lect.9

$5\cos(5t+\frac{\pi}{3})$

A ↑ ↑ ω

$\Rightarrow 5\angle\frac{\pi}{3} \Rightarrow$ phasor

$5e^{j\frac{\pi}{3}}$

F⇒f(t) $Re\{F\cdot e^{j\omega t}\}$ time invariant & Linear LTI system.

$e^{j\frac{\pi}{3}} = \cos\frac{\pi}{3} + j\sin\frac{\pi}{3}$

$Re\{5e^{j(\omega t+\frac{\pi}{3})}\} \Rightarrow 5\cos(5t+\frac{\pi}{3})$

求导法则: Derivative Principle

$f(t) = Re\{F\cdot e^{j\omega t}\}$ $F \xrightarrow[\text{phasor of}]{} f(t)$ $\frac{df(t)}{dt} = ?$

$\frac{df(t)}{dt} = \frac{d\,Re\{F\cdot e^{j\omega t}\}}{dt} = Re\{\frac{d\,Fe^{j\omega t}}{dt}\}$ complex number const.

$= Re\{F\frac{de^{j\omega t}}{dt}\} = Re\{F\,j\omega\,\frac{e^{j\omega t}dt}{dt}\}$

$= Re\{j\omega F\,e^{j\omega t}\}$

$f(t) = Re\{Fe^{j\omega t}\}$

$\frac{df(t)}{dt} = Re\{j\omega F\,e^{j\omega t}\}$

⇒ $f(t) \backsim F$

$\frac{df(t)}{dt} \backsim j\omega F$

$\frac{d^2f(t)}{dt^2} \backsim (j\omega)^2 F$

⋮

$\frac{d^nf(t)}{dt^n} = (j\omega)^n F$

⇒ other Phasors.

Example.

Particular sol. of

$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 5\sin(6t)$ $5\cos(6t-\frac{\pi}{2})$ trig identity ↳ $5\angle -90°$

$y(t) \xrightarrow{\text{phasor}} Y$ $j\omega = 6j$

$(j6)^2Y + 3(j6)Y + 2Y = 5e^{-j\frac{\pi}{2}}$ ⇒ algebric eq.

$\Rightarrow -36Y + 18jY + 2Y = -5j$ ⇒ $Y = \frac{-5j}{-34+j\cdot 18}$ (分母有理化)

$\Rightarrow (-34+18j)Y = -5j$

$\Rightarrow Y = \frac{-5j(-34-j\cdot 18)}{(-34+j\cdot 18)(-34-j\cdot 18)} = 0.13\angle 117.9°$

$y_p(t) = 0.13\cos(6t+117.9°)$

simplify sols.

ODE for higher order

→ $I(t) = Re\{Ie^{j\omega t}\}$ $V(t) = I(t)\cdot R$

$V(t) = Re\{Ve^{j\omega t}\}$ $Re\{Ve^{j\omega t}\} = Re\{Ie^{j\omega t}\}\cdot R$

↳ VI for a resistor?

⇒ V=IR stands for phasor

+ L −

$I(t) = Re\{Ie^{j\omega t}\}$ $V(t) = L\frac{dI(t)}{dt}$

$V(t) = Re\{Ve^{j\omega t}\}$ ↓ ↓ V $L(j\omega)I$

impedance ↳ complex number $\frac{V}{I}$

$V = (j\omega L)I$

reactance

+ | | −

$I(t) = Re\{Ie^{j\omega t}\}$ $I(t) = C\frac{dV(t)}{dt}$ ⇒ $V = \frac{I}{j\omega C}$

$V(t) = Re\{Ve^{j\omega t}\}$ ↓ ↓ I $C\cdot(j\omega)\cdot V$

L : $V = j\omega LI$

C : $\frac{1}{j\omega C}I$

$V = RI$

i(t)

I ↓ ↓

$V = RI$ $V\cdot\frac{1}{j\omega C}\cdot I$

total: $V = V_R + V_C = RI + \frac{1}{j\omega C}I$

$= (R + \frac{1}{j\omega C})I$

$\frac{V}{I}$ = complex

Resistance reactance

Impedance.

$\frac{I}{V} = Y$ ⇒ admittance $= R + jX$ 电纳 susptance.

电导 conductance 导纳

$Z = \begin{cases} j\omega L & \text{for inductor} \\ \frac{1}{j\omega C} & \text{for capacitor} \\ R & \text{for resistance} \end{cases}$ 通低阻高 通高阻低

Impedance 阻抗

Impede → slow sth down.

admittance ⇒ 电导

admit

① ω - signal freq.

② Z - impedance measures in unit of Ω

$\frac{V}{I}$ V to I ratio

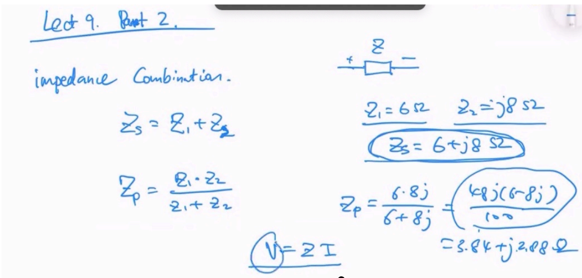
- unlike resistance/reactance ⇒ Z a complex number

- img part of Z called reactance, $\frac{\text{Ind}}{\text{Cad}}$ ad

- real part of Z called resistance. cap only inducors. reactances

③ using the generalized V - I relationship Z all KCL KVL in phasor form we can express LTI ckt + C with in an algebraic form of a sinusoidal signal.

lect 9 part 2



$V = |V| e^{j\theta_V}$

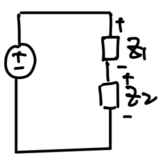
$I = |I| e^{j\theta_I}$ $\Rightarrow Z = |Z| e^{j\theta_Z}$

$|V| = |Z||I|$

$\theta_V = \theta_I + \theta_Z$ ⇒ 变非线性变线性，简化

Phase shift → complex

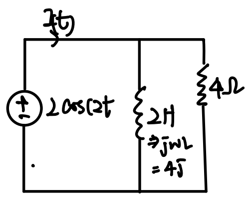
voltage & current division

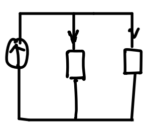


$V_1 = \frac{Z_1}{Z_1 + Z_2} V$

$V_2 = \frac{Z_2}{Z_1 + Z_2} V$

← resistive





$I_1 = I \frac{Z_2}{Z_2 + Z_1}$

$I_2 = I \frac{Z_1}{Z_2 + Z_1}$

$I = \frac{2}{\frac{4j \cdot 4}{4+4j}} = \frac{8+8j}{16j} = \frac{1+j}{2j}$

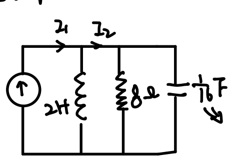
$= \frac{(1+j)(-j)}{2j(-j)} = \frac{1-j}{2}$

$= \frac{1}{\sqrt{2}} e^{-j\frac{\pi}{4}} \cdot A$

$\Rightarrow I(t) = \frac{\sqrt{2}}{2} \cos(2t - \frac{\pi}{4})$ $\boxed{\text{lagging}}$

I

Example:



$2\cos(4t - \frac{\pi}{3})$
$\Rightarrow \omega = 4$

$\frac{1}{j\omega C}$

$\Rightarrow Z_p = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = -8j$

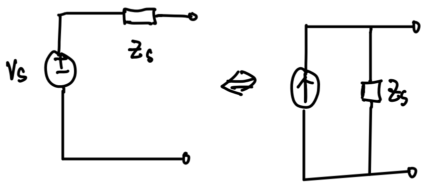
$I_2 = \frac{8}{8-8j} \cdot I$ ⇒ 画图
(phasor)

$C = a + jb$ $|C| = \sqrt{a^2+b^2}$ $|C| \angle \phi$

$\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$ $\Rightarrow$ $|C| \cdot e^{j\phi_1} \cdot |d| \cdot e^{j\phi_2}$

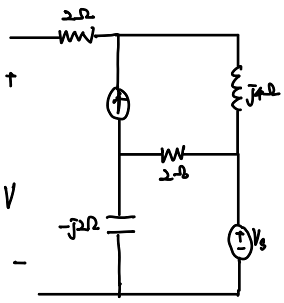
$\Rightarrow |C| \cdot |d| \cdot e^{j(\phi_1 + \phi_2)}$

Source transformation/source suppression.



$I_S = \frac{V_s}{Z_s}$

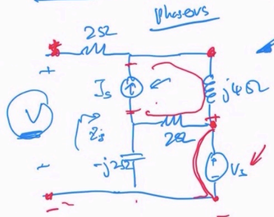
Example



example

phasors

Thevenin equivalent of the network

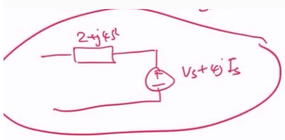


suppress (I_s).

$V = V_s$

suppress V_s $V = 4j I_s$

$\boxed{V = V_s + 4j I_s}$



$\frac{V_s}{I_s} Z$