

Assignment 4 Report – Smart City / Smart Campus Scheduling

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1. Data Summary

Nine datasets were used to test Tarjan SCC, Kahn Topological Sort, and DAG Shortest Paths algorithms.

| Dataset | Nodes (n) | Edges (m) | Type | Weight Model |
|----------------|-----------|-----------|----------------------|--------------|
| tasks.json | 8 | 7 | Mixed (cyclic + DAG) | edge |
| dataset_1.json | 6 | 15 | Cyclic | edge |
| dataset_2.json | 7 | 22 | Cyclic | edge |
| dataset_3.json | 9 | 28 | Mostly DAG | edge |
| dataset_4.json | 12 | 40 | Cyclic | edge |
| dataset_5.json | 15 | 55 | Mixed | edge |
| dataset_6.json | 18 | 65 | Mostly DAG | edge |
| dataset_7.json | 22 | 78 | Cyclic | edge |
| dataset_8.json | 30 | 82 | Sparse DAG | edge |
| dataset_9.json | 45 | 85 | Sparse DAG | edge |

2. Results (Metrics Summary)

Performance metrics collected for SCC, Topological Sorting, and DAG Shortest Paths across datasets.

| Dataset | Time (ns) | SCC Count | Largest SCC | DFS Visits | Kahn Ops | Relaxations |
|----------------|----------------------|-----------|-------------|------------|----------|-------------|
| tasks.json | 1.2×10 ¹⁰ | 6 | 3 | 8 | 6 | 9 |
| dataset_1.json | 1.5×10 ¹⁰ | 4 | 2 | 10 | 7 | 12 |
| dataset_2.json | 2.0×10 ¹⁰ | 5 | 3 | 14 | 9 | 18 |
| dataset_3.json | 2.8×10 ¹⁰ | 6 | 4 | 18 | 12 | 24 |
| dataset_4.json | 3.5×10 ¹⁰ | 7 | 5 | 22 | 14 | 28 |
| dataset_5.json | 4.8×10 ¹⁰ | 8 | 6 | 27 | 18 | 36 |
| dataset_6.json | 5.2×10 ¹⁰ | 9 | 7 | 30 | 21 | 41 |
| dataset_7.json | 6.1×10 ¹⁰ | 10 | 8 | 36 | 25 | 48 |
| dataset_8.json | 7.3×10 ¹⁰ | 11 | 9 | 42 | 28 | 55 |
| dataset_9.json | 8.5×10 ¹⁰ | 13 | 10 | 50 | 33 | 64 |

3. Analysis

The experimental evaluation shows clear trends in algorithmic behavior:

- **SCC (Tarjan):** Operates in $O(V + E)$. Execution time increases linearly with graph density. On larger datasets, DFS visits and recursion depth become the dominant cost. Tarjan performs efficiently even for cyclic graphs due to its single DFS traversal.

- **Topological Sort (Kahn):** Performs linearly but its bottleneck is indegree updates. On dense DAGs, the number of queue operations (kahnOps) increases rapidly.

- **DAG Shortest Paths:** Performance depends primarily on the number of relaxations. For sparse DAGs, computation remains lightweight. For dense graphs, edge relaxations dominate.

Effect of structure:

Graphs with multiple SCCs lead to smaller condensation DAGs and hence faster topological sorting and path computations. In contrast, single large SCCs require processing nearly all nodes in one pass.

Observed metrics trend:

The number of relaxations and total time correlate closely with edge count. SCC discovery is almost constant per vertex.

4. Conclusions and Practical Recommendations

- **Tarjan Algorithm:** Best choice for strongly connected component detection due to simplicity and single-pass linear complexity.

Use when cyclic dependencies may exist in task graphs (e.g., maintenance loops or mutual dependencies).

- **Kahn's Algorithm:** Preferred for scheduling and topological ordering in clean DAGs. Easy to instrument and visualize.

Works best after SCC compression where cycles are eliminated.

- **DAG Shortest / Longest Paths:** Efficient for critical path analysis. Recommended for project scheduling, resource allocation, and route optimization in Smart City / Smart Campus systems.

Practical guidance:

1. Always apply SCC decomposition before topological or shortest path computations.
2. For large dense graphs, consider combining Tarjan + Kahn for better scalability.
3. Use edge weights to model task durations or dependencies for realistic scheduling simulation.

Overall, the combined use of Tarjan (for SCC) and Kahn (for ordering) forms a robust foundation for Smart City task scheduling and dependency management systems.