ZJU-UIUC Institute

ECE-210 ANALOG SIGNAL PROCESSING: MIDTERM

& ECE-211 ANALOG SIGNAL PROCESSING: FINAL EXAM

Instructor: Prof. Yang Xu & Prof. Songbin Gong

April 7th (Thursday), 2022

Time: 100 minutes (10:00AM – 11:40AM)

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INSTRUCTIONS

1. **The first page of this paper should be filled clearly.**
2. **Calculator is permitted.**
3. **Access to internet is not permitted in addition to asking questions in the Wechat group, downloading and submitting the paper.**
4. **Solutions should be written clearly either on paper or on Pad.**
5. **Convert the finished paper to PDF version and submit it in the Blackboard. The submission deadline is 11:50am.**

Question #	Full Marks	Your Score
1. KCL & KVL	15	
2. Thevenin Theorems & Max Power Transfer	15	
3. Op-amplifier	15	
4. Op-amplifier & 1st Order RC Circuit	20	
5. Impedance/Admittance & Phasors Diagrams	20	
6. Superposition in Phasors	15	
Total	100	

Question 1:

Determine the power of each current sources and voltage sources in the following circuit in *Figure 1.*

For V_4:

$I_1+I_2+I_3+I_6+I_7=0$

$\Rightarrow 2+2+3+\frac{V_4-8}{3}+\frac{V_4-(V_5-3)}{2}=0$

For V_5:

$I_1+I_7=I_4+I_8$

$2+\frac{V_4-(V_5-3)}{3}=4+\frac{V_5-8-V_6}{2}$

For Node GND:

$I_2+I_6=I_5$

$2+\frac{V_4-8}{3}=5$

For Node V_6:

$I_3+I_5+I_4+I_8=0$

$\frac{V_5-8-V_6}{2}+4+3+5=0$

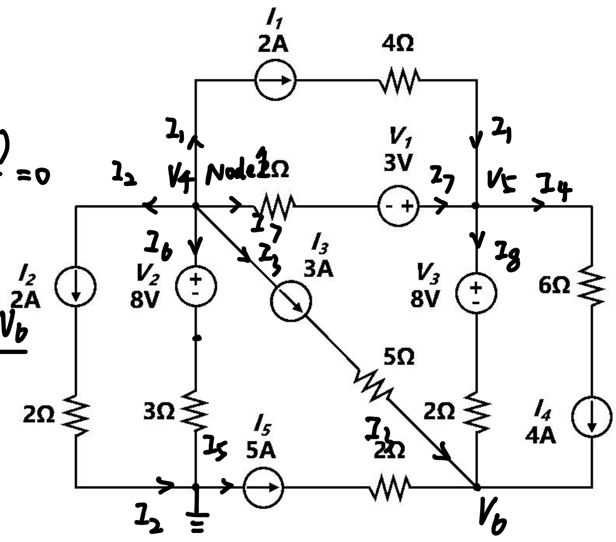


Figure 1

$\Rightarrow \begin{cases} V_4=17V \\ V_5=40V \\ V_6=56V \end{cases}$

$\Rightarrow I_6=3A$

$I_7=-10A$

$I_8=-12A$

$\Rightarrow P_{V_1}=I_7\cdot 3V=30W$ (absorbing pwr)

$P_{V_2}=I_6\cdot 8V=24W$ (absorbing pwr)

$P_{V_3}=I_8\cdot 8V=-96W$ (supplying pwr)

$P_{I_1}=31\cdot 2=-62W$ (supplying pwr)

$P_{I_2}=42W$ (absorbing pwr)

$P_{I_3}=-162W$ (supplying pwr)

$P_{I_4}=-160W$ (supplying pwr)

$P_{I_5}=-330W$ (supplying pwr

Question 2:

In the following circuit in *Figure 2*, R_L is an adjustable resistor. It is observed that for $i = 2A$, $v = 6\ V$ and $R_L = R$, $i_L = 1\ A$, while for $i = 4\ A$, $v = 4\ V$ and $R_L = 2R$, $i_L = 9/8\ A$.

(a): $\begin{cases} \dfrac{V_1-6}{R} = \dfrac{V_2-V_1}{R} \\ \dfrac{V_2-V_1}{R} + \dfrac{V_2-V_3}{R} = I = 2A \\ \dfrac{V_2-V_3}{R} = \dfrac{V_3}{2R} + \dfrac{V_3}{R} \\ \dfrac{V_3}{R} = 1A \end{cases} \Rightarrow \begin{cases} V_1 = 8V \\ V_2 = 10V \\ V_3 = 4V \\ R = 4\Omega \end{cases}$

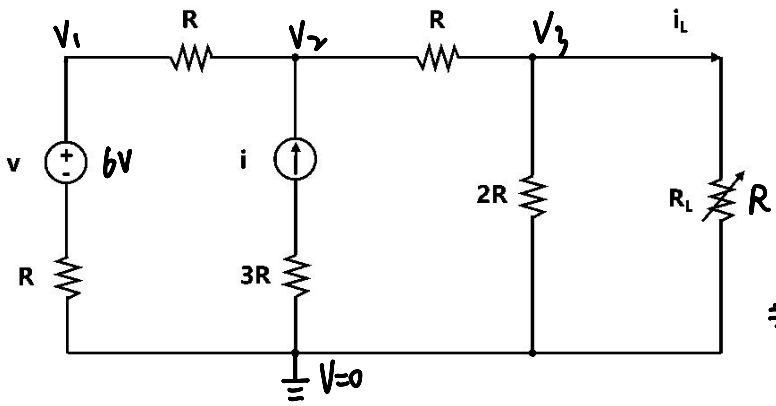


Figure 2

(a) Determine the value of resistances R.

(b) Determine i_L when $i = 3\ A$, $v = 2\ V$ and $R_L = 2R$.

(c) Determine the value of resistances R_L in order to get maximum power in R_L when $i = 3\ A$ and $v = 2\ V$.

I short:

(b) $\begin{cases} \dfrac{V_1-2}{4} = \dfrac{V_2-V_1}{4} \\ \dfrac{V_2-V_1}{4} + \dfrac{V_2-V_3}{4} = 3A \\ V_3 = 0 \end{cases} \Rightarrow \begin{cases} 2V_1 - V_2 = 2 \\ -V_1 + 2V_2 - V_3 = 12 \\ V_3 = 0 \end{cases}$

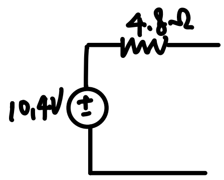
$\Rightarrow \begin{cases} V_1 = \frac{16}{3}\ V \\ V_2 = \frac{26}{3}\ V \end{cases} \Rightarrow I_{short} = \dfrac{V_2 - V_3}{4} = \dfrac{26}{12} = \dfrac{13}{6} A$

Vopen:

$\begin{cases} \dfrac{V_1-2}{4} = \dfrac{V_2-V_1}{4} \\ \dfrac{V_2-V_1}{4} + \dfrac{V_2-V_3}{4} = 3A \\ \dfrac{V_2-V_3}{4} = \dfrac{V_3}{8} \end{cases} \Rightarrow \begin{cases} 2V_1 - V_2 = 2 \\ -V_1 + 2V_2 - V_3 = 12 \\ 2V_2 - 3V_3 = 0 \end{cases} \Rightarrow \begin{cases} V_1 = \frac{44}{5} V \\ V_2 = \frac{78}{5} V \\ V_3 = \frac{52}{5} V \end{cases}$

$\Rightarrow V_{open} = 10.4V$

$\Rightarrow$



$\Rightarrow$ If $R_L = 2R = 8\Omega$

$\therefore I_L = \dfrac{10.4}{8+4.8} = \dfrac{13}{16} A$

(C) $R_L = 4.8\Omega$

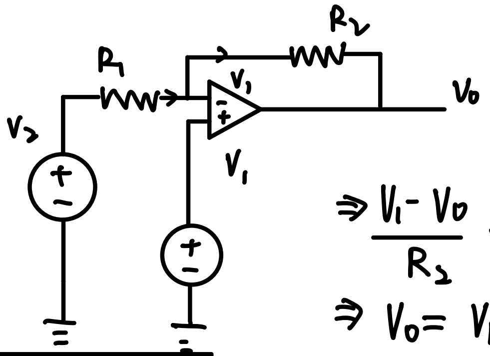
Question 3:

Design an op-amp circuit that performs the following operation

$$\mathbf{v_0 = 2v_1 - 3v_2}$$

Notice:

1. **Only one operational amplifier that you can use in this circuit.**
2. **All resistances must $\geq 1k\Omega$ and $\leq 100k\Omega$.**



$$\Rightarrow \frac{V_1 - V_0}{R_2} = \frac{V_2 - V_1}{R_1}$$

$$\Rightarrow V_0 = V_1 - \frac{R_2}{R_1}(V_2 - V_1)$$

$$= V_1 - \frac{R_2}{R_1}V_2 + \frac{R_2}{R_1}V_1$$

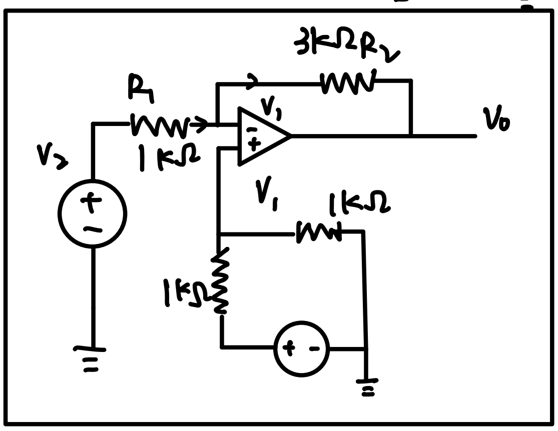
$$= \left(1 + \frac{R_2}{R_1}\right)V_1 - \frac{R_2}{R_1}V_2$$

$$\Rightarrow 1 + \frac{R_2}{R_1} = 2 \quad \frac{R_2}{R_1} = 3$$

cannot operate such operation

$$\Rightarrow k\left(1 + \frac{R_2}{R_1}\right) = 2$$

$$\Leftarrow \frac{R_2}{R_1} = 3 \qquad \Rightarrow k = \frac{1}{2}$$



Question 4:

Write the 1st order ODE in the RC circuit, and determine $v_o(t)\ for\ t > 0$ in *Figure 4*.

Let $v_{in} = 3u(t)\ V$, and assume that capacitor is initially uncharged. ($u(t)$ is unit step function)

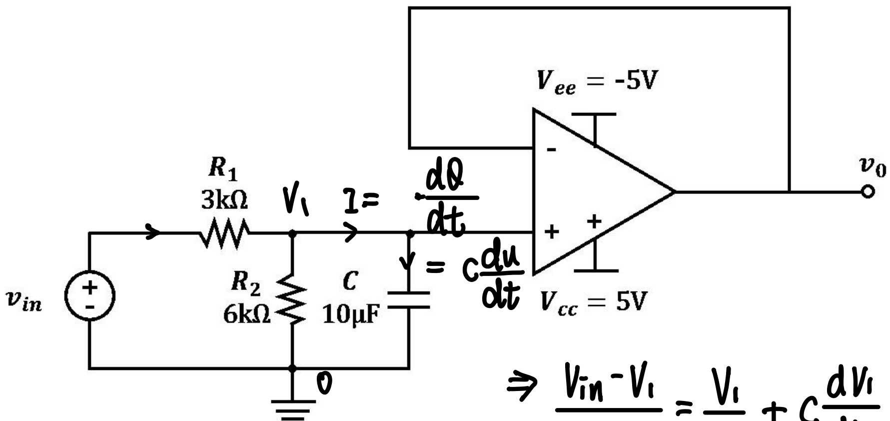


Figure 4

$$\Rightarrow \frac{V_{in}-V_1}{3k\Omega} = \frac{V_1}{6k\Omega} + C\frac{dV_1}{dt}$$

(ODE) $10\times10^{-6}\frac{dV_1}{dt} + 3\cdot\frac{V_1}{6\times10^3} = \frac{V_{in}}{3\times10^3}$

$$\Rightarrow \frac{dV_1}{dt} + 50V_1 = \frac{100}{3}V_{in}$$

$$\Rightarrow \frac{dV_1}{dt} + 50V_1 = 100u(t)$$

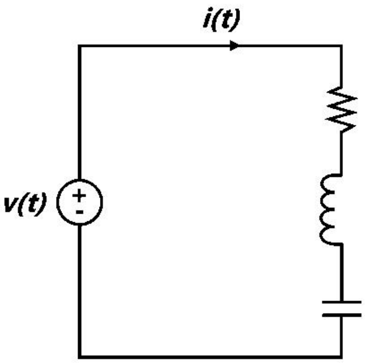
$$V_1 = \left(\int g(t)e^{\int 50\,dt}dt + C\right)e^{-\int 50\,dt}$$

$$= \left(\int 100u(t)e^{50t}dt + C\right)e^{-50t}$$

$V_1 = V_0(t) \Rightarrow \quad V_0(t) = \left(\int_0^t 100u(t)e^{50t}dt\right)\cdot e^{-50t}$

Question 5:

5.1 In *Figure 5(a)*, determine the equivalent impedance of the network if the frequency f=60 Hz. Then, compute the current $i(t)$ if the voltage source is $v(t) = 50\cos(\omega t + 30°)$.



R = 10Ω $\Rightarrow Z = 10\Omega$

L = 53mH $\Rightarrow Z = j\omega L = 53\times10^{-3}\omega j$

C = 1mF $\Rightarrow Z = \frac{1}{j\omega C} = -\frac{1000}{\omega} j$

$f = 60Hz \Rightarrow \omega = 2\pi \cdot f = 120\pi$

Figure 5(a)

$\Rightarrow Z_L = \frac{159}{25}\pi \cdot e^{i\frac{\pi}{2}}$

$Z_C = 2.6526\, e^{i - \frac{\pi}{2}}$

$V(t) = 50\, e^{i(120\pi t + \frac{\pi}{6})}$

$\Rightarrow V = 50\, e^{i\frac{\pi}{6}}$

$\Rightarrow I = \frac{V}{Z_R + Z_L + Z_C} \approx 2.5\, e^{i - 0.5238 rad}.$

$\Rightarrow i(t) = 2.5\, e^{i -0.523} \cdot e^{i \cdot 120\pi t}$

$= 2.5 \cos(120\pi t - 0.523)$

5.2 In *Figure 5(b)*, $i(t) = 10\cos(500t)$. Find the value of C such that *v*(*t*) and *i*(*t*) are in phase. Then sketch the phasor diagram for the network shown in *Figure 5(b)*. (Hint: Here 'in phase' refers to $\theta_v = \theta_i$)

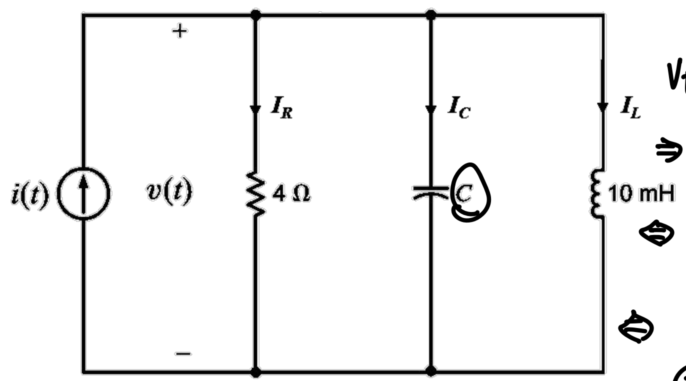


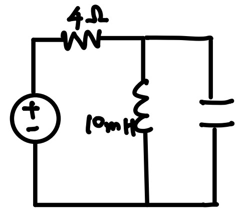
Figure 5(b)

V_t & I_t in phase

⇒ Reasonance.

⇔ $I_C + I_L = Re\{I_C + I_L\}$

⇔



⇒ $Z_H = jwL = 500 \cdot 10^{-2} \cdot j = 5j$

$Z_C = -5j$

⇒ $\frac{1}{jwC} = -5j$

∴ $\frac{+j}{500 \cdot C} = +5j$

∴ $C = \frac{1}{2500}$

Question 6:

In *Figure 6*,

1) Use superposition to find V_o in the network in *Figure 6*.
2) Express time-domain output voltage $V_o(t)$.

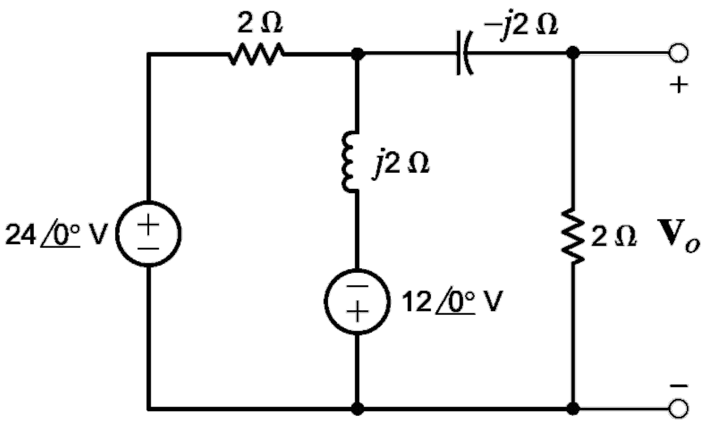
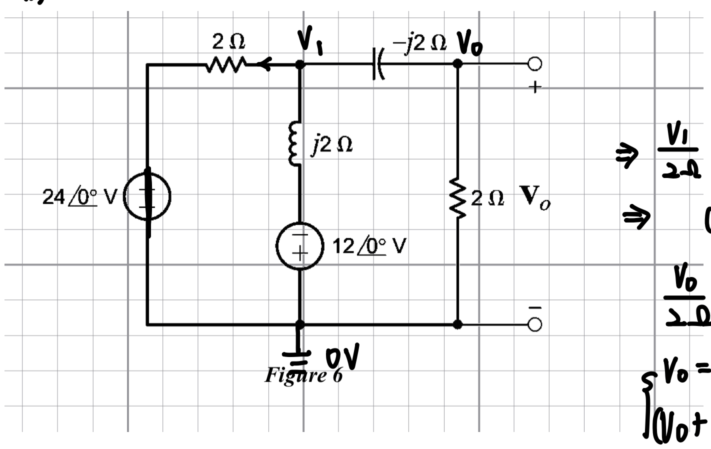


Figure 6

1)

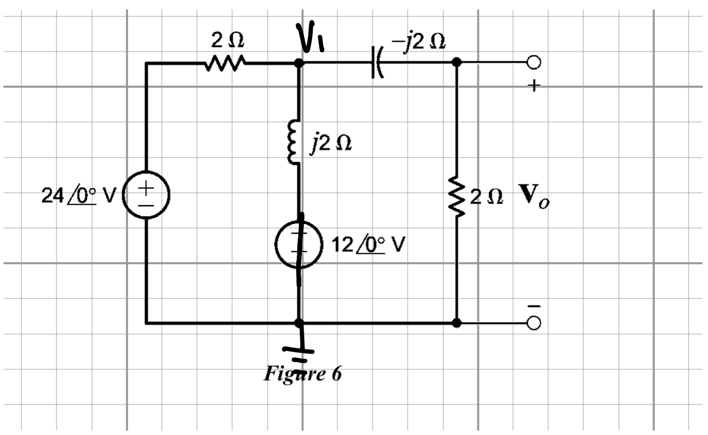


$$\Rightarrow \frac{V_1}{2\Omega} + \frac{V_1 - V_0}{-2j} = \frac{-12 - V_1}{2j}$$

$$\Rightarrow (V_0 + 12)j = V_1$$

$$\frac{V_0}{2\Omega} = \frac{V_1 - V_0}{-j2\Omega}$$

$$\begin{cases} V_0 = (V_1 - V_0)j \\ (V_0 + 12)j = V_1 \end{cases} \Rightarrow V_0 = -\frac{24}{5} + \frac{12}{5}j$$



$$\frac{V_1}{2j} + \frac{V_1 - V_0}{-2j} = \frac{24}{2}$$

$$\Rightarrow \frac{V_0}{2j} = 12$$

$$\Rightarrow V_0 = 24j$$

$$\Rightarrow V_0 = \frac{-24}{5} + 26.4j$$

$$= 12\sqrt{5}\, e^{j1.175}$$

$$\Rightarrow V(t) = 12\sqrt{5} \cos(\omega t + 1.175 \text{ rad})$$

Formula Sheet

Table 1. Suggestions for particular solutions of $\frac{dy}{dt} + ay(t) = bf(t)$ with various source functions $f(t)$.

	Source function $f(t)$	Particular solution of $\frac{dy}{dt} + ay(t) = bf(t)$
1	constant D	constant K
2	Dt	$Kt + L$ for some K and L
3	De^{pt}	Ke^{pt} if $p \neq -a$ Kte^{pt} if $p = -a$
4	$\cos(\omega t)$ or $\sin(\omega t)$	$H\cos(\omega t + \theta)$, where H and θ depend on ω, a, and b