| $f(t)$, period $T = \frac{2\pi}{\omega_o}$ | Form | Coefficients |
|---|---|---|
| $\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$ | Exponential | $F_n = \frac{1}{T}\int_T f(t)e^{-jn\omega_o t}dt$ |
| $\frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$ | Trigonometric | $a_n = F_n + F_{-n}$<br>$b_n = j(F_n - F_{-n})$ |
| $\frac{c_o}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$ | Compact for real $f(t)$ | $c_n = 2\|F_n\|$<br>$\theta_n = \angle F_n$ |

Table 1: Fourier series forms.

| | Name: | Condition: | Property: |
|---|---|---|---|
| 1 | Scaling | Constant $K$ | $K\, f(t) \leftrightarrow K\, F_n$ |
| 2 | Addition | $f(t) \leftrightarrow F_n$, $g(t) \leftrightarrow G_n$, ... | $f(t) + g(t) + \ldots \leftrightarrow F_n + G_n + \ldots$ |
| 3 | Time shift | Delay $t_o$ | $f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$ |
| 4 | Derivative | Continuous $f(t)$ | $\frac{df}{dt} \leftrightarrow jn\omega_o F_n$ |
| 5 | Hermitian | Real $f(t)$ | $F_{-n} = F_n^*$ |
| 6 | Even function | $f(-t) = f(t)$ | $f(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$ |
| 7 | Odd function | $f(-t) = -f(t)$ | $f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$ |
| 8 | Average power | | $P \equiv \frac{1}{T}\int_T \|f(t)\|^2 dt = \sum_{n=-\infty}^{\infty} \|F_n\|^2$ |

Table 2: Fourier series properties

| | Name: | Condition: | Property: |
|---|---|---|---|
| 1 | Amplitude scaling | $f(t) \leftrightarrow F(\omega)$, constant $K$ | $Kf(t) \leftrightarrow KF(\omega)$ |
| 2 | Addition | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$, $\cdots$ | $f(t) + g(t) + \cdots \leftrightarrow F(\omega) + G(\omega) + \cdots$ |
| 3 | Hermitian | Real $f(t) \leftrightarrow F(\omega)$ | $F(-\omega) = F^*(\omega)$ |
| 4 | Even | Real and even $f(t)$ | Real and even $F(\omega)$ |
| 5 | Odd | Real and odd $f(t)$ | Imaginary and odd $F(\omega)$ |
| 6 | Symmetry | $f(t) \leftrightarrow F(\omega)$ | $F(t) \leftrightarrow 2\pi f(-\omega)$ |
| 7 | Time scaling | $f(t) \leftrightarrow F(\omega)$ , real $s$ | $f(st) \leftrightarrow \frac{1}{\lvert s\rvert}F(\frac{\omega}{s})$ |
| 8 | Time shift | $f(t) \leftrightarrow F(\omega)$ | $f(t-t_o) \leftrightarrow F(\omega)e^{-j\omega t_o}$ |
| 9 | Frequency shift | $f(t) \leftrightarrow F(\omega)$ | $f(t)e^{j\omega_o t} \leftrightarrow F(\omega - \omega_o)$ |
| 10 | Modulation | $f(t) \leftrightarrow F(\omega)$ | $f(t)\cos(\omega_o t) \leftrightarrow \frac{1}{2}F(\omega-\omega_o) + \frac{1}{2}F(\omega+\omega_o)$ |
| 11 | Time derivative | Differentiable $f(t) \leftrightarrow F(\omega)$ | $\frac{df}{dt} \leftrightarrow j\omega F(\omega)$ |
| 12 | Freq derivative | $f(t) \leftrightarrow F(\omega)$ | $-jtf(t) \leftrightarrow \frac{d}{d\omega}F(\omega)$ |
| 13 | Time convolution | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$ | $f(t) * g(t) \leftrightarrow F(\omega)G(\omega)$ |
| 14 | Freq convolution | $f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$ | $f(t)g(t) \leftrightarrow \frac{1}{2\pi}F(\omega) * G(\omega)$ |
| 15 | Compact form | Real $f(t)$ | $f(t) = \frac{1}{2\pi}\int_0^\infty 2\lvert F(\omega)\rvert\cos(\omega t + \angle F(\omega))d\omega$ |
| 16 | Parseval, Energy W | $f(t) \leftrightarrow F(\omega)$ | $W \equiv \int_{-\infty}^{\infty}\lvert f(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert F(\omega)\rvert^2 d\omega$ |

Table 3: Important properties of the Fourier transform.

| | $f(t) \leftrightarrow F(\omega)$ | | |
|---|---|---|---|
| 1 | $e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}$, $a>0$ | 14 | $\delta(t) \leftrightarrow 1$ |
| 2 | $e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}$, $a>0$ | 15 | $1 \leftrightarrow 2\pi\delta(\omega)$ |
| 3 | $e^{-a\lvert t\rvert} \leftrightarrow \frac{2a}{a^2+\omega^2}$, $a>0$ | 16 | $\delta(t-t_o) \leftrightarrow e^{-j\omega t_o}$ |
| 4 | $\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a\lvert\omega\rvert}$, $a>0$ | 17 | $e^{j\omega_o t} \leftrightarrow 2\pi\delta(\omega-\omega_o)$ |
| 5 | $te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}$, $a>0$ | 18 | $\cos(\omega_o t) \leftrightarrow \pi[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)]$ |
| 6 | $t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}$, $a>0$ | 19 | $\sin(\omega_o t) \leftrightarrow j\pi[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)]$ |
| 7 | $\text{rect}(\frac{t}{\tau}) \;\leftrightarrow\; \tau\text{sinc}(\frac{\omega\tau}{2})$ | 20 | $\cos(\omega_o t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega-\omega_o)+\delta(\omega+\omega_o)] + \frac{j\omega}{\omega_o^2-\omega^2}$ |
| 8 | $\text{sinc}(Wt) \;\leftrightarrow\; \frac{\pi}{W}\text{rect}(\frac{\omega}{2W})$ | 21 | $\sin(\omega_o t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega+\omega_o)-\delta(\omega-\omega_o)] + \frac{\omega_o}{\omega_o^2-\omega^2}$ |
| 9 | $\triangle(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2}\text{sinc}^2(\frac{\omega\tau}{4})$ | 22 | $\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$ |
| 10 | $\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W}\triangle(\frac{\omega}{2W})$ | 23 | $u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$ |
| 11 | $e^{-at}\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{(a+j\omega)^2+\omega_o^2}$, $a>0$ | 24 | $\sum_{n=-\infty}^{\infty}\delta(t-nT) \leftrightarrow \frac{2\pi}{T}\sum_{n=-\infty}^{\infty}\delta(\omega-n\frac{2\pi}{T})$ |
| 12 | $e^{-at}\cos(\omega_o t)u(t) \leftrightarrow \frac{a+j\omega}{(a+j\omega)^2+\omega_o^2}$, $a>0$ | 25 | $\sum_{n=-\infty}^{\infty}f(t)\delta(t-nT) \leftrightarrow \sum_{n=-\infty}^{\infty}\frac{1}{T}F(\omega-n\frac{2\pi}{T})$ |
| 13 | $e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$ | | |

Table 4: Important Fourier transform pairs. The left-hand column includes only "energy signals" $f(t)$ , while the right-hand column includes "power signals" and distributions.

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| Name | Property |
| --- | --- |
| Commutative | $h(t) * f(t) = f(t) * h(t)$ |
| Distributive | $f(t) * (g(t) + h(t)) = f(t) * g(t) + f(t) * h(t)$ |
| Associative | $f(t) * (g(t) * h(t)) = (f(t) * g(t)) * h(t)$ |
| Shift | $h(t) * f(t) = y(t) \Rightarrow h(t - t_0) * f(t) = h(t) * f(t - t_0) = y(t - t_0)$ |
| Derivative | $h(t) * f(t) = y(t) \Rightarrow (\frac{d}{dt} h(t)) * f(t) = h(t) * (\frac{d}{dt} f(t)) = \frac{d}{dt} y(t)$ |
| Reversal | $f(t) * h(t) = y(t) \Rightarrow h(-t) * f(-t) = y(-t)$ |
| Start-point | If $h(t) = 0$ for $t < t_{sh}$ and $f(t) = 0$ for $t < t_{sf}$ then $y(t) = h(t) * f(t) = 0$ for $t < t_{sy} = t_{sh} + t_{sf}$. |
| End-point | If $h(t) = 0$ for $t > t_{eh}$ and $f(t) = 0$ for $t > t_{ef}$ then $y(t) = h(t) * f(t) = 0$ for $t > t_{ey} = t_{eh} + t_{ef}$. |
| Width | $h(t) * f(t) = y(t) \Rightarrow T_y = T_h + T_f$ where $T_h$, $T_f$, and $T_y$ denote the widths of $h(t)$, $f(t)$, and $y(t)$. |

Table 5: Convolution properties.

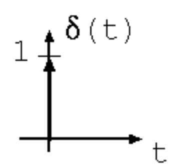
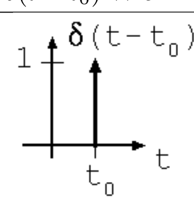
| Name | Impulse properties | Shifted-impulse properties |
| --- | --- | --- |
| Convolution | $\delta(t) * f(t) = f(t)$ | $\delta(t - t_0) * f(t) = f(t - t_0)$ |
| Sifting | $\int_{-\infty}^{\infty} \delta(t) f(t) dt = f(0)$ and $\int_a^b \delta(t) f(t) dt = \begin{cases} f(0) & \text{if } a < 0 < b \\ 0, & otherwise \end{cases}$ | $\int_{-\infty}^{\infty} \delta(t - t_0) f(t) dt = f(t_0)$ and $\int_a^b \delta(t - t_0) f(t) dt = \begin{cases} f(t_0) & \text{if } a < t_0 < b \\ 0, & otherwise \end{cases}$ |
| Sampling | $f(t)\delta(t) = f(0)\delta(t)$ | $f(t)\delta(t - t_0) = f(t_0)\delta(t - t_0)$ |
| Symmetry | $\delta(-t) = \delta(t)$ | $\delta(t_0 - t) = \delta(t - t_0)$ |
| Scaling | $\delta(at) = \frac{1}{\lvert a \rvert}\delta(t),\ a \neq 0$ | $\delta(a(t - t_0)) = \frac{1}{\lvert a \rvert}\delta(t - t_0),\ a \neq 0$ |
| Area | $\int_{-\infty}^{\infty} \delta(t) dt = 1$ and $\int_a^b \delta(t) dt = \begin{cases} 1 & \text{if } a < 0 < b \\ 0, & otherwise \end{cases}$ | $\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$ and $\int_a^b \delta(t - t_0) dt = \begin{cases} 1 & \text{if } a < t_0 < b \\ 0, & otherwise \end{cases}$ |
| Definite integral | $\int_{-\infty}^{t} \delta(\tau) d\tau = u(t)$ | $\int_{-\infty}^{t} \delta(\tau - t_0) d\tau = u(t - t_0)$ |
| Unit-step derivative | $\frac{d}{dt} u(t) = \delta(t)$ | $\frac{d}{dt} u(t - t_0) = \delta(t - t_0)$ |
| Derivative | $(\frac{d}{dt}\delta(t)) * f(t) = \frac{d}{dt} f(t)$ | $(\frac{d}{dt}\delta(t - t_0)) * f(t) = \frac{d}{dt} f(t - t_0)$ |
| Fourier transform | $\delta(t) \leftrightarrow 1$ | $\delta(t - t_0) \leftrightarrow e^{-j\omega t_0}$ |
| Graphical symbol |   |   |

Table 6: Properties of the impulse and shifted impulse.

| | | | |
|---|---|---|---|
| 1 | $\delta(t) \leftrightarrow 1$ | 7 | $\delta'(t) \leftrightarrow s$ |
| 2 | $e^{pt}u(t) \leftrightarrow \frac{1}{s-p}$ | 8 | $u(t) \leftrightarrow \frac{1}{s}$ |
| 3 | $te^{pt}u(t) \leftrightarrow \frac{1}{(s-p)^2}$ | 9 | $tu(t) \leftrightarrow \frac{1}{s^2}$ |
| 4 | $t^n e^{pt}u(t) \leftrightarrow \frac{n!}{(s-p)^{n+1}}$ | 10 | $t^n u(t) \leftrightarrow \frac{n!}{s^{n+1}}$ |
| 5 | $\cos(\omega_o t)u(t) \leftrightarrow \frac{s}{s^2+\omega_o^2}$ | 11 | $\sin(\omega_o t)u(t) \leftrightarrow \frac{\omega_o}{s^2+\omega_o^2}$ |
| 6 | $e^{-\alpha t}\cos(\omega_d t)u(t) \leftrightarrow \frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$ | 12 | $e^{-\alpha t}\sin(\omega_d t)u(t) \leftrightarrow \frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$ |

Table 7: Laplace transforms pairs $h(t) \leftrightarrow \hat{H}(s)$ involving frequently encountered causal signals — $\alpha$, $\omega_o$, and $\omega_d$ stand for arbitrary real constants, $n$ for non-negative integers, and $p$ denotes for an arbitrary complex constant.

| | Name: | Condition: | Property: |
|---|---|---|---|
| 1 | Multiplication | $f(t) \to \hat{F}(s)$, constant $K$ | $Kf(t) \to K\hat{F}(s)$ |
| 2 | Addition | $f(t) \to \hat{F}(s)$, $g(t) \to \hat{G}(s) \cdots$ | $f(t) + g(t) + \cdots \to \hat{F}(s) + \hat{G}(s) + \cdots$ |
| 3 | Time scaling | $f(t) \to \hat{F}(s)$ , real $a > 0$ | $f(at) \to \frac{1}{a}\hat{F}(\frac{s}{a})$ |
| 4* | Time delay | $f(t) \leftrightarrow \hat{F}(s)$, $t_o \geq 0$ | $f(t-t_o) \leftrightarrow \hat{F}(s)e^{-st_o}$ |
| 5 | Frequency shift | $f(t) \to \hat{F}(s)$ | $f(t)e^{s_o t} \to \hat{F}(s-s_o)$ |
| 6 | Time derivative | Differentiable $f(t) \to \hat{F}(s)$ | $f'(t) \to s\hat{F}(s) - f(0^-)$<br>$f''(t) \to s^2\hat{F}(s) - sf(0^-) - f'(0^-)$<br>$\cdots$<br>$f^{(n)}(t) \to s^n\hat{F}(s) - \cdots - f^{(n-1)}(0^-)$ |
| 7 | Time integration | $f(t) \to \hat{F}(s)$ | $\int_{0^-}^{t} f(\tau)d\tau \to \frac{1}{s}\hat{F}(s)$ |
| 8 | Freq. derivative | $f(t) \to \hat{F}(s)$ | $-tf(t) \to \frac{d}{ds}\hat{F}(s)$ |
| 9* | Time convolution | $h(t) \leftrightarrow \hat{H}(s)$, $f(t) \leftrightarrow \hat{F}(s)$ | $h(t) * f(t) \leftrightarrow \hat{H}(s)\hat{F}(s)$ |
| 10 | Freq. convolution | $f(t) \to \hat{F}(s)$, $g(t) \to \hat{G}(s)$ | $f(t)g(t) \to \frac{1}{2\pi j}\hat{F}(s) * \hat{G}(s)$ |
| 11 | Poles | $f(t) \to \hat{F}(s)$ | Values of $s$ such that $\lvert\hat{F}(s)\rvert = \infty$ |
| 12 | ROC | $f(t) \to \hat{F}(s)$ | Portion of $s-$ plane to the right of rightmost pole $\neq \infty$ |
| 13* | Fourier transform | $f(t) \leftrightarrow \hat{F}(s)$ | $F(\omega) = \hat{F}(j\omega)$ if and only if ROC includes $s = j\omega$ |
| 14 | Final value | Poles of $s\hat{F}(s)$ in LHP | $f(\infty) = \lim_{s\to 0} s\hat{F}(s)$ |
| 15 | Initial value | Existence of the limit | $f(0^+) = \lim_{s\to\infty} s\hat{F}(s)$ |

Table 8: Important properties of the *one-sided* Laplace transform. Properties marked by * in the first column hold only for causal signals.