ZJU-UIUC INSTITUTE

Zhejiang University – University of Illinois at Urbana-Champaign Institute

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ECE-210 Analog Signal Processing Spring 2022
Homework #10: Submission Deadline 4th May (10:00 PM)

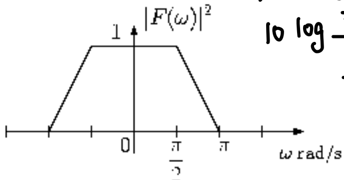
1. Determine the 3-dB bandwidth and the 95%-bandwidth of signals $f(t)$ and $g(t)$ with the following energy spectra:

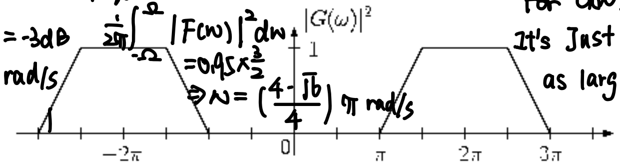
For: $F(w)$ ⇒ 3-dB band-width

$10\log\frac{\frac{2}{\pi}(\pi-w)}{1}=-3\text{dB}$ ⇒ $w=\frac{3}{4}\pi$ rad/s

95% band-width

$\frac{1}{2\pi}\int_{-\Omega}^{\Omega}|F(w)|^2dw=0.95\times\frac{3}{2}$ ⇒ $w=\left(\frac{4-\sqrt{6}}{4}\right)\pi$ rad/s





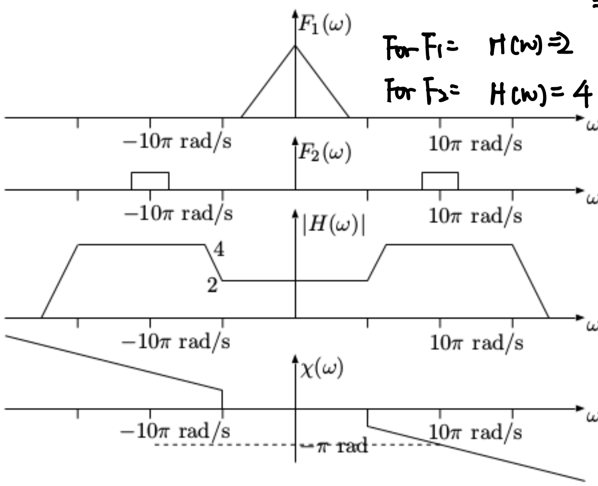
For $G(w)$
It's Just twice as large as first one
For 3dB: $\frac{3\pi}{2}$ rad/s
For 95% = $\left(\frac{4-\sqrt{6}}{2}\right)\pi$ rad/s

2. Let $f(t) = f_1(t) + f_2(t)$ such that $f_1(t) \leftrightarrow F_1(\omega)$ and $f_2(t) \leftrightarrow F_2(\omega)$, and let $H(\omega) = |H(\omega)|e^{j\chi(\omega)}$. The functions $F_1(\omega)$, $F_2(\omega)$, $H(\omega)$ and $\chi(\omega)$ are given graphically below. The signal $f(t)$ is the input to an LTI system with a frequency response $H(\omega)$. Express the output $y(t)$ of the system as a superposition of scaled and/or shifted versions of $f_1(t)$ and $f_2(t)$.

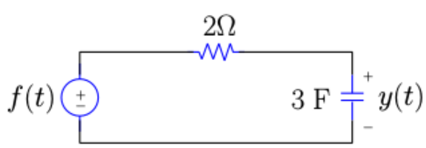
$Y(w)=H(w)\cdot F(w) = H(w)[F_1(w)+F_2(w)]$
$= H(w)F_1(w) + H(w)F_2(w)$

For F_1: $H(w) \Rightarrow 2$ ⇒ $y_1(t)=2f_1(t)$
For F_2: $H(w)=4e^{-j\frac{1}{10}t}$ ⇒ $y_2(t)=4f_2(t-\frac{1}{10})$

⇒ $y(t)=2f_1(t)+4f_2(t-\frac{1}{10})$



3. Determine the response $y(t)$ of the circuit shown below with an arbitrary input $f(t)$ in the form of an inverse Fourier transform and then evaluate $y(t)$ for the case $f(t) = e^{-\frac{t}{6}}u(t)$ V.



LTI $H(w)$ = As $H(w)=\frac{Y(w)}{F(w)}$

$y(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}Y(w)\cdot e^{jwt}dw$

$Z_R=2\Omega$ $Z_C=\frac{1}{jw3}$

$\because e^{-at}u(t)=\frac{1}{a+jw}$

⇒ $H(w)=\frac{1}{3jw}\cdot\frac{1}{\frac{1}{3jw}+2}=\frac{1}{1+6jw}$

$\therefore F(w)=\frac{1}{\frac{1}{6}+jw}$

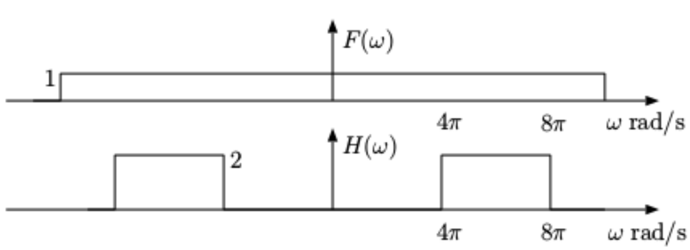
⇒ $Y(w)=\frac{1/6}{(\frac{1}{6}+jw)^2}$ → $y(t)=\frac{1}{6}te^{-\frac{1}{6}t}u(t)$

4. Given that $f(t)\cos(\omega_o t) \leftrightarrow \frac{1}{2}F(\omega-\omega_0)+\frac{1}{2}F(\omega+\omega_0)$, determine the Fourier transform of $g(t) = f(t+\frac{\theta}{\omega_o})\cos(\omega_o t+\theta)$ in terms of scaled and/or shifted versions of $F(\omega)$. (Hint: use the time-shift property.)

5. An LTI system with frequency response $H(\omega)$ is excited with an input $f(t) \leftrightarrow F(\omega)$. $H(\omega)$ and $F(\omega)$ are plotted below:

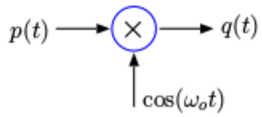
Ex4: $g(t)=f(t+\frac{\theta}{w_0})\cos(w_0(t+\frac{\theta}{w_0}))$

⇒ Using Time shift proporty.
$g(t+t_0)=G(w)e^{jwt_0}$
⇒ $g(t)\rightarrow G(w)=G'(w)\cdot e^{jw\frac{\theta}{w_0}}$
$=\frac{1}{2}[F(w-w_0)+F(w+w_0)]\cdot e^{jw\frac{\theta}{w_0}}$

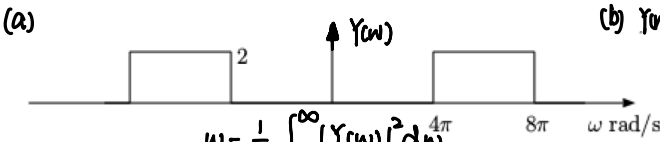


(a) Sketch the Fourier transform $Y(\omega)$ of the system output $y(t)$ and calculate the energy W_y of $y(t)$.

(b) It is observed that output $q(t)$ of the following system equals $y(t)$ determined in part (a).



(a)



(b) $Y(\omega) = \dfrac{P(\omega+\omega_0) + P(\omega-\omega_0)}{2}$

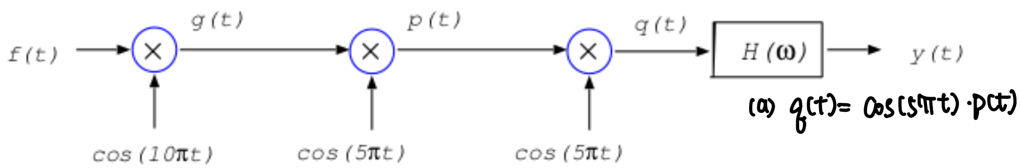
$\Rightarrow \omega_0 = 6\pi$ rad/s

Sketch $P(\omega)$ and determine ω_o.

$W = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(\omega)|^2 d\omega$

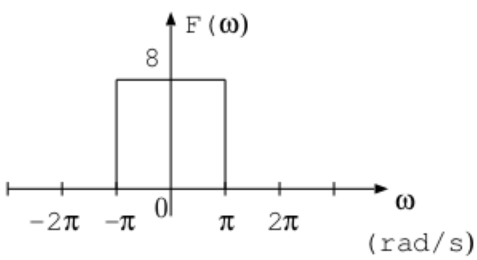
$= \frac{1}{2\pi} \times 4 \times 4\pi \times 4 = 16J$

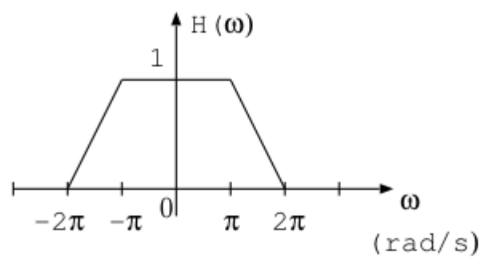
6. Consider the system



(a) $q(t) = \cos(5\pi t)\cdot p(t)$

where $F(\omega)$ and $H(\omega)$ are as follows:





(a) Express $q(t)$ in terms of $p(t)$.

(b) Sketch the Fourier transforms $G(\omega)$, $P(\omega)$, $Q(\omega)$, and $Y(\omega)$.

(c) Express $y(t)$ in terms of $f(t)$. (c) $y(t) = \frac{1}{4}f(t)$ as $Y(\omega) = \frac{1}{4}F(\omega)$