Mensuration

Mensuration is the branch of mathematics which deals with the study of Geometric shapes, their area, volume and related parameters.

Learning Outcomes

Through this Topic Candidates should be able to

- understand the basic concepts of mensuration
- observe the data given and interpret it from the given problem
- apply the Concepts to solve Company Specific Aptitude tests
- apply the Concepts to solve questions asked in various government exams like SSC CGL, Banking Sector etc.

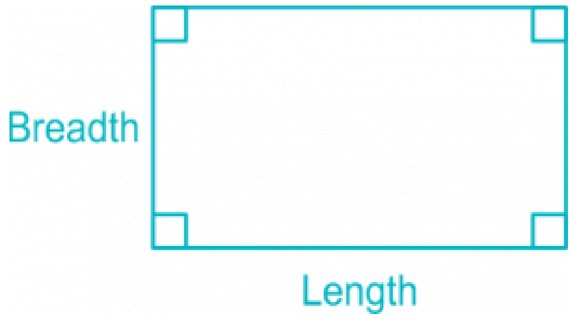
Contents

- **2D Figures: Area, Perimeter.**
- **3D Figures: Volume, Curved surface Area and Total surface Area.**

2D Figures	**3D Figures**
Rectangle	Cuboid
Square	Cube
Triangle	Cylinder
Circle	Cone
Parallelogram	Sphere
Trapezium	Hemisphere

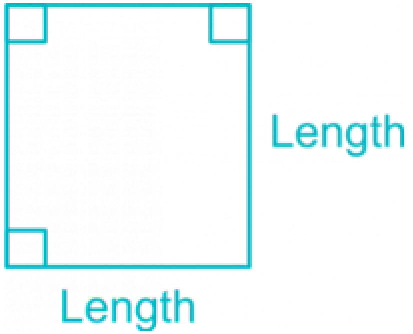
2D Figures:

Rectangle



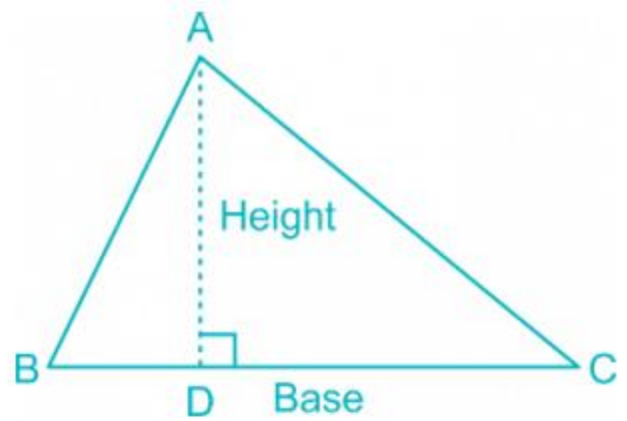
- Area of Rectangle = Length × Breadth.
- Perimeter of a Rectangle = 2 × (Length + Breadth)
- Length of the Diagonal = $\sqrt{(\text{Length}^2 + \text{Breadth}^2)}$

Square



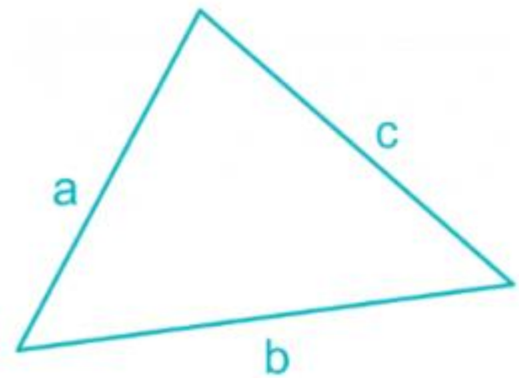
- Area of a Square = Length × Length = $(\text{Length})^2$
- Perimeter of a square = 4 × Length
- Length of the Diagonal = $\sqrt{2}$ × Length

Triangle



- Area of a triangle=(1/2)(Base × Height)=(1/2)(BC×AD)

Triangle

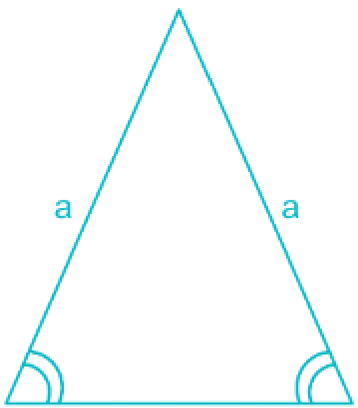


For a triangle with sides measuring a, b and c, respectively:

- Perimeter = a + b + c
- s = semi perimeter = perimeter/2 = (a+b+c)/2
- Area of Triangle, A=$\sqrt{s(s-a)(s-b)(s-c)}$

  (This is also known as "Heron's formula")

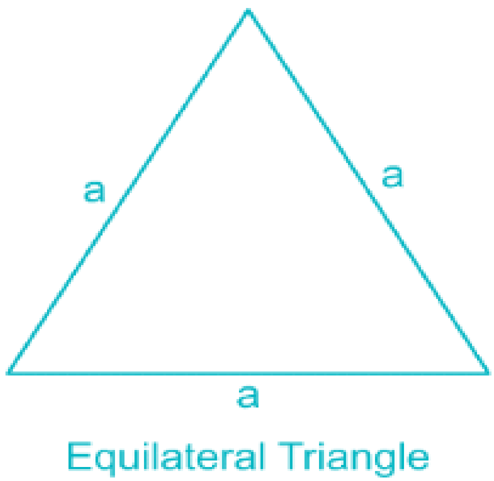
Isosceles Triangle



- Area of Isosceles Triangle: $\frac{b}{4}\sqrt{4a^2 - b^2}$

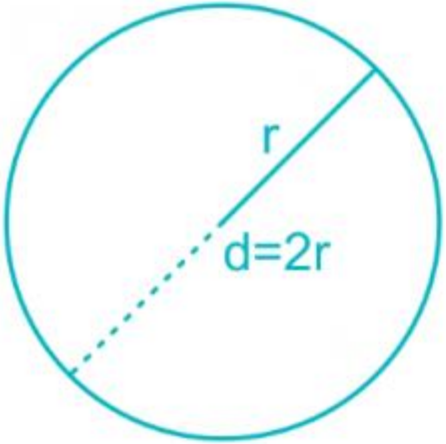
(Where a = length of two equal side, b = length of base of isosceles triangle.)

Equilateral Triangle



- Area of Equilateral Triangle : $\frac{\sqrt{3}}{4} \times a^2$

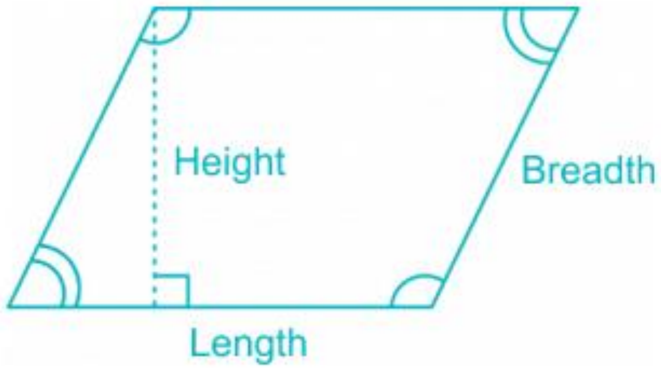
Circle & Semicircle



- Area of a circle = $\pi r^2 = \pi d^2/4$
- Circumference of a circle = $2\pi r = \pi d$
- Circumference of a semicircle = $\pi r + 2r$
- Area of semicircle = $\pi r^2/2$

(In the following formulae, r = radius and d = diameter of the circle)

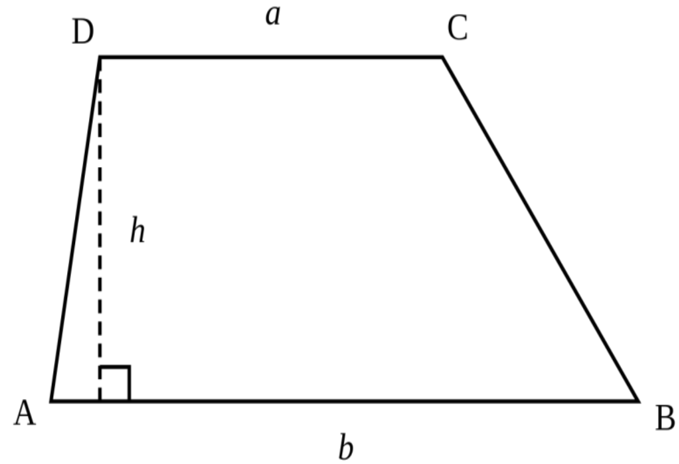
Parallelogram



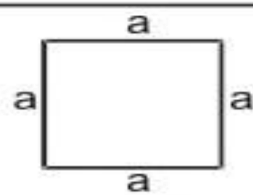
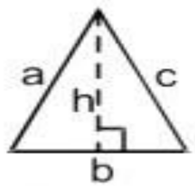
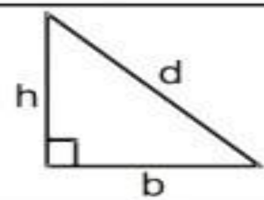
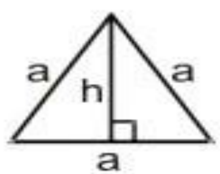
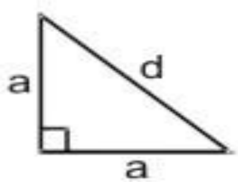
- Area of a Parallelogram = Length × Height

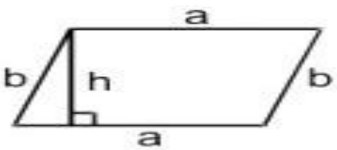
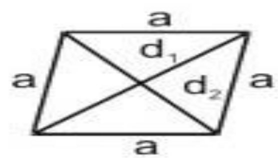
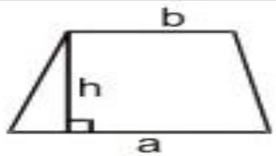
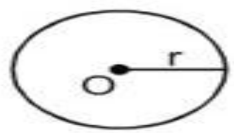
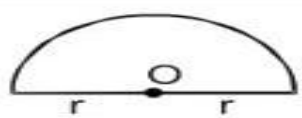
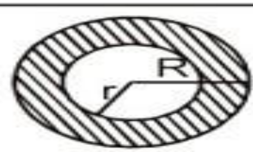
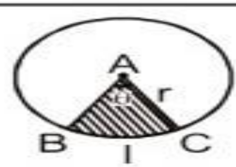
- Perimeter of a Parallelogram = 2 × (Length + Breadth)

Trapezium



- Area of a trapezium =(1/2)× sum of parallel sides × distance between parallel sides

Name	Figure	Perimeter	Area
Rectangle		$2(a+b)$	ab
Square		$4a$	a^2
Triangle		$a+b+c=2s$	1 = $\frac{1}{2} \times b \times h$ 2 = $\sqrt{s(s-a)(s-b)(s-c)}$
Right triangle		$b+h+d$	$\frac{1}{2}bh$
Equilateral triangle		$3a$	1. $\frac{1}{2}ah$ 2. $\frac{\sqrt{3}}{4}a^2$
Isosceles right triangle		$2a+d$	$\frac{1}{2}a^2$

Parallelogram		2 (a + b)	ah
Rhombus		4a	$\frac{1}{2} d_1 d_2$
Trapezium		Sum of its four sides	$\frac{1}{2} h (a + b)$
Circle		$2\pi r$	πr^2
Semicircle		$\pi r + 2r$	$\frac{1}{2} \pi r^2$
Ring (shaded region)		----	$\pi (R^2 - r^2)$
Sector of a circle		$l + 2r$ where $l = (\theta/360) \times 2\pi r$	$\theta/360° \times \pi r^2$

Practice Questions

1. The area of a square is 4096 sq. cm. Find the ratio of the breadth and length of a rectangle whose length is twice the side of the square and breadth is 24 cm less than the side of the square

A. 5:32

B. 7:16

C. 5:16

D. None of these

2. A wire in the form of a circle of radius 3.5 m is bent in the form of a rectangle whose length and breadth in the ratio of 6:5. What is the area of rectangle.

A. 30

B. 60

C. 120

D. None of these

3. The circumference of two circles are 264 m and 352 m. Find the difference between area of the larger and smaller circles.

A. 4123

B. 8642

C. 4312

D. 2612

4. What would be the cost of building 7 m wide garden around a circular field with diameter equal to 280m, if the cost per sq. m for building the garden is Rs. 21

A. Rs. 156242

B. Rs. 248521

C. Rs. 132594

D. None of these

5. A cow is tied on the corner of a rectangular field of size 30m*20m by a 14 m long rope. The area of the region that she can graze is

A. 350 sq. m

B. 196 sq. m

C. 154 sq. m

D. 22 sq. m

6. At each corner of a triangular field of side 26m, 28m and 30m a cow is tethered by a rope of length 7m. The area un-grazed by the cow is

A. 336 sq. m

B. 259 sq. m

C. 154 sq. m

D. 77 sq. m

7. The radius of a circular field is equal to the side of a square field. If the difference between the perimeter of the circular field and that of the square field is 32m, what is the perimeter of the square field?

A. 84m

B. 95m

C. 56m

D. 28m

8. Two equal maximum sized circular plates are cut-off from a circular paper-sheet of circumference 352 cm. The circumference of each circular plate is.

A. 176 cm

B. 180 cm

C. 165 cm

D. 150 cm

9. A wire, when bent in the form of a square, encloses a region having area 121 cm2. If the same wire is bent into the form of a circle, then the area of the circle is?

A. 144 sq. cm

B. 180 sq. cm

C. 154 sq. cm

D. 176 sq. cm

10. Four circles having equal radii are drawn with centres at the four corners of a square. Each circle touches the other two adjacent circle. If remaining area of the square is 168 cm square, what is the size of the radius of the circle?

A. 1.4 cm

B. 14 cm

C. 35 cm

D. 21 cm

11. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is

A. 6 cm

B. 12 cm

C. 36 cm

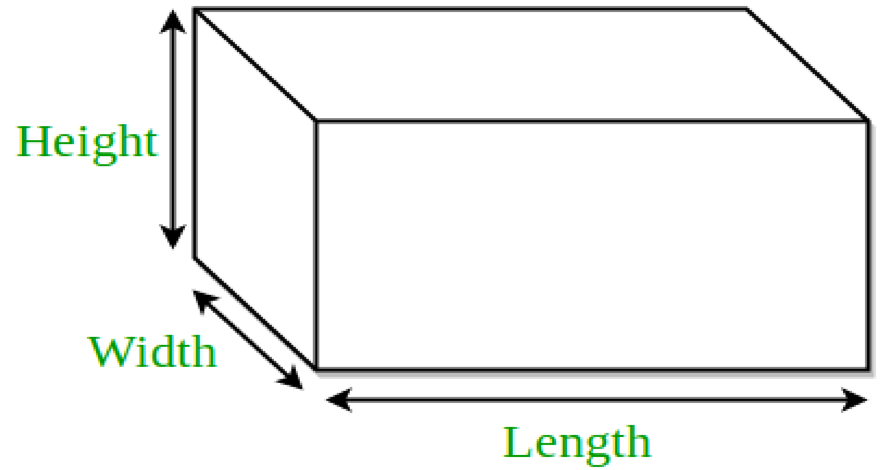
D. 72 cm

Answer key

1 C	2 A	3 C
4 C	5 C	6 B
7 C	8 A	9 C
10 B	11 D	

3D Figures:

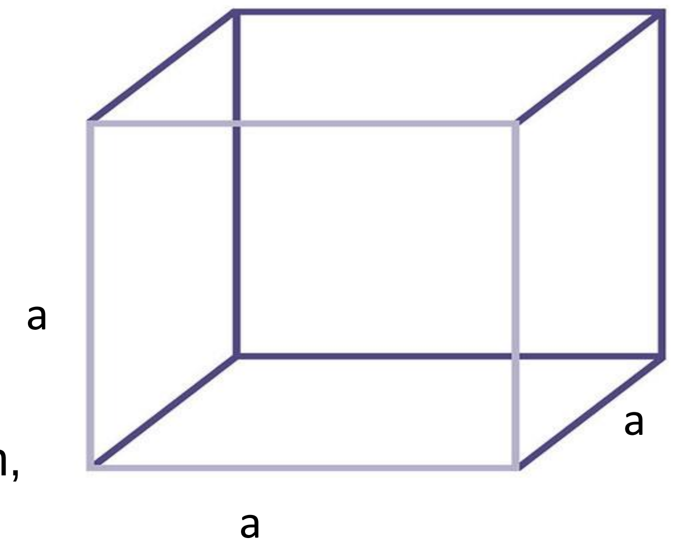
Cuboid



Let
l= length
b= breadth
h= height. Then,

- **Volume** = (*l* x *b* x *h*) cubic units.
- **Lateral Surface area** = 2(*b* + *l*) h sq. units.
- **Surface area** = 2(*lb* + *bh* + *lh*) sq. units.
- **Diagonal** = $\sqrt{(l^2 + b^2 + h^2)}$ units.

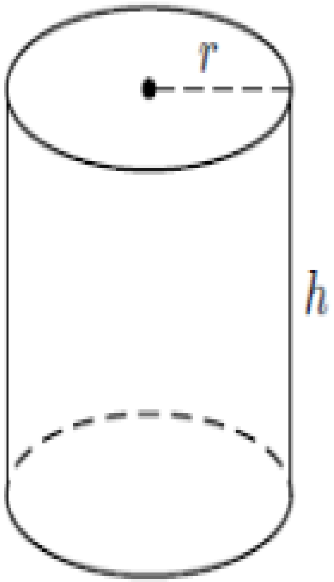
Cube



Let each edge of a cube be of length *a*. Then,

- **Volume** = a^3 cubic units.
- **Lateral Surface area** = $4a^2$ sq. units
- **Surface area** = $6a^2$ sq. units.
- **Diagonal** = $\sqrt{3}a$ units.

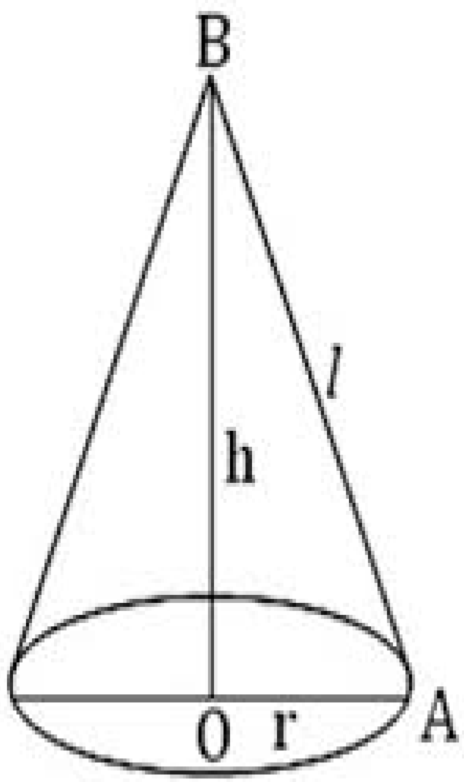
Cylinder



Let radius of base = r and Height (or length) = h. Then,

- **Volume** = $(\pi r^2 h)$ cubic units.
- **Curved surface area =** $(2\pi r\, h)$ sq. units.
- **Total surface area** = $2\pi r(h + r)$ sq. units.

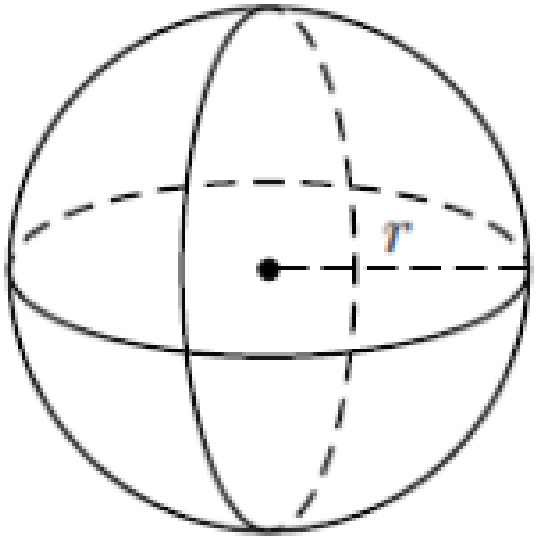
Cone



Let radius of base = r and Height = h. Then,

- **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
- **Volume** = $(1/3)\,\pi r^2 h$ cubic units.
- **Curved surface area** = (πrl) sq. units.
- **Total surface area** = $(\pi rl + \pi r^2)$ sq. units.

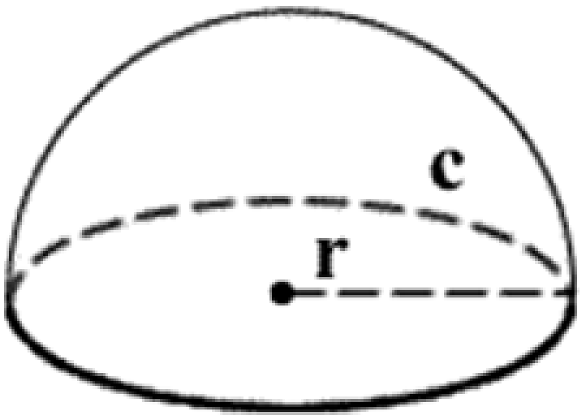
Sphere



Let the radius of the sphere be *r*. Then,

- **Volume** = (4/3) π r^3 cubic units.
- **Surface area** = (4 π r^2) sq. units.

Hemisphere



Let the radius of a hemisphere be *r*. Then,

- **Volume** = (2/3) πr^3 cubic units.
- **Curved surface area** = ($2 \pi r^2$) sq. units.
- **Total surface area** = ($3 \pi r^2$) sq. units.

Practice Questions

1. If the areas of the three adjacent faces of a cubical box are 120 cm square, 72 cm square and 60 cm square respectively, then the volume of the box is:

A. 800 cm cube

B. 680 cm cube

C. 700 cm cube

D. 720 cm cube

2. A wooden box measures 20 cm × 12 cm × 10 cm. the thickness of the wood is 1 cm. The volume of the wood required to make the box is:

A. 960 cm cube

B. 900 cm cube

C. 1000 cm cube

D. 1100 cm cube

3. The size of a wooden block is (15 cm × 12 cm × 20 cm). How many such blocks will be required to construct a solid wooden cube of minimum size?

A. 50

B. 40

C. 60

D. 55

4. Two solid cylinders of radii 4 cm and 5 cm and lengths 6 cm and 4 cm, respectively are recast into cylindrical disc of thickness 1 cm. The radius of the disc is

A. 7 cm

B. 14 cm

C. 21 cm

D. 28 cm

5. The radius of cross-section of a solid cylindrical rod of iron is 50 cm. The cylinder is melted down and formed into 6 solid spherical balls of the same radius as that of the cylinder. The length of the rod (in m) is

A. 0.8

B. 2

C. 3

D. 4

6. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:

A. $(\sqrt{3}+1) : 3 : 4$

B. $(\sqrt{2}+1) : 7 : 8$

C. $(\sqrt{2}+1) : 3 : 4$

D. None of these

7. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller spheres over that of the large sphere?

A. 200%

B. 400%

C. 500%

D. can’t be determined

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube (in cm) is:

A. 23

B. 24

C. 25

D. 26

9. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the Centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of:

A. 3 : √3 : 2

B. 3 : 2 : 1

C. 1 : 2 : 3

D. 2 : 3 : 1

10. The height of a cone is 40 cm. The cone is cut parallel to its base such that the volume of the small cone is 1/64 of the cone. Find at which height from base the cone is cut?

A. 20 cm

B. 30 cm

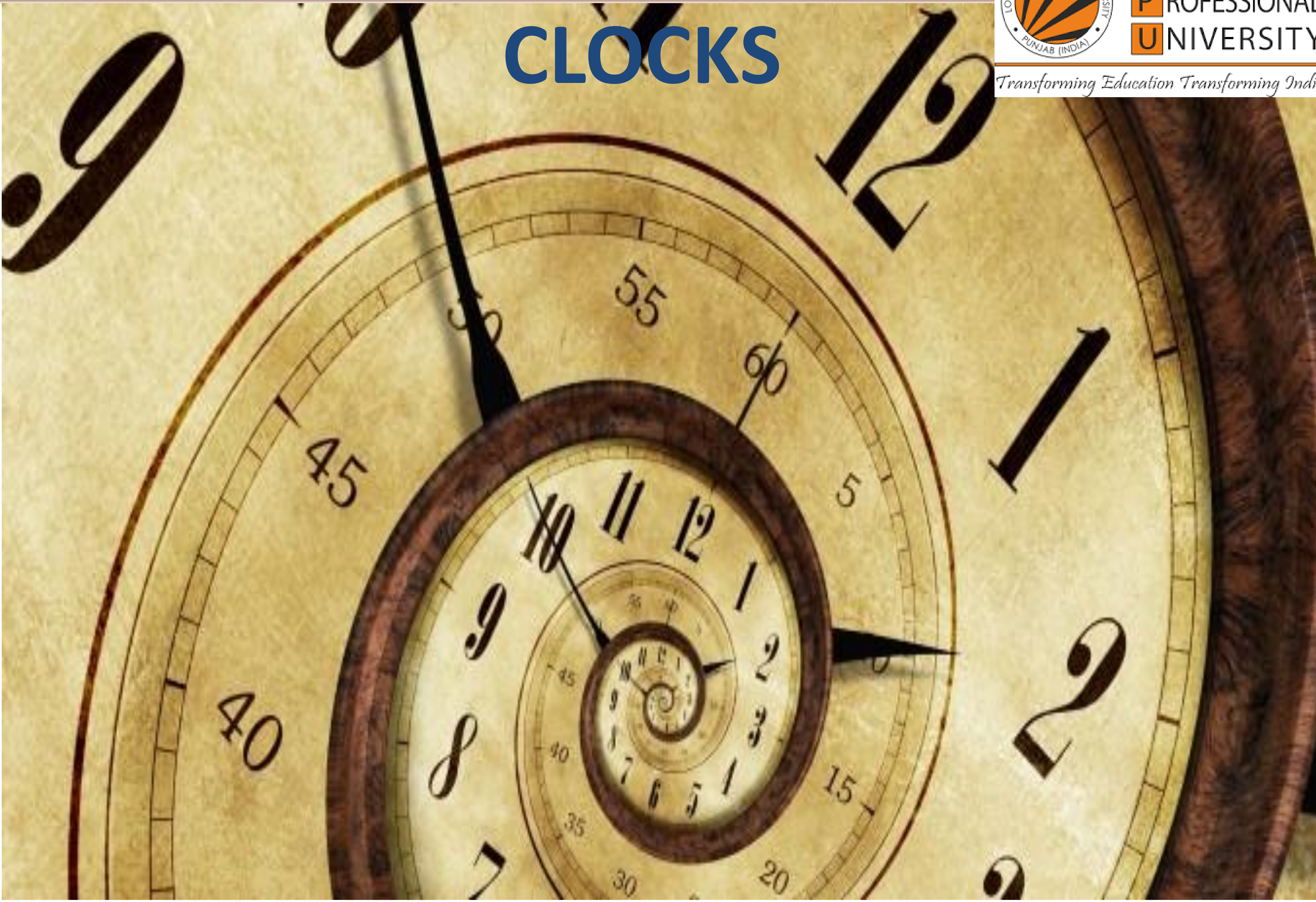
C. 25 cm

D. 22.5 cm

Answer Key

1 D	2 A
3 C	4 B
5 D	6 C
7 B	8 C
9 B	10 B

CLOCKS



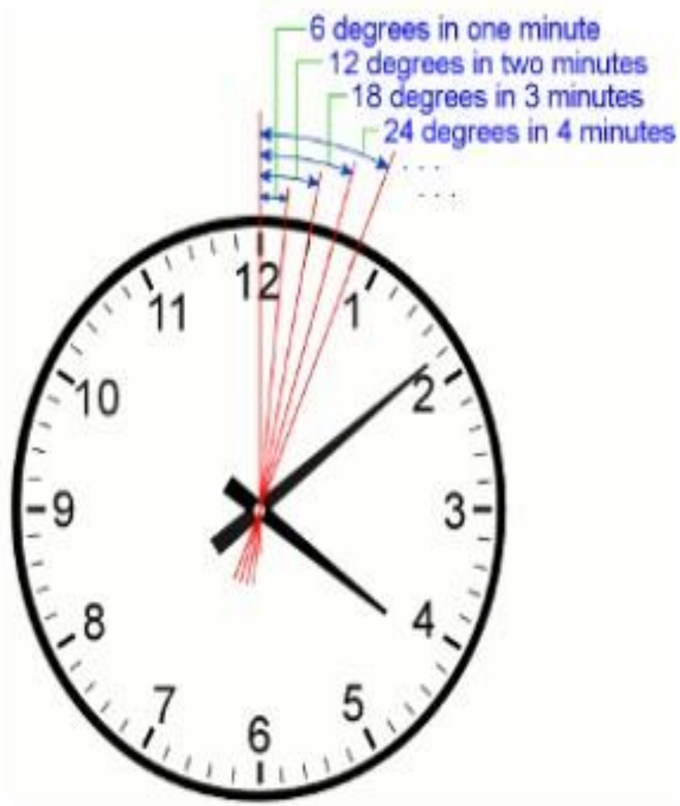
CONCEPT BASE

DIAL:

- The face or dial of a clock is a circle whose circumference is divided into 60 equal parts, called minute spaces.
- They are marked by short lines in the face of the clock. However, the end of every fifth minute space is marked longer than the others and the 60 minute spaces are represented as 12 divisions or 12 hour spaces.

As the minute hand takes a complete round in one hour it covers 360° in 60 min.

In 1 minute it covers 360/60 = 6°/ minute.



HANDS OF THE CLOCK:

- A clock has two hands; the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand as shown in the figure above.
- The hands of the clock are the indicators of time.
- The time is read based on the positions of the hands with respect to minute or hour spaces.

UNIT OF TIME:

- The unit of time in clocks is seconds.
- A second, the SI unit of time, is defined as the natural periodicity of the radiation of a caesium-133 atom.
- A duration of 60 seconds is defined as a minute.
- In clocks, it is also defined as the time taken by the minute hand to move across a minute space.
- A duration of 60 minutes is defined as an hour.
- In clocks, it is also defined as the time taken by the hour hand or short hand to move across five minute spaces or one hour space.
- A day is also defined as the duration of 24 hours
- The time is measured owing to the movement of the hands of a clock. A complete rotation of any of the hands would cover a measure of 360 degrees (circular displacement) .
- Hour hand covers an angle of 360° in 12 hours. So, the hour hand in one hour will cover 360 /12 = 30°.
- So for every minute, the hour hand moves through 30 /60 = 0.5 °.

- Minute hand covers an angle of 360° in 1 hour. So for every minute, the minute hand moves through 360/60 = 6 °.
- Both the hands of the clock move in same direction.
- So, their relative displacement for every minute is 5.5° or 11/2°.
- This 5.5° movement constitutes the movements of both the hands.
- So for every minute, both the hands give a relative displacement of 5.5°.

FORMULA TO FIND THE ANGLE BETWEEN THE HANDS OF THE CLOCK:

- Let us say in a race, Sam gives a head-start of 120 metres to Anand.
- Sam and Anand run at the speed of 10 m/s and 6 m/s respectively.
- Running in the same direction, the relative displacement is $10 - 6 = 4$ metres for every second.
- Now, let us calculate the time taken by

  (i) Sam to catch up with Anand

  (ii) Sam to lead Anand by 20 metres

(i) Time taken by Sam to catch up with Anand

Once they begin to run, the initial gap of 120 metres starts decreasing at the rate (relative speed) of 4 m/s.

Therefore, it takes

$4 \times t = 120$ (Relative Speed × Time = Relative distance or gap)

$t = 30$ secs to catch up with Anand or to say that they are at the same point.

(ii) Time taken by Sam to lead Anand by 20 metres:

We know that Sam needs 30 secs to catch up with Anand and thereby it can be considered that both of them are running, starting at the same point after 30 secs.

Though Sam runs at 10 m/s, the gap between them increases at the rate of their relative speed i.e. 4 m/s.

Hence, it takes $4 \times t = 20$, $t = 5$ secs to lead

Anand by 20 metres when they start from the same point.

Thus, from the starting point Sam takes

$30 + 5 = 35$ secs to lead Anand by 20 metres.

However, the answer can be directly found by taking the relative distance to be

$120 + 20 = 140$ metres

$4 \times t = 140$

$t = 35$ secs.

From (i) and (ii), we can think of the hands of the clock as two runners running in a circular track of 360°. Here, the angle between the hands of the clock should be considered as the relative distance between them.

Therefore, we can formulate the following

Angle between the hands of the clock = Initial gap between the hands of the clock ~ Relative displacement for a period of time

$\theta = 30°H - 11/2M$

(when the minute hand is trailing the hour hand)

$\theta = 11/2M - 30°H$

(when the minute hand is leading the hour hand)

- θ = Angle between the hands of the clock
- $30°H$ = Initial gap; H - the division pointed by the hour hand which indicates the gap between the minute hand and hour hand initially when multiplied by 30°.
- $11/2M$ = Relative displacement for a period of time; 11/2 - Relative speed of the hands of the clock; M - no. of minute spaces moved by the minute hand.

COINCIDENCE OF THE HANDS OF THE CLOCK:

- At 12 o' clock, the hands of the clock overlap i.e. the angle formed between them is 0°.
- To find the next time they overlap, the gap between them should be 360° so that they are at the same point. Hence it takes,

$$11/2 \times t = 360$$

$$t = 720/11 = 65\ \ 5/11 \text{ minutes}$$

Thus, it can be easily concluded that the hands of the clock coincide every 65 5/11 minutes.

FREQUENCY OF FORMATION OF ANGLES BETWEEN THE HANDS OF THE CLOCK:

- With the relative displacement, it is easier to find the frequency at which the hands of a clock form a particular angle.
- Let's find the no. of times the hands of the clock form 0° in a day.
- We know that the hands of the clock coincide every 65 5/11 or 720/11 minutes. So, in a day there are 24 × 60 = 1440 minutes
- Therefore, in a day the hands of the clock coincide (1440 × 11) / 720 = 22 times.
- Thus, it can be inferred that the hands of the clock coincide once in every hour except for two occasions i.e. from 11 to 1 o' clock they coincide only once at 12 o' clock.
- So, it is 11 times in 12 hours and thus 22 times in 24 hours.
- The finding can be extended to other angles by applying the formula,

$$\theta = 30^\circ H - 11/2M \text{ or}$$

$$\theta = 11/2M - 30^\circ H$$

for every hour in the clock.

GAIN AND LOSS OF TIME IN AN INCORRECT CLOCK:

- A normal clock becomes incorrect when there is a change in the speeds of the hands of the clock.
- As seen already, duration of a day is measured by the time taken by the hour hand to move across the 12 divisions twice.
- So, when the speed of the hands of the clock increases they complete their rotation for a day sooner.
- Thus, they result in loss of time when actually compared with the duration for a day in a correct clock .
- Similarly, when the hands of the clock move slower than usual, they take more time to complete the rotations for a day.
- Thus, they result in gaining more time for a day when compared with that in a correct clock.

KEY POINTS:

- In 60 minutes, the minute hand gains 55 minutes on the hour hand
- The hands of the clock coincide every 65 5/11 minutes and for every hour, both the hands coincide once
- The hands are in the same straight line when they are coincident or opposite to each other
- When the two hands are at right angles, they are 15 minute spaces apart
- When the hands are in opposite directions, they are 30 minute spaces apart
- The hands of the clock form an angle of 0° or 180° − 22 times a day
- The hands of the clock form an angle of 1° to 179° − 44 times a day
- If a watch or a clock indicates 8:15, when the correct time is 8, it is said to be 15 minutes faster than the correct time. On the other hand, if it indicates 7:45, when the correct time is 8, it is said to be 15 minutes slow.

PRACTICE QUESTIONS

1. Find the angle between the hands of a clock when the time is 5:40.

(a) 80º
(b) 160º
(c) 70º
(d) 120º

2. Find the angle between the minute hand and the hour hand of a clock when the time is 7:20.

(a) 80º

(b) 90º

(c) 100º

(d) 110º

3. The reflex angle between the hands of a clock at 10:25 is

(a) 180º
(b) 162.5º
(c) 165º
(d) 197.5º

4. Find the angle between the hour hand and the minute hand of a clock when the time is 15:25.

(a) 47.5 º

(b) 45.5 º

(c) 50 º

(d) None of these

5. At what time between 5 p.m. and 6 p.m., do the hands of a clock coincide?

(a) 8 hour 29 3/11 min

(b) 9 hour 33 8/11 min

(c) 5 hour 27 3/11 min

(d) None of these

8. Find at what time (in minutes) past 8 o' clock but before 9 o' clock will the hands of a clock be in the same straight line but not together.

(a) 100/11
(b) 110/11
(c) 120/11
(d) None of these

9. How many times in a day do the hands of a clock form 60º?

(a) 22
(b) 33
(c) 44
(d) 55

10. A few times per day, the minute hand of a clock is exactly above (or below) the hour hand. How many times per day does this occur?

(a) 11 times

(b) 24 times

(c) 22 times

(d) 44 times

11. At how many times between 12 o'clock and 1 o' clock are the minute hand and the hour hand of a clock at an angle of 90 degrees to each other?

(a) 4

(b) 6

(c) 3

(d) 2

12. How much does a watch gain per day, if its hands coincide every 64 minutes?

(a) 32 8/11 min
(b) 36 5/11 min
(c) 90 min
(d) 96 min

13. My watch was 8 minutes behind at 8 p.m. on Sunday but it was 7 minutes ahead of time at 8 p.m. on Wednesday. During this period, at which time has this watch shown the correct time?

(a) Tuesday 10.24 a.m.

(b) Wednesday 9.16 p.m.

(c) Tuesday 10.24 p.m.

(d) Wednesday 9.16 a.m.

14. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose in minutes?

(a) 1440/143

(b) 1444/143

(c) 1400/143

(d) 4440/143

15. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 p.m. on the following day?

(a) 48 min past 12

(b) 48 min past 11

(c) 45 min past 12

(d) 45 min past 11

16. At what time between 4 and 5 o'clock, will the hands of a watch point in opposite directions?

(a) 54 min past 4

(b) 53 7/11 min past 4

(c) 54 7/11 min past 4

(d) 54 6/11 min past 4

17. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is

(a) 4 p.m.

(b) 6 p.m.

(c) 5 p.m.

(d) 7 p.m.

18. At what time between 5.30 and 6 o' clock, will the hands of a clock be at right angles?

(a) 43 5/11 min past 5
(b) 43 7/11 min past 5
(c) 5 p.m.
(d) 7 p.m.

19. Find the reflex angle between the hour hand and the minute hand of a clock when the time is 11:25.

(a) 192.5 º

(b) 230 º

(c) 45 º

(d) 72 º

20. How many minutes is it until six o’clock if fifty minutes ago it was four times as many minutes past three o’clock?

(a) 26 min
(b) 27 min
(c) 30 min
(d) None of these

Answer Key

1 C	2 C	3 D	4 A	5 C
6 C	7 B	8 C	9 C	10 C
11 D	12 A	13 A	14 A	15 A
16 D	17 A	18 B	19 A	20 A

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CALENDAR

Content

I. Introduction

The Calendar which we currently follow is called the **Gregorian Calendar**. It is named after Pope Gregory XIII, who introduced it in October 1582.

The calendar was a refinement to the Julian calendar amounting to a. The motivation for the reform was to stop the drift of the calendar and set the date for Easter celebrations. Transition to the Gregorian calendar would restore the holiday to the time of the year in which it was celebrated when introduced by the early Church.

The reform was adopted initially by the Catholic countries of Europe. Protestants and Eastern Orthodox countries continued to use the traditional Julian calendar and adopted the Gregorian reform after a time, for the sake of convenience in international trade.

The last European country to adopt the reform was Greece, in 1923.

The Gregorian reform modified the Julian calendar's scheme of leap years as follows:

1. Every year that is exactly divisible by four is a leap year, except for years that are exactly divisible by 100, but these centurial years are leap years if they are exactly divisible by 400. For example, the years 1700, 1800, and 1900 are not leap years, but the years 1600 and 2000 are.

2. In addition to the change in the mean length of the calendar year from 365.25 days (365 days 6 hours) to 365.2425 days (365 days 5 hours 49 minutes 12 seconds

II. What are Ordinary and Leap years??

The time taken by the Earth to make one complete revolution is said to be a complete Year, which is equivalent to exactly

365.2425 days.

*But ignoring the **.2425** we generalize an **ordinary** year to be of **365** days.*

*Further for approximate calculation we assume the 365.2425 days to be **365.25** days.*

*This extra **0.25** days in **4 years** becomes **1 full day**. Hence that extra day is added to a year **every 4 years**. That particular year is called a **leap** year, which has one extra day than the ordinary year, which means a leap year has **366 days** in total.*

365.2425

0.0025 when ignored

365.24

0.24 approximated to 0.25 for simpler calculation

365.25

The 0.25 becomes 1 extra day in 4 years

Only the 365 days taken

Ordinary year
365 days

Leap year
=Ordinary year+(0.25x4)
=365+1
=**366 days**

How to check for a leap year?

1. Non Century Years:

If any non-century year (not ending with “00”) is divisible by 4 (last two digits of the year should be divisible by 4), then it is said to be a leap year.

Reason: As explained in the previous slide, the 0.25 extra days of 365.25 days become one extra day every 4 years (0.25x4=1).

Hence every 4 years we have one leap year.

Ex: 1844, 1892, 1996, 2004, 2012, 2016 etc

2. Century Years:

But in case of a century year(Years ending with “00”,ex: 1400,1600,2000,etc), the year will be a leap year only if it is divisible by 400.

Reason:

365.2425

↓

365.25

Let us take an example. The 100th year is divisible by 4. But it is not a leap year.

Why???

The approximation we make while assuming 365.24 days to be 365.25 days (increasing the value by 0.01), leads to an extra 1 day which does not exist in real. Hence this extra 1 day which is forming every 100 year should be removed from the 100th year. So, although the hundredth year is divisible by 4 and should be a leap year, we remove that extra one day of the leap year from the 100th year. Hence it ends up as an ordinary year only.

But why is the 400th year a leap year then???

So, till now we learnt that the 100th year (Ex: 100,200,300 etc) is not a leap although it is divisible by 4.

But why is 400th year a leap year then???

This answer also lies in the value **365.2425** days which is the time taken by the earth for one revolution.

In 365.24**25** we initially ignore **0.0025.** which is a negligible value for one year. But in 400 years, this **0.0025** becomes 1 complete day **(0.0025x4=1) and we add this extra day every 400th year.**

Although like every 100th year, 400th year also should have been an ordinary year, but because of the addition of this extra day(mentioned above), every 400th year is a leap year.

And that is why every century leap year is divisible by 400.

Example: 400,1200,1600,2000 etc.

Let us take a quick quiz(Ask the students)

Q1. Is 1996 a leap year? Why?

Q2. Are all the years divisible by 4 leap years? Why?

Q3. How do you check for a century year, whether it is a leap year or not? Why?

Answers

- 1) Yes, as it is completely divisible by 4
- 2) No, the century year should be divisible by 400. For ex 100 is not a leap year.

3) The century year is completely divisible by 400 then it is a leap year, otherwise not.

III. Odd day concept

If we are supposed to find the day of the week on a given date, we use the concept of 'odd days'. **In a given period, the number of days more than the complete weeks are called odd days .**

For example the number of odd days in 15 total days = 1, since remainder when 15 is divided by 7 is 1.

Similarly the number of odd days in the month of January is 3. Since January has 31 days,

31/7= R(3).

Note: The odd day value will always lie from 0 to 6. If the odd day value is more than 6, keep dividing it by 7 until the remainder value comes below 7. The remainder value smaller than 7 is the final odd day value.

Finding odd days

1. In a single year

Now that you know what are odd days, can you find out how many odd days are there in an ordinary year and a leap year respectively???

1. Ordinary year:

Total no. of days=365 days

365/7= R(1)

Odd days = 1

2. Leap year:

Total number of days= 366

366/7= R(2)

Odd days= 2

Finding odd days

2. Through out a century/year span

Let us see through examples how to find the number of odd days through out a century or year span.

Ex: Find the number of odd days in the first 35 years in a century.

Step1: Let us first find the number of leap years and ordinary years in these 35 years. 35 years have **8 leap years** (4x8=32, closest to 35).

Therefore no. of ordinary years= 35-8 = **27 years**.

Step2: Odd days in 8 leap years= 8x2=**16** (Since 1 leap year has 2 odd days)

Odd days in 27 ordinary years= 27x1=**27** (Since 1 ordinary year has 1 odd day)

Step3: Total number of odd days=(16+27)=43= 43/7=R(1)

Therefore, number of odd days in 35 years = **1**

Following the same steps, find the number of odd days in the first century(first 100 years).

100

(No. of leap years) **24** **76** **(No. of ordinary years)**

(Odd days in 24 leap years) **48** **76** **(Odd days in 76 ordinary years)**

Total no. of odd days=(48+76)=124

124/7=R(5)

Hence, the first 100 years(century) have 5 odd days in total**.**

Note: The first 100 years have 24 leap years, not 25.

Because 100th year is not a leap year as discussed in **slide no.8.**

Odd days in the centuries

Since first 100 years is having 5 odd days, we can say:

First 200 years have 5x2=10, 10/7= R(3)

First 300 years have 5x3=15, 15/7= R(1)

First 400 years have 5x4=20, 20/7=R(6) + 1= 7= R(0)

And the cycle keeps repeating for the next centuries from here.

Note: 1 extra day has been repeated for 400 years, since 400^{th} year is a leap year as mentioned in earlier slides.

Until Century	Up to Year	Odd days	Similar centuries	
1^{st}	100	5	500	>
2^{nd}	200	3	600	>
3^{rd}	300	1	700	>
4^{th}	400	0	800	>

Odd days in the months

Month	No. of days	Odd days
January	31	3
February	28,29	0,1
March	31	3
April	30	2
May	31	3
June	30	2
July	31	3
August	31	3
September	30	2
October	31	3
November	30	2
December	31	3

Concept Review Question

Q. If May 10, 1997 was a Monday, what will be the day on Oct 10, 2001?

A. Wednesday
B. Thursday
C. Friday
D. Saturday

Solution:

In this question the reference point is May 10, 1997 and we have to find the number of odd days from May 10, 1997 up to Oct 10, 2001.

Now, from May 11, 1997 - May 10, 1998 = 1 odd day
May 11, 1998 - May 10, 1999 = 1 odd day
May 11, 1999 - May 10, 2000 = 2 odd days (2000 was leap year)
May 11, 2000 - May 10, 2001 = 1 odd day

Thus, the total number of odd days up to May 10, 2001 = 5.

Now, the remaining 21 days of May will give 0 odd days.

In June, we have 2 odd days; in July, 3 odd days; in August, 3 odd days; in September,2 odd days and up to 10^{th} October, we have 3 odd days. Hence, total number of odd days = 18 i.e. 4 odd days.

Since, May 10, 1997 was a Monday, then 4 days after Monday will be Friday. So, Oct 10, 2001 would be a Friday.

IV. 1. a) Finding the day for a particular date

1. *Odd days in the year(Actual year-1)*
2. *Odd days in the month(Actual month-1)*
3. *Odd days in the date*

Get the sum of the odd days and the final odd day value from it.

Ex: 15/07/2016

1. *Odd days in 2015*
2. *Odd days in January to June*
3. *Odd days in the date(15/7= R(1))*

Sum/7=Odd day value

Odd day value	1	2	3	4	5	6	7 or 0
Day of the week	Mon	Tue	Wed	Thu	Fri	Sat	Sun

Concept review Questions:

Using the above mentioned conventional method, find the day for the following dates:

1. Independence day of India

2. Republic Day of India

IV. 2 b) Shortcut to find the day

1. Month Code:

1	4	4		0	2	5		0	3	6		1	4	6
Jan	Feb	Mar		Apr	May	Jun		July	Aug	Sept		Oct	Nov	Dec

2. Century Code

Years between	Code/Odd days
1600-1699	6
1700-1799	4
1800-1899	2
1900-1999	0
2000-2099	6

Steps:
Find the sum of:
1. Date
2. Last 2 digits of the year
3. Quotient of last two digits of the year when divided by 4
4. Code of month
5. Odd days of the year

The odd days in the above sum value will give the day

Note:
Incase of months of January and February in a leap year, subtract one odd day from the total odd days.

Concept review Questions:

Using the Shortcut-method, find the day for the following dates:

1. Independence day of India
2. Republic Day of India

IV.3. Finding years with similar calendars

For two different years having the same calender, the following conditions should be satisfied:

1. *Both years must be of the same type. i.e., both years must be ordinary years or both years must be leap years.*

2. *1^{st} January of both the years must be the same day of the week.*

It simply means that for a year to have the same calendar with X year ,the total odd days from X should be 0.

Let us understand with an example.

Example:

For a year to have the same calendar with 2007 ,the total odd days from 2007 should be 0.

As we know how to recognize an ordinary and leap year, and also we know the number of odd days they have, we can make the below table:

Year	:	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Odd Days	:	1	2	1	1	1	2	1	1	1	2	1

Since up to 2017 the number of odd days become 0, **hence the next year(2018)** will have the same calendar as 2007.
And also both the years are ordinary years, hence the first condition is also satisfied.

Shortcut (Only when you are not crossing a an ordinary century year) :
Leap year calendar repeats every 28 years.

LY	1ST	2ND	3RD	LY
28	6	11	11	28

Here 28 is distributed as 6+11+11.

Rules:

a) If given year is at 1st position then next repeated calendar year is Given+6.

b) If given year is at 2nd position then next repeated calendar year is Given+11.

c) If given year is at 3rd position then next repeated calendar year is Given+11.

Example:

Find the year which as same calendar as that of 2007 after it.

Example:

Find the year which as same calendar as that of 2007 after it.

Sol:

LY	1ST	2ND	3RD	LY
28	6	11	11	28

Given year is 2007

According to the above Rule:
2007 is at the 3rd position. So, add 11 yrs.

2007+11=2018
so, the same Calendar after 2007 is 2018.

Concept Review Question

1) Find the year after 2015 which has the same calendar as 2015.

A. 2019
B. 2023
C. 2026
D. 2029

Concept Review Question

2) The year after 1996 having the same calendar as of 1996 will be

A. 1999

B. 1998

C. 2001

D. None of these

Answer Key (Concept review question)

1 C	2 D

PRACTICE QUESTION

Q1. If today is Monday, what will be the day one year and 50 days from now? (Tech Mahindra-2011)

A. Tuesday
B. Wednesday
C. Thursday
D. Can not be determined

Q3. If day before Yesterday it was Monday, What day will fall on day after tomorrow ? (Tech Mahindra-2013)

a. Sunday
b. Tuesday
c. Saturday
d. Friday

Q4. Radha remembers that her father's birthday is after 16th but before 21st of March, While her brother Mangesh remembers that his father's birthday is before 22nd but after 19th of March. On which date is the birthday of their father?
(Tech Mahindra-2015)

a) 19th
b) 20th
c) 21st
d) Cannot be determined
e) None of these

Q5. 1.12.91 is the first Sunday. Which is the fourth Tuesday of December 91? (TCS-2015)

(a) 31.12.91
(b) 24.12.91
(c) 17.12.91
(d) 26.12.91

Q6. If the third day of the month is Monday, Which of the following will be the fifth day from 21st of that month? (Tech Mahindra-2015)

a) Tuesday
b) Monday
c) Wednesday
d) Thursday
e) None of these

Q7. If 8th Dec 2007 was Saturday, then what day of the week was it on 8th Dec 2006? (Capegemini-2015)

(a) Monday

(b) Thursday

(c) Friday

(d) Sunday

Q8. Two brothers were expected to return here on the same day. Rajat returned 3 days earlier, but Rohit returned 4 days later. If Rajat returned on Thursday, what was the expected day when both the brothers were to return home and when did Rohit Return?(Infosys 2015)

a) Wednesday, Sunday
b) Thursday, Monday
c) Sunday, Thursday
d) Monday, Friday

Q9. 8th Dec 2009 was Tuesday, what day of the week was it on 8th Dec 2006? (SSC,2016)

A. Sunday
B. Tuesday
C. Friday
D. Tuesday

Q11. What was the day of the week on January 1, 1998? (Tech Mahindra-2016)

a. Wednesday
b. Monday
c. Friday
d. Thursday

Q13. The day of week on July 1,2000 was
(Tech Mahindra-2014)

a) Monday
b) Friday
c) Saturday
d) Tuesday
e) None of the above

Q14. On what dates of August 1980 did Monday fall? (TCS-2014)

A. 4th, 11th, 18th, 25th
B. 3rd, 10th, 17th, 24th
C. 6th, 13th, 20th, 27th
D. 9th, 16th, 23rd, 30th

Q15. If we suppose the 60th independence day of India was on Thursday, then the 85^{th} independence day would have been on?

A. Monday
B. Wednesday
C. Friday
D. Sunday

Q16. If 1 Jan 2015 is Thursday, then probability of 22nd October to be Wednesday will be? (Infosys-2015)

A. 0
B. 0.5
C. 1
D. Can not be determined

Q17. The calendar for 1992 is the same as for (Tech Mahindra-2012,2015)

a) 1997
b) 2020
c) 2016
d) 1994

Q18. The calendar of year 1982 is same as which year?

A. 1988
B. 1990
C. 1992
D. 1993

Q21. History Professor Nagarajan was talking to the students about a century which has started with a Monday. What day India would be witnessing on the last day of the century, the Professor was posing a question. Incidentally he posed a question that the last day of the century cannot be:

(a)Monday (b) Tuesday

(c)Wednesday (d) Friday

Can you answer the Professor ‘s question?

Answer key (Practice Questions)

1 D	2 B	3 D	4 B	5 B	6 C	7 C	8 C	9 C	10 D
11 D	12 B	13 C	14 A	15 A	16 A	17 B	18 D	19 C	20 B
21 B									


THANKS
FOR
LISTENING