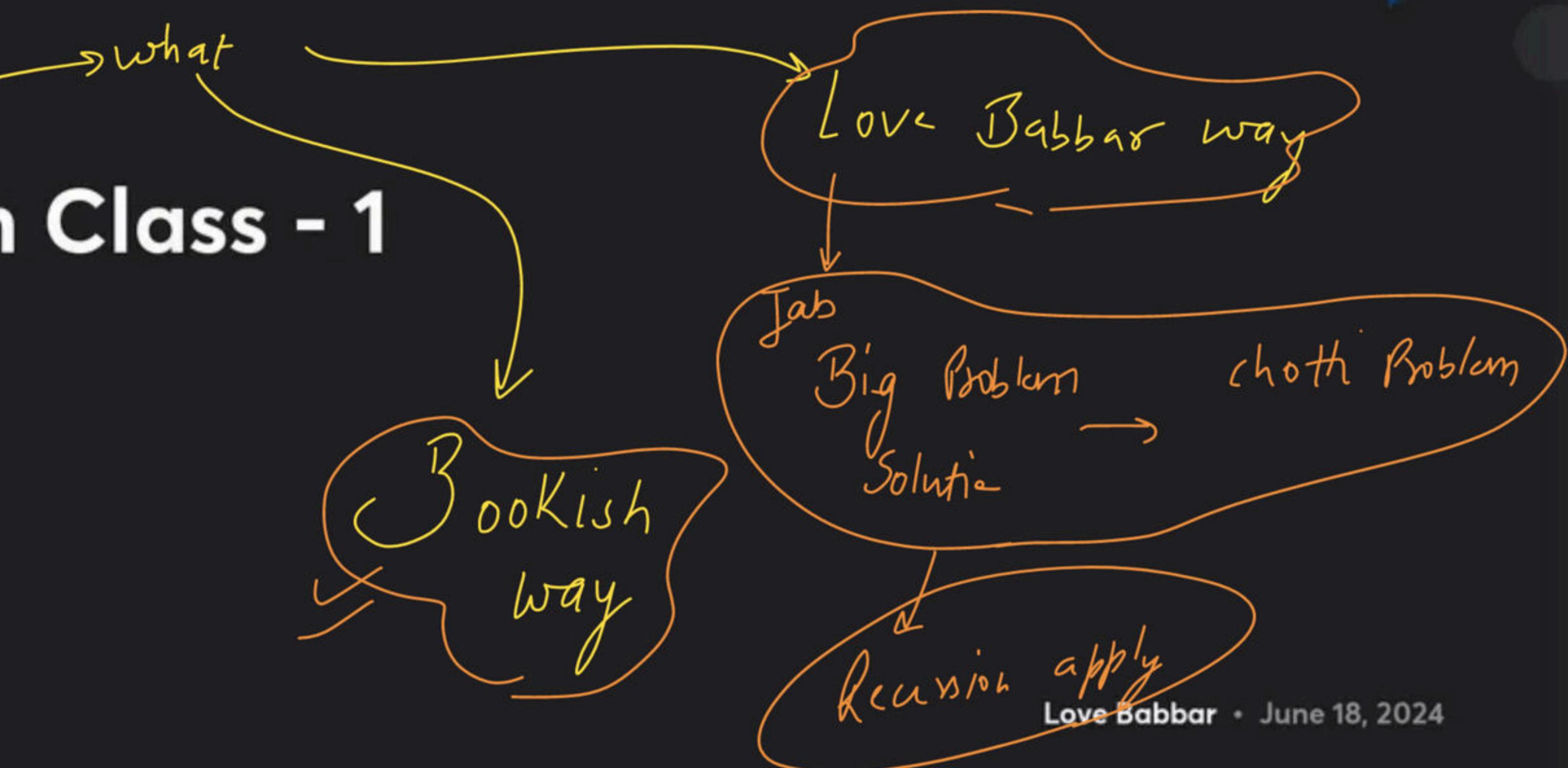
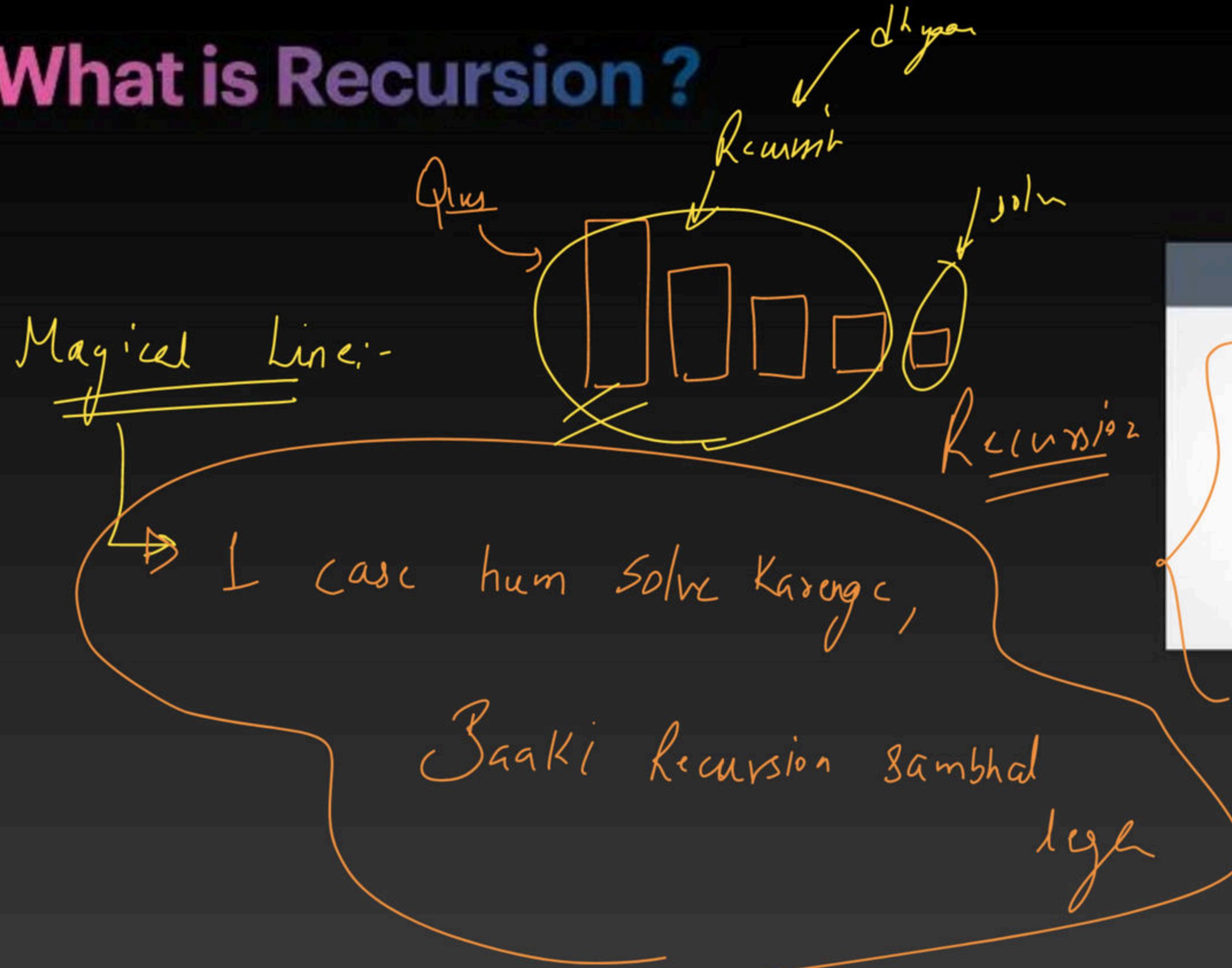


# Recursion Class - 1

Special class



# What is Recursion ?



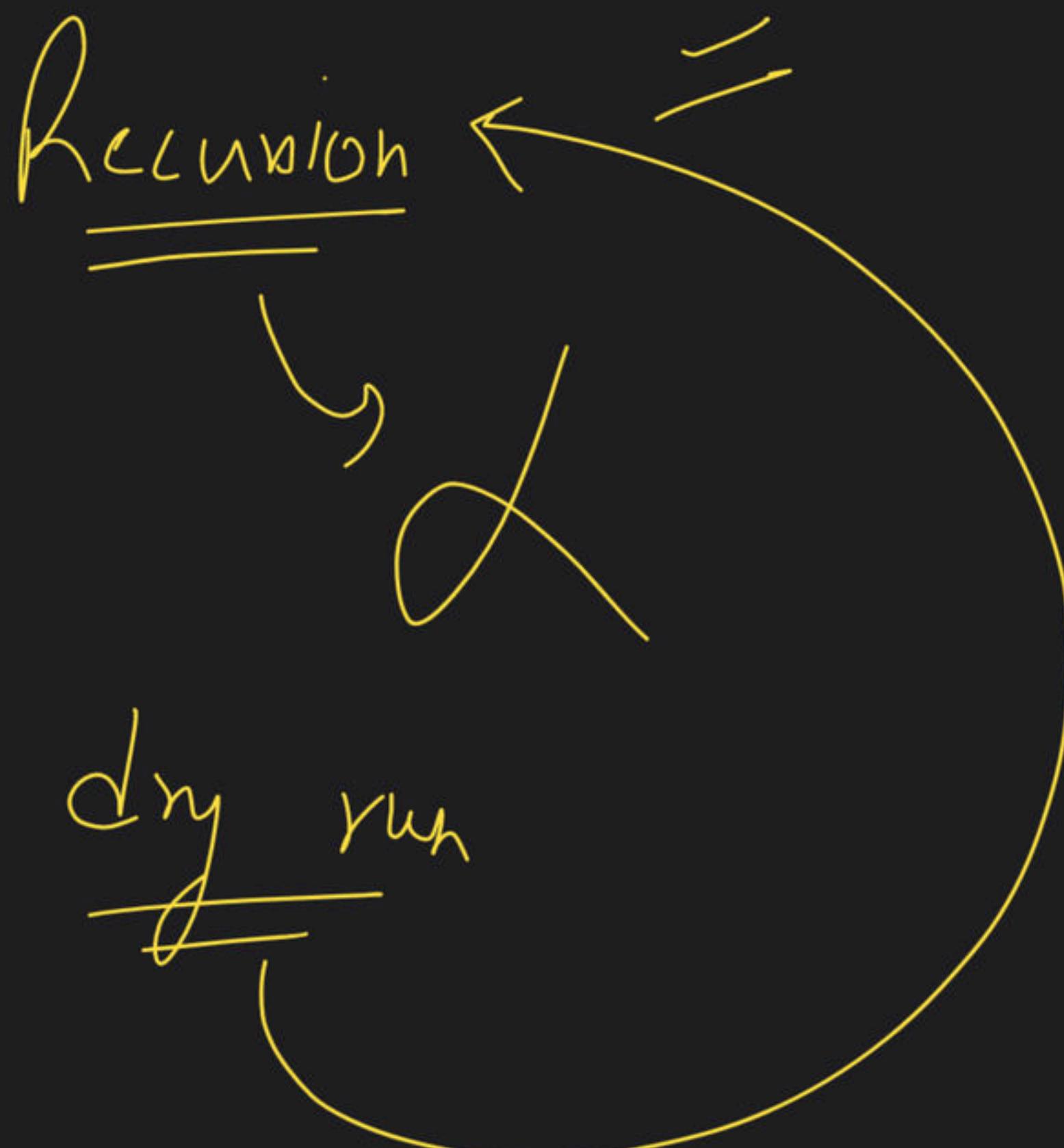
TRUST

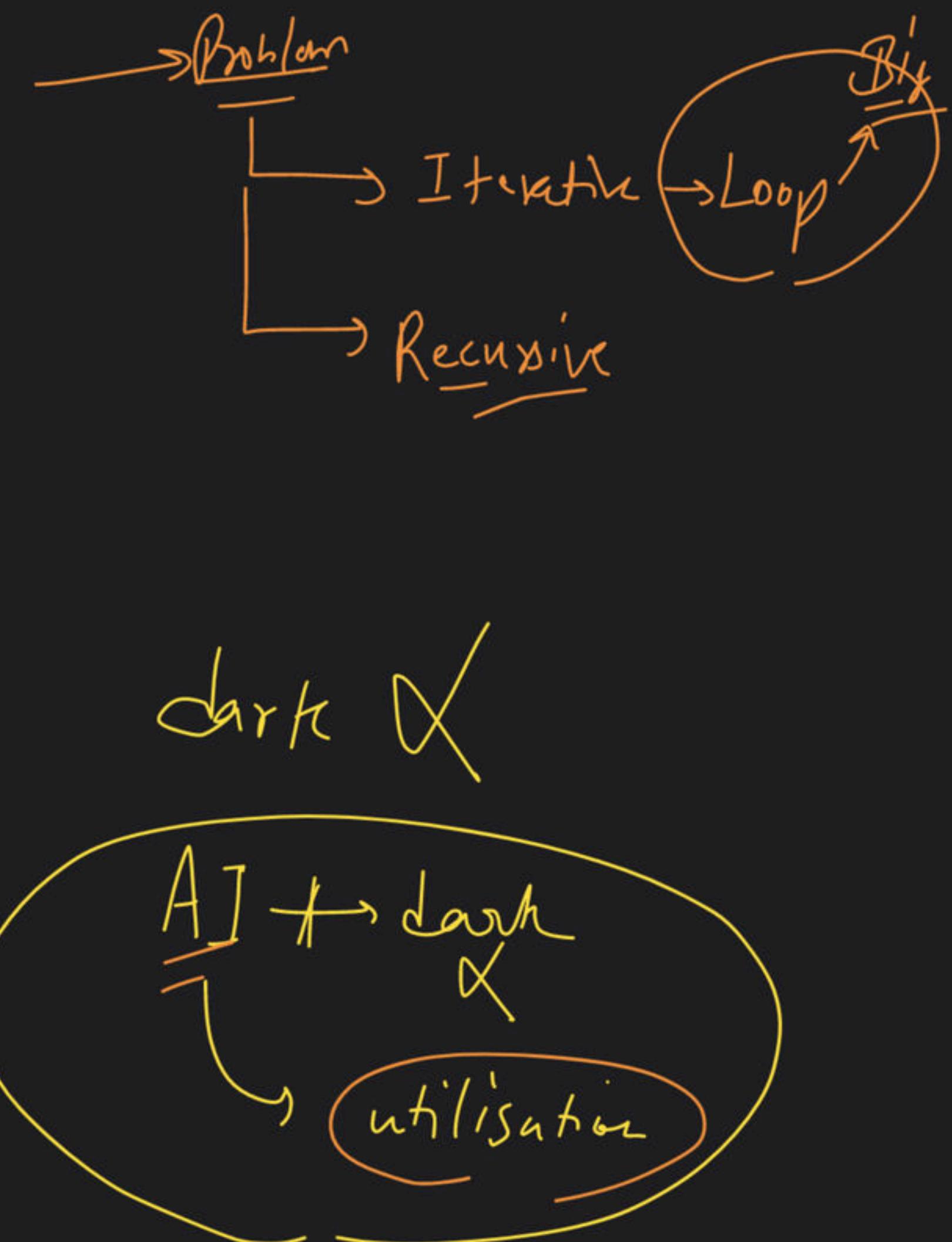
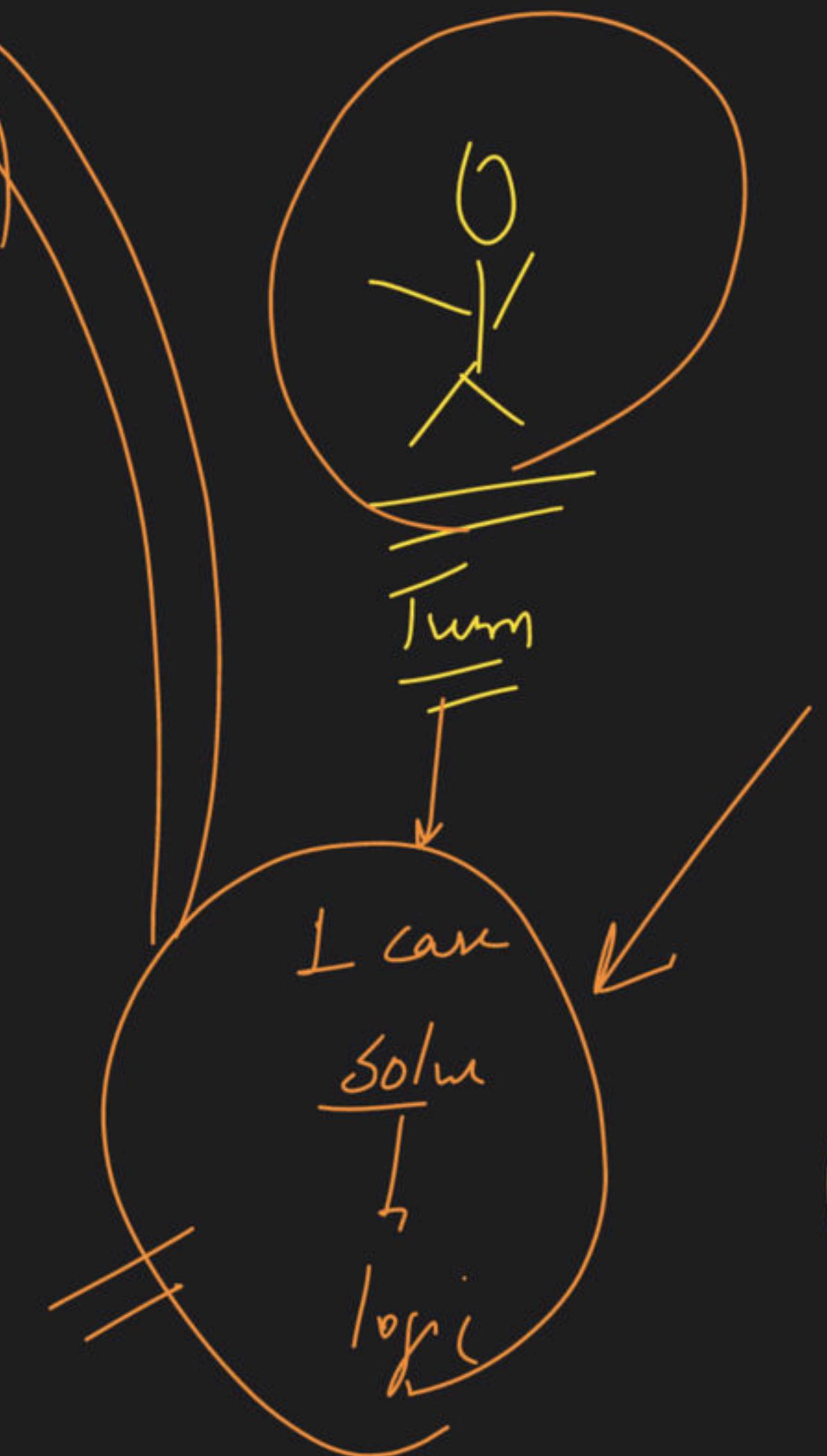
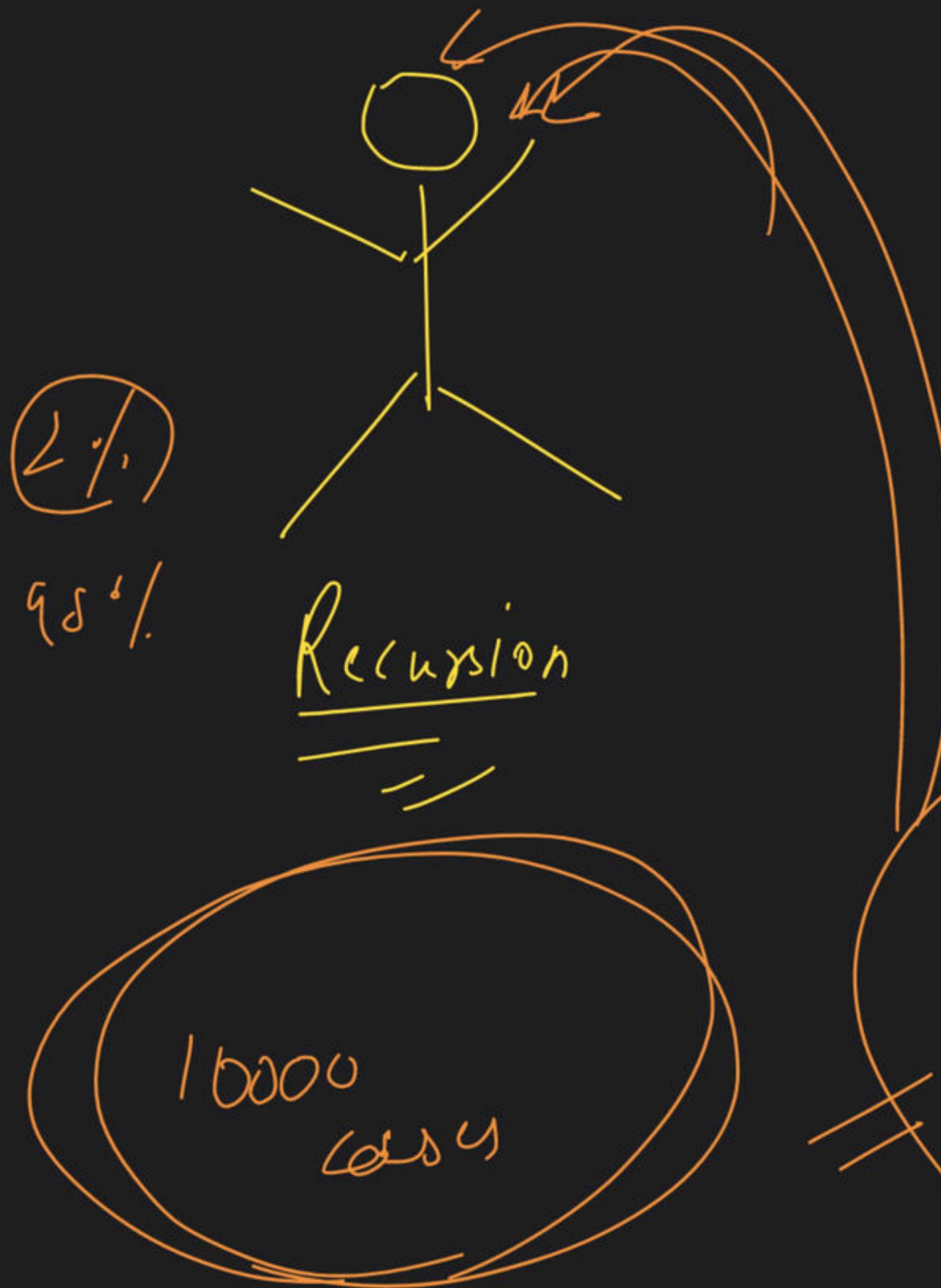
How does recursion work?

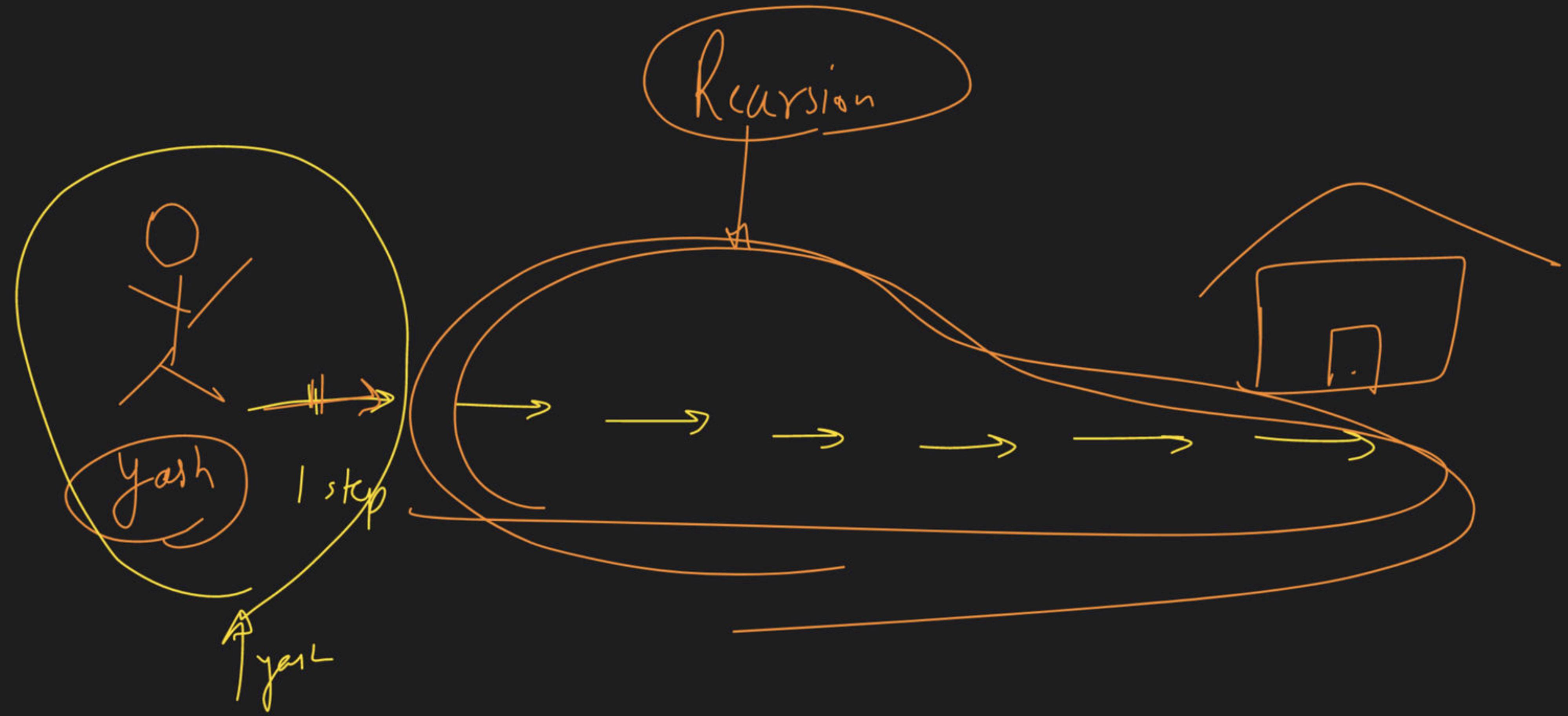
```
void recurse() {  
    .....  
    recurse();  
    .....  
}  
  
int main()  
{  
    .....  
    recurse();  
    .....  
}
```

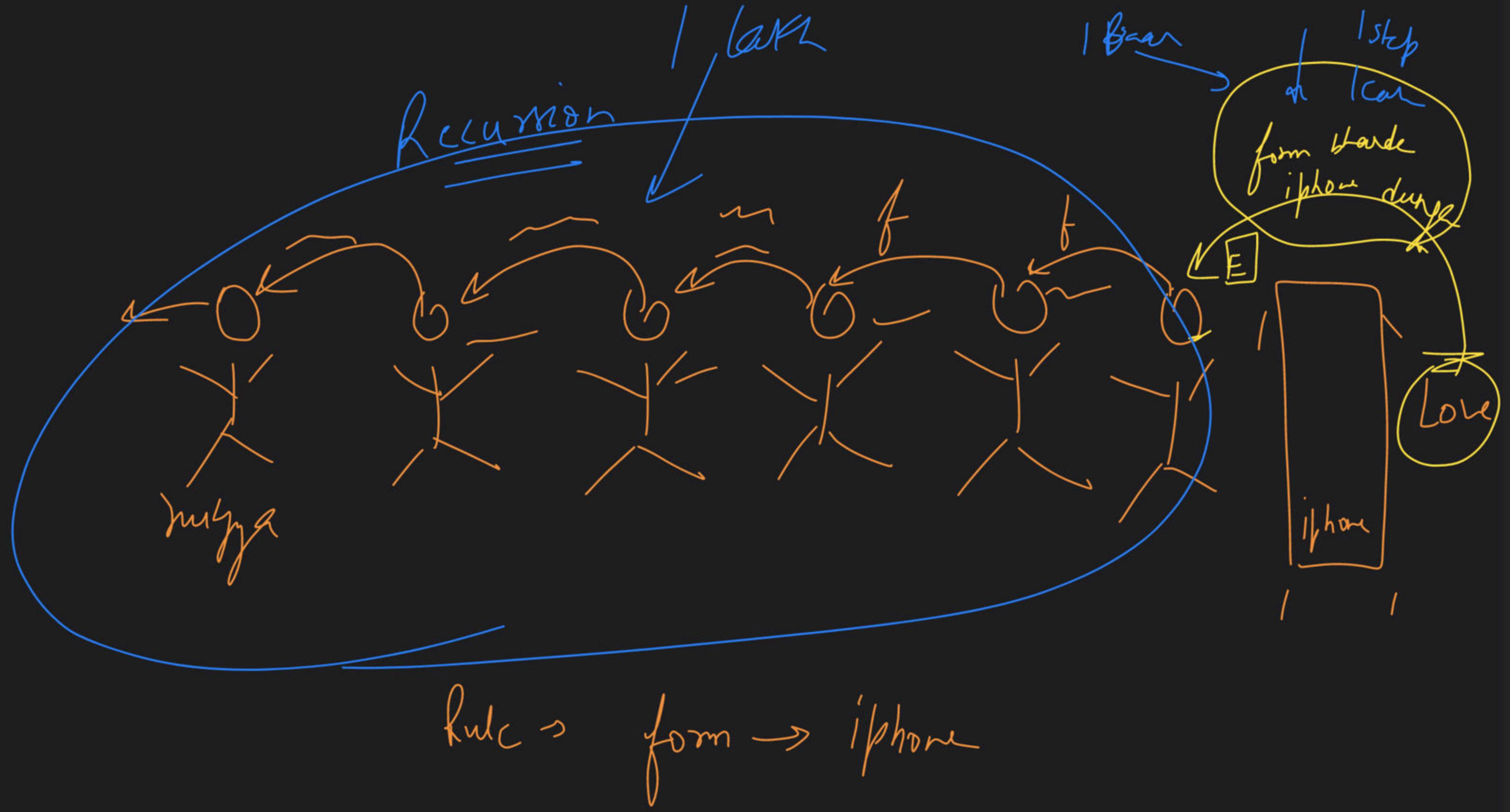
A callout from the handwritten text 'Recursion 2' points to the recursive call 'recurse();' in the code.

what ?  
when to apply?  
How to apply?



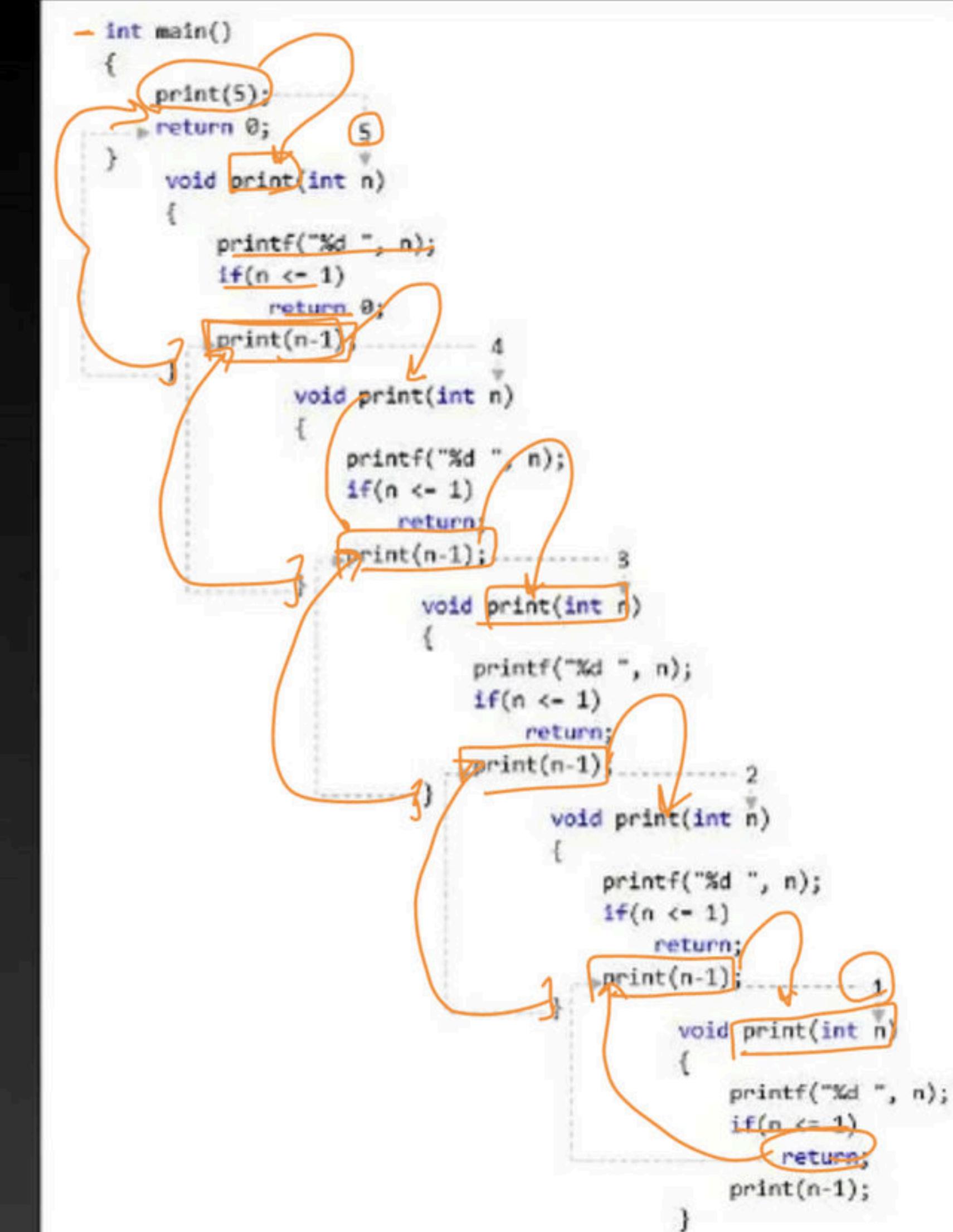






# What is Recursion?

Big  
→ ( both )



factorial

5(1)

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

Big  
7!

= 7 \* 6!  
-  
Small

$$n=7$$

$$\text{factorial}(7) = 7 * \text{factorial}(6)$$

$$\text{factorial}(n) = n * \text{factorial}(n-1)$$

Power

( $2^5$ )

$$2^3 = 2 \times 2 \times 2$$

$$2^7 = 2 \times \overbrace{2 \times 2 \times 2 \times 2 \times 2 \times 2}^{2^6}$$

$$\text{Power}(7) = 2 \times \text{Power}(6)$$

Recursive / Recurrence relation

$$\text{factorial}(7) = 7 * \text{factorial}(6)$$

$$\text{factorial}(n) = n * \text{factorial}(n-1)$$

Big O  
maine  
Kya  
chahi  
Recursion

$$\text{Power}(7) = 2 * \text{Power}(6)$$

chock

$$= 2 * \underline{\underline{64}}$$

Record

$$2^6 \rightarrow$$

2

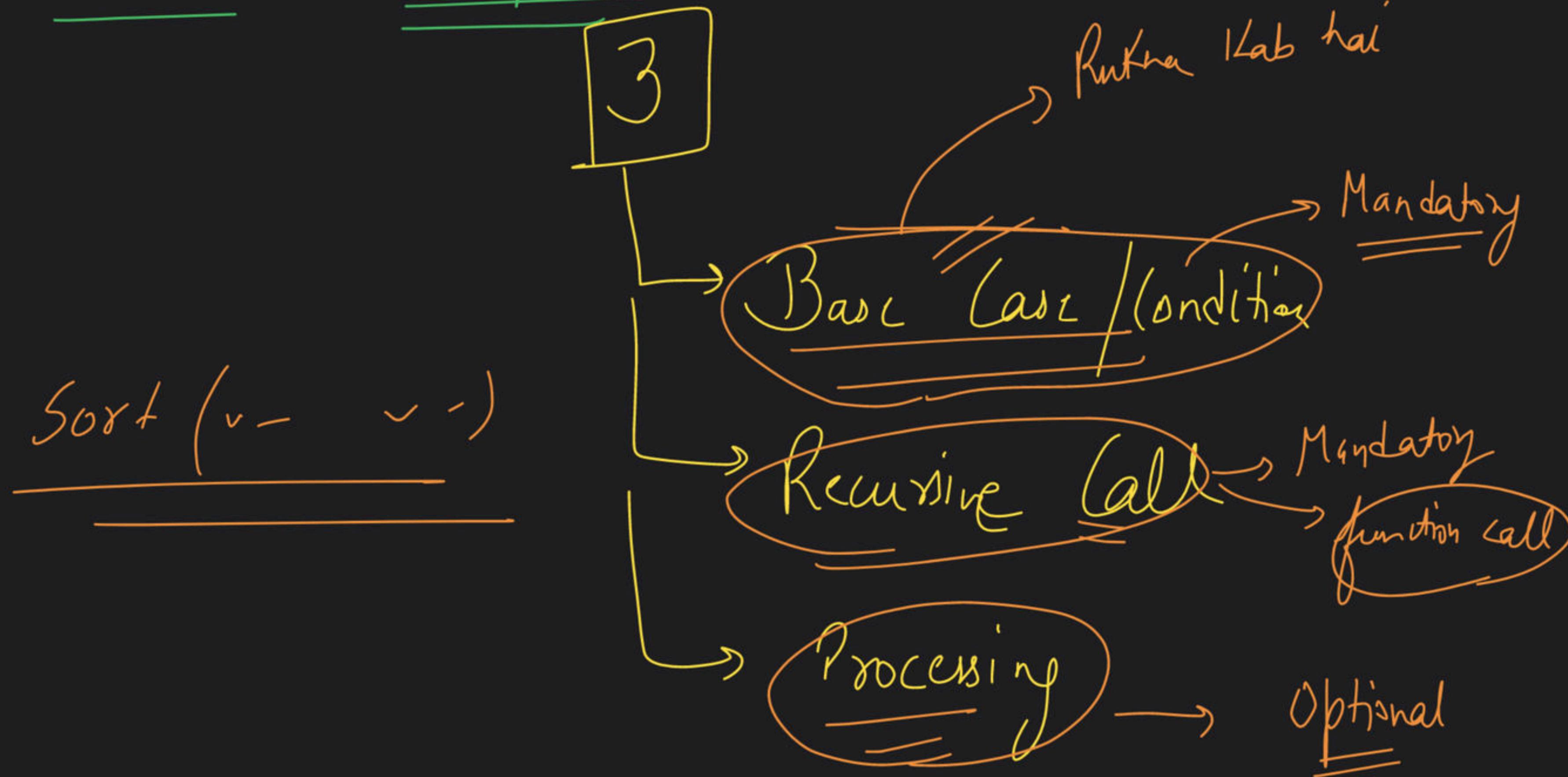
128

10.1

90 ✓

## Recursion

### Components :-



factorial  $\rightarrow n! = n * (n-1) * (n-2) * (n-3) * \dots * 3 * 2 * 1$

$n!$   $\downarrow$   $\downarrow$   $\downarrow$

$n! = n * (n-1)!$

$(n-1)!$

$$\text{fact}(n) = n * \text{fact}(n-1)$$

$\text{fact}(n)$   $\downarrow$   $\downarrow$   $\text{Small}$

$\rightarrow$  Relation

Let  $n = 5$

main  
 $\text{fact}(5) = 5 \times \text{fact}(4)$   
 $\text{fact}(4) = 4 \times \text{fact}(3)$   
 $\text{fact}(3) = 3 \times \text{fact}(2)$   
 $\text{fact}(2) = 2 \times \text{fact}(1)$   
 $\text{fact}(1) = 1 \times \text{fact}(0)$   
 $\text{fact}(0) = 1$

$$\text{fact}(n) = n \star \text{fact}(n-1)$$

$$\text{fact}(4) = 4 \times \frac{6}{2} \text{fact}(3)$$

$$\text{fact}(3) = 3 \times \frac{2}{6} \text{fact}(2)$$

$$\text{fact}(2) = 2 \times \frac{1}{2} \text{fact}(1)$$

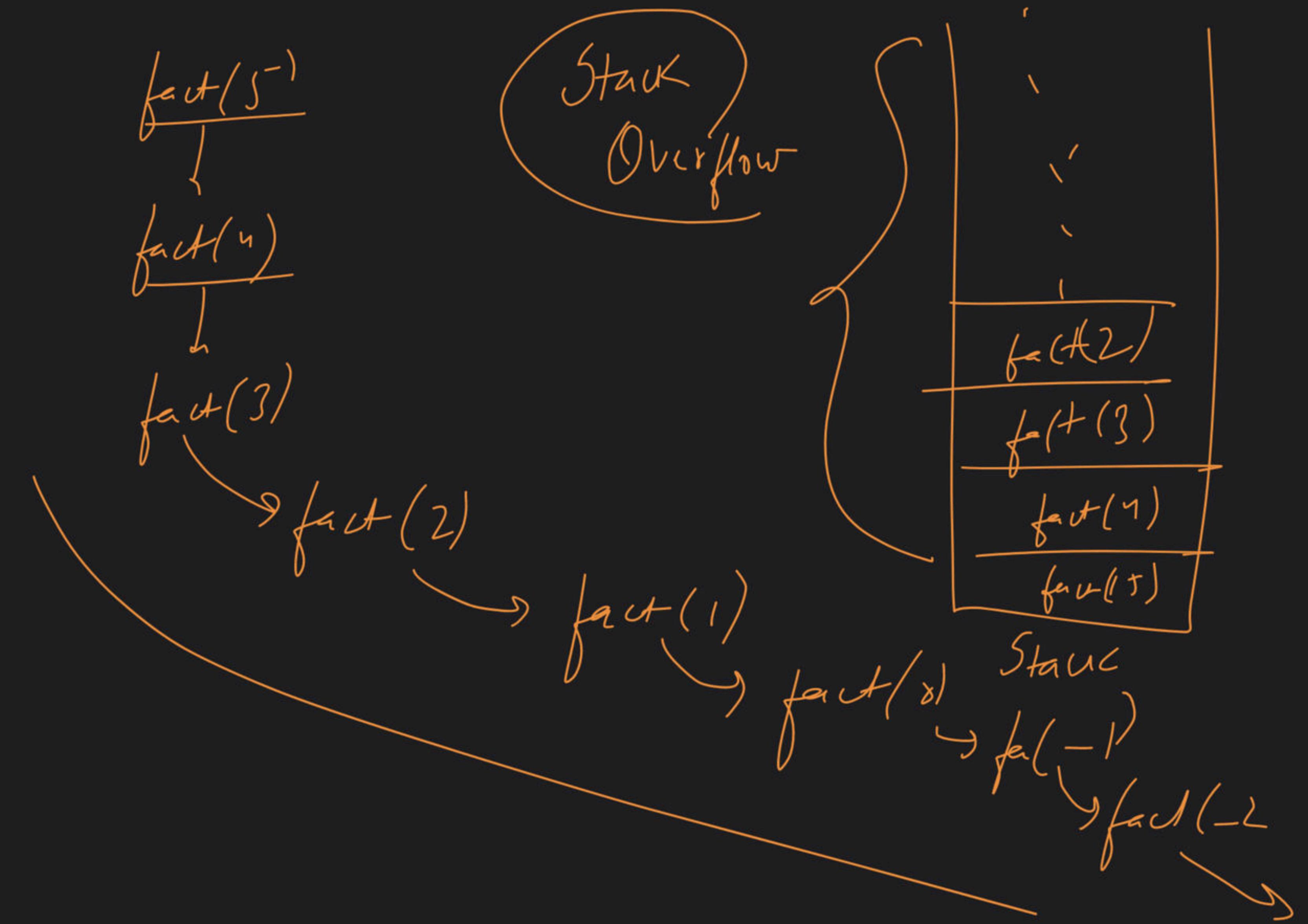
$$\text{fact}(1) = 1 \times \frac{1}{1} \text{fact}(0)$$

Jagah

Ruk Jaw -> Basulan

$$0! = 1$$
$$\text{fact}(0) = 1$$

Rechnung



```
int getFactorial(int n) {  
    if(n == 0 || n == 1) return 1;  
    int recursionAns = getFactorial(n - 1);  
    int finalAns = n * recursionAns;  
    return finalAns;
```

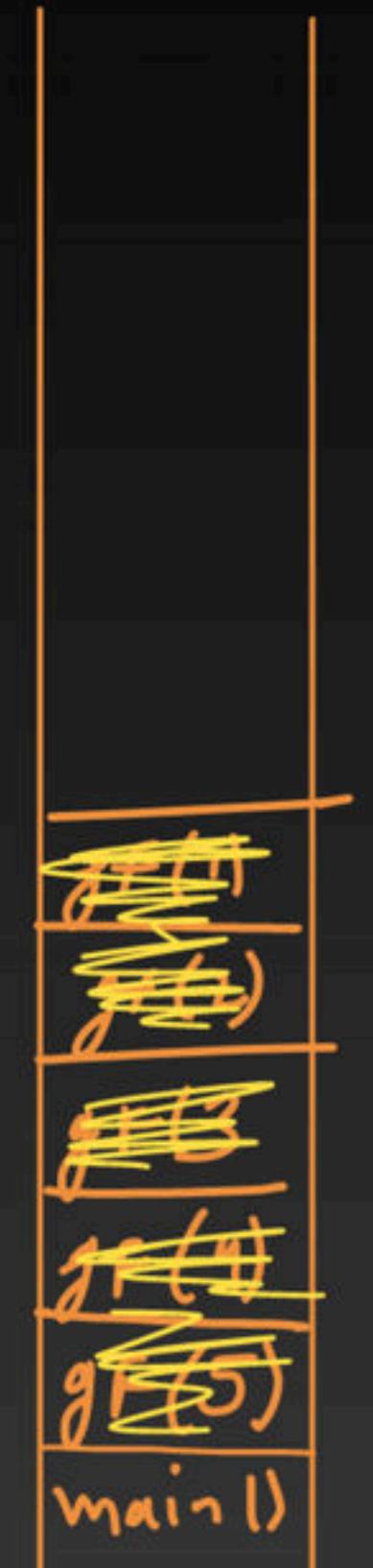
```
int getFactorial(int n) {  
    if(n == 0 || n == 1) ✗  
        return 1; 6  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns; 4 6
```

```
int getFactorial(int n) {  
    → if(n == 0 || n == 1) ↗  
        return 1; ↗  
    → int recursionAns = getFactorial(n-1); ↗  
    int finalAns = n * recursionAns; ↗  
    return finalAns; ↗
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;
```

```
int getFactorial(int n) {  
    → if(n == 0 || n == 1)  
        return 1;  
  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1) {  
        return 1; }  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns; }
```



## Call Stack

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    }
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    }
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

3

6

2

~~n=3~~

31

1

0

1

2

R.C

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

```
int getFactorial(int n) {
```

```
    if(n == 0 || n == 1)
```

```
        return 1;
```

```
    int recursionAns = getFactorial(n-1);
```

```
    int finalAns = n * recursionAns;
```

```
    return finalAns;
```

```
}
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

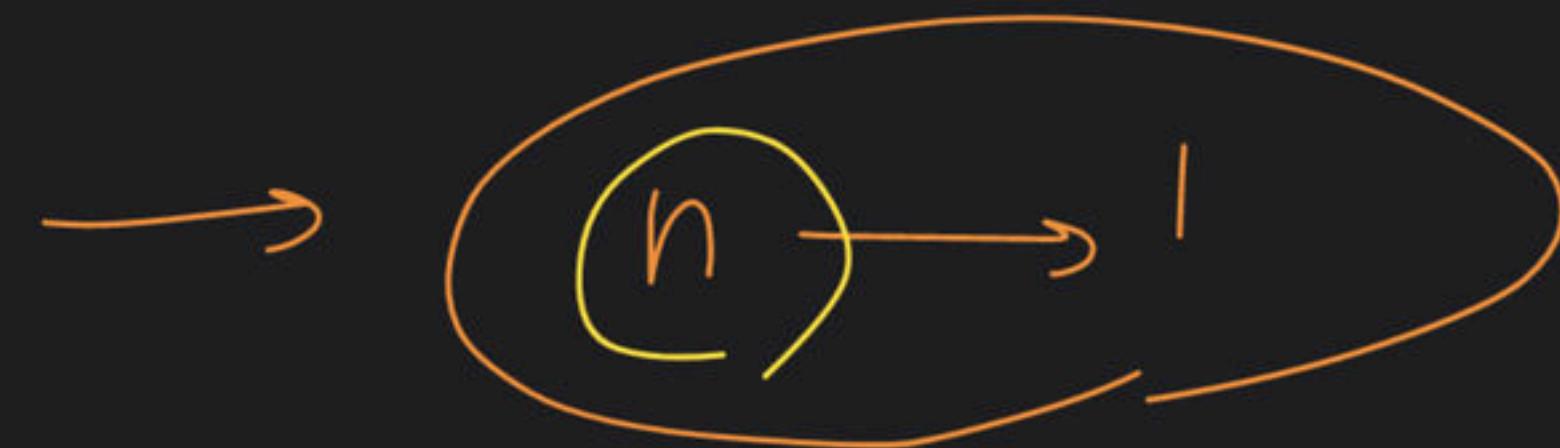
```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

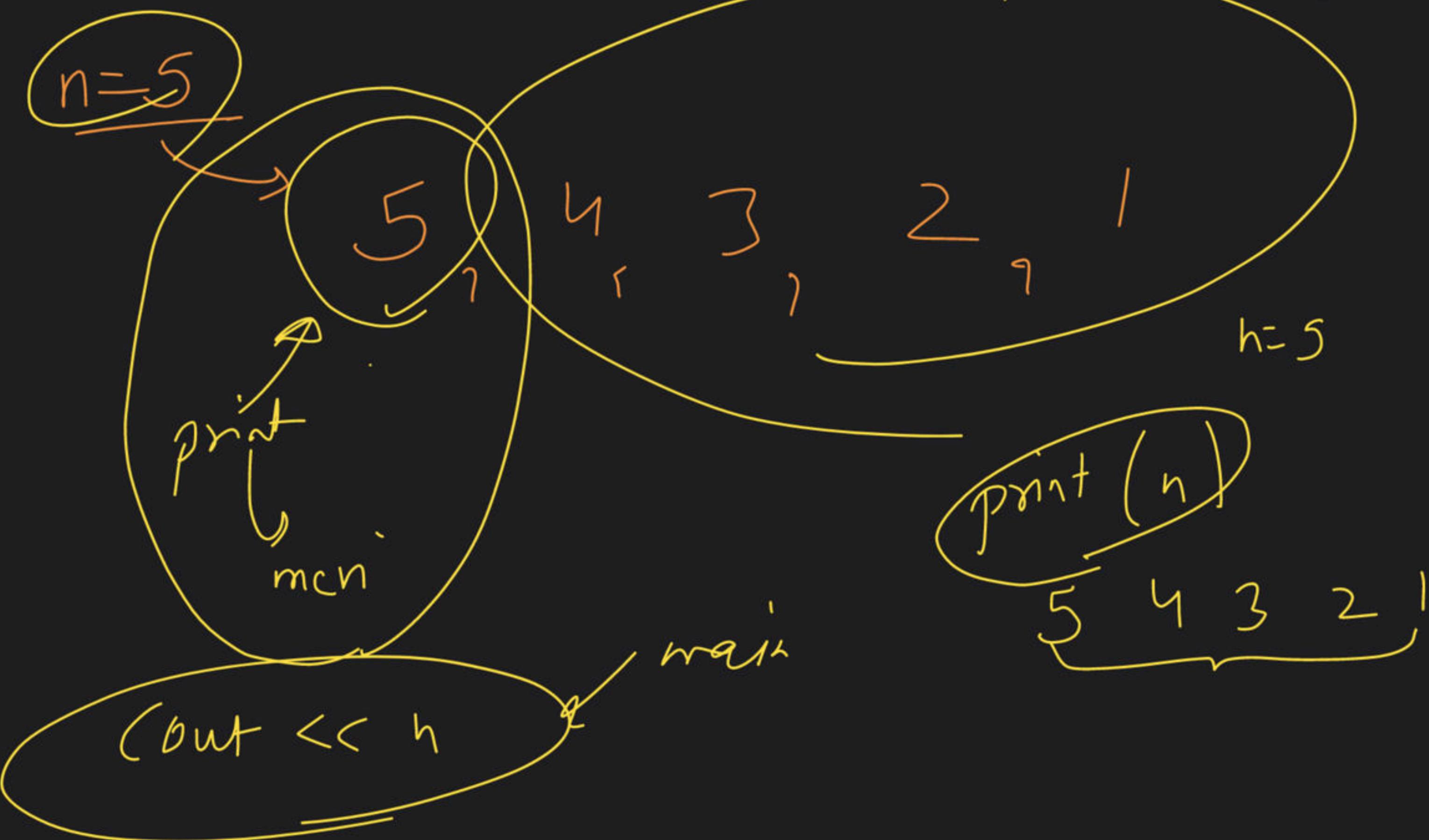
```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

```
int getFactorial(int n) {  
    if(n == 0 || n == 1)  
        return 1;  
    int recursionAns = getFactorial(n-1);  
    int finalAns = n * recursionAns;  
    return finalAns;  
}
```

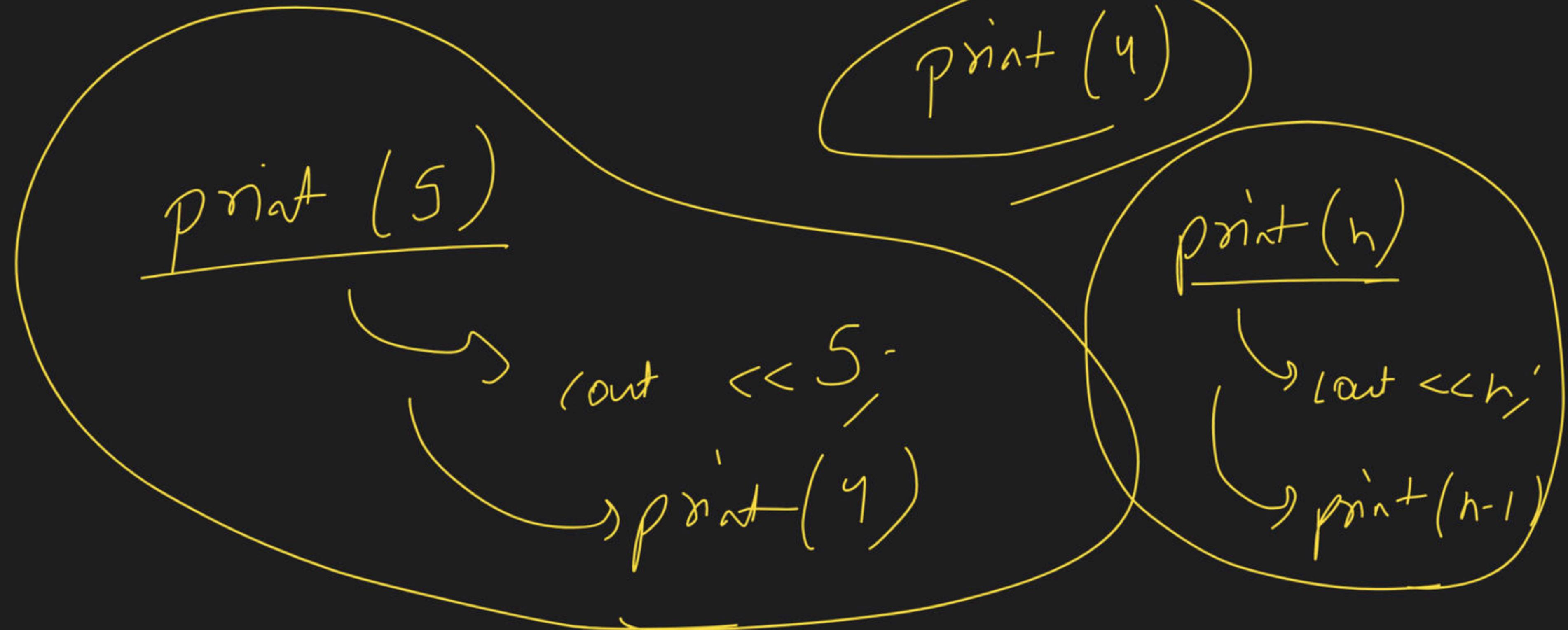
Counting



Recursion

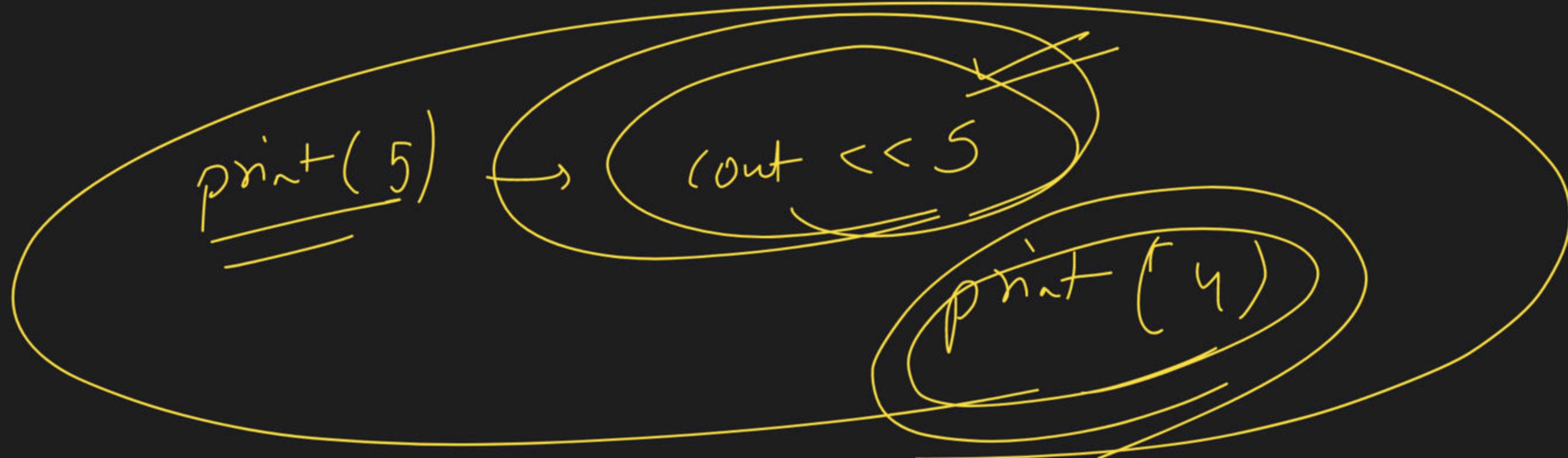


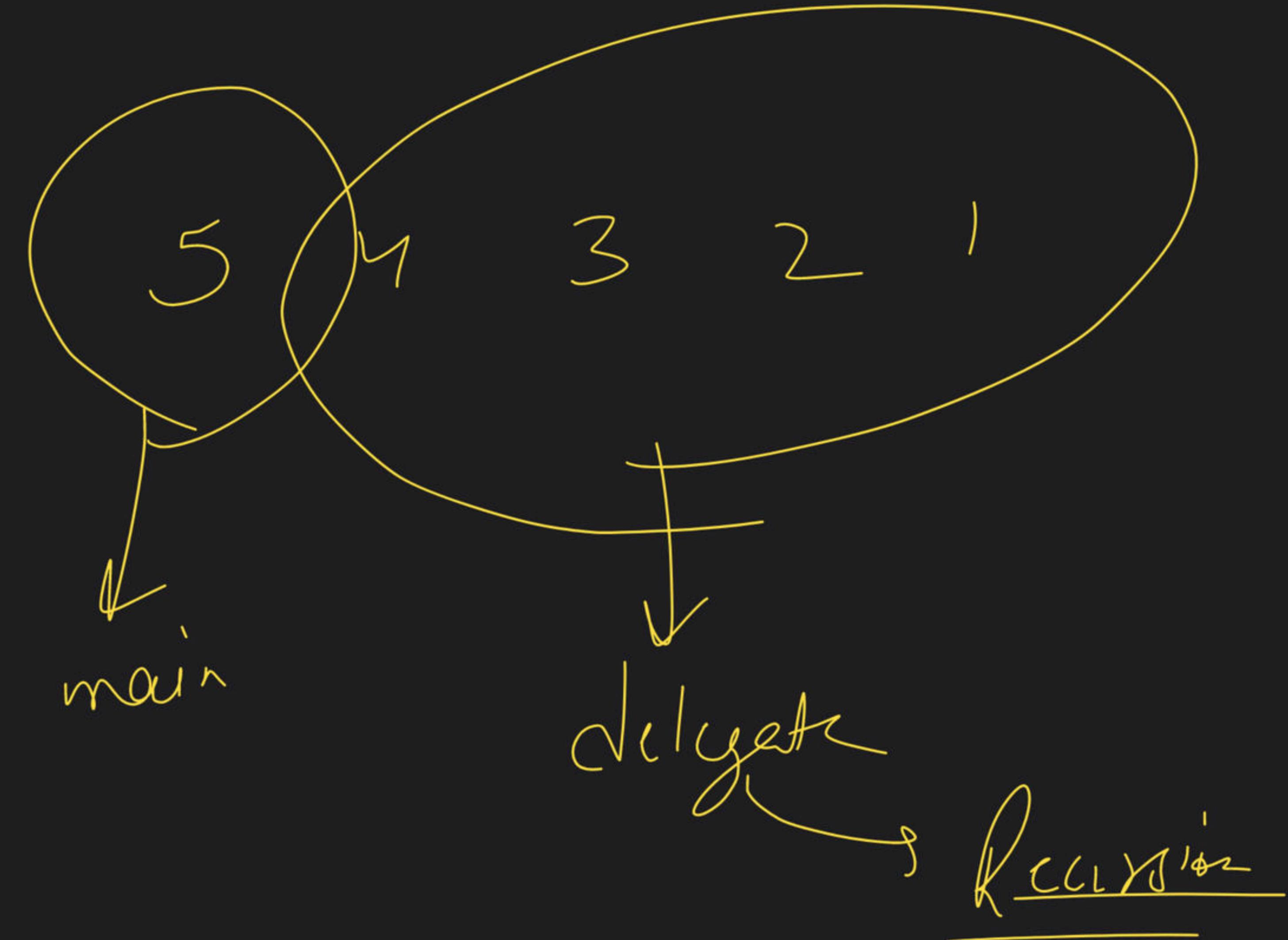
`print(5)` → 5 4 3 2 1

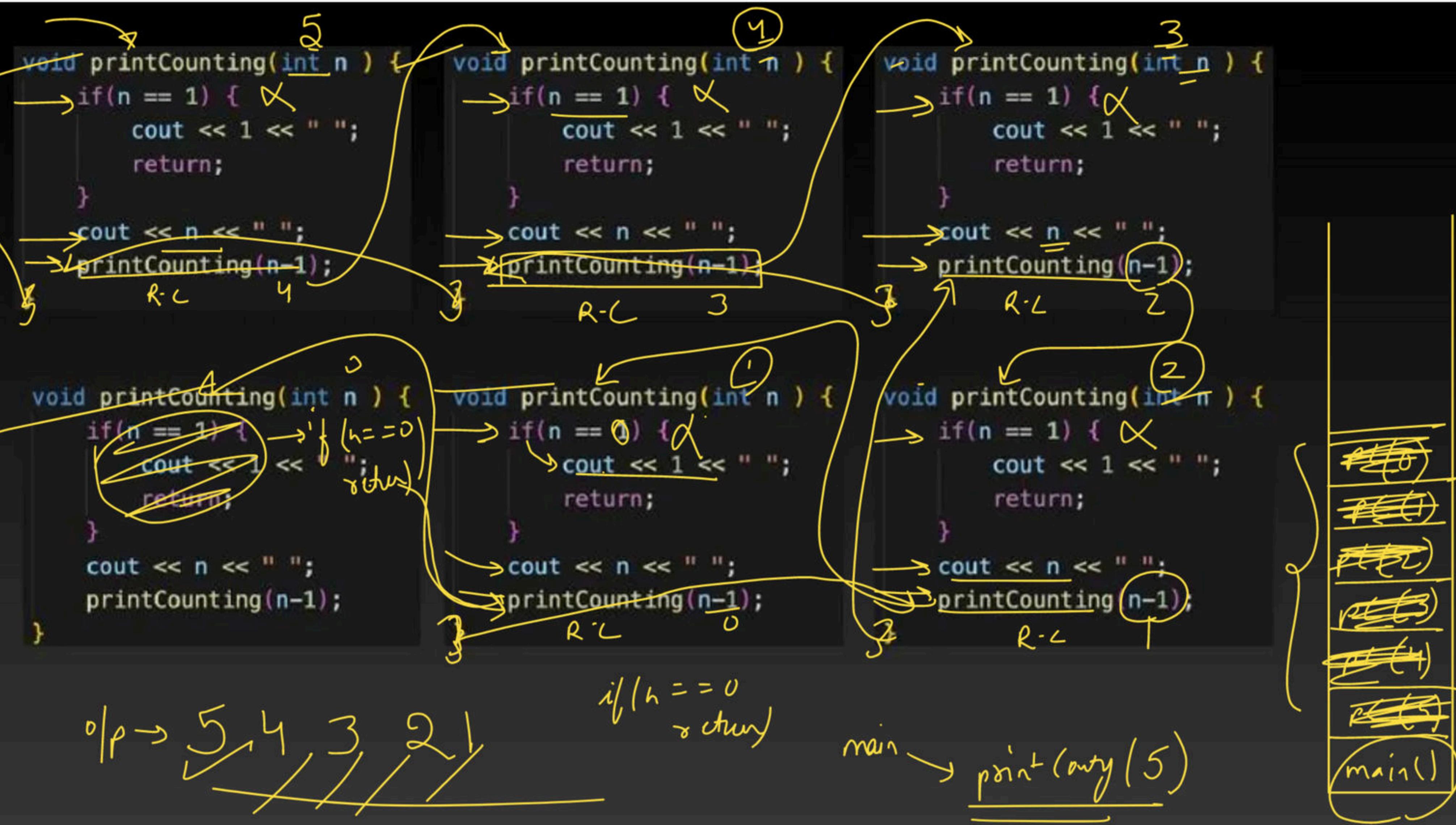


`print(5) → 5 4 3 2 1`

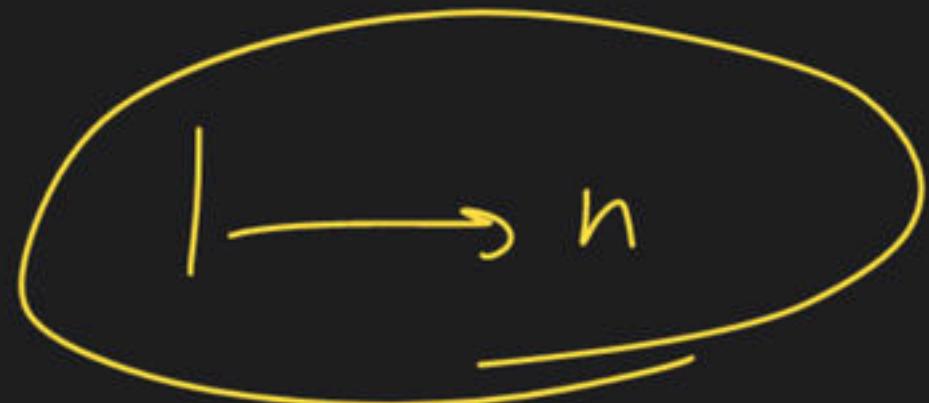
`print(n) → 4 3 2 ↴, → Rec`







Counting  $\rightarrow$



$$\underline{\underline{n=5}}$$

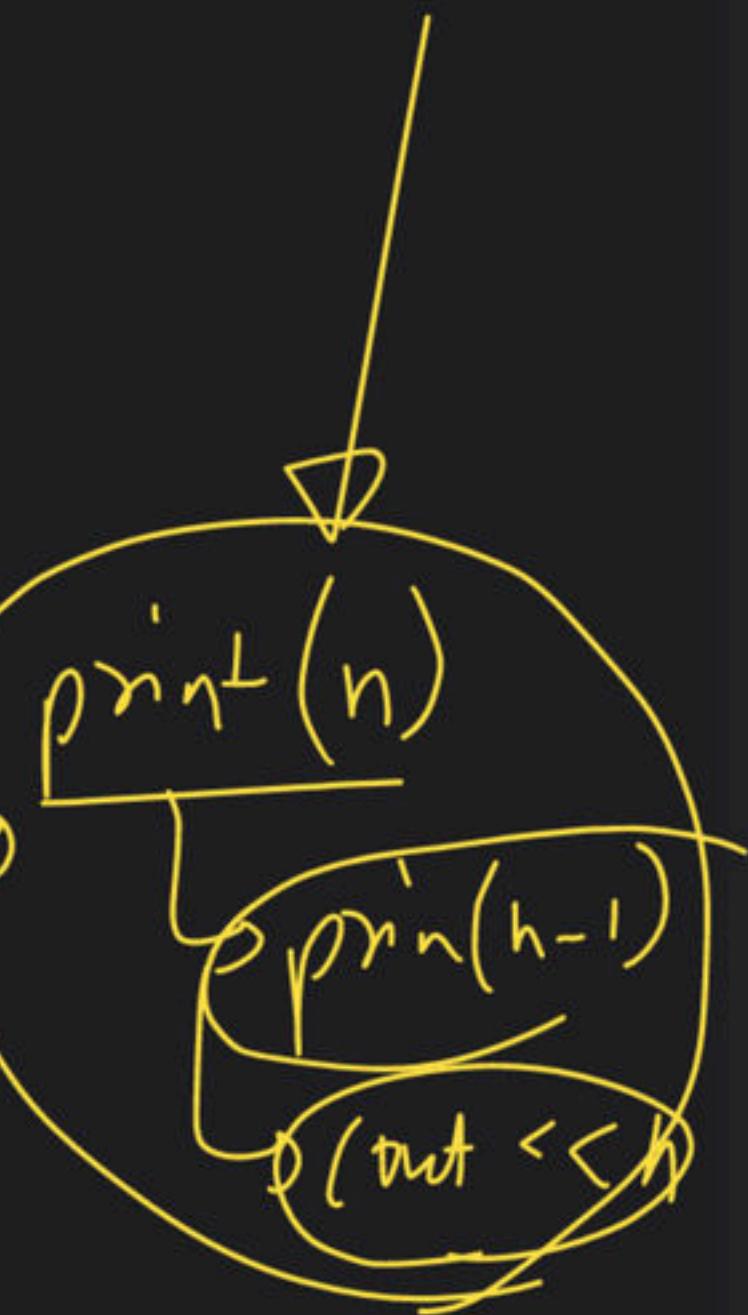
1 2 3 4 5

$\text{print}(5)$  =



$\text{print}(4)$

$\text{print}(5)$   
 $\text{print}(4)$   
 $\text{cout} \ll 5$



$\text{print}(n)$

$\text{print}(n-1)$

$\text{cout} \ll 1$

```
void printCounting(int n ) {    void printCounting(int n ) {    void printCounting(int n ) {  
    if(n == 1) {          if(n == 1) {          if(n == 1) {  
        cout << 1 << " ";      cout << 1 << " ";      cout << 1 << " ";  
        return;            return;            return;  
    }                      }                      }  
    cout << n << " ";    cout << n << " ";    cout << n << " ";  
    printCounting(n-1);  printCounting(n-1);  printCounting(n-1);  
}  
  
void printCounting(int n ) {    void printCounting(int n ) {    void printCounting(int n ) {  
    if(n == 1) {          if(n == 1) {          if(n == 1) {  
        cout << 1 << " ";      cout << 1 << " ";      cout << 1 << " ";  
        return;            return;            return;  
    }                      }                      }  
    cout << n << " ";    cout << n << " ";    cout << n << " ";  
    printCounting(n-1);  printCounting(n-1);  printCounting(n-1);  
}  
}
```

```
void printCounting(int n ) {    void printCounting(int n ) {    void printCounting(int n ) {  
    if(n == 1) {          if(n == 1) {          if(n == 1) {  
        cout << 1 << " ";      cout << 1 << " ";      cout << 1 << " ";  
        return;            return;            return;  
    }                      }                      }  
    cout << n << " ";    cout << n << " ";    cout << n << " ";  
    printCounting(n-1);  printCounting(n-1);  printCounting(n-1);  
}  
  
void printCounting(int n ) {    void printCounting(int n ) {    void printCounting(int n ) {  
    if(n == 1) {          if(n == 1) {          if(n == 1) {  
        cout << 1 << " ";      cout << 1 << " ";      cout << 1 << " ";  
        return;            return;            return;  
    }                      }                      }  
    cout << n << " ";    cout << n << " ";    cout << n << " ";  
    printCounting(n-1);  printCounting(n-1);  printCounting(n-1);  
}  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
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    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

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    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

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    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

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        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
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        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
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    if(n == 1) {  
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        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n ) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

```
void printCounting(int n) {  
    if(n == 1) {  
        cout << 1 << " ";  
        return;  
    }  
    printCounting(n-1);  
    cout << n << " ";  
}
```

if (n == 0)  
 Return

O/P  
1 2 3 4

main

print(out(y))

PS  
PS  
PS  
PS  
PS  
main()

$$2^0$$

$$n=5$$

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

$$\text{if } (b == 0)$$

$$\text{if } (k == 1)$$

$$2^n \rightarrow 2^1 \rightarrow 2$$

$$2^5 = 2 \times 2^4$$
$$\text{pow}(5) = 2 \times \text{pow}(4)$$

$$\text{pow}(n) = 2 \times \text{pow}(n-1)$$

$$\text{pow}(n) = 2 * \text{pow}(n-1)$$

$$\text{pow}(5)$$

$$\text{pow}(4)$$

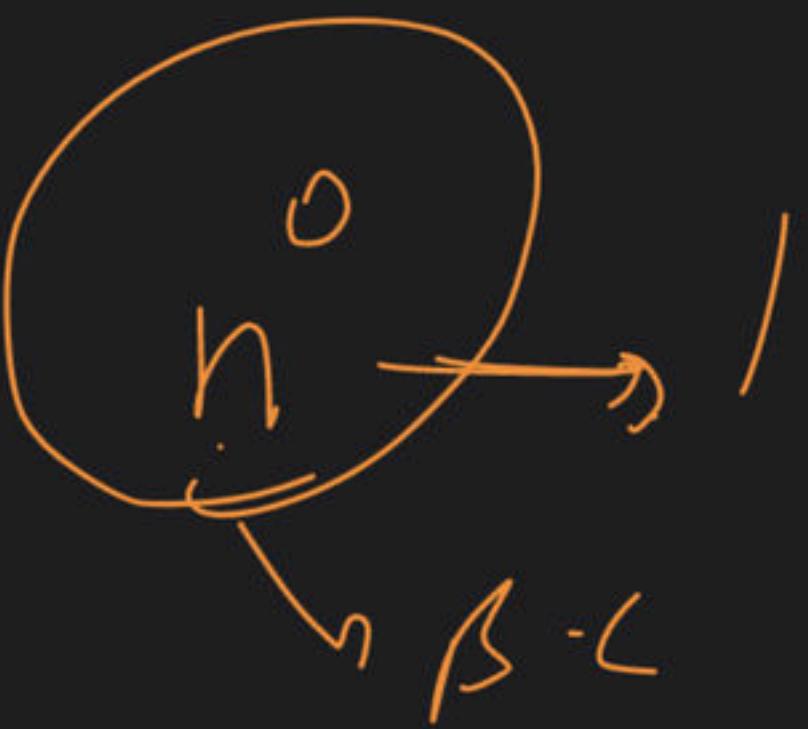
$$\text{pow}(3)$$

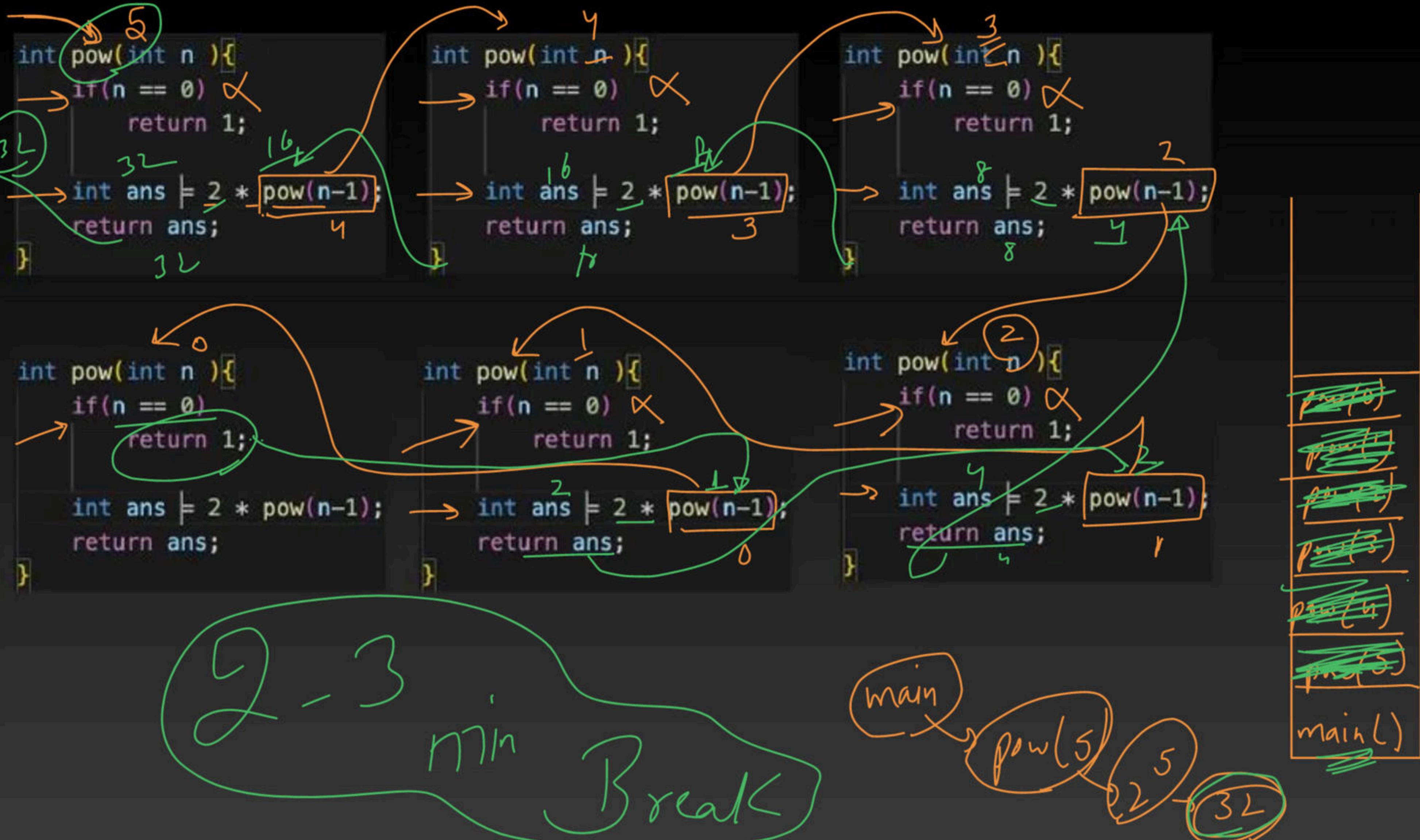
$$\text{pow}(2)$$

$$\text{pow}(1)$$

$$\text{pow}(0)$$

$$\text{pow}(-1)$$





```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

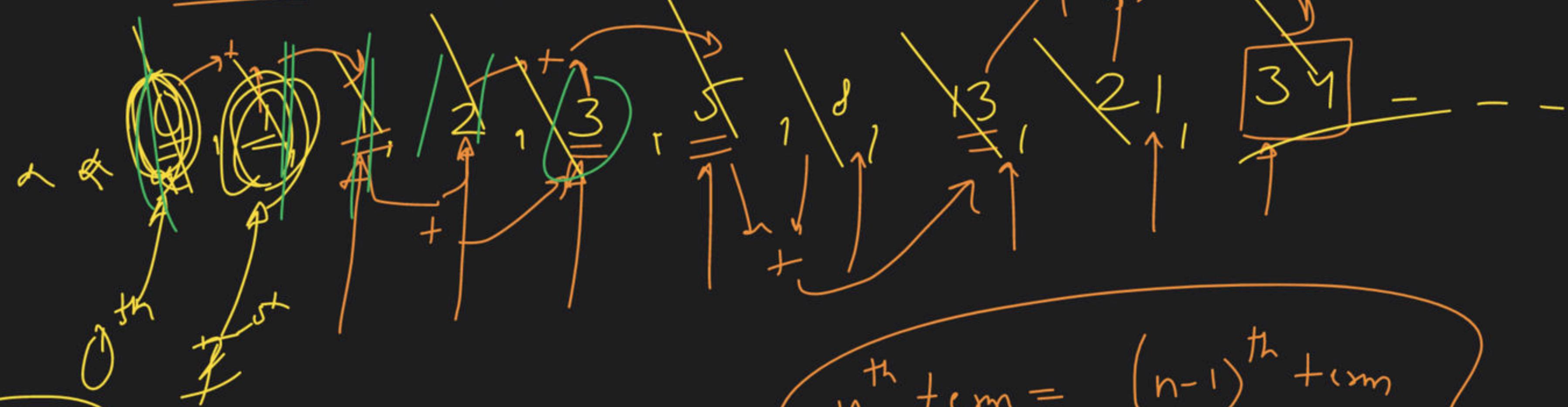
```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

```
int pow(int n ){  
    if(n == 0)  
        return 1;  
  
    int ans = 2 * pow(n-1);  
    return ans;  
}
```

→ Fibonacci

Series: →



3. C

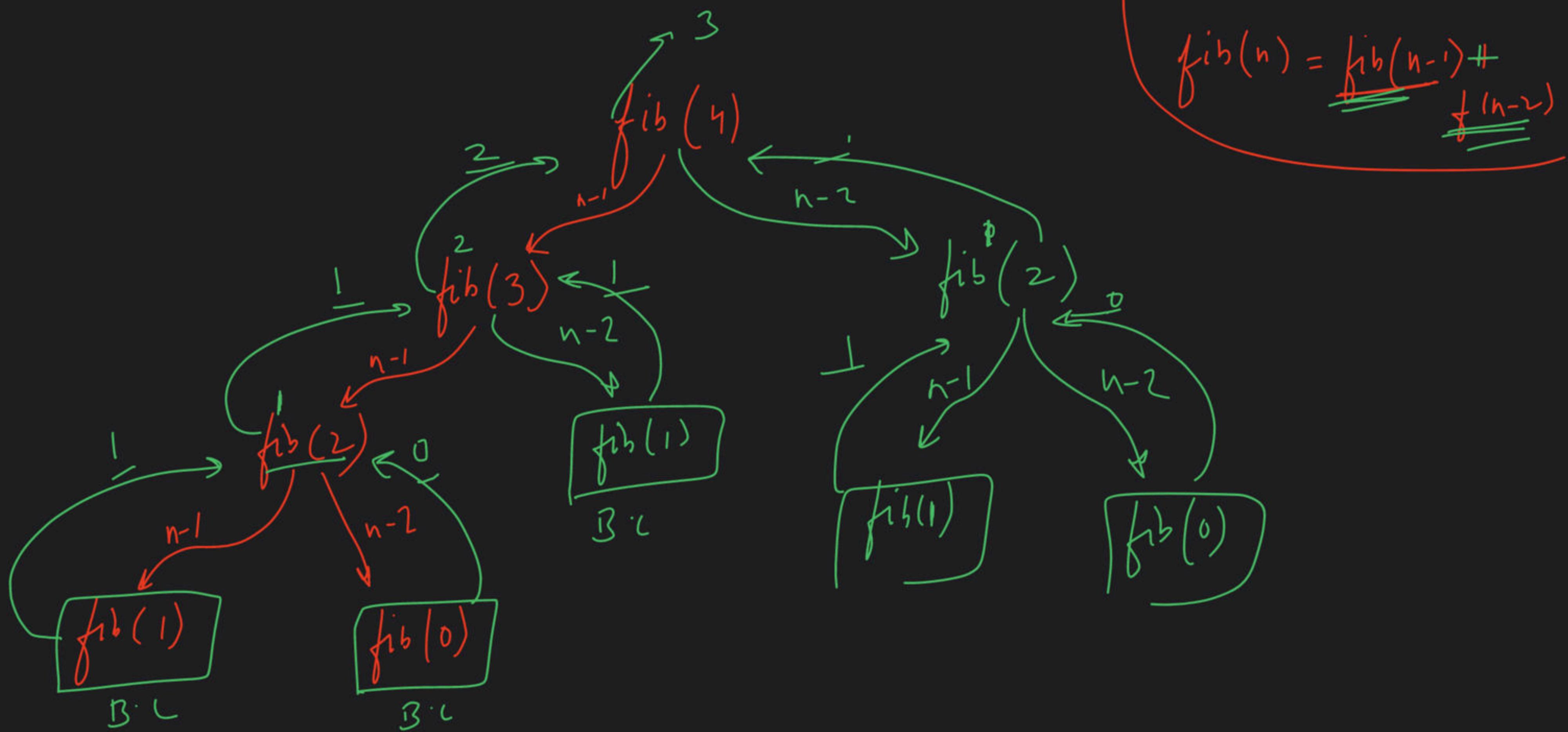
$n = -0$   
→ 0<sup>th</sup> turn → 0

$n = -1$   
→ 1<sup>st</sup> turn → 1

$$n^{\text{th}} + \text{turn} = (n-1)^{\text{th}} + \text{turn}$$

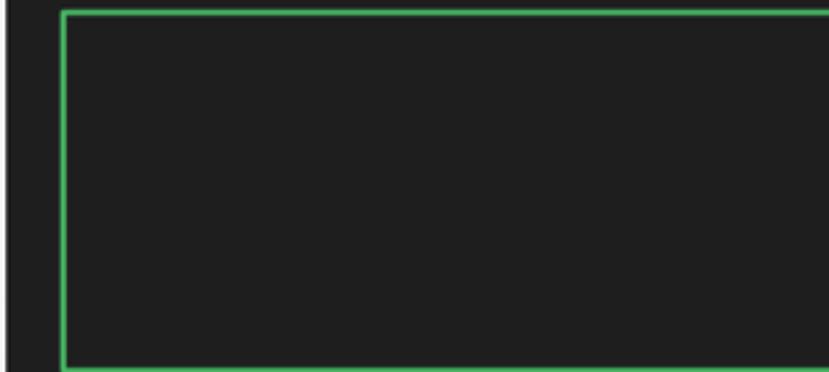
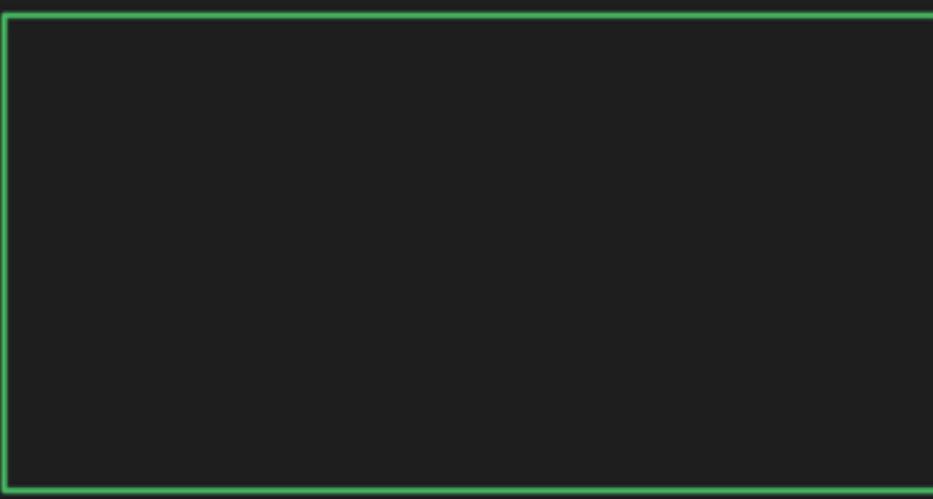
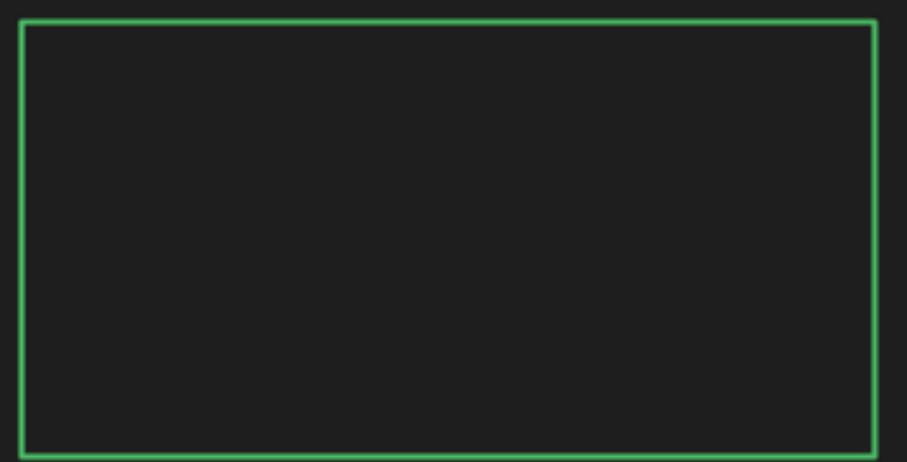
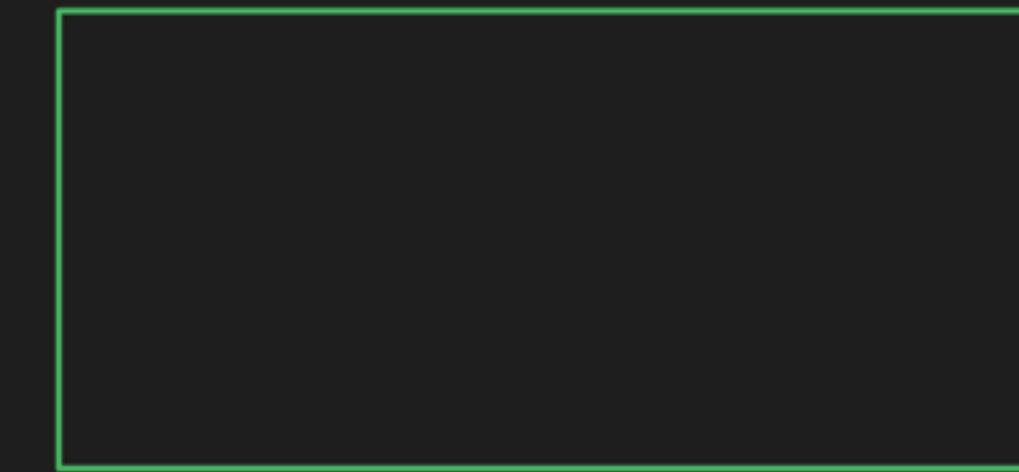
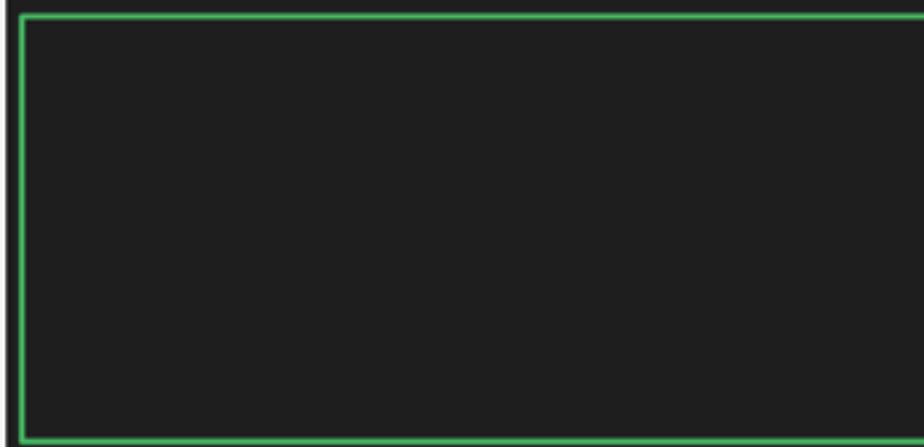
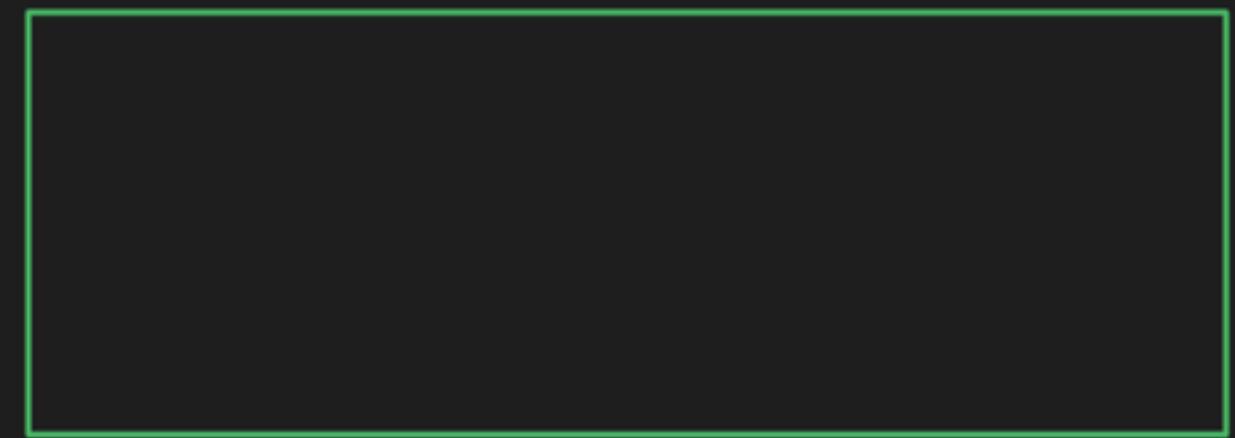
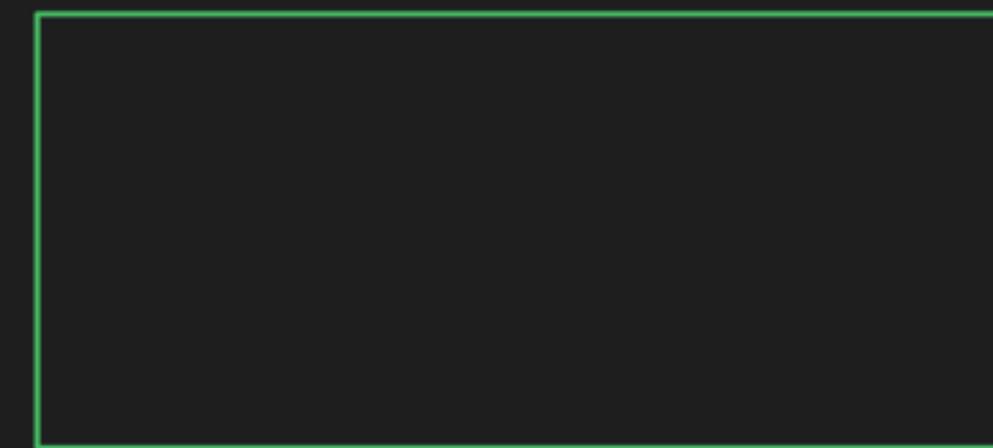
$$(n-2)^{\text{th}} + \text{turn}$$

$$\boxed{\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)}$$



```
int fib (int n)
{
    if (n == 0 || n == 1) return n;
    int ans = fib(n-1) + fib(n-2);
}
```

93 /,



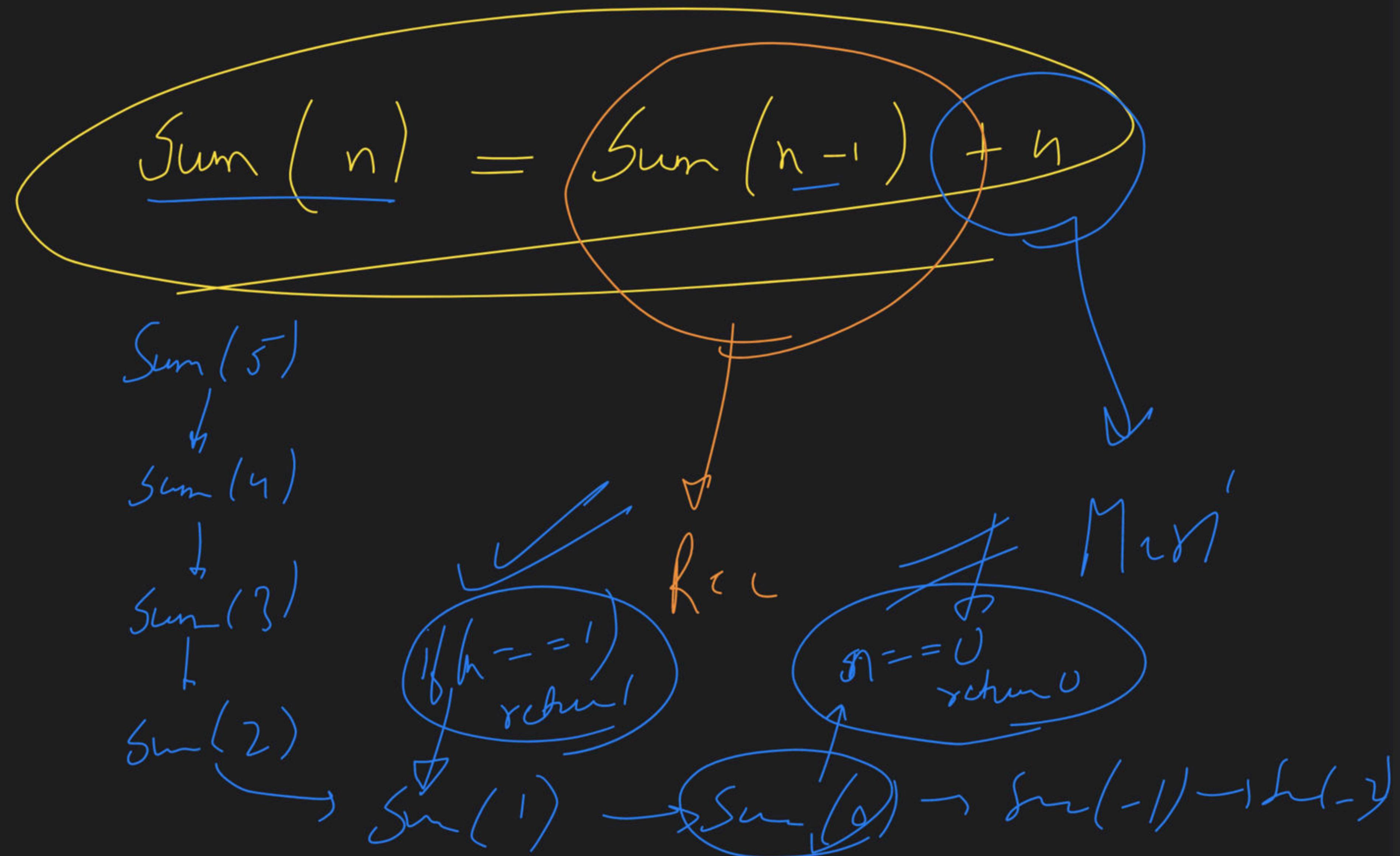


$\mathcal{B} \quad \text{getSum}(5) \rightarrow 1 + 2 + 3 + 4 + 5$

$\text{getSum}(4)$

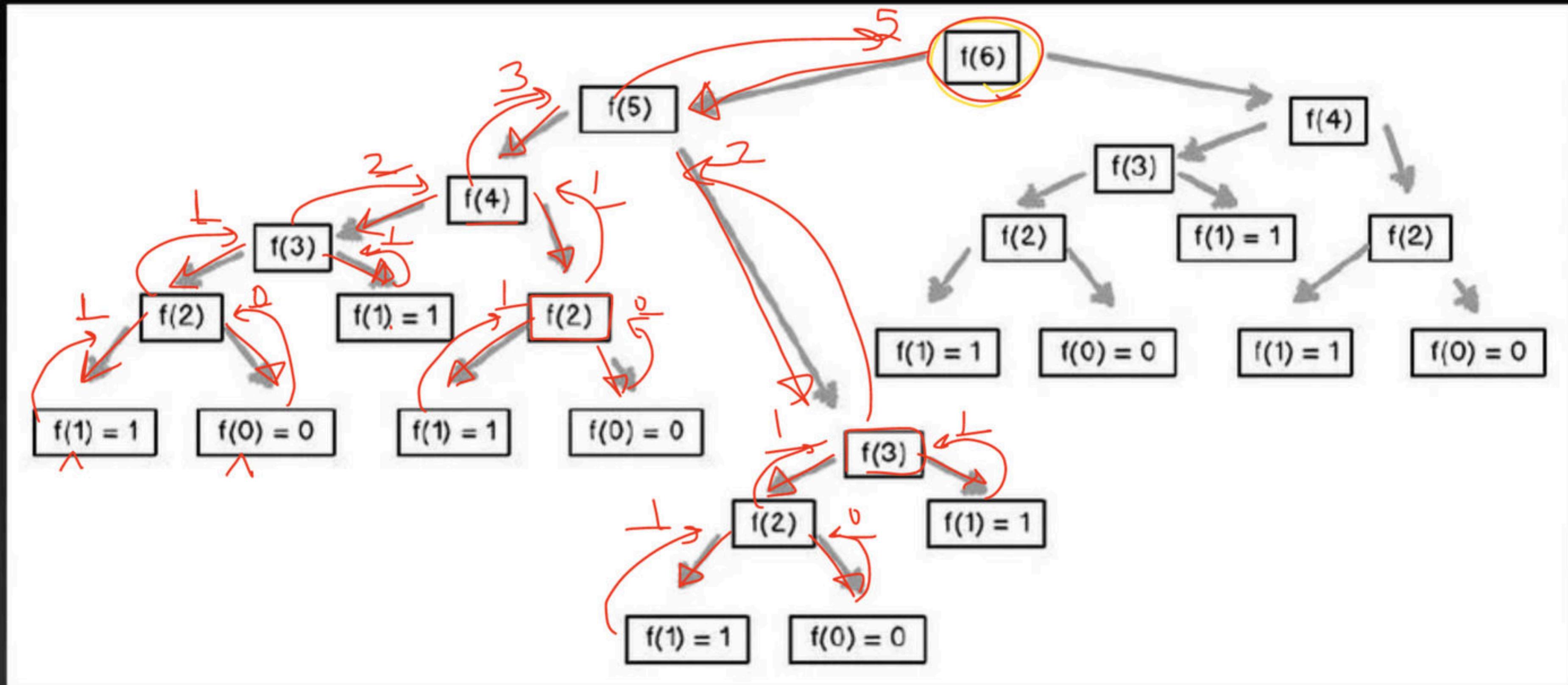
$\text{getSum}(5) = \text{getSum}(4) + 5$

$\text{getSum}(n) = \overbrace{\text{getSum}(n-1)}^+ + n$

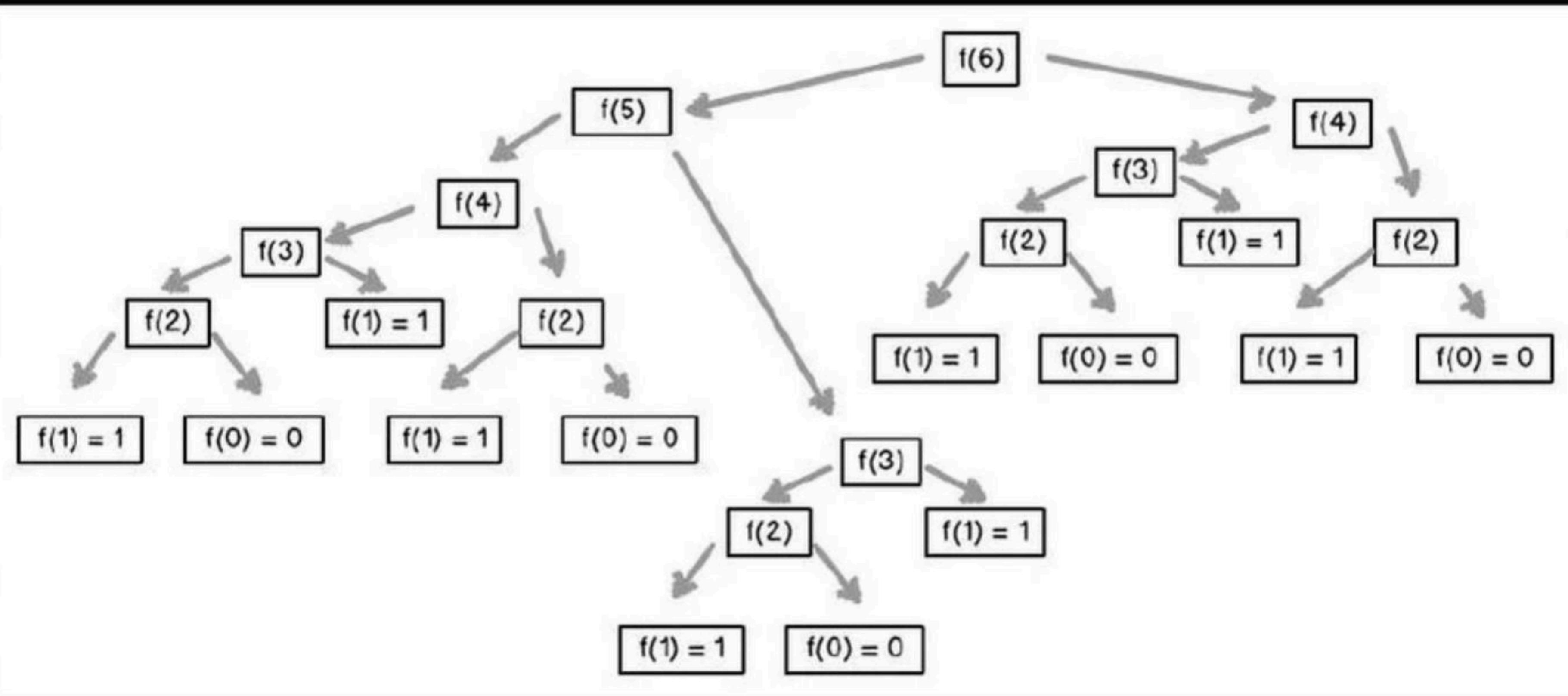


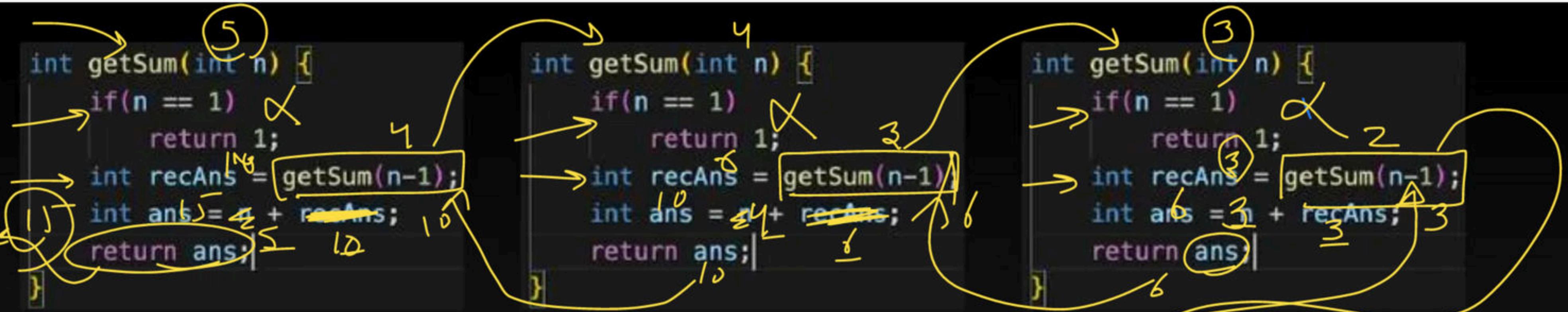
$n \rightarrow 6$

~~if ( $n == 0 \text{ || } n == 1$ ) return  $n$~~   
 $\rightarrow \underline{\text{int ans} = \underline{\text{fib}(n-1)} + \underline{\text{fib}(n-2)}}$   
 $\rightarrow \text{return ans}$



$$\left. \begin{array}{l} \text{fib}(0) = 0 \\ \text{fib}(1) = 1 \end{array} \right\} \rightarrow \text{B.C}$$





```
int getSum(int n) {
    if(n == 1)
        return 1;
    int recAns = getSum(n-1);
    int ans = n + recAns;
    return ans;
}
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Magical  
line

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```

main  
↓  
Sum (5)  
→ 15

main()  
↓  
get(3)  
↓  
get(4)  
↓  
get(5)  
↓  
main()

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