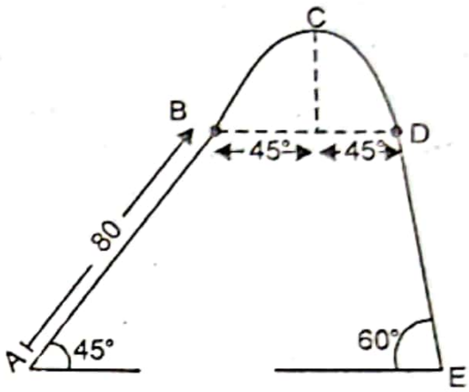
$$I_{G_3} = 2\times\left[\frac{10\times 160^3}{12}\right] = I_{G3} = 6826666$$

M.O.I of whole built up section about axis C_G & | | to $x-x$ axis

$$I_{xx} = I_{xx1}+I_{xx2}+I_{G3}$$

$$36115000 + 14466664 + 6826666 = 57408\ 330$$

Q.3. (b) Determine Centroid of bent wire ABCDE



Ans.

Length of $AB = l_1 = 80$

C_1 is at centre of line

$$x_1 = 80 \cos 45 = 56.56$$

$$y_1 = 80 \sin 45 = 56.56$$

Length of semicircular BC

$$l_2 = \pi\times r = \pi\times 45 = 141.37$$

Centroid $\quad C_2 = \frac{2r}{\pi} = \frac{2\times 45}{\pi} = 28.247$

Coordinate of Centroid C_2 of semi-Circular portion

$$x_2 = 80 \cos 45 + 45 = 101.56$$

$$y_2 = 80 \sin 45 + 45 = 101.65$$

$$x_3 = 80 \cos 45 + 90 + 80 \cos 60 = 186.56$$

$$y_3 = 80 \sin 45 + 90 + 80 \sin 60 = 215.8$$

$$\bar{x} = \frac{x_1 l_1 + l_2 x_2 + l_3 x_3}{l_1 + l_2 + l_3}$$

$$\bar{x} = \frac{56.56\times 80 + 141.37\times 101.56 + 80\times 186.56}{80+80+141.37}$$

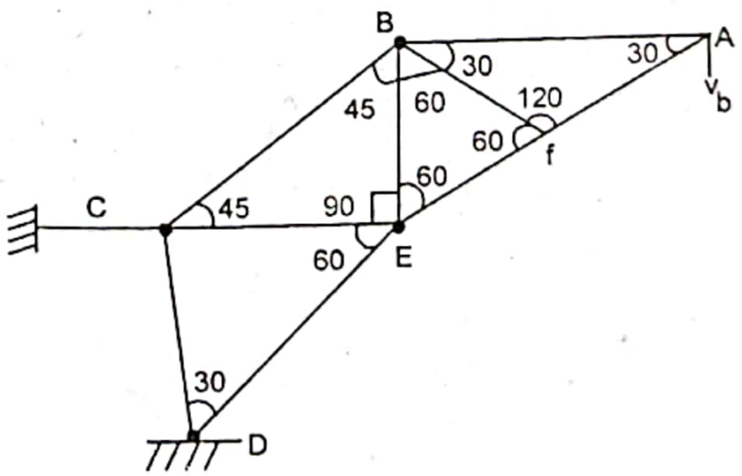
$$\bar{x} = 112.178$$

$$\bar{y} = y_1 l_1 + y_2 l_2 + y_3 l_3/(l_1+l_2+l_3)$$

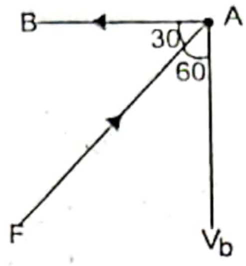
$$= \frac{56.56\times 80 + 101.56\times 141.37 + 215.8\times 80}{80+80+141.37}$$

$$\bar{y} = 119.94.$$

Q.4. (a) A truss is loaded and supported as shown in Fig. Compute axial force in all the members of truss.

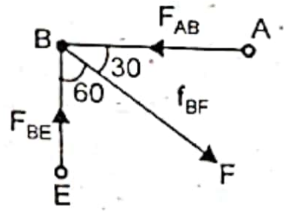


Ans. Joint A



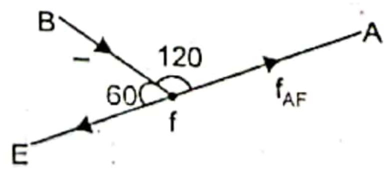
$$\Sigma H = 0$$
$$F_{AB} = F_{AF}\cos 30$$
$$F_{AB} = 10.39$$
$$\Sigma V = 0$$
$$6 = F_{AF}\cos 60$$
$$F_{AF} = \frac{6}{\cos 60} = 12$$

Joint B



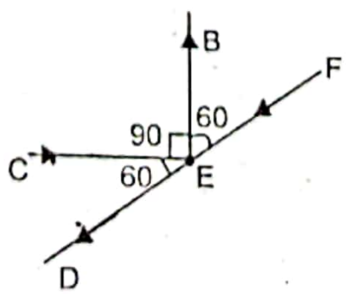
$$\Sigma x = 0$$
$$F_{AB} = F_{BF}\cos 30$$
$$F_{BF} = 12$$
$$\Sigma V = 0$$
$$F_{BE} = F_{BF}\cos 60$$
$$F_{BE} = 6$$

Joint F



$$F_{BF} = F_{AF}$$
$$12 = 12$$

Joint E



$$\Sigma V = 0$$
$$F_{BE} = F_{FE}\cos 60 + F_{ED}\sin 60$$

$$6 = 12 \times .5 + F_{ED} \sin 60]$$

$$\cancel{6} = \cancel{6} + F_{ED} \sin 60$$

$$F_{ED} = 0$$

$$F_{CD} = 0$$

As $\Sigma x = 0$

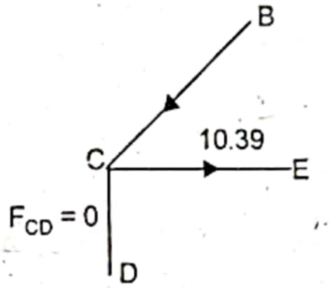
$$F_{CE} = F_{FE} \sin 60 + F_{ED} \cos 60°$$

$$F_{CE} = 12 \sin 60 + F_{ED} \times .5$$

$$F_{CE} = 10.39 + F_{ED} \times .5 \qquad [\text{As } F_{ED} = 0]$$

$$F_{CE} = 10.39$$

Point C



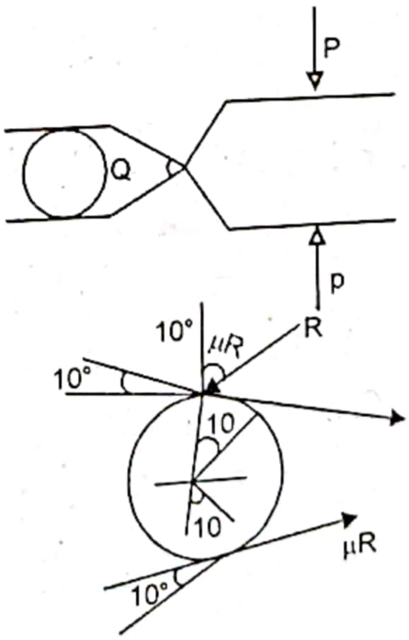
$$\Sigma x = 0$$

$$F_{BC} \cos 45 = 10.39$$

$$F_{BC} = 14.69$$

Q.4. (*b*) The tongs are used to handle hot steel tubes that are being heat treated in an oil bath. For a 20° jaw opening, what is the coefficient of static friction between the jaws & the tube that will enable the tongs to grip the tube without slipping? (3)

Ans.



$$\Sigma f_x = 0$$

$$\mu R \cos 10 + \mu R \cos 10 - R \sin 10 - R \sin 10 = 0$$

$$2\mu R \cos 10 - 2R \sin 10 = 0$$

$$\mu = \tan (10)$$

$$\mu = .176$$

FIRST TERM EXAMINATION [FEB. 2017]
SECOND SEMESTER [B.TECH.]
ENGINEERING MECHANICS
[ETME–110]

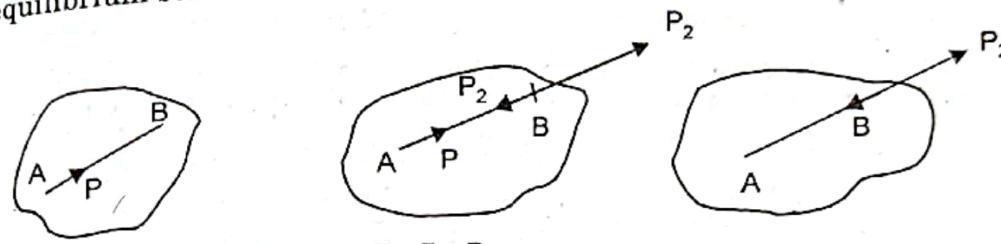
M.M. : 30

Time : 1½ hrs.

Note: Q.No. 1 is compulsory. Attempt any two more Questions from the rest.

Q.1. (a) Describe the 'principle of transmissibility of force' with an example. **(2)**

Ans. When the point of application of force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there is no change in the equilibrium state of the body

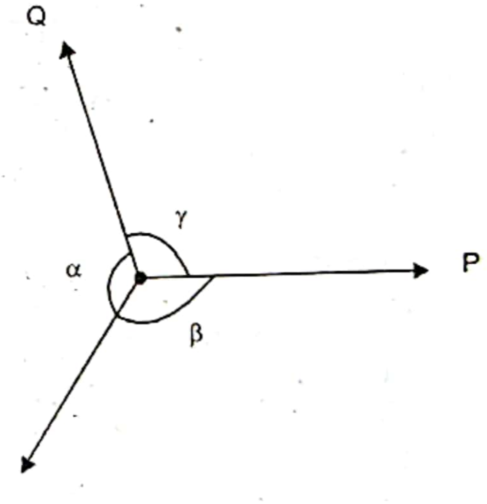


$P_1 = P_2 = P$

Q.1. (b) State the Lame's theorem for the equilibrium of a body. **(2)**

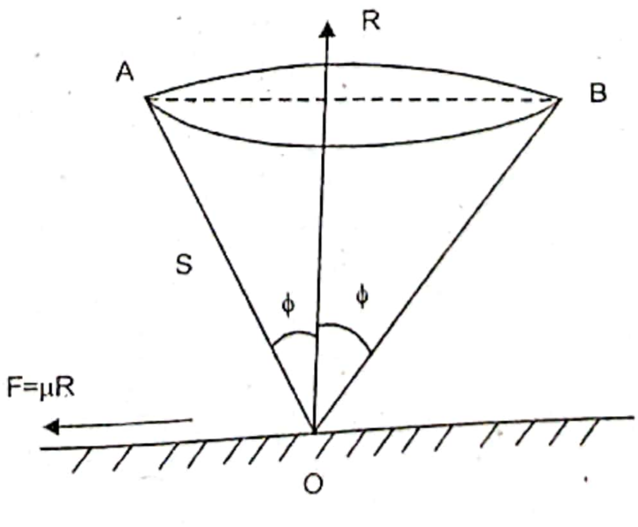
Ans. If three forces acting a point are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



Q.1. (c) Describe "cone of friction". **(2)**

Ans.



Normal reaction (R) and frictional force (F) meet at 0. R = $\sqrt{F^2 + R^2}$ and it make an angle ϕ with the normal reaction. O is the vertex of cone whose axis is R and sem vectex angle is ϕ. Such a cone is called the cone of friction.

Q.1. (*d*) State the assumptions in the analysis of perfect truss. (2

Ans. Assumptions of a truss

(1) Load is applied only at the joints

(2) All the members are straisht two force members. Forces are collinear wit centerline of the members

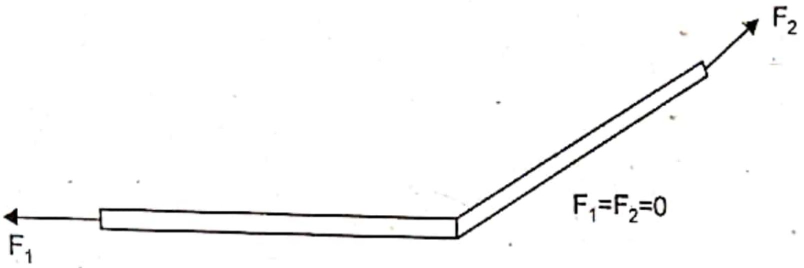
(3) Connection b/w members are frictionless and pin connections

(4) Truss is statically determinate

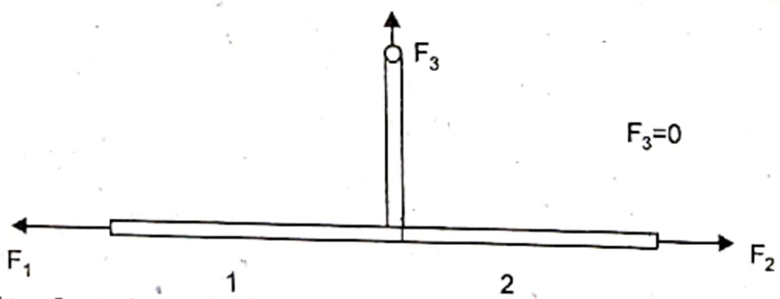
(5) Weight of the members is negligibly small : unless mentioned.

Q.1. (*e*) What is a zero force member in a truss and how it is identified? (

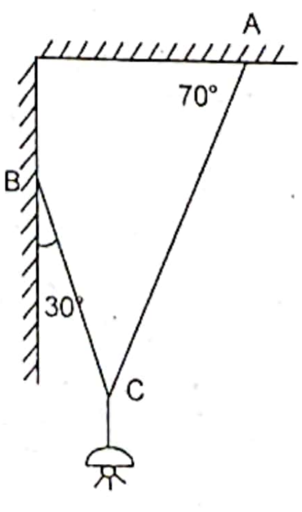
Ans. (1) When two members meeting at a joint are not collinear and there is external force acting on the joint, then the forces in both the members are zero.



(2) When there are 3 membes meeting at a joint of which two are collinear and th be at an angle and if there is no load at the joint the force in the third member



Q.2. (*a*) An electric light fixture weighing 20N hangs from point C, by t strings AC and BC as shown in Fig. Determine the tension in the strings AC a BC.



Ans.

$$\Sigma F_x = 0$$

$$T_1 \cos^\circ 70 - T_2 \cos 60^\circ = 0$$

$$T_1 \cos 70 = T_2 \cos 60^\circ$$

$$T_1 = T_2 \frac{\cos 60^\circ}{\cos 20^\circ}$$

$$T_1 = 8.9297 \times \frac{0.5}{.3420}$$

$$= 13.055 \text{ N}$$

$$\Sigma F_y = 0$$

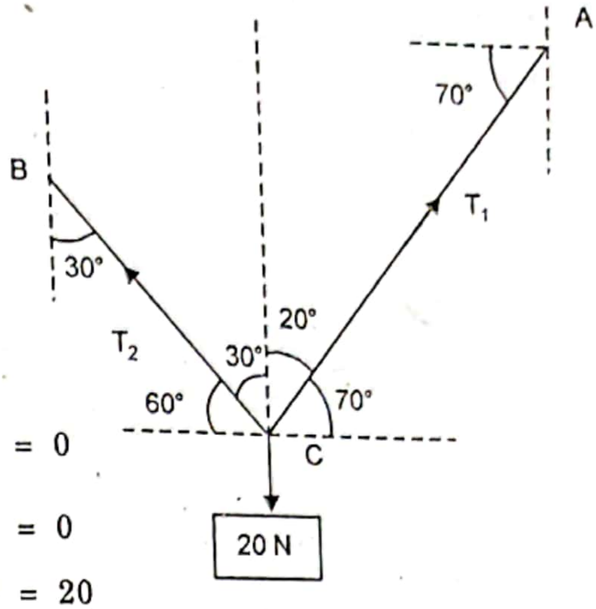
$$T_1 \sin 70 + T_2 \sin 60^\circ - 20 = 0$$

$$\therefore \frac{T_2 \cos 60}{\cos 70} \times \sin 70^\circ + T_2 \sin 60^\circ - 20 = 0$$

$$T_2 \times 0.5 \times 2.7474 + T_2 \times 8660 = 20$$

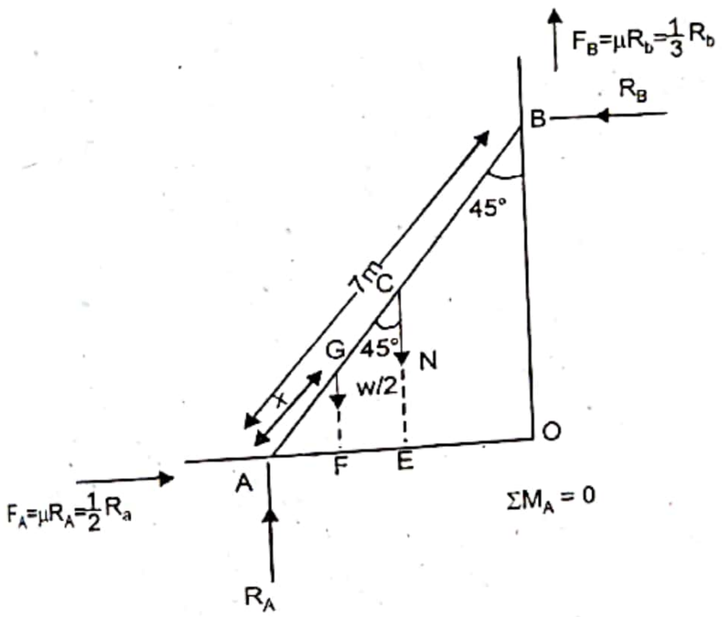
$$T_2 = 20/2.2397$$

$$= 8.9297 \text{ N}$$



Q.2. (b) A 7.0 m long ladder rests against a vertical wall with it makes an angle of 45° and on floor, if a man whose weight is one half of that of the ladder climbs it, at what distance along the ladder will he be, when the ladder is about to slip. μ = 1/3 for wall and ladder and μ = ½ for ladder and floor. (7)

Ans.



$$\cos 45^\circ = \frac{OB}{AB}$$

$$OB = 7 \times \cos 45^\circ$$

$$= 4.9497 \text{ N}$$

$$\sin 45^\circ = \frac{OA}{AB}$$

$$OA = 4.9497 \text{ N}$$

$$\sin 45^\circ = \frac{AE}{AC}$$

$$AE = 2.4748$$

$$\sin 45^\circ =$$

$$AF = x \sin 45^\circ$$

$$= .707x$$

$$-R_b \times OB - \frac{1}{3} R_b \times OA = W \times 2.4748 + \frac{W}{2} \times .707x$$

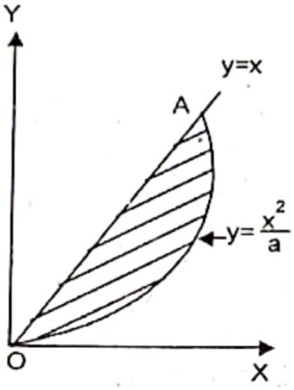
$$-R_b \times 4.9497 - \frac{1}{3} R_b \times 4.9497 = 2.4748 \times W + \frac{W}{2} \times .707x$$

$$-6.5996 \text{ Rb} = 2.4748 \times W + \frac{W}{2} \times .707x \quad \ldots(1)$$

$$R_A + \frac{1}{3} R_b = W + \frac{W}{2} = \frac{3W}{2} \quad \ldots(2)$$

$$\frac{3R_A + R_B}{3} = \frac{3W}{2}$$

Q.3. (*a*) Determine the coordinates of the centroid of the shaded area formed by the intersection of a straight line y = x and a parabola y = x²/a, as shown in Fig. **(10)**



Ans.

$$y = x \qquad \text{...(1)}$$

$$y = \frac{x^2}{a} \qquad \text{...(2)}$$

Area of strip = (pq) dx

$y_1 \rightarrow$ lies on straight line OA

$y_1 \rightarrow$ lies on parabola oA

$$dA = y'dx$$

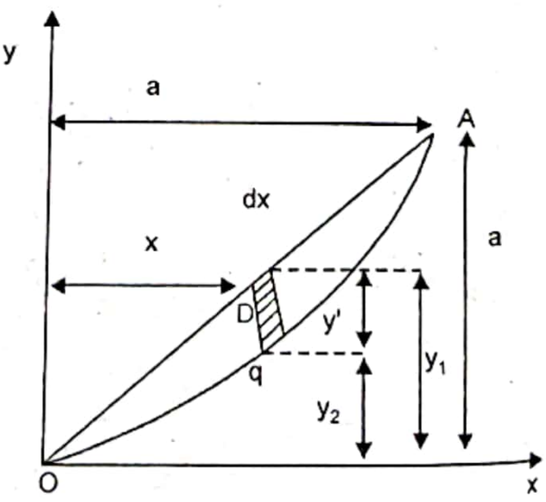
$$dA = (y_1 - y_2)\,dx$$

$$y_1 = x;\ y_2 = \frac{x^2}{a}$$

$$\text{Area of strip } dA = \left(x - \frac{x^2}{a}\right)dx$$

Distance of the centroid of the strip from the y-axis = x

$$x_c = \frac{\int x\,dA}{\int dA} = \frac{\int_0^a x\left(x - \frac{x^2}{a}\right)dx}{\int_0^a \left(x - \frac{x^2}{a}\right)dx} = \frac{a}{2}$$



$$y_c = \frac{\int y\,dA}{\int dA} < \frac{\int\left(\frac{y_1 + y_2}{2}\right)(y_1 - y_2)\,dx}{\int (y_1 - y_2)\,dx}$$

$$y_1 = x,\ y_2 = \frac{x^2}{a}$$

$$y_c = \frac{\int_0^a \frac{1}{2}\left(x - \frac{x^2}{a}\right)\left(x + \frac{x^2}{a}\right)dx}{\int_0^a \left(x - \frac{x^2}{2}\right)dx} = \frac{2a}{5}$$

Q.4. Determine the forces in each members of the truss as shown in fig.3. **(10)**

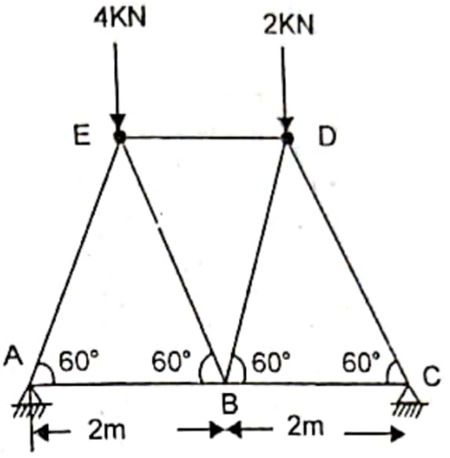
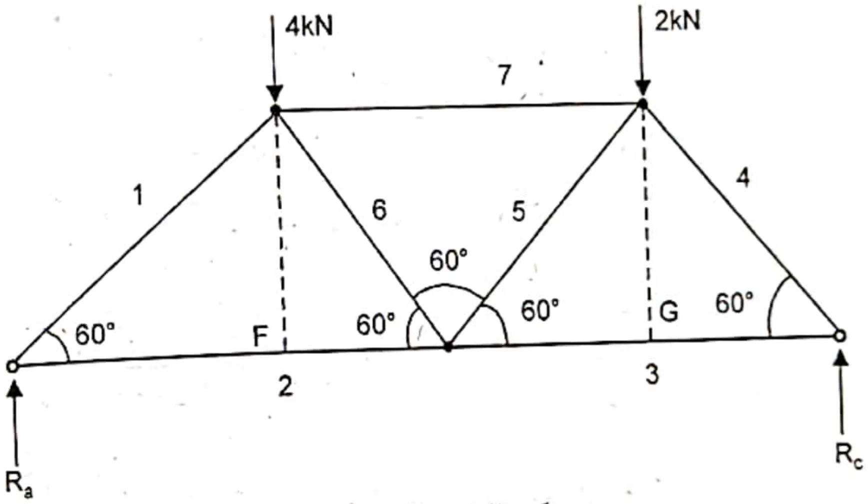


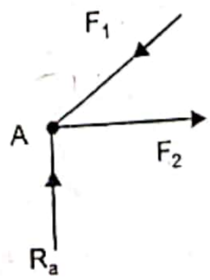
fig.3

Ans.



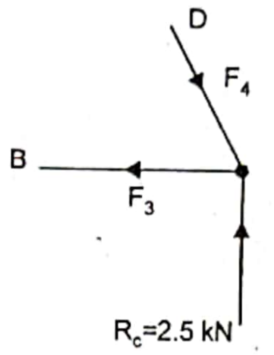
$$AF = AE\cos 60° = 2 \times 0.5 = 1\text{ m}$$
$$AG = AB + BG = AB \perp BD\cos 60°$$
$$= 2 \perp 2 \times 0.5 = 3$$
$$\Sigma m_n = 0$$
$$R_c \times 4 = 2 \times 3 + 4 \times 1 = 10\ R_c = 2.5\text{ kN}$$
$$R_c = (4 + 2) - 2.5 = 3.5\text{ kN}$$

Joint A



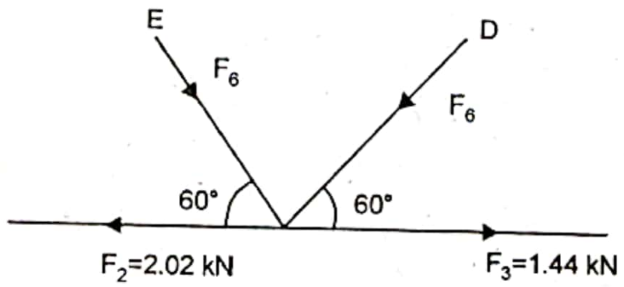
$$\Sigma F_x = 0\ F_2 - F_1\cos 60° = 0$$
$$\Sigma F_y = 0\ R_a - F_1\sin 60° = 0$$
$$F_1 = Ra/\sin 60° = 3.5/866$$
$$= 4.04\text{ kN (Compresive)}$$
$$F_1 = F_1\cos 60° = 4.04 \times 0.5 = 2.02\text{ kN (Tensile)}$$

Joint C



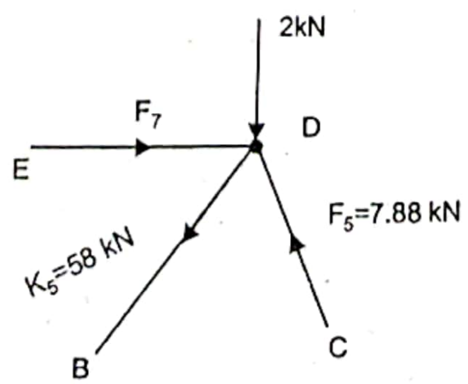
$$\Sigma F_x = 0 \; F_4 \cos 60° - F_3 = 0$$
$$R_c - F_y \sin 60° = 0$$
$$F_4 = \frac{R_c}{\sin 60°} = \frac{2.5}{866} = 2.88 \text{ kN (compresive)}$$
$$F_3 = F_4 \cos 60° = 2.88 \times 0.5 = 1.44 \text{ kN (Tensile)}$$

Joint B



$$\Sigma F_x = 0$$
$$1.44 - 202 + F_5 \cos 60° + F_6 \cos 60°$$
$$F_5 + F_6 = 1.16$$
$$\Sigma F_y = 0$$
$$F_5 \sin 60° - F_6 \sin 60° = 0$$
$$F_5 = F_6$$
$$F_5 = F_6 = 0.58 \text{ KN}$$

Joint D



$$\Sigma F_1 = 0 \; F_7 - 0.58 \cos 60° - 2.88 \cos 60°$$
$$F_7 = 1.73 \text{ kN (compresive)}$$

FIRST TERM EXAMINATION [FEB. 2018]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME–110]

Time : 1.5 hrs. **M.M. : 30**

Note: Q. No. 1 which is compulsory. Attempt any two more questions from the rest.

Q. 1. (a) State and explain Lami's theorem. (2)

Ans. Refer Q.1. (b) First Term Examination 2017.

Q. 1. (b) State "principle of moments" (or Varignon's principle). (2)

Ans. Refer Q.1. (b) End Term Examination 2017.

Q. 1. (c) Explain the terms: perfect frame, imperfect frame, deficient frame and redundant frame. (2)

Ans. Perfect, deficient and redundant frames: The structure is said to be perfect if the number of members is just sufficient to prevent its distortion of shape when subjected to external loads. For a perfect frame, the following correlation exists between the number of joints j and the number of members m,

$$m = 2j - 3$$

The number of joints and the number of members in the frames shown in Fig.1 satisfy the identity $m = 2j - 3$ and as such these are perfect structures (frames). A perfect structure is statically determinate. Equations of static equilibrium are sufficient to determine the forces in its members without the need of considering their deformation.

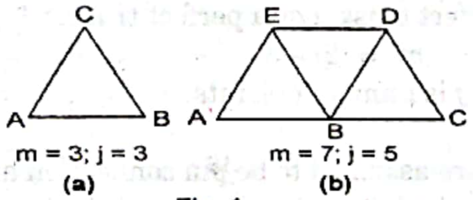


Fig. 1.

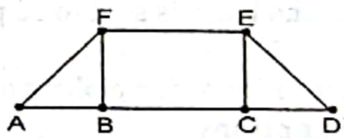


Fig. 2.

A structure is termed imperfect or deficient frame if the number of members in it is less than that required for a perfect frame. Refer Fig. 2. which shows a frame having 6 joints and 8 members. The number of members required for the frame to be perfect is

$$m = 2j - 3 = 2 \times 6 - 3 = 9$$

Since there are only 8 members (one less than that required for a perfect frame), the given frame is deficient or imperfect. An imperfect frame is under rigid or collapsible. Such a frame cannot prevent geometrical distortion when loaded.

A structure is termed redundant frame if the number of members in it is more than that required for a perfect frame. Refer Fig. (2) which shows a frame which has 7 joints and 12 members. The number of members required for the frame to be perfect is

$$m = 2j - 3 = 2 \times 7 - 3 = 11$$

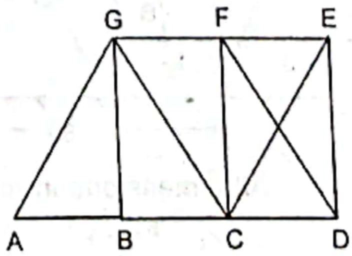


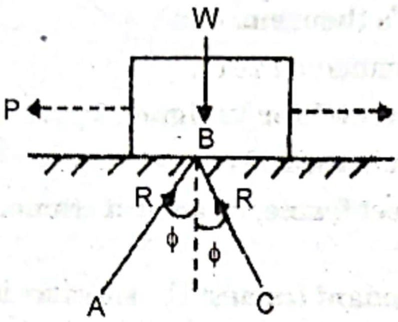
Fig. 3.

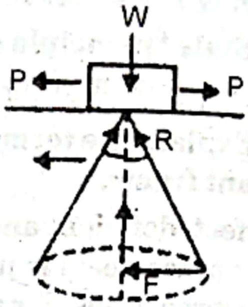
indeterminate. Further, each extra member adds one degree of indeterminancy. Thus the frame depicted in Fig. 3. is redundant to a single degree because one member is extra.

The frame depicted in Fig. 3. would remain stable even if one of the members CE or DF is removed.

Q. 1. (d) Explain "Cone of friction". (2)

Ans. Cone of friction: Consider a block of weight W resting on horizontal surface and acting upon fay force P. When we consider coplanar forces, in order for motion not to occur in any direction, resultant R must lie within angle ABC, where ϕ is angle of friction





Consider force P is gradually changed through 360°. For motion not to occur, resultant reaction R must be contained with in the cone generated by revolving line AB about normal BN.

The inverted so formed with semi-central angle equal to angle of friction ϕ is called cone of friction. Now, for motion to occur resultant R will be on the surface of cone.

Q. 1. (e) State the assumptions made for the analysis of truss. (2)

Ans. Perfect Truss: When the truss is non collapsible on the removal of external supports, the truss is said to be perfect truss. For a perfect truss.

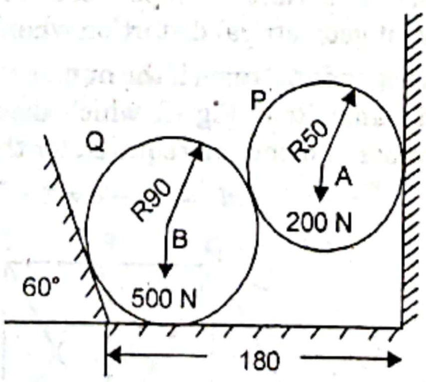
$$m = 2j - 3$$

where m *is no.* of members and j is number of joints.

Assumptions:

1. The joints of a simple truss are assumed to be pin connection and frictionless.
2. The loads on the truss are applied at joints only.
3. The members of a truss are straight two force members with the forces acting collinear with the centre line of the members.
4. The weights of the members are negligible unless otherwise mentioned.
5. The trusse is statically determinate.

Q. 2. (a) Two cylinders P and Q rest in a channel as shown in Fig. 1. Determine the forces at all the four points of contact. (5)



All dimensions in mm

Fig. 1

Ans. First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A *i.e.* the centre of the cylinder P as shown in Fig. 1. (a).

1. Weight of the cylinder (200 N) acting downwards.
2. Reaction (R_1) of the cylinder P at the vertical side.
3. Reaction (R_2) of the cylinder P at the point of contact with the cylinder Q.

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder P} = \frac{100}{2} = 50\,\text{mm}$$

Similarly $$BF = \text{Radius of cylinder Q} = \frac{180}{2} = 90\,\text{mm}$$

and $$\angle BCF = 60^\circ$$

$\therefore$ $$CF = BF \cot 60^\circ = 90 \times 0.577 = 52 \text{ mm}$$

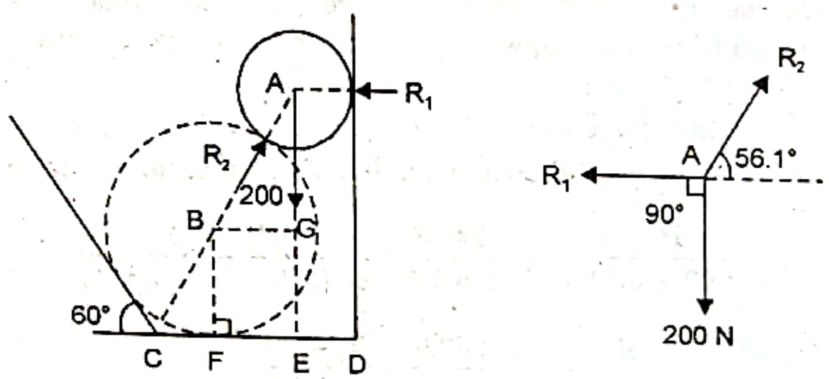
$\therefore$ $$FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and $$AB = 50 + 90 = 140 \text{ mm}$$

$\therefore$ $$\cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or $$\angle ABG = 56.1^\circ$$

The system of forces at A is shown in Fig. 1. (b).



(a) Free body diagram (b) Force diagram

Fig. 1.

Applying Lami's equation at A,

$$\frac{R_1}{\sin(90^\circ + 56.1^\circ)} = \frac{R_2}{\sin 90^\circ} = \frac{200}{\sin(180^\circ - 56.1^\circ)}$$

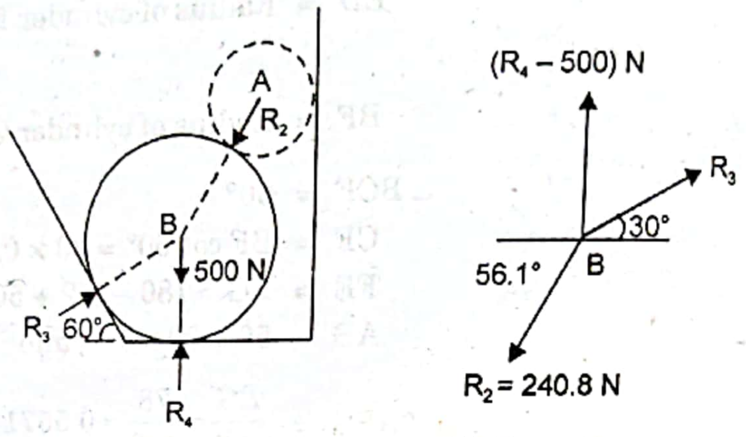
$$\frac{R_1}{\cos 56.1^\circ} = \frac{R_2}{1} = \frac{200}{\sin 56.1^\circ}$$

$\therefore$ $$R_1 = \frac{200 \cos 56.1^\circ}{\sin 56.1^\circ} = \frac{200 \times 0.5571}{0.830} = 134.2\,N$$

and $$R_2 = \frac{200}{\sin 56.1^\circ} = \frac{200}{0.8300} = 240.8\,N$$

Now consider the equilibrium of the cylinder Q. it is in equilibrium under the of the following four forces, which must pass through the centre of the cylinder as in Fig. 2. (a).

1. Weight of the cylinder Q (500 N) acting downwards.
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.
3. Reaction R_3 of the cylinder Q on the inclined surface.
4. Reaction R_4 of the cylinder Q on the base of the channel.



(a) Free body diagram (b) Force diagram

Fig. 2.

A little consideration will show, that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$\therefore$ Net downward force = $(R_4 - 300)$ N

The system of forces is shown fig. 2. (b) Applying Lami's equation at B.

$$\frac{R_3}{\sin(90° + 56.1°)} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin(180° + 30° - 56.1°)}$$

$$\frac{R_3}{\cos 56.1°} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin 26.1°}$$

$$\therefore \quad R_3 = \frac{240.8 \times \cos 56.1°}{\sin 60°} = \frac{240.8 \times 0.5577}{0.866} = 155\,N$$

and

$$R_4 - 500 = \frac{240.8 \times \sin 26.1°}{\sin 60°} = \frac{240.8 \times 399}{0.866} = 122.3\,N$$

$$R_4 = 122.3 + 500 = 622.3 \text{ N}$$

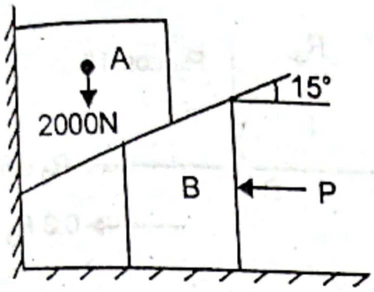
Q. 2. (b) For a flat belt drive, prove that $\frac{T_1}{T_2} = e^{\mu\theta}$. (5)

where T_1 = Tension on the tight side of the belt, T_2 = Tension on the slack side of the belt, μ = coefficient of friction between the belt and the pulley surface and θ = angle of contact between belt and pulley.

Ans. Refer Q.2. of End Term Exam. 2017.

Q. 3. (a) A block overlying a 15° wedge on a horizontal floor and leaning against vertical wall having weight 2000 N is to be raised by applying a horizontal force to the wedge. Assume coefficient of friction between all the surfaces to be 0.3, determine the minimum horizontal force to be applied to raise the block. (5)

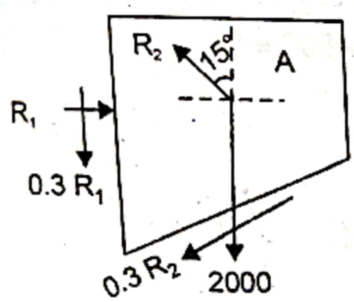
Ans.

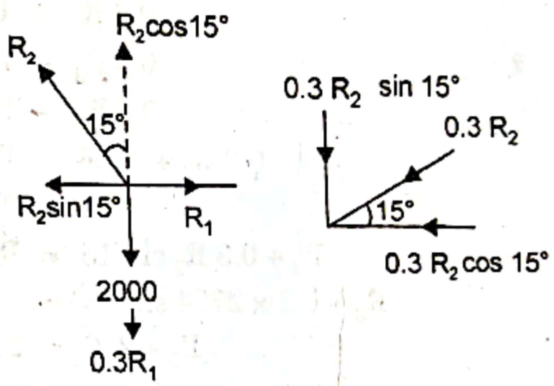


Given,

$\mu = 0.3, \theta = 15°, W_A = 2000, W_B = 0$

F.B.D. of block A.





Applying the condition of equilibrium.

$$\Sigma f_x = 0.$$

$$R_1 - 0.3 R_2 \cos 15 - R_2 \sin 15° = 0$$

$$R_1 - 0.289 R_2 - R_2 (0.25) = 0$$

$$\boxed{R_1 = 0.539 R_2} \quad ...(i)$$

$$\Sigma f_y = 0$$

$$R_2 \cos 15 - 0.3 R_2 \sin 15° - 2000 - 0.3R_1 = 0$$

$$R_2 \cos 15° - 0.078 R_2 - 2000 - 0.3 R_1 = 0$$

$$0.88 R_2 - 2000 = 0.3 R_1$$

From eq. (i)

$$0.88 R_2 - 2000 = 0.3 \times 0.539 R_2$$

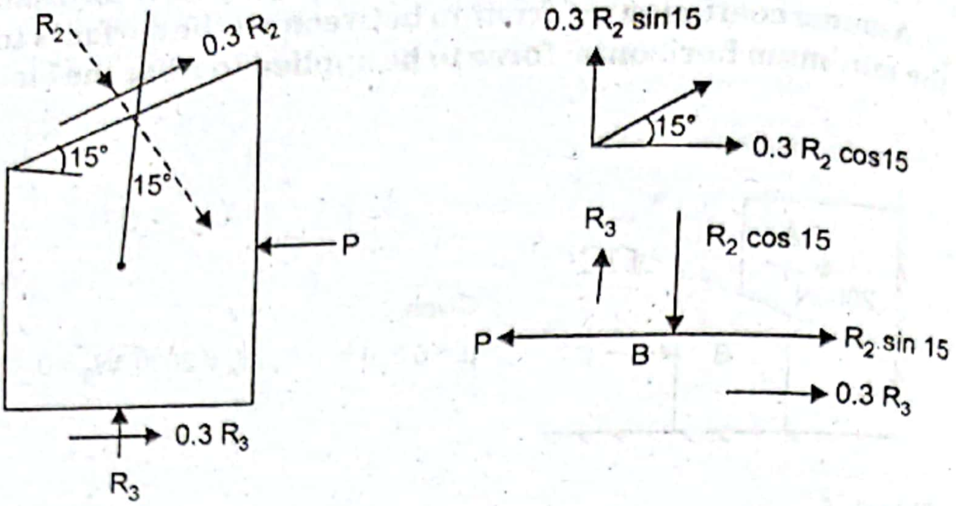
$$0.88 R_2 - 2000 = 0.1617 R_2$$

$$0.718 R_2 = 2000$$

$$\boxed{R_2 = 2784} N$$

$$\boxed{R_1 = 750} N$$

Block B



$$\Sigma Fx = 0.$$
$$0.3\ R_3 - P + R_2 \sin 15° + 0.3\ R_2 \cos 15 = 0$$
$$0.3\ R_3 - P + 2784 \sin 15 + 0.3 \times 2784 \cos 15 = 0$$
$$0.3\ R_3 - P + 720 + 806 = 0$$
$$0.3\ R_3 + 1526 = P \qquad \text{...(ii)}$$
$$\Sigma Fy = 0.$$
$$R_3 + 0.3\ R_2 \sin 15 = R_2 \cos 15$$
$$R_3 + 0.3 \times 2784 \sin 15 = 2784 \cos 15$$
$$R_3 + 216 = 2689$$
$$\boxed{R_3 = 2473\,N}$$

From eq. (ii)

$$\boxed{P = 2268\,N}$$

Q. 3. (b) A beam is loaded as shown in fig. 2. Determine the reaction at both ends. (5)

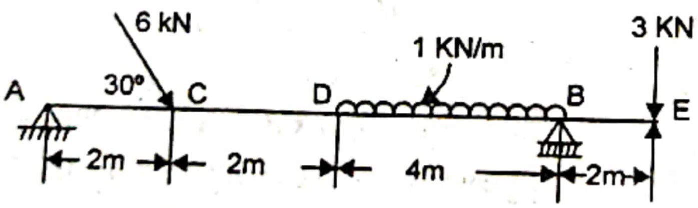
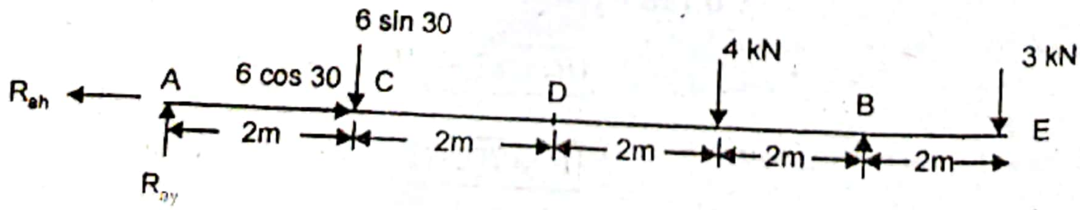


Fig. 3.

$$\mathbf{R_{ah} = 5.19\ kN}$$
$$\mathbf{R_{ay} = 2.5\ kN}$$
$$\mathbf{R_B = 7.5\ kN}$$

Ans. Free body diagram of the beam



Applying the condition of equilibrium. [∵ converting U.D.L. to point Load from D to B.]

$$\Sigma M = 0;\ \Sigma F_x = 0\ ;\ \Sigma F_y = 0$$

Taking moment about point A.

$$6 \sin 30 \times 2 + 4 \times 6 - R_B \times 8 + 3 \times 10 = 0$$

$$6 + 24 - 8R_B + 30 = 0$$

$$\therefore \quad R_B = \frac{60}{8} = 7.5$$

$$\boxed{R_B = 7.5\,KN} \quad \ldots(i)$$

$$\Sigma F_x = 0;$$

$$R_{ah} = 6 \cos 30° = 5.19 \text{ kN}$$

$$\Sigma F_y = 0;$$

$$R_{ay} - 6 \sin 30 - 4 + R_B - 3 = 0$$

or

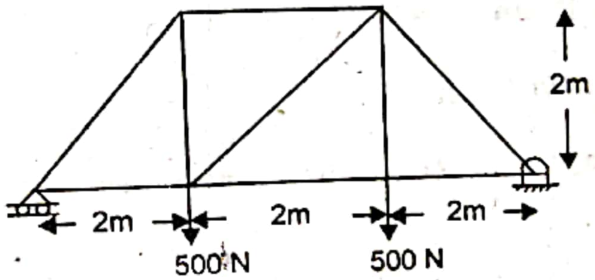
$$R_{ay} + R_B = 6 \sin 30 + 7 = 10 \text{ kN}$$

From equation (i)

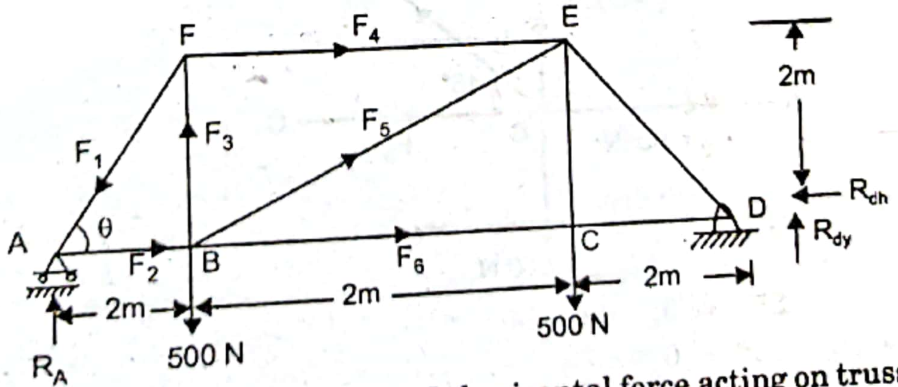
$$R_{ay} = 10 - 7.5 = 2.5$$

$$\boxed{R_{ay} = 2.5\,KN}\ \boxed{R_{ah} = 5.19\,KN},\ \boxed{R_B = 7.5\,KN}$$

Q. 4. (a) Determine the reactions and the forces in each member of a truss, supporting two load as shown in fig. 3. **(5)**



Ans.



In hinged support, $R_{Dh} = 0$ because No horizontal force acting on truss. So both the reaction point A and D acting vertically. Now, condition of equilibrium.

Taking moments about point A.

$$0 + 500 \times 2 + 500 \times 4 = R_{Dy} \times 6$$

$$R_{Dy} = \frac{3000}{6} = 500\,kN$$

$$R_A = (1000 - 500) = 500 \text{ kN} \quad [\because R_A + R_{Dy} = 500 + 500]$$

$R_{Dy} \rightarrow$ Reaction at D point in vertical direction]

In Δ AFB,

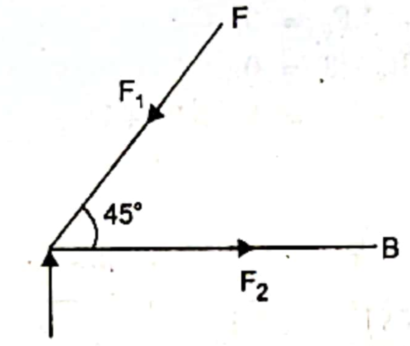
$$\tan\theta = \frac{FB}{AB} = \frac{2}{2} = 1$$

$$\boxed{\theta = 45°}$$

Now, Finding the forces in each members. Consider the joint method of the truss.

Joint A. (F.B.D.)

from equation of equilibrium

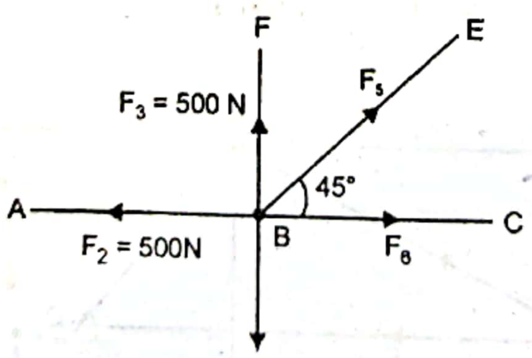


$$\Sigma F_x = 0; \; F_2 - F_1 \cos 45° = 0$$
$$\Sigma F_y = 0; R_A - F_1 \sin 45° = 0$$

$$\therefore \quad F_1 = \frac{R_A}{\sin 45°} = 707.21N \qquad \text{(Compression)}$$

$$F_2 = F_1 \cos 45° = 707.21 \times 0.707 = 500 \text{ N} \qquad \text{(Tension)}$$

Joint B.



$$\Sigma Fx = 0;$$
$$F_6 + F_5 \cos 45° - 500 = 0$$
$$\Sigma Fy = 0; 500 - 500 + F_5 \sin 45° = 0$$
$$\therefore \boxed{F_5 = 0}$$

Now, $\boxed{F_6 = 500 N}$ (Tension)

The zero value for force F_5 signifies that no force is induced in the member BE.

Member	AF, DE	AB,CD	BF,CE	FE	BE	BC
Force	707.21N	500N	500N	500N	0	500N
Nature	C	T	T	C	–	T

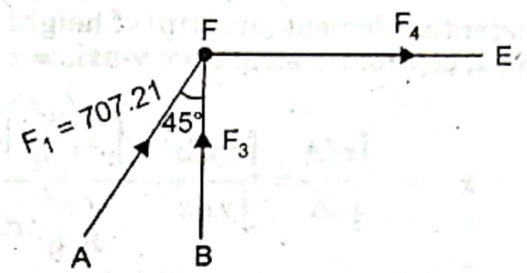
Since the Truss is symmetrical, we have

Force is member CE = force in member BF = 500 N (T)

$$F_{CD} = F_{AB} = 500 \text{ (T)}$$

$$F_{DE} = 707.21 \text{ (C)}.$$

Joint F



$$\Sigma Fx = 0;$$

$$F_4 + 707.21 \sin 45° = 0$$

$$\therefore \quad F_4 = -707.21 \times 0.707 = -500 \text{ N}$$

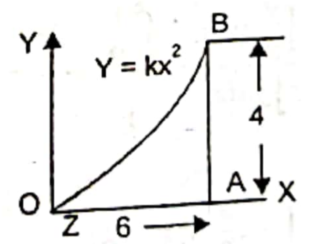
$$\Sigma F_y = 0; F_3 + 707.21 \cos 45° = 0$$

$$\therefore \quad F_3 = -707.21 \times 0.707 = -500 \text{ N}$$

Note: The –ve sign with the magnitude of forces F_3 and F_4 shows that a wrong choice has been made while assuming their directions. Obviously the assumed direction of the forces in members FB and FE need to be reversed

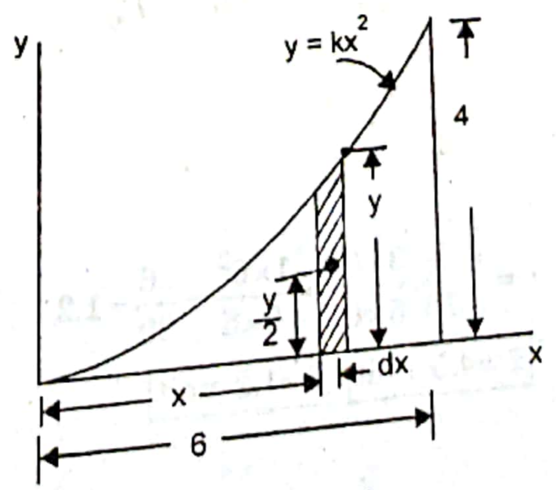
Therefore the member FB is a tension member and the member FE is a compression member.

Q. 4. (b) Determine the coordinates of the C.G of the area OAB shown in fig. 4, if the curve OB represents the equation of parabola, given by $y = kx^2$ in which OA = 6 units and AB = 4 units. (5)



Ans. The equation of parabola is ...(i)

$$y = kx^2$$



Now, The value of constant k is determined by substituting the coordinates (6, 4) of a point on the curve in equation of parabola.

$$4 = k6^2$$

or, $$k = \frac{4}{36} = \frac{1}{9}$$

Now from eq. (i)

$$y = k.x^2$$

$$y = \frac{1}{9}.x^2$$

Or, $$x^2 = 9y \Rightarrow x = 3\sqrt{y}$$

Consider a vertical differential element (or strip) of height y and width dx.
Area of the strip $dA = y.dx$, centroid of strip from y-axis $= x$

$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{\int x.y dx}{\int y.dx} = \frac{\int_0^6 x\left(\frac{x^2}{9}\right)dx}{\int_0^6 \frac{x^2}{9}.dx}$$

$$= \frac{\left[\frac{1}{9}.\frac{x^4}{4}\right]_0^6}{\left[\frac{1}{9}.\frac{x^3}{3}\right]_0^6} = \frac{\cancel{1/9}\left(\frac{6^4}{4}\right)}{\cancel{1/9}\left(\frac{6^3}{3}\right)} = \frac{3}{4}\left[\frac{6^4}{6^3}\right]$$

$$= \frac{3}{4}.6^1$$

$$= \frac{18}{4} = 4.5$$

Distance of the centroid of the strip from the x-axis $= \frac{y}{2}$

$$\bar{y} = \frac{\int \frac{y}{2}.y dx}{\int y.dx} = \frac{\int_0^6 \frac{1}{2}y^2 dx}{\int_0^6 y\, dx} = \frac{\int_0^6 \frac{1}{2}\left(\frac{x^2}{9}\right)^2 dx}{\int_0^6 \left(\frac{x^2}{9}\right).dx}$$

$$= \frac{\int_0^6 \frac{1}{2}.\frac{x^4}{81}dx}{\int_0^6 \frac{x^2}{9}dx} = \frac{\frac{1}{2}\left[\frac{1}{81}\left(\frac{x^5}{5}\right)\right]_0^6}{\left[\frac{1}{9}\left(\frac{x^3}{3}\right)\right]_0^6}$$

$$= \frac{1}{2}\times\frac{\cancel{9}}{\cancel{81}}.\frac{\left[\frac{6^5}{5}\right]}{\left(\frac{6^3}{3}\right)}$$

$$= \frac{1}{18}.\frac{3\times 6^5}{5\times 6^3} = \frac{1\times 6^2}{6\times 5} = \frac{36}{30} = 1.2$$

Now, $\boxed{\bar{x} = 4.5 \text{ unit}}$ $\boxed{\bar{y} = 1.2 \text{ unit}}$

MID TERM EXAMINATION [FEB-2019]
SECOND SEMESTER [B.TECH]
ENGINEERING MECHANICS [ETME-110]

Time : 1.30 hrs. **M.M. : 30**

Note :- *Attempt Q.No. 1.which is compulsory and any two more question from remaining.*

Q.1. Define the following :

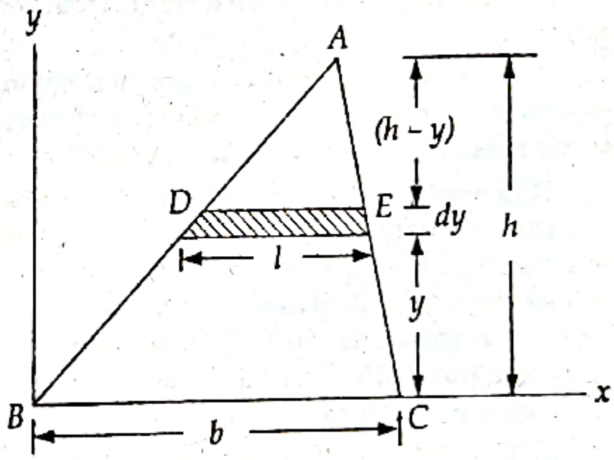
Q.1. (a) Derive an expression for center of gravity of a triangular lamina of height (H) and base (B), about its base using method of integration. **(2.5)**

Ans. Let ABC be the triangle of base width b and height h. Consider an elementary strip of width l, thickness dy and located at distance y from base BC of the triangle. For this elemental strip,

$$\text{area} = l\,dy$$

$$\text{moment about } x\text{-axis} = l\,dy \times y$$

Since the integration is to be done with respect to y within the limits 0 to h, it is necessary to express l in terms of y. For that we obtain the following correlation from the similarity of triangles ADE and ABC,



$$\frac{l}{b} = \frac{h-y}{h};\; l = b\left(1-\frac{y}{h}\right)$$

$\therefore$ Moment of elemental strip about x-axis

$$= b\left(1-\frac{y}{h}\right)y\,dy$$

area of triangle ABC $= \frac{1}{2}bh$

If $\bar{y}$ is the distance of the centroid from the base, then from the moment principle

$$\frac{1}{2}bh \times \bar{y} = \int_0^h b\left(1-\frac{y}{h}\right)y\,dy = b\int_0^h\left(y-\frac{y^2}{h}\right)dy$$

$$= b\left|\frac{y^2}{2}-\frac{y^3}{3h}\right|_0^h = b\left(\frac{h^2}{2}-\frac{h^2}{3}\right) = \frac{bh^2}{6}$$

$$\therefore \quad \bar{y} = \frac{bh^2}{6}\times\frac{2}{bh} = \frac{h}{3}$$

Thus the centroid of a triangle of height h is at a distance $h/3$ from the base or $2h/3$ from the apex.

Q.1. (b) Angle of friction and Cone of friction. **(2.5)**

Ans. Angle of friction : The angle of friction ϕ is a measure of the limiting position of total reaction between the two contacting surfaces. It is defined as the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.

With reference to Fig.

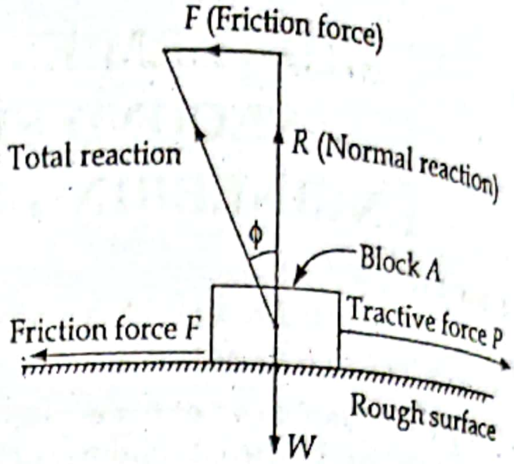
R = normal reaction

F = limiting force of friction

$S = \sqrt{R^2 + F^2}$

= total or resultant reaction

$\tan\phi = \frac{F}{R}$ where f is the angle of friction



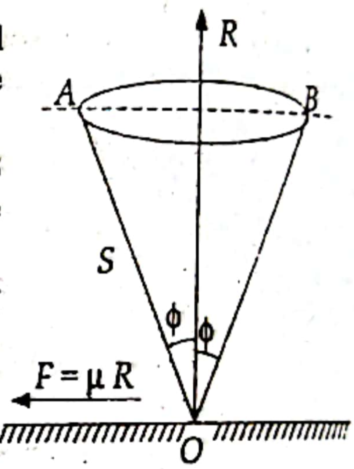
The ratio F/R is also called the coefficient of friction, m.

$\therefore \quad \mu = \tan\phi$ ...(1)

The above correlation does suggest that angle of friction in radians is equal to the coefficient of friction provided the angle has a small value.

$$\mu = \tan\phi \simeq \phi \text{ in radians}$$

Cone of friction : With reference to Fig., the lines of action of normal reaction R and the frictional force F meet at point O. The magnitude of total reaction is $\sqrt{F^2 + R^2}$ and it makes an angle ϕ with the normal reaction.



Obviously O is the vertex of a cone whose axis is R and semi-vertex angle is ϕ. Such a right circular cone is called the cone of friction.

Apparently the cone of friction is the imaginary cone AOB generated in the case of non-coplanar forces by revolving the static resultant reaction S about the normal OR.

It is worthwhile to point out that in case of impending motion, the friction force is maximum and the total reaction lies on the surface of cone. When the friction force is less than the limiting friction, the total reaction would lie within the cone. In that case, the forces acting on the body are not sufficient to cause motion. This aspect forms the working principle for self-locking mechanisms.

Q.1. (c) Define Pappus and Guldinus theorem. **(2.5)**

Ans. Pappus-Guldinus Theorems: When a plane curve rotates about a fixed axis, it generates a surface called the surface of revolution.

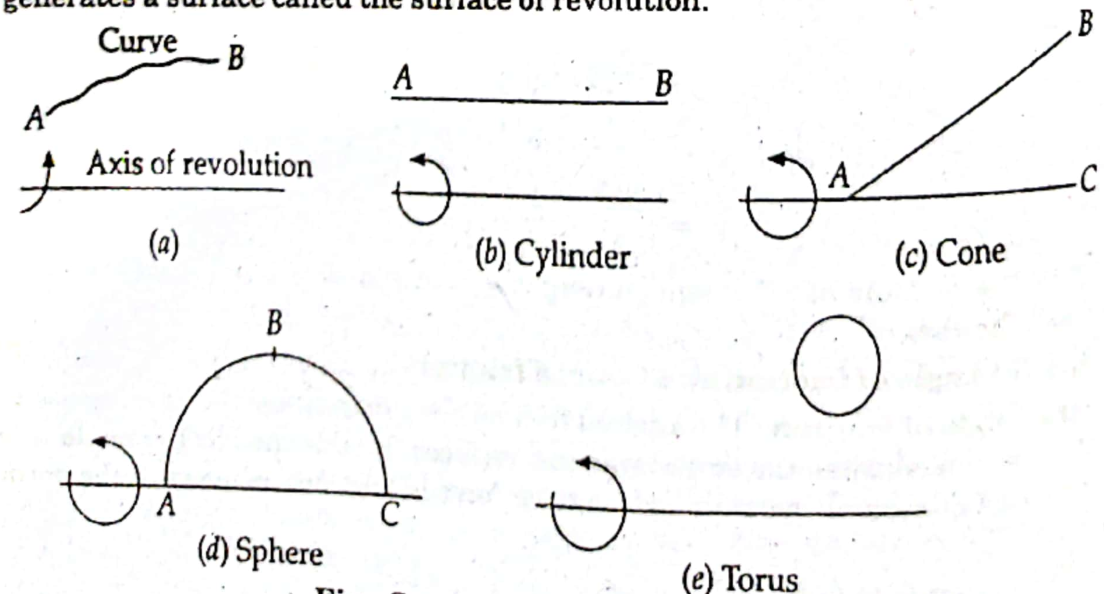
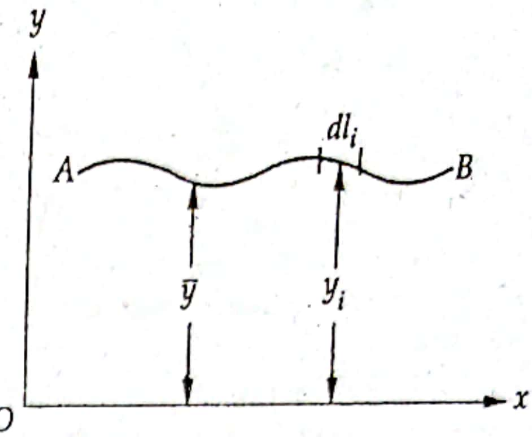


Fig : Generation of surface of revolution

Theorem I. The area of the surface generated by rotating any plane curve about a non-intersecting axis in its plane is equal to the product of the length of the curve and the distance travelled by its centroid.



Let AB be any plane curve of length l that lies in the x-y plane and does not intersect the x-axis. Consider an elemental length dl_i so short that it may be thought to be a minute portion of a straight line. Let y - coordinate of its mid point be y_i and that of the centroid of the entire curve be $\bar{y}$.

When this curve is rotated about the x-axis, then area generated by the elementary length $= 2\pi y_i dl_i$

area generated by the entire line $= \int 2\pi y_i \, dl_i$

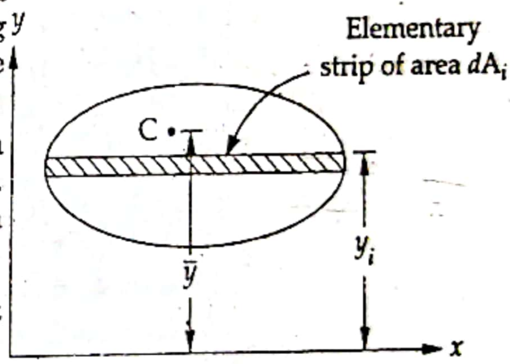
The integral $\int y_i \, dl_i$ is equal to $\bar{y}\, l$ and therefore area generated $= l \times 2\pi\bar{y}$

where $2\pi y$ is the distance travelled by the centroid of the curve

$\therefore$ area generated = (length of the generating curve) × (distance travelled by centroid of the curve)

It is to be noted that the generating curve does not cross the axis about which it is rotated. If it does so, then the two sections on either side of the axis would generate areas of opposite signs and the theorem would not apply.

Theorem II. The volume of the solid generated by rotating any plane figure about a non-intersecting axis in its plane is equal to the product of area of the figure and the distance travelled by its centroid.



Consider area A of any plane figure divided into a large number of very thin strips parallel to the x-axis. Let the y-coordinate of the middle of any strip of area dA_i be y_i and the centroid of the entire curve be $\bar{y}$.

When the plane of the figure is rotated about the x-axis, then

Volume generated by the elementary area $= 2\pi y_i \, dA_i$

Volume generated by the entire area $= \int 2\pi y_i dA_i$

The integral $\int y_i dA_i$ is equal to y A and therefore

Volume generated V $= A \times 2\pi y$

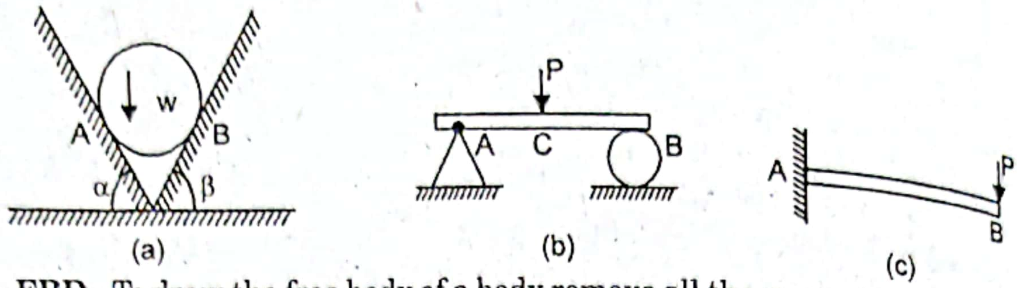
The parameter $(2\pi y)$ is the distance travelled by the centroid of the area being rotated.

$\therefore$ Volume generated = (area of the figure) × (distance travelled by the centroid)

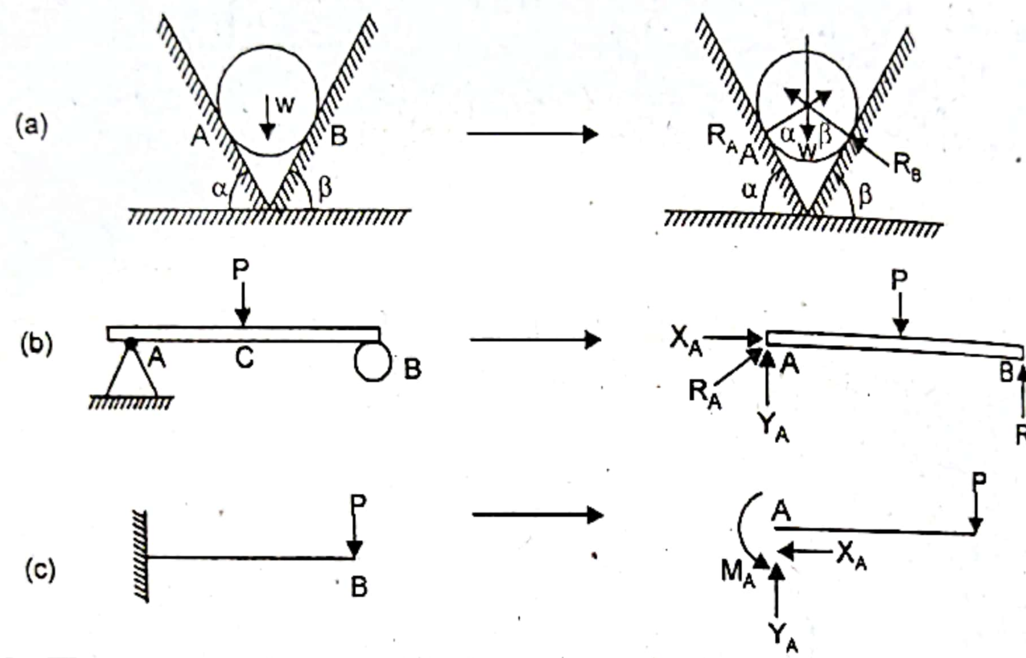
Utility: The theorems of Pappus-Guldinus offer a simple way to compute the areas of surfaces of revolution and the volumes of bodies of revolution.

Conversely their application lies in determining the centroid of a plane curve when the area of the surface generated by the curve is known or to determine the centroid of a plane area when the volume of the body generated by the area is known.

Q.1. (d) Define free body diagram and draw the free body diagram for the supports as shown in Fig. (2.5)



Ans. FBD - To draw the free body of a body remove all the supports (like wall, floor, hinge or any other body) and replace them by the reactions which these supports exert on the body.



Q.2. The spanner shown in Fig. is used to rotate a shaft. A pin fits in a hold at A, while a flat frictionless surface support the shaft at B. If a 300 N force P is exerted on the spanner at D, find the reactions at A and B. **(10)**

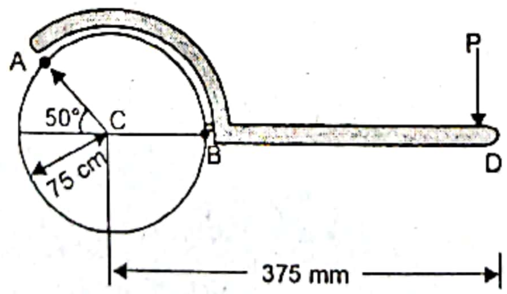
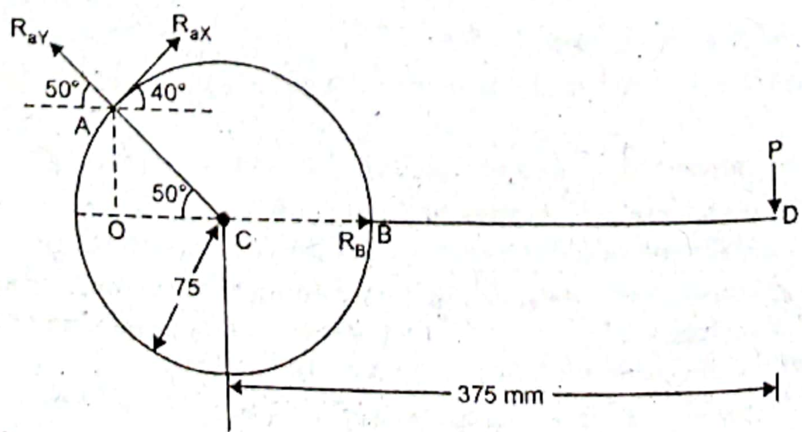


Fig.

Ans.



From ΔAOC,

$$AO = 57.4 \text{ mm}; \quad OC = 48.2 \text{ mm}$$

$\Sigma fx = 0$

$$R_{ax}\cos 40 + R_B = R_{ay}\cos 50 \quad ...(1)$$

$\Sigma fy = 0,$

$$R_{ax}\sin 40 + R_{ay}\sin 50 = 300 = p \quad ...(2)$$

$\Sigma M_A = 0$

$$R_B \times AO = P(48.2 + 375)$$

$$R_B = \frac{300(423.2)}{57.4} = 2211$$

$$\boxed{R_B = 2211\text{ N}}$$

$\Sigma M_C = 0$

$$R_{ax} \times 75 = P \times 375$$

$$\boxed{R_{ax} = 1500\text{ N}}$$

From eq (2)

$$1500\sin 40 + R_{ay}\sin 50 = 300$$

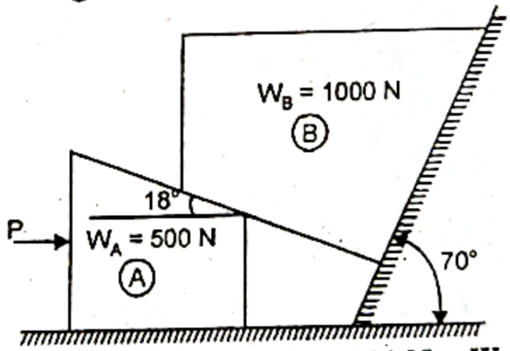
$$964.1 + R_{ay}\sin 50 = 300$$

$$R_{ay} = -\frac{664.1}{\sin 50} = -867\text{ N}$$

$$R_A = \sqrt{R_{ax}^2 + R_{ay}^2} = \sqrt{1500^2 + 867^2}$$

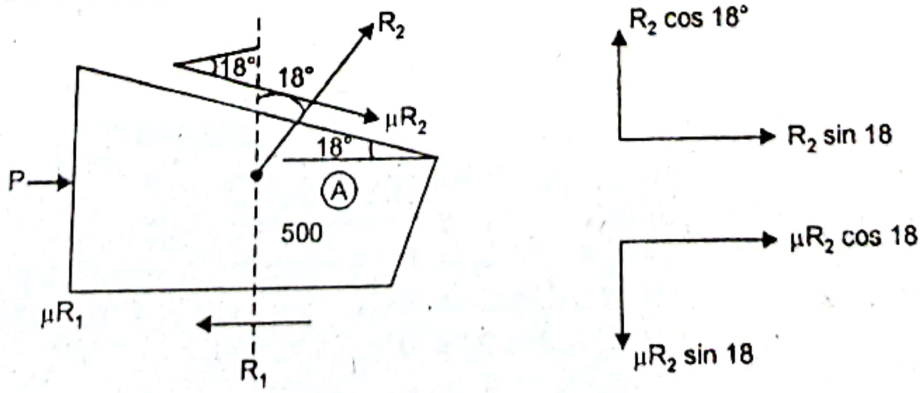
$$= \boxed{R_A = 1732\text{ N}}$$

Q.3. Determine the force P required to be applied on wedge A(W_A = 500 N) to lift the load B as shown in Fig.3. Take μ = 0.24 at all contact surface. (10)



Ans. Given, $\mu = 0.24$ at contact surface; $W_A = 500$ N; $W_B = 1000$ N

FBD for Block (A)



$\Sigma f_x = 0,$

$$P - \mu R_1 + \mu R_2\cos 18 + R_2 \sin 18 = 0$$

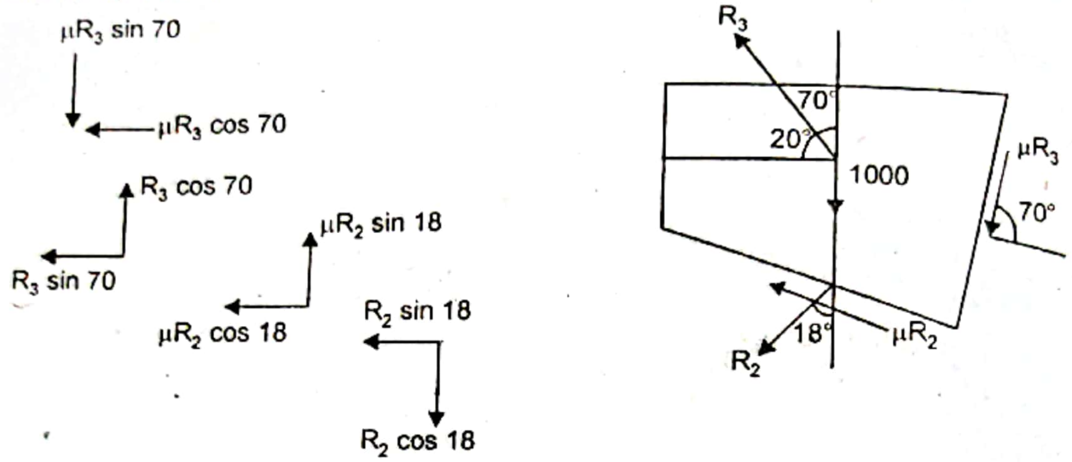
$$P - 0.24\,R_1 + 0.24\,R_2\cos 18 + R_2\sin 18 = 0$$

$P - 0.24\ R_1 + 0.53\ R_2 = 0$ ...(1)

$\Sigma f_y = 0$

$$R_1 + R_2 \cos 18 - \mu R_2 \sin 18 = 500$$
$$R_1 + 0.95\ R_2 - 0.074\ R_2 = 500$$
$$R_1 + 0.878\ R_2 = 500 \quad ...(2)$$

FBD of Block B



$\Sigma f_x = 0$

$\mu R_3 \cos 70 + R_3 \sin 70 + \mu R_2 \cos 18 + R_2 \cos 18 = 0$

$0.24\ R_3 \cos 70 + R_3 \sin 70 + 0.24\ R_2 \cos 18 + R_2 \sin 18 = 0$

$$0.08\ R_3 + 0.528\ R_2 = 0$$

$$R_3 = -\frac{0.528R_2}{0.08} = -6.6\ R_2 \quad ...(3)$$

$\Sigma f_y = 0$

$R_3 \cos 70 - \mu R_3 \sin 70 - R_2 \cos 18 + \mu R_2 + \sin 18 - 1000 = 0$

$0.34\ R_3 - 0.24\ R_3 \sin 70 - 0.95\ R_2 + 0.24\ R_2 \sin 18 = 1000$

$0.34\ R_3 - 0.225\ R_3 - 0.95\ R_2 + 0.06\ R_2 = 1000$

$$0.12R_3 - 0.89R_2 = 1000$$
$$R_3 - 7.41R_2 = 8333.3 \quad ...(4)$$

From eq (3)

$$-6.6R_2 - 7.41\ R_2 = 8333.3$$
$$-14.01R_2 = 8333.3$$
$$\boxed{R_2 = -594.8\ N}$$
$$\boxed{R_3 = 3925\ N}$$

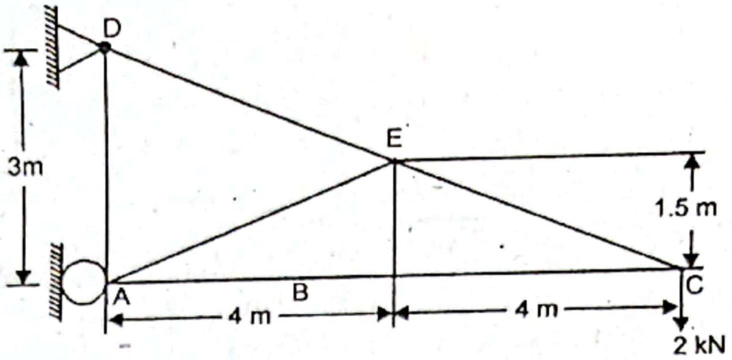
From eq (2)

$$R_1 + 0.878\ R_2 = 500$$
$$R_1 = 500 - 0.878\ (-594.8)$$
$$\boxed{R_1 = 1022\ N}$$

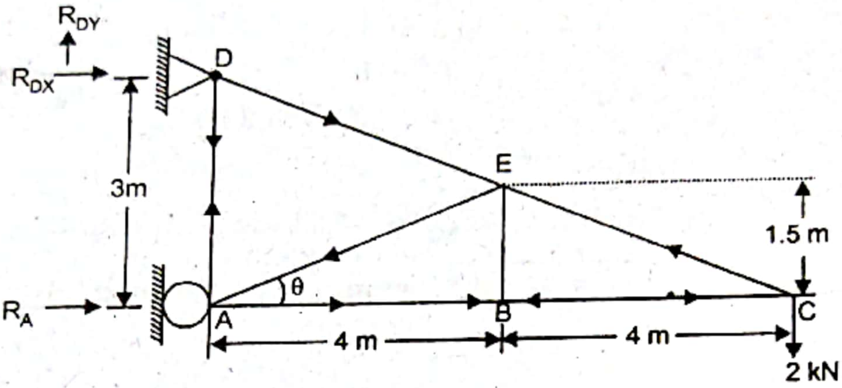
From eq. (1) $P - 0.24R_1 + 0.53R_2 = 0$

$$P - 0.24\ (1022) + 0.53\ (-594.8) = 0$$
$$P - 245.28 - 315.2 = 0$$
$$\boxed{P = 560\ N}$$

Q.4. Find the forces induced in all the members due to a load of 2 KN acting at joint C of the truss as shown in the Fig. (10)



Ans.



Δ AEB,

$$\tan\theta = \frac{1.5}{4}$$

$$\Rightarrow \boxed{\theta = 20.5°}$$

$$\Sigma f_x = 0$$

$$R_A + R_{DX} = 0 \quad \ldots(1)$$

$$\Sigma f_y = 0$$

$$R_{DY} = 2 \text{ KN} \quad \ldots(2)$$

$$M_D = 0 \quad \text{(Taking moment about D point)}$$

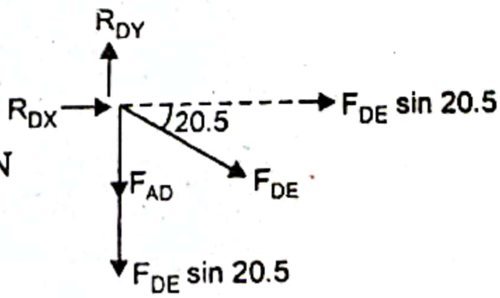
$$R_A \times 3 = 2 \times 8$$

$$R_A = 16/3 = 5.33 \text{ KN}$$

$$\Rightarrow \boxed{R_A = 5.33 \text{ KN}}$$

$$\Rightarrow \boxed{R_{DX} = -5.33 \text{ KN}}$$

Joint D



$$\Sigma f_x = 0$$

$$R_{DX} + F_{DE}\cos 20.5° = 0$$

$$-5.33 + F_{DE}\cos 20.5° = 5.69 \text{ KN}$$

$$\boxed{F_{DE} = 5.69 \text{ KN}}$$

$$\Sigma f_y = 0$$

$$R_{DY} = F_{AD} + F_{DE}\sin 20.5°$$

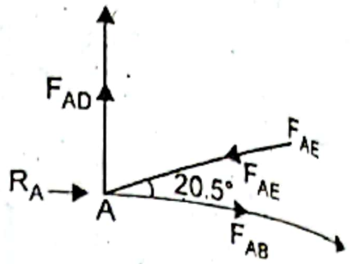
$$2 = F_{AD} + 5.69 \sin 20.5$$

$$2 = F_{AD} + 2$$

$$\boxed{F_{AD} = 0}$$

Joint A

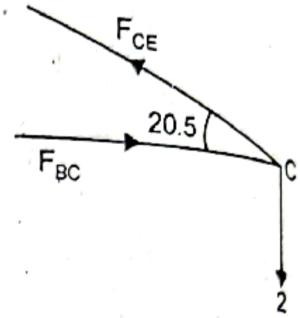
$$\Sigma f_x = 0$$
$$F_{AE}\cos 20.5 = R_A + F_{AB}$$
$$\Sigma f_y = 0$$
$$F_{AD} = F_{AE}\sin 20.5$$
$$0 = F_{AE}\sin 20.5$$
$$\boxed{F_{AE} = 0}$$
$$R_A = -F_{AB}$$



$\therefore$
$$\boxed{F_{AB} = -5.33\ \text{KN}}$$

Joint C

$$\Sigma f_x = 0,$$
$$-F_{CE}\cos 20.5 = F_{BC}$$
$$\Sigma f_y = 0,$$
$$2 = F_{CE}\sin 20.5$$
$$\boxed{F_{CE} = 5.71\ \text{KN}}$$



Now,
$$F_{BC} = -5.71\cos 20.5$$
$$\boxed{F_{BC} = -5.34\ \text{KN}}\ \text{(wrong chosen assumption)}$$

Joint B

$$\Sigma f_y = 0,$$
$$F_{BE} = 0$$
$$\Sigma f_x = 0$$
$$F_{BC} = F_{AB}$$
$$= -5.34\ \text{KN.}$$