$y = Ae^{-at} + y_p(t)$

$y(t) = y(0^-)e^{-at} + y_p(t) - y_p(0)e^{-at} = [y(0^-) - y_p(0)]e^{-at} + y_p(t)$

$y_{transient}(t) = [y(0^-) - y_p(0)]e^{-at}$

$y_{steady}(t) = y_p(t)$

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$y_{zero\ input}(t) = y(0^-)e^{-at}$

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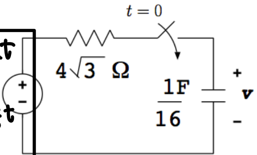
$y_{zero\ state}(t) = y_p(t) - y_p(0)e^{-at}$

ECE-210 Analog Signal Processing Spring 2022

Homework #5: Submission Deadline 23rrd March (10:00 PM)

1. Consider the following circuit with $v(0^-) = 1\,V$ and let $f(t) = 2e^{-t/3}$ V. For $t > 0$, obtain:



As $V(0^-) = 1V$ and $f(t) = 2e^{-t/3}$

$\Rightarrow C\frac{dv}{dt} \cdot 4\sqrt{3} + V = f(t)$

$\Rightarrow \frac{dv}{dt} + \frac{4}{\sqrt{3}}V = \frac{4f(t)}{\sqrt{3}} = \frac{8e^{-t/3}}{\sqrt{3}}$

$\Rightarrow y = [\int \frac{8e^{-t/3}}{\sqrt{3}} \cdot e^{\int \frac{4}{\sqrt{3}}dt}dt + C]e^{-\int \frac{4}{\sqrt{3}}dt}$

$y|_{t=0} = 1V$

$y = \frac{96+8\sqrt{3}}{47}e^{-\frac{1}{3}t} + \frac{-49-8\sqrt{3}}{47}e^{-\frac{4}{\sqrt{3}}t}$

c) $V_{tr} = [y(0) - y_p(0)]e^{-at}$
$= [1 - \frac{96+8\sqrt{3}}{47}]e^{-\frac{4}{\sqrt{3}}t}$
$= -\frac{49+8\sqrt{3}}{47}e^{-\frac{4}{\sqrt{3}}t}$

(a) the zero-state voltage across the capacitor's terminals, $v_{ZS}(t)$,

(b) the zero-input voltage across the capacitor's terminals, $v_{ZI}(t)$,

(c) the transient voltage across the capacitor's terminals, $v_{tr}(t)$,

(d) the steady state voltage across the capacitor's terminals, $v_{ss}(t)$, and

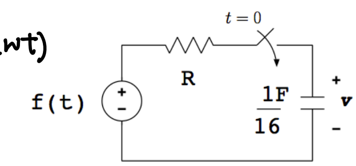
(e) the total voltage across the capacitor's terminals, $v(t)$.

d) $V_{ss} = \frac{96+8\sqrt{3}}{47}e^{-\frac{t}{3}}$

a) $V_{ZS} = y_p(t) - y_p(0)e^{-at}$
$= \frac{96+8\sqrt{3}}{47}e^{-\frac{1}{3}t} - \frac{96+8\sqrt{3}}{47}e^{-\frac{4}{\sqrt{3}}t}$

b) $V_{ZI} = y(0^-)e^{-at}$
$= e^{-\frac{4}{\sqrt{3}}t}$

2. Consider the following circuit with $f(t) = 10\cos(\omega t)$ volts and $v(0^-) = v_0$ volts.



$f(t) = 10\cos(wt)$

(a) $\frac{R}{16}\frac{dv}{dt} + V = f(t)$

$\Rightarrow \frac{dv}{dt} + \frac{16}{R}V = \frac{16}{R}f(t)$

$\frac{dv}{dt} + \frac{16}{R}V = \frac{160}{R}\cos(wt)$

It is known that for $t > 0$, $v(t) = Ae^{-t} + B\cos(2t) + C\sin(2t)$ volts.

(b) for 1st order linear ODE.

$y(t) = (\int q(t)e^{\int p(t)dt}dt + C) \cdot e^{-\int p(t)dt}$

only term have exponential term is $e^{-\int p(t)dt}$

$e^{-\int p(t)dt} = e^{-\int \frac{16}{R}dt} = e^{-t} \Rightarrow \boxed{R = 16\Omega}$

c) $y = (\int 10\cos(wt) \cdot e^t dt + C) \cdot e^{-t}$
$= (10\int e^{t(1+iw)}dt + C) \cdot e^{-t}$
$= (\frac{10e^{(1+iw)t}}{1+iw} + C)e^{-t}$
$= \frac{10e^{iwt}}{1+iw} + Ce^{-t}$
$= \frac{10(1-iw)e^{iwt}}{1+w^2} + Ce^{-t}$
$= \frac{10(1-iw)(\cos wt + i\sin wt)}{1+w^2} = \frac{10}{1+w^2}\cos(wt) + Ce^{-t}$
$= \frac{10}{1+w^2}(w\sin wt + \cos wt)$

For IVP $\{ L[V] = k \cdot f(t);\ V(0^-) = V_0 \}$

(a) Write the ODE that governs this system for $t > 0$ in terms of R, $v(t)$, and ω.

(b) Find the value of R.

(c) What is the value of ω?

(d) What are the values of B and C?

(e) If $v(t)$ is the zero-state response, what is the value of A?

(f) Identify $v_{tr}(t)$, the transient component of $v(t)$.

(g) Identify $v_{ss}(t)$, the steady-state component of $v(t)$.

(h) What is steady-state phasor V ?

$w = 2$ c)
$C = 4$ d)
$B = 2$
$A = V_0 - 2$

h) Steady state Phasor
$2\sqrt{5}\angle -\phi \quad \tan\phi = 2$

e) Zero state $\Leftrightarrow y(0^-) = 0$
$\Rightarrow V = 2\cos(2t) - 2e^{-t}V + 4\sin(2t)$

f) & g) $V_{tr} = [y(0) - y_p(0)]e^{-at}$
$= -2e^{-t}$

$V_{ss} = 2\sqrt{5}\cos(2t - \phi),\ \tan\phi = 2$

$V_{ss} = y_p(t) = 2\cos(2t) + 4\sin(2t) = 2\sqrt{5}(\frac{\sqrt{5}}{5}\cos(2t) + \frac{2\sqrt{5}}{5}\sin(2t))$

3. The different parts of this problem are unrelated:

(a) Show that $\frac{e^{j4t}+e^{-j4t}}{2} = \sin(4t + \frac{\pi}{2})$.

(b) Express $\frac{e^{-j2t}-e^{j2t}}{j}$ in terms of a sine function.

(c) Express $\mathrm{Re}\{2e^{j\frac{\pi}{3}}e^{-j5t}\}$ in terms of a cosine function.

(d) Determine the phasor F of $f(t) = -2\sin(2t - \frac{\pi}{3})$. Express F in both polar and rectangular coordinates.

(e) Determine the phasor F of $f(t) = \cos(3t - \frac{\pi}{2})$. Express F in both polar and rectangular coordinates.

(f) Express the phasor $F = 2 - j2$ in terms of a cosine function $f(t)$ having frequency $\omega = 3\ \frac{\text{rad}}{\text{s}}$.

(g) Express the phasor $F = 3e^{-j\frac{\pi}{3}}$ in terms of a cosine function $f(t)$ having frequency $\omega = 3\ \frac{\text{rad}}{\text{s}}$.

a) $\frac{1}{2}[e^{j4t} + e^{-j4t}]$
$= \frac{1}{2}[\cos 4t + j\sin 4t + \cos 4t - j\sin 4t]$
$= \cos 4t = \sin(4t + \frac{\pi}{2})$
Q.E.D.

(c) $\mathrm{Re}\{2e^{j(\frac{\pi}{3} - 5t)}\}$
$= 2\cos(\frac{\pi}{3} - 5t)$

d) $f(t) = -2\sin(2t - \frac{\pi}{3})$
$= 2\cos(2t - \frac{\pi}{3} + \frac{\pi}{2})$
$= 2\cos(2t + \frac{\pi}{6}) = \mathrm{Re}\{2e^{j(2t + \frac{\pi}{6})}\}$
$\Rightarrow F = 2e^{j\frac{\pi}{6}} = 2\angle\frac{\pi}{6} = 2\cos\frac{\pi}{6} + 2j\sin\frac{\pi}{6} = \sqrt{3} + j$

e) $f(t) = \cos(3t - \frac{\pi}{2}) = e^{j(3t - \frac{\pi}{2})}$
$F = e^{-j\frac{\pi}{2}} = \angle -\frac{\pi}{2}$
$= -j$

b) $\frac{e^{-j2t} - e^{j2t}}{j} = \frac{1}{j}(\cos 2t - j\sin 2t - \cos 2t - j\sin 2t)$
$= -2\sin 2t$

f) $F = 2\sqrt{2}(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j)$
$= 2\sqrt{2}(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4})$
$= 2\sqrt{2}e^{-j\frac{\pi}{4}}$
$\Rightarrow f(t) = \mathrm{Re}\{2\sqrt{2}e^{-j\frac{\pi}{4}}e^{j \cdot 3t}\}$
$= 2\sqrt{2}\cos(3t - \frac{\pi}{4})$

g) $F = 3e^{-j\frac{\pi}{3}}$
$f(t) = \mathrm{Re}\{3e^{-j\frac{\pi}{3}} \cdot e^{j \cdot 3t}\}$
$= 3\cos(3t - \frac{\pi}{3})$

4. Use the phasor method to express the following signals in terms of a single cosine function:

(a) $f(t) = -3\sin(3t) + 3\cos(3t)$.

(b) $g(t) = 2\left[\sin(2t) - \sin(2t + \pi/2)\right]$.

(a) $f(t) = 3\cos(3t) - 3\sin(3t)$

$= 3\sqrt{2}\left[\frac{\sqrt{2}}{2}\cos(3t) - \frac{\sqrt{2}}{2}\sin(3t)\right]$

$= 3\sqrt{2}\left(\cos\left(3t + \frac{\pi}{4}\right)\right)$

$= \mathrm{Re}\left\{3\sqrt{2}\, e^{\bar{i}\left(3t + \frac{\pi}{4}\right)}\right\} = \mathrm{Re}\left\{3\sqrt{2}\, e^{\bar{j}\frac{\pi}{4}}\, e^{3t}\right\}$

$F = 3\sqrt{2}\, e^{\bar{j}\frac{\pi}{4}}$, $w = 3$ rad/s

(b) $f(t) = -2\left[\cos(2t) - \sin(2t)\right]$

$= -2\sqrt{2}\cos\left(2t + \frac{\pi}{4}\right)$

$= \mathrm{Re}\left\{-2\sqrt{2}\, e^{\bar{j}\frac{\pi}{4}}\, e^{2t}\right\}$

$\Rightarrow F = -2\sqrt{2}\, e^{\bar{j}\frac{\pi}{4}}$, $w = 2$ rad/s