Algorithms Analysis and Design

Week 12 - Diary

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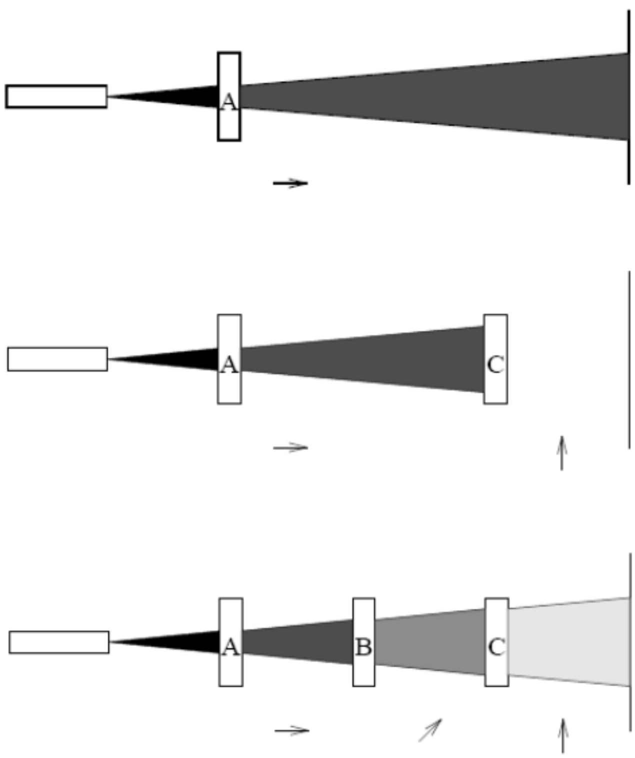
Lecture 17 : Quantum Algorithms

Problems for the class :

- Photon Experiment
- Qubits
- Quantum Mechanics
- No-cloning Theorem
- Quantum Entanglement
- Quantum gates and circuits
- Quantum Teleportation

Photon Experiment



- We know that if light travels through two polarising filters at 90 degrees to each other, no light will pass through the second filter.
- When a third filter is placed in front of the second, light passes through the third filter, albeit at a lower intensity than the original.

As a result of this study, we can deduce that light acts differently than ordinary fluids, and that traditional physics and mathematics are insufficient to explain this phenomena.

Qubits

- It's a unit vector in a 2-D complex vector space with a fixed basis. The linear superposition of a qubit's two orthonomal basis vectors can be used to express its general quantum state.
- These vectors are

  $$\{|0>, 1>\}$$

  where,

  $$|0>= \begin{bmatrix}1\\0\end{bmatrix}$$
  $$|0>= \begin{bmatrix}0\\1\end{bmatrix}$$

- A pure qubit state/vector is a coherent superposition of the basis vectors. This means that a single qubit can be described by a linear combination of these 2 vectors:

  $$|\psi>= \alpha|0> +\beta|1>$$

  where,

  $$|\alpha|^2 + |\beta|^2 = 1$$

  here, α and β are complex numbers, called as **Probability amplitudes.**

  1. probability that measure values is $|0>= |\alpha|^2$ after which the state collapses to $|0>$.
  2. probability that measure values is $|1>= |\beta|^2$ after which the state collapses to $|1>$.
- Individual state spaces of n particles combine classically through the cartesian product, but quantum states however combine through tensor product.
  - The space of all the states that may be created from its constituent states is the **Tensor product** of two states.
    - For example ,

      $|00> = |0> \otimes |0> = \begin{bmatrix}1\\0\end{bmatrix} \otimes \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1& 1\\1&0\\0&1\\0&0\end{bmatrix} $

      An n qubit system has 2^n basis vectors.

Quantum mechanics

1. Superposition Postulate

If a physical system can be in any of a variety of configurations, the most generic state is a mixture of all of them, with the amount of each arrangement being described by a complex number.

2. Measurement Postulate

In any measurement of a qubit $|\psi>$ using the operator $\hat{A}$, the only values that will be observed are the eigenvalues a, which satisfy the following eigenvalue equation.

$$\hat{A}\psi = a\psi$$

3. Collapse Postulate

A wave function collapse occurs in quantum mechanics when a wave equation that is initially in a superposition of several eigenstates is reduced to a single eigenstate as a result of an interaction with the outside world, also known as an observation.

4. Evolution Postulate

The wavefunction or state function of a system evolves with time according to the time dependent Schrödinger equation

$$\hat{H}\psi(r,t) = i\hbar\frac{\partial\psi}{\partial t}$$

No-cloning theorem

The no-cloning theorem states that it is impossible to create an independent and identical copy of an arbitrary unknown quantum state.

Proof:

Assume that U is a unitary transformation that clones, in that $U(|a0>) = |aa›$ for all quantum states $|a>$.

Consider $|c>= (1/\sqrt{2})(|a>+|b>)$ Then,

$$U(|c0>) = 1/\sqrt{2}(U(|a0>) + U(|b0>)) = 1/\sqrt{2}(|aa>+|bb>)$$

But if U is a cloning transformation then

$$U(|c0>) = |cc>= 1/2(|aa>+|ab>+|ba>+|bb>)$$

Thus, there is a contradiction and the theorem is proved.

- Quantum states combine through the tensor product rather than the cartesian product. An n qubit system has 2^n basis vectors, so quantum states have 8 basis vectors.
- Now it is easy to see the exponential growth of the state space with the number of quantum particles.

Quantum Entanglement

It's a physical phenomena that occurs when a collection of particles is formed, interacts, or shares a shared space in such a way that each particle's quantum state cannot be described independently of the state of the other particles, even when they're separated by a significant distance.

Example: the state |00> + |11>, it cannot be described in terms of its qubit components.

Quantum Gates and Circuits

Quantum gates are represented as matrices that when applied to qubits changes their state.

Pauli X gate (NOT)

$$|0> \rightarrow |1>$$

$$|1> \rightarrow |0>$$

$$\alpha|0> + \beta|1> \rightarrow \alpha|1> + \beta|0>$$

The gate can be represented as

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and

$$X|0> = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1>$$

and the same can be proved that $X|1> = |0>$

Identity gate

$$|0> \rightarrow |0>$$

$$|1> \rightarrow |1>$$

$$\alpha|0> + \beta|1> \rightarrow \alpha|0> + \beta|1>$$

Let X denote the NOT-gate,

$$X|0> = |1>$$

$$X|1> = X|0>$$

We can say that applying X twice on a qubit would give us the original qubit itself as the product.

$$XX|0> = X|1> = |0>$$

$$XX|1> = X|0> = |1>$$

$$XX = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Therefore, XX here is the identity gate.

Hamdard Gate

$$|0> \rightarrow \frac{|0> + |1>}{\sqrt{2}}$$

$$|1> \rightarrow \frac{|0> - |1>}{\sqrt{2}}$$

$$\alpha|0> + \beta|1> \rightarrow \frac{\alpha+\beta}{\sqrt{2}}|0> + \frac{\alpha-\beta}{\sqrt{2}}|1>$$

Let H represent the Hamdard gate,

$$H = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, H\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{bmatrix}, H\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}\frac{1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{bmatrix}$$

we get,

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

CNOT gate

Here, the 1st qubit is the control qubit and the 2nd is the target qubit. Only when the control qubit is $|1>$, then only the target qubit will get reversed, or else it is given as output as it is.

$$|00> \rightarrow |00>$$

$$|01> \rightarrow |01>$$

$$|10> \rightarrow |11>$$

$$|11> \rightarrow |10>$$

Here the matrix that we obtain will be a 4x4 matrix. Using the above conditions one by one and making equations with solving, we get

$$C_{not} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

Pauli Y gate

$$|0> \rightarrow -|1>$$

$$|1> \rightarrow |0>$$

$$\alpha|0> + \beta|1> \rightarrow \alpha|1> + \beta|0>$$

$ Y = \begin{bmatrix}0&1\\-1&0\end{bmatrix}$

Pauli Y gate

$$|0> \rightarrow -|1>$$

$$|1> \rightarrow |0>$$

$$\alpha|0> + \beta|1> \rightarrow \alpha|1> + \beta|0>$$

$ Y = \begin{bmatrix}0&1\\-1&0\end{bmatrix}$

Pauli Z gate

$$|0> \rightarrow -|1>$$

$$|1> \rightarrow |0>$$

$$\alpha|0> + \beta|1> \rightarrow \alpha|0> - \beta|1>$$

$ Y = \begin{bmatrix}1&0\\0&-1\end{bmatrix}$

Quantum Teleportation

The objective is to transmit the quantum state of a particle using classical bits and reconstruct the exact quantum state at the receiver. It follows the no-cloning theorem.

There is a certain protocol which is required to be followed in teleportation:

- A Bell state is generated with one qubit sent to location A and the other sent to location B.
- A Bell measurement of the Bell state qubit and the qubit to be teleported ($|\Phi >$) is performed at location A. This yields one of four measurement outcomes which can be encoded in two classical bits of information. Both qubits at location A are then discarded.
- Using the classical channel, the two bits are sent from A to B. (This is the only potentially time-consuming step after step 1 since information transfer is limited by the speed of light.)
- As a result of the measurement performed at location A, the Bell state qubit at location B is in one of four possible states. Of these four possible states, one is identical to the original quantum state $|\Phi >$, and the other three are closely related. The identity of the state actually obtained is encoded in two classical bits and sent to location B. The Bell state qubit at location B is then modified in one of three ways, or not at all, which results in a qubit identical to $|\Phi >$, the state of the qubit that was chosen for teleportation.