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Zhejiang University – University of Illinois at Urbana-Champaign Institute

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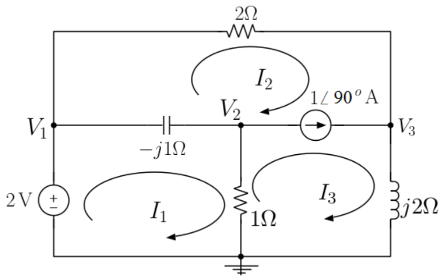
ECE-210 Analog Signal Processing Spring 2022
Homework #6: Solution

1. In the following circuit determine the node-voltage phasors V_1, V_2, and V_3 and express them in polar form.



Solution:

V_1 can be determined directly from the circuit,

$$V_1 = 2\,\mathrm{V}{=}2\angle 0^o\,V.$$

Applying KCL at V_2 gives

$$\frac{V_2}{1} + \underbrace{1\angle 90^o}_{j} + \frac{V_2 - V_1}{-j} = 0.$$

Likewise, the KCL equation at V_3 is

$$\frac{V_3}{j2} + \frac{V_3 - V_1}{2} = \underbrace{1\angle 90^o}_{j}.$$

From the first KCL equation, we obtain

$$\begin{aligned} V_2 + j + j(V_2 - 2) &= 0 \\ V_2 &= \frac{j}{1+j} = \frac{e^{j\frac{\pi}{2}}}{\sqrt{2}e^{j\frac{\pi}{4}}} \\ V_2 &= \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}\,\mathrm{V}{=}\frac{\sqrt{2}}{2}\angle 45^o\,\mathrm{V}. \end{aligned}$$

From the second KCL equation, we have

$$\begin{aligned} -jV_3 + V_3 - 2 &= j2 \\ V_3 &= \frac{2+j2}{1-j} = 2j\,\mathrm{V} = 2\angle 90^o\,V. \end{aligned}$$

2. In the circuit shown for Problem 1, determine the loop-current phasors I_1, I_2, and I_3 and express them in polar form.

Solution:

From the circuit we notice that

$$V_3 = 2j{\cdot}I_3 = 2j$$

Therefore

$$I_3 = 1\,A = 1\angle 0^o\,A$$

Applying KCL at V_3 yields

$$I_3 - I_2 = \underbrace{1\angle 90^o}_{j}$$

Plugging in I_3we have

$$\begin{aligned} I_2 &= 1 - j = \sqrt{2}e^{-j\frac{\pi}{4}}\,\mathrm{A}. \\ &= \sqrt{2}\angle -45^o\,A \end{aligned}$$

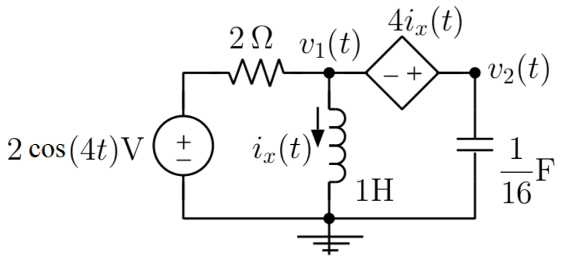
By applying KCL at the bottom node (ground), we have

$$I_3 + \frac{V_2}{1} = I_1$$

Plugging in I_3 and V_2 we obtain I_1

$$\begin{aligned} I_1 &= \frac{3+j}{2} \\ &= \frac{\sqrt{10}}{2} e^{j \arctan \frac{1}{3}} A \\ &= \frac{\sqrt{10}}{2} \angle 18.4^o A \end{aligned}$$

3. Use the phasor method to determine $v_1(t)$ in the following circuit:



Solution:

From the voltage source we know $w = 4rad/s$. Inpedence of the capacitor is given by

$$Z_C = \frac{1}{jwC} = \frac{4}{j} = -4j$$

Inpedence of the inductor is given by

$$Z_L = jwL = 4j$$

Writing the KCL equation for the super-node on top of the circuit we can obtain

$$\frac{V_1 - 2}{2} + \frac{V_1}{4j} + \frac{V_1 + 4I_x}{-4j} = 0,$$

Meanwhile from the $V - I$ relation for the inductor's impedance we have

$$I_x = \frac{V_1}{4j},$$

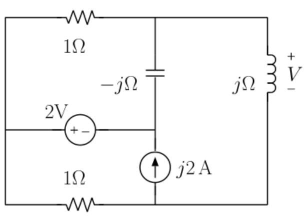
Plugging in I_x to the previous equation yields

$$\begin{aligned} i_x &= \frac{1}{3j} \\ V_1 &= \frac{4}{3} \end{aligned}$$

Therefore the steady-state voltage $v_1(t)$ can be express as

$$v_1(t) = \frac{4}{3} \cos(4t) \,\mathrm{V}.$$

4. In the following circuit determine the phasor V and express it in polar form:



Solution:

Denote the intersection node in the middle of this circuit by V_1. The KCL equation at the top node is given by

$$\frac{V-(V_1+2)}{1}+\frac{V-V_1}{-j}+\frac{V}{j}=0,$$

which yields

$$jV+(1-j)V_1=2j$$

While for the bottom node, the KCL equation is give by

$$\frac{V}{j}+\frac{V_1+2}{1}=2j$$

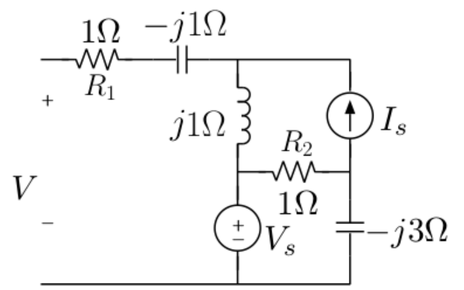
which yields

$$jV_1+V \quad = \quad -2-2j$$

We can obtain V by solving these two equations, which yields

$$\begin{aligned} V &= -\frac{4+2j}{5} \\ &= \frac{2}{\sqrt{5}}e^{j\,(\arctan(\frac{1}{2})-\pi)}\,\text{V} = \frac{2}{\sqrt{5}}\angle-153.4^o\,\text{V}. \end{aligned}$$

5. Use the following network to answer (a) through (d):



(a) Determine the phasor V when $I_s=0$.
(b) Determine the phasor V when $V_s=0$.
(c) Determine V when $V_s=4\,\text{V}$ and $I_s=-2\,\text{A}$, and calculate the average power absorbed in the resistors.
(d) What is the Thevenin equivalent and the available average power of the network when $V_s=4\,\text{V}$ and $I_s=-2\,\text{A}$?

Solution:

(a) When $I_s=0$, there is no current flowing through the inductor or the capacitor or the resistor R_1, therefore

$$V=V_s.$$

(b) When $V_s=0$ the only current flowing through the inductor is I_s, consequently

$$V=I_s(j1)=jI_s.$$

(c) By superposition,

$$V=V_s+jI_s=(4-j2)\text{V}$$

For the resistor R_1, the average absorbed power is ,

$$P_{R_1}=0\text{W},$$

since no current is flowing through it. For the other resistor R_2,

$$P_{R_2} = \frac{|V_{R_2}|^2}{2R_2}.$$

Denote the node on top of the capacitor as V_1, applying KCL at V_1 gives us

$$\frac{V_s - V_1}{1} = I_s + \frac{V_1}{3j}.$$

which yields

$$V_1 = \frac{18}{3+j}$$

Therefore the voltage across resistor R_2 is given by

$$V_{R_2} = V_s - V_1 = \frac{-7+9j}{5} = \frac{\sqrt{130}}{5} e^{j \arctan -\frac{9}{7}}.$$

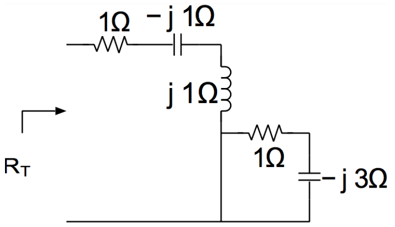
Plugging V_{R_2}into the previous equation and we can obtain P_{R_2}

$$\begin{aligned} P_{R_2} &= \frac{|V_{R_2}|^2}{2R_2} = \frac{|\frac{\sqrt{130}}{5}|^2}{2} \\ &= 2.6\,\mathrm{W}. \end{aligned}$$

(d) From part (c) we have the Thevenin Voltage

$$V_T = (4 - j2)\,\mathrm{V} = 2\sqrt{5}\mathrm{e}^{\mathrm{j}\arctan(-\frac{1}{2})}\,\mathrm{V}$$

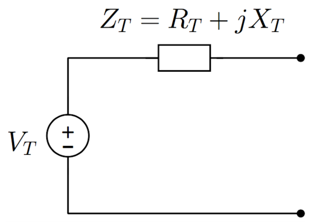
For the Thevenin impedance we suppress the independent sources, yielding to the following circuit:



The Thevenin impedance is

$$Z_T = 1\,\Omega.$$

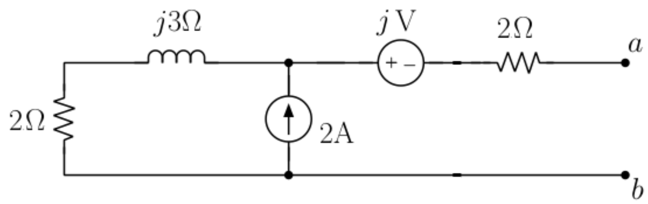
Then the Thevenin equivalent can be represented as



where $R_T = 1$ and $X_T = 0$. Finally the available average power is given by

$$P_a = \frac{|V_T|^2}{8R_T} = \frac{(2\sqrt{5})^2}{8} = 2.5\,\mathrm{W}.$$

6. Determine the impedance Z_L of a load that is matched to the following network at terminals a and b, and determine the net power absorbed by the matched load:



Solution:

To determine Z_L we first need to calculate the Thevenin impedance Z_T. Suppressing the independent sources we have

$$Z_T = j3 + 4\,\Omega$$

Therefore, the matched load can be obtained as follows,

$$Z_L = Z_T^* = (4 - j3)\Omega.$$

We can obtain the Thevenin Voltage by calculating the open-circuit voltage as follows

$$V_T = 2 \cdot (j3 + 2) - j = 4 + 5j\,V.$$

Thus, the net power absorbed by the matched load is the average available power,

$$P_a = \frac{|V_T|^2}{8R_T} = \frac{16 + 25}{8 \times 4} = \frac{41}{32}\,\text{W}.$$