CS 246:
Artificial Intelligence

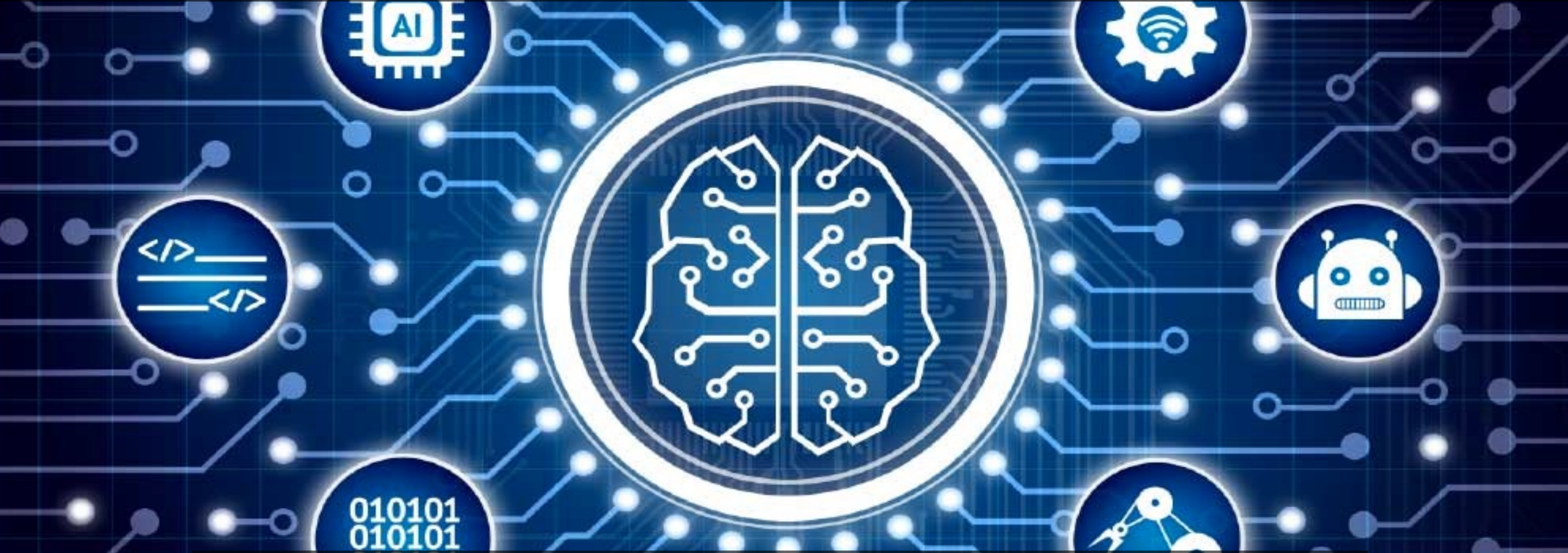

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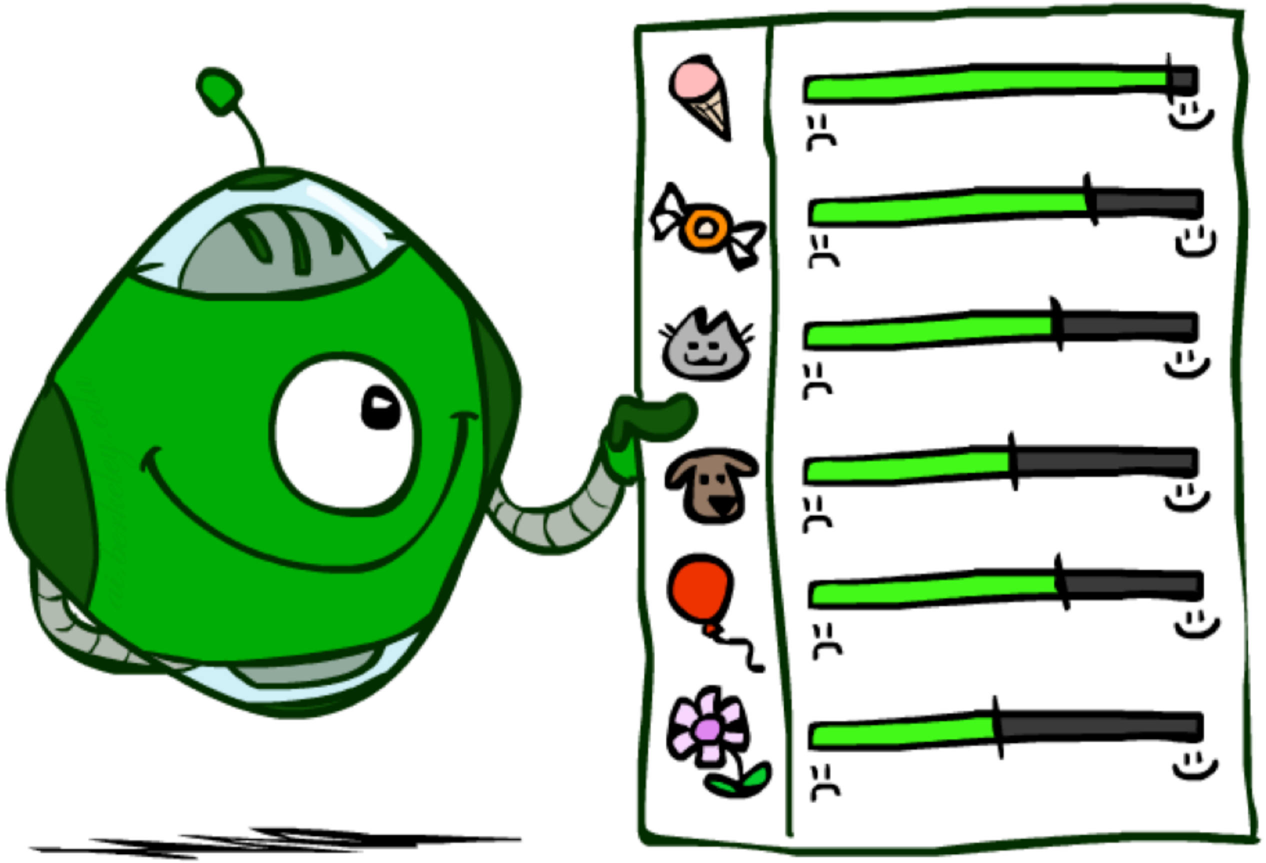
Image credit
https://futureoflife.org/

[slides adapted from Dan Klein, Pieter Abbeel, Sergey Levine & Stuart Russel (University of California, Berkeley)]

Om Saha Naav[au]-Avatu
Saha Nau Bhunaktu
Saha Viiryam Karavaavahai
Tejasvi Naav[au]-Adhiitam-
Astu Maa Vidvissaavahai
Om Shaantih Shaantih
Shaantih

Om, May we all be protected
May we all be nourished
May we work together with great energy
May our intelect be sharpened (may our study be effective)
Let there be no Animosity amongst us
Om, peace (in me), peace (in nature), peace (in divine forces)

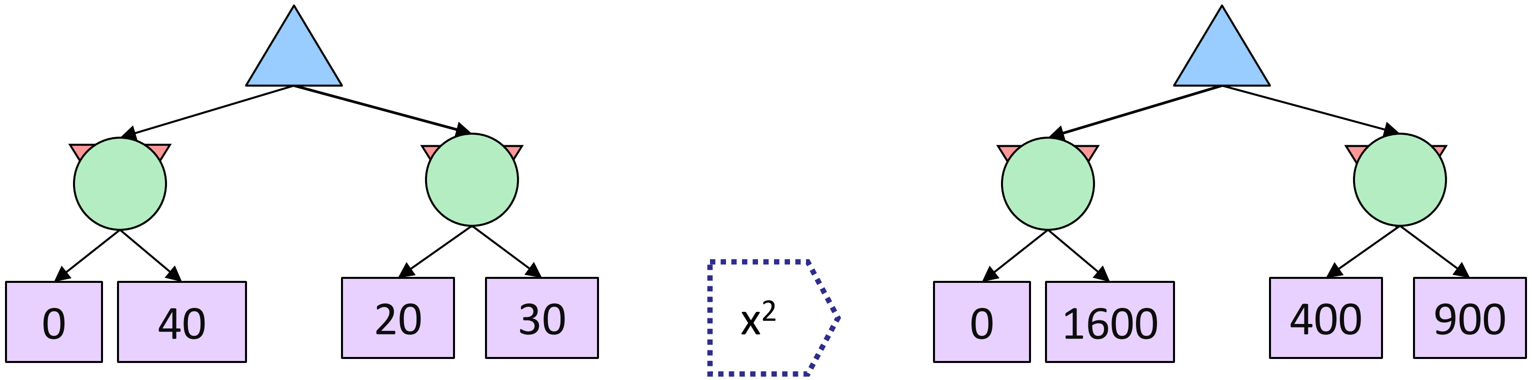
Utilities



Maximum Expected Utility

- Why should we average utilities? Why not minimax?
- Principle of maximum expected utility:
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge
- Questions:
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

What Utilities to Use?

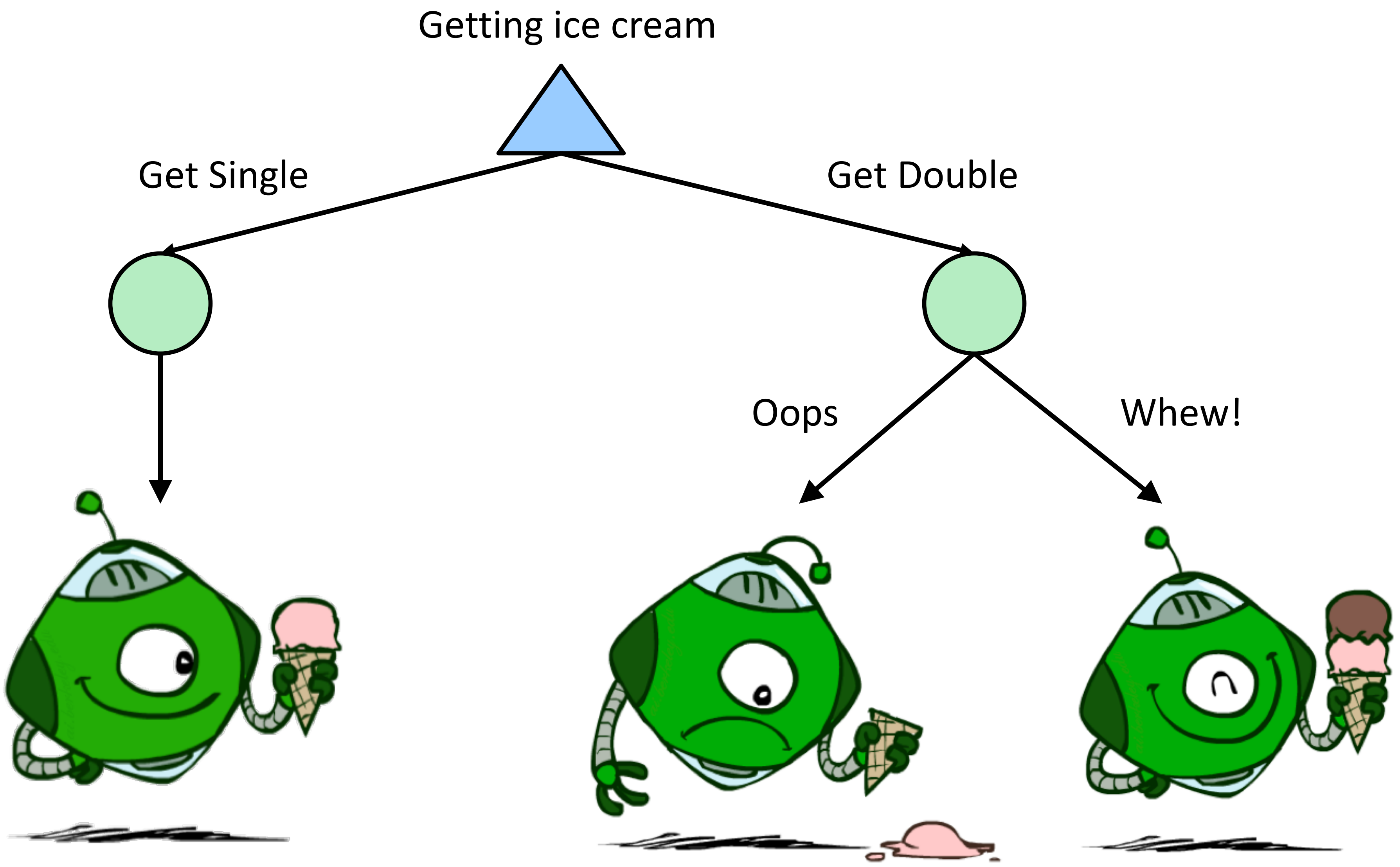


- For worst-case minimax reasoning, terminal function scale doesn't matter
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- For average-case expectimax reasoning, we need *magnitudes* to be meaningful

Utilities

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences
- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function
- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?

Utilities: Uncertain Outcomes

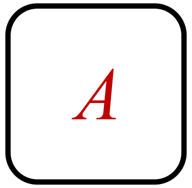


Preferences

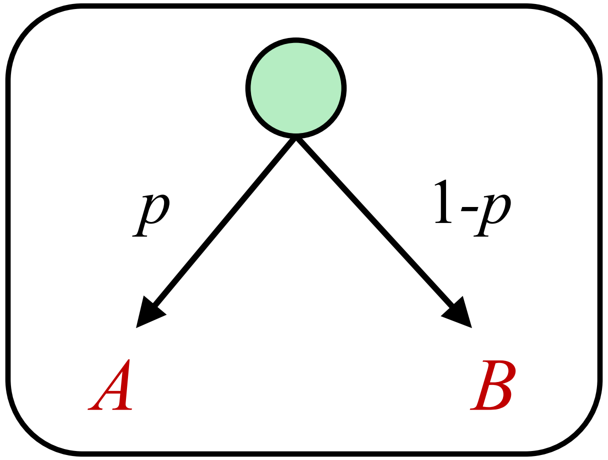
- An agent must have preferences among:
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ \ (1-p), B]$$

A Prize

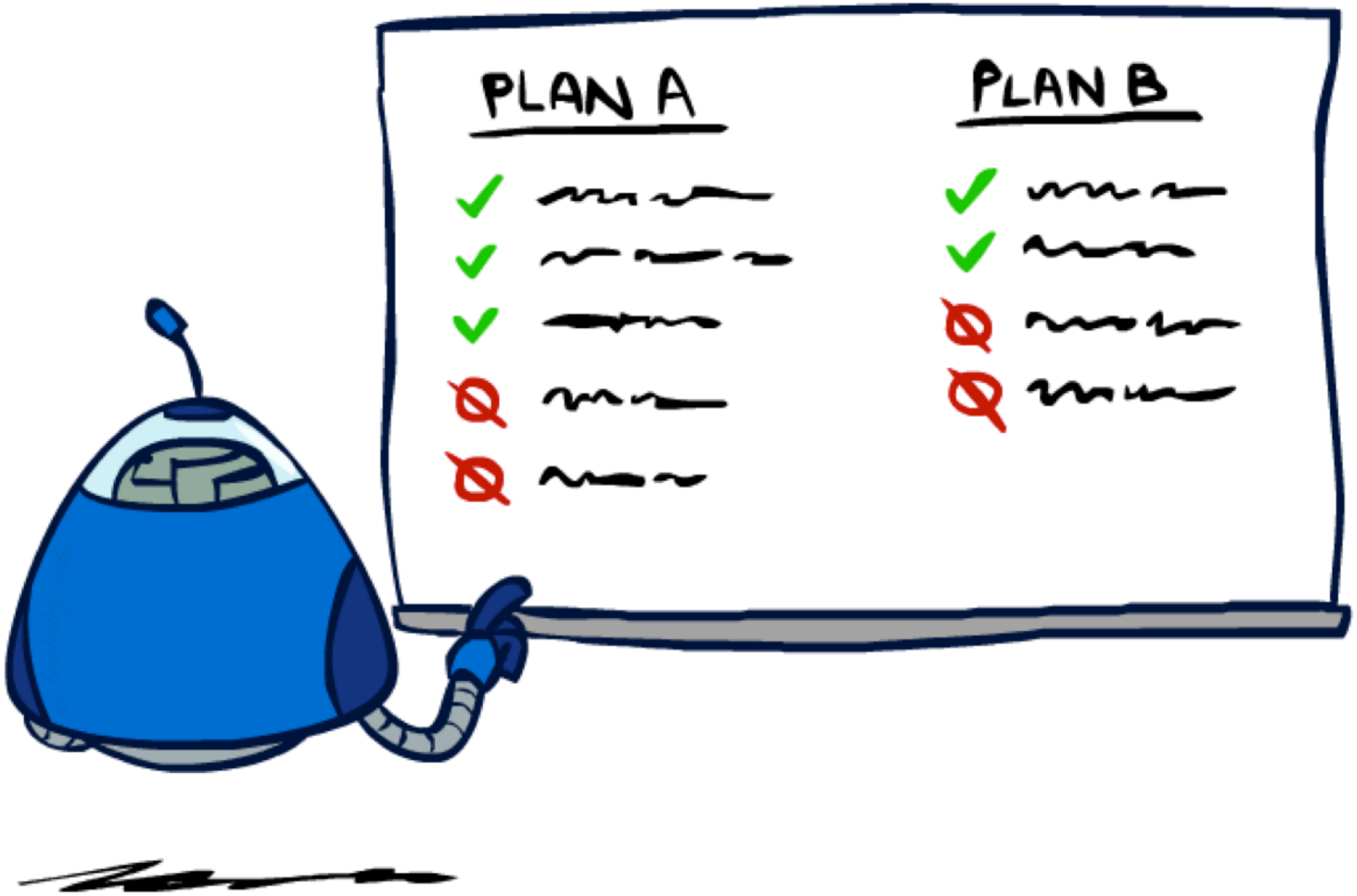


A Lottery



- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

Rationality

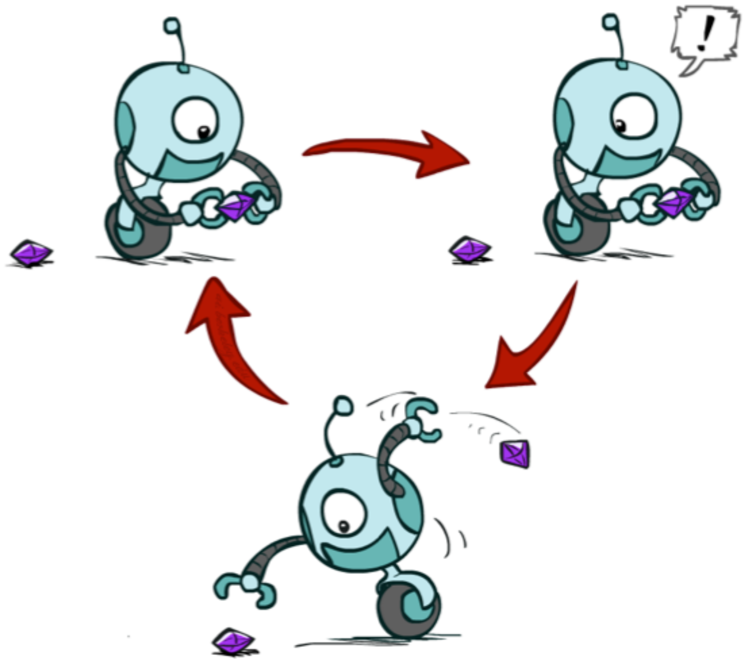


Rational Preferences

- We want some constraints on preferences before we call them rational, such as:

$$\text{Axiom of Transitivity: } (A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

- For example: an agent with intransitive preferences can be induced to give away all of its money
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



Rational Preferences

The Axioms of Rationality

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p \; [p, A; \; 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A; \; 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A; \; 1-p, B] \succeq [q, A; \; 1-q, B])$$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

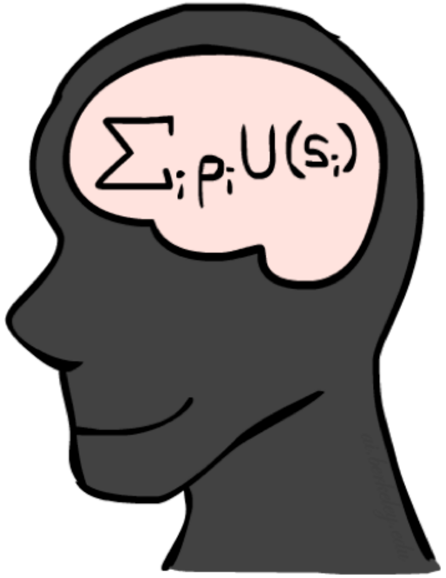
MEU Principle

- Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succeq B$$

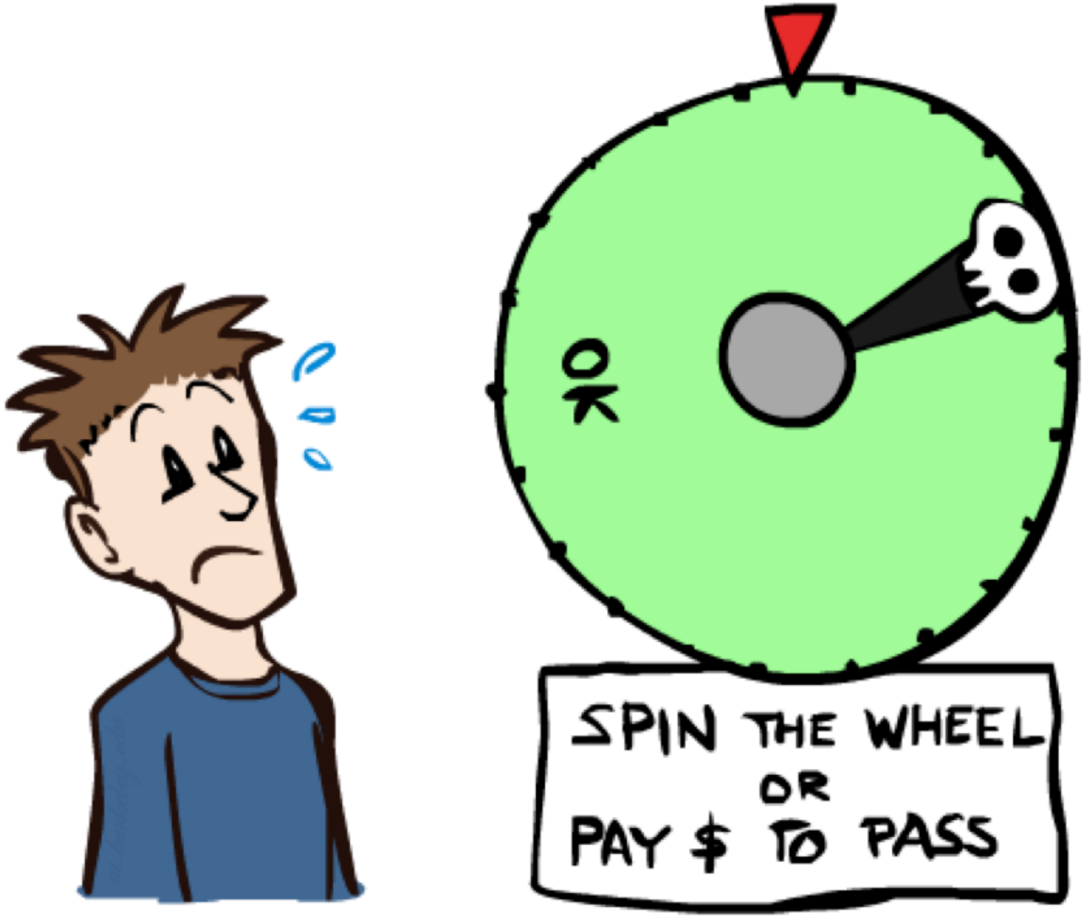
$$U([p_1, S_1; \;\ldots\; ; \; p_n, S_n]) = \sum_i p_i U(S_i)$$

  - I.e. values assigned by U preserve preferences of both prizes and lotteries!



- Maximum expected utility (MEU) principle:
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

Human Utilities



Maximum Expected Utility

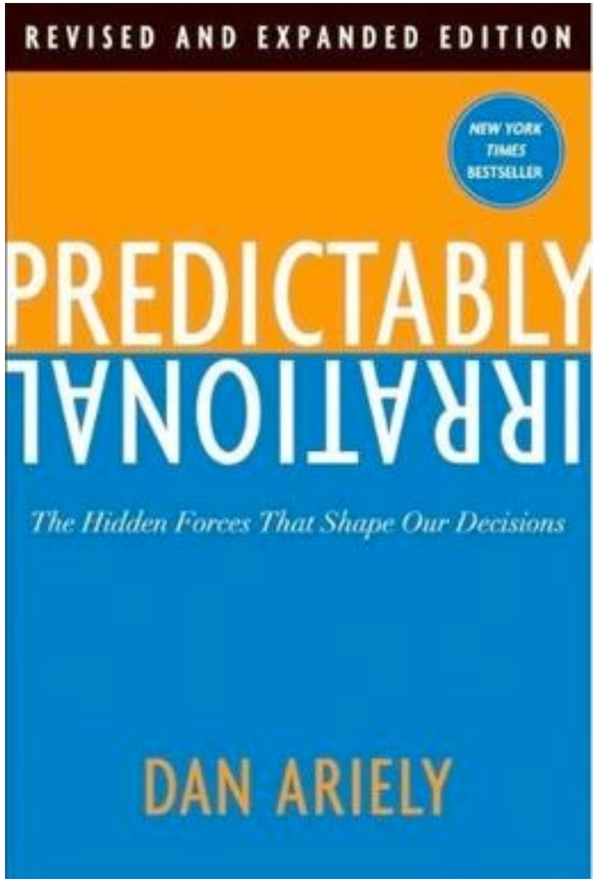
- A rational agent should choose the action that maximizes the agent’s expected utility.

- If the agent prefers A over B ,
- then we can write:
  - $\boldsymbol{A} \succ \boldsymbol{B}$

Preference Lead to Utility

- $A \succ B \;\Leftrightarrow U(A) > \; U(B)$
- Expected utility of gamble: $\sum_i p_i U(S_i)$

Humans are predictably irrational!



Why only predict?
Why not measure?

Experiment 1

Gamble 1A		Gamble 1B	
Winnings	Chance	Winnings	Chance
$1 million	100%	$1 million	89%
		Nothing	1%
		$5 million	10%

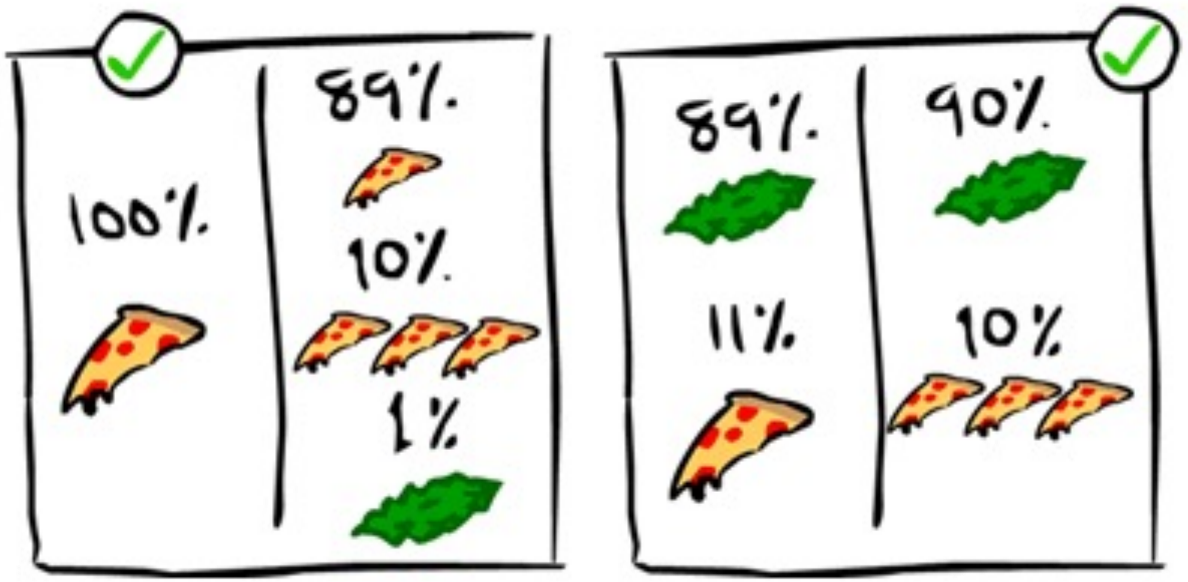
http://etc.ch/5fn6

Experiment 2

Gamble 2A		Gamble 2B	
Winnings	**Chance**	**Winnings**	**Chance**
Nothing	89%	Nothing	90%
$1 million	11%		
		$5 million	10%

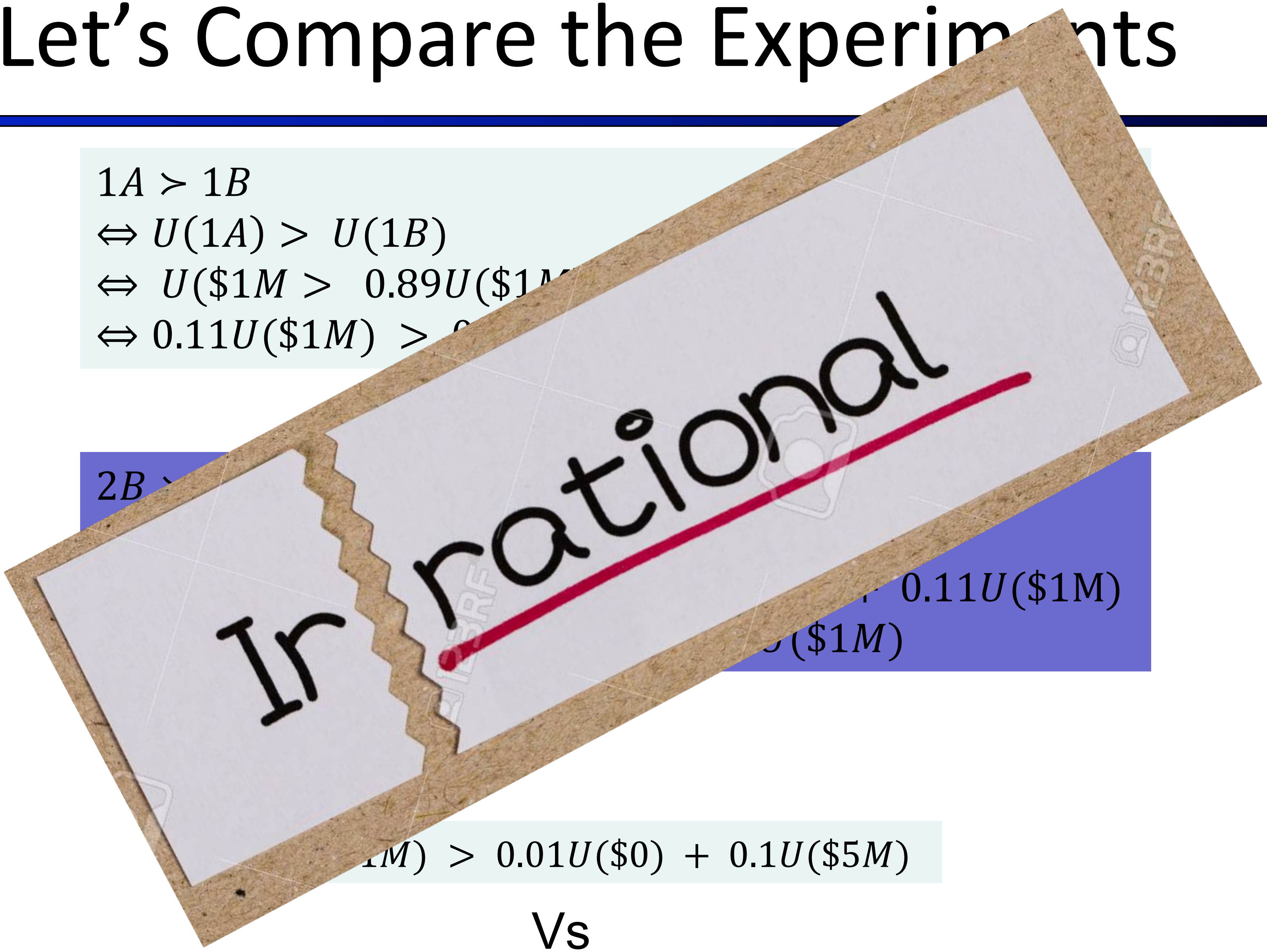
http://etc.ch/5fn6

Allais paradox (1953)



Let's Compare the Experim nts

$1A \succ 1B$
$\Leftrightarrow U(1A) > U(1B)$
$\Leftrightarrow U(\$1M > 0.89U(\$1$
$\Leftrightarrow 0.11U(\$1M) > 0$



$2B \succ$

$+ 0.11U(\$1M)$
$U(\$1M)$

$1M) > 0.01U(\$0) + 0.1U(\$5M)$

Vs

$0.01U(\$0) + 0.1U(\$5M) > 0.11U(\$1M)$

Utility Scales

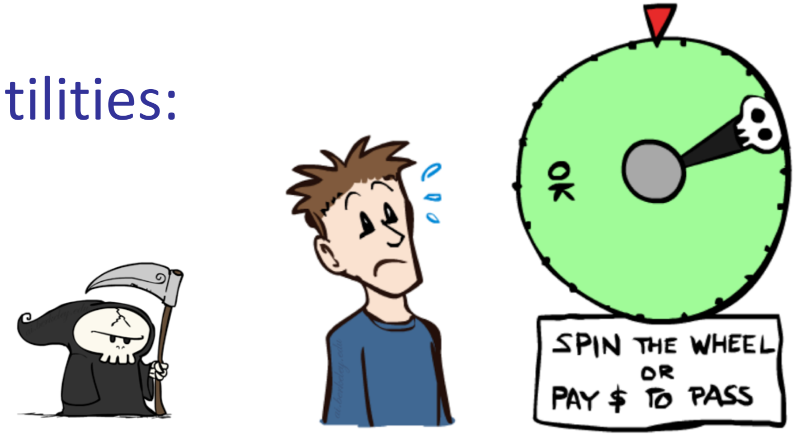
- Normalized utilities: $u_+ = 1.0$, $u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

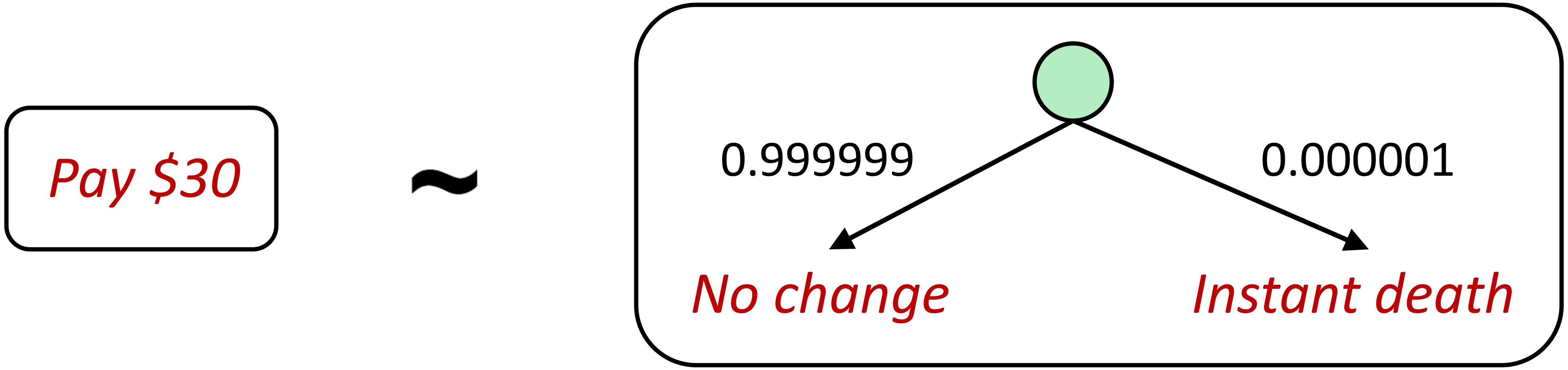
$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Human Utilities

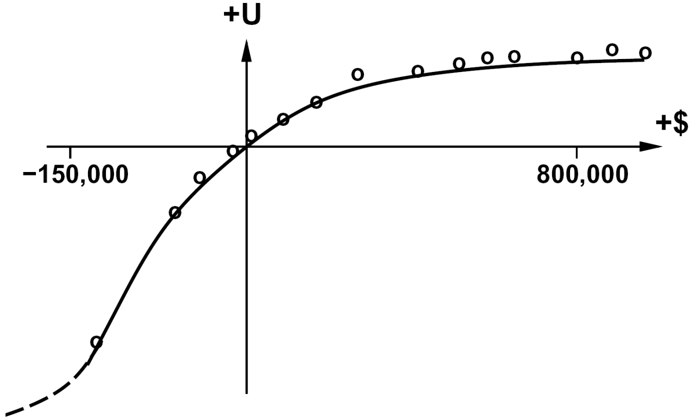
- Utilities map states to real numbers. Which numbers?
- Standard approach to assessment (elicitation) of human utilities:
  - Compare a prize A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until indifference: A ~ L_p
  - Resulting p is a utility in [0,1]





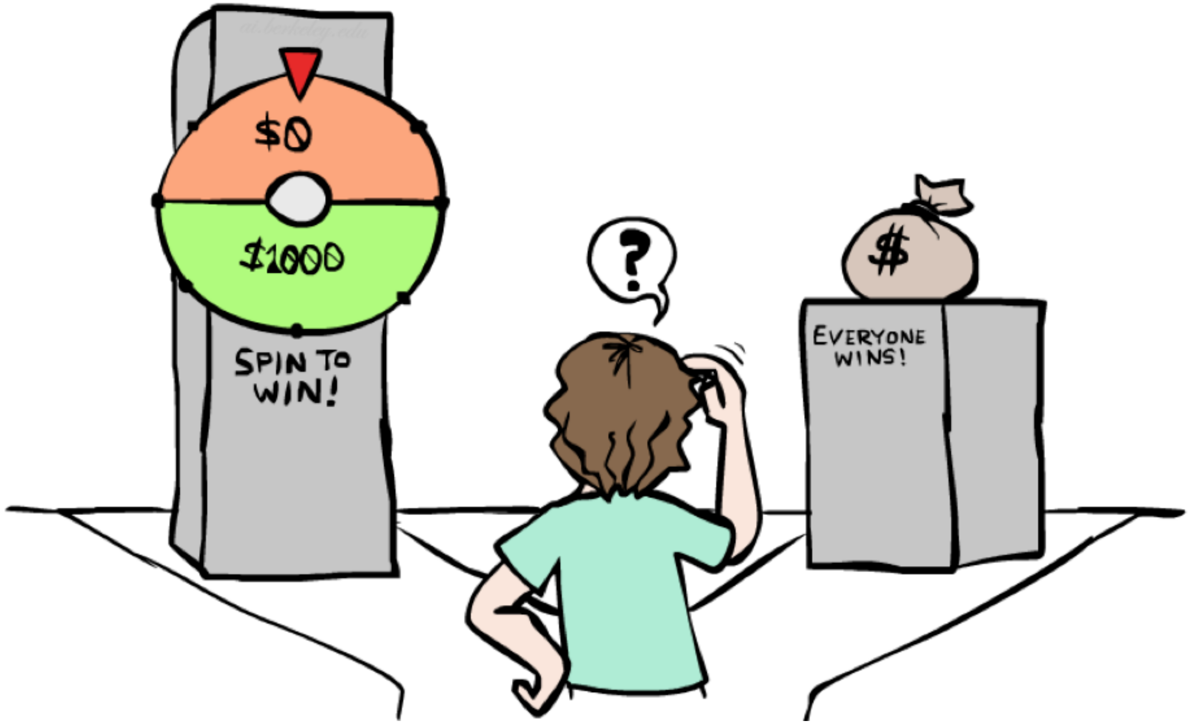
Money

- Money <u>does not</u> behave as a utility function, but we can talk about the utility of having money (or being in debt)
- Given a lottery L = [p, $X; (1-p), $Y]
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L))
  - In this sense, people are risk-averse
  - When deep in debt, people are risk-prone



Example: Insurance

- Consider the lottery [0.5, $1000; 0.5, $0]
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - Monetary value acceptable in lieu of lottery
    - $400 for most people
  - Difference of $100 is the insurance premium
    - There's an insurance industry because people will pay to reduce their risk
    - If everyone were risk-neutral, no insurance needed!
  - It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is flat and they have many lotteries)



Example: Human Rationality?

- Famous example of Allais (1953)
  - A: [0.8, $4k; 0.2, $0]
  - B: [1.0, $3k; 0.0, $0]
  - C: [0.2, $4k; 0.8, $0]
  - D: [0.25, $3k; 0.75, $0]
- Most people prefer B > A, C > D
- But if U($0) = 0, then
  - B > A $\Rightarrow$ U($3k) > 0.8 U($4k)
  - C > D $\Rightarrow$ 0.8 U($4k) > U($3k)