Algorithms Analysis and Design

Week 6 - Diary

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Lecture 10 : Edit Distance

Problems for the class :

We discussed more about Dynamic Programming and tried to expand our understanding to solve the **"Edit Distance"** Problem.

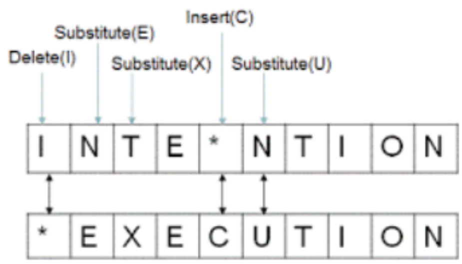
Edit Distance

Problem :

- Given two strings A and B , we need to find the minimum number of insert, delete or replace operations in order to convert A to B.
- It is equivalent to asking the extent to which these two strings align or match.

Example :

Assume $A = INTENTION$ and $B = EXECUTION$



- We can delete the 1st I , substitute N , T and so on and we can observe that we can convert the string A into B using a minimum of 5 operations.

More ways are possible but this one has least number of operations.

Let's have a look on how to solve this :

DP solution :

First, Lets check the sub-problem property and sub-structure property here.

1. Sub-problem property

We can very easily define a sub-problem for the given problem at hand.

- Say the length of string A is n and that of B is m , then we can ask a problem like what are the minimum number of delete, insert and replace operations to convert come prefix of A say $A[i]$ (prefix of first i letters of A) and similarly B say $B[j]$ (prefix of first j letters of B) .

2. Sub-structure property

Since we have already defined the sub - problem , we have following options for each i, j.

- Delete a character from the string.
- Insert a character into string.
- Substitute one character into another .

Now, Lets see the algorithm involved here,

So, Let's consider $dp[i][j]$ as the minimum number of operations that we have to perform to convert the prefix of first i character of A into the prefix of first j character of B.

1. When $A[i] == B[j]$, we don't need to perform any of those of operations so
$$dp[i][j] = dp[i-1][j-1]$$
2. When they don't match,
    - If we are inserting a character in A, then
    $$dp[i][j] = 1 + dp[m][n-1]$$
    - If we remove a character from A, then
    $$dp[i][j] = 1 + dp[i-1][j]$$
    - If we replace a character in A, then
    $$dp[i][j] = 1 + dp[i-1][j-1]$$

Therefore, To calculate $dp[i][j]$, we need to take minima of the above mentioned 3 options.

Pseudo code :

```
for i in (0,1,....m):
    dp[i][0] = i
for j in (0,1,....n):
    dp[j][0] = j
for i in (1,2,....m):
    for j in (1,2,....n):
        dp[i][j] = min{(1 + dp[i-1][j]),(1 + dp[i][j-1]),(dp[i-1][j-1] +
check(i,j))}

return dp[m][n]
```