ZJU-UIUC INSTITUTE

Zhejiang University – University of Illinois at Urbana-Champaign Institute

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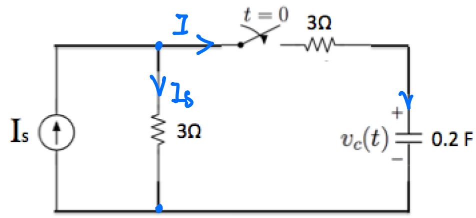
ECE-210 Analog Signal Processing Spring 2022
Homework #4: Submission Deadline 16 th March (10:00 PM)

1. In the next circuit, $v(t) = 4\,\text{V}$ for $t < 0$. Determine $v(t)$ for $t > 0$ after the switch is closed, and identify the zero-state and zero-input components of $v(t)$. (Hint: I_s donates a DC current source)



$Q = CU \Rightarrow \frac{dQ}{dt} = i$

$(I_s - C\frac{dV_c(t)}{dt})\cdot 3\Omega = C\frac{dV_c(t)}{dt}\cdot 3\Omega + V_c(t)$

$\Rightarrow 6C\frac{dV_c(t)}{dt} + V_c(t) = 3I_s$

$\Rightarrow \frac{dV_c(t)}{dt} + \frac{1}{6C}V_c(t) = 3I_s$

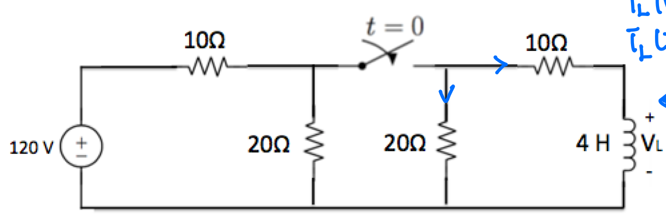
zero input $\Leftrightarrow I_s = 0$.

$\Rightarrow V_c(t) = Ae^{-\int\frac{1}{6C}dt}$
$= Ae^{-\frac{t}{6C}}$

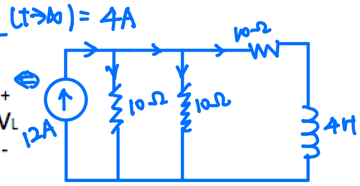
$$\boxed{\begin{aligned} &= Ae^{-\frac{5}{6}t} \\ &A = V_c(0) \Rightarrow V_c(t) = 4e^{-\frac{5}{6}t}\,\text{V}\end{aligned}}$$

2. The circuit shown below is in DC steady-state before the switch flips at $t = 0$
   (a) Find $V_L(0^-)$, $V_L(t \to \infty)$, $i_L(0)$, and $i_L(t \to \infty)$.
   (b) Find $i_L(t)$ and $V_L(t)$.
   (c) Sketch $i_L(t)$ and $V_L(t)$. Identify discontinuities.

a) $V_L(0^-) = 60\,V$
$V_L(t\to\infty) = 0\,V$
$I_L(0) = 0A$
$I_L(t\to\infty) = 4A$



$\Leftrightarrow$



b) $L\frac{di}{dt} + 10\Omega\cdot i = 5(12 - i)$

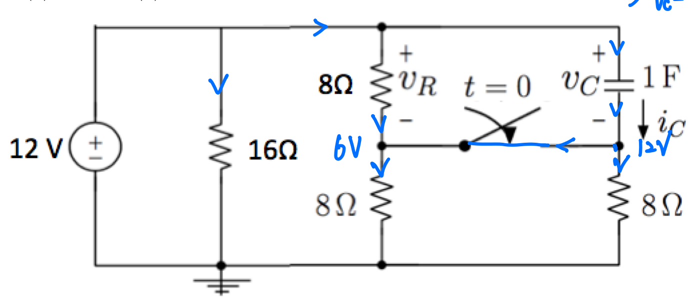
$\Leftrightarrow L\frac{di}{dt} + 15\Omega\cdot i = 60 \Rightarrow \frac{di}{dt} + \frac{15}{4}i = 15$

$\Rightarrow i_L(t) = [\int 15e^{\frac{15}{4}t}dt + C]\cdot e^{-\frac{15}{4}t}$
$= [\int \frac{15}{4}\times 4e^{\frac{15}{4}t}dt + C]\,e^{-\frac{15}{4}t}$
$= [4e^{\frac{15}{4}t} + C]\,e^{-\frac{15}{4}t}$
$= 4 + Ce^{-\frac{15}{4}t}$

$$\boxed{\begin{aligned} &i_L|_{t=0} = 0 \Rightarrow C = -4 \\ &\therefore i_L(t) = 4 - 4e^{-\frac{15}{4}t} \\ &V_L(t) = L\frac{di}{dt} = 4\times(-4\cdot -\tfrac{15}{4})e^{-\frac{15}{4}t} \\ &= 60e^{-\frac{15}{4}t}\end{aligned}}$$

3. The circuit shown below has been in DC steady-state before the switch closes at t=0s. Find and sketch $V_R(t)$, $V_c(t)$ and $i_c(t)$.

$\hookrightarrow V_c = 12V$



$c\frac{dV_c}{dt} + \frac{V_R}{8\Omega} = \frac{12 - V_R}{8\Omega} + \frac{12 - V_c}{8\Omega}$

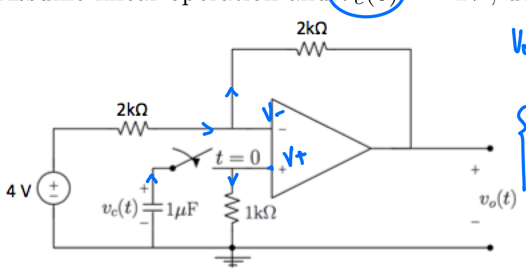
$c\frac{dV_c}{dt} + \frac{3V_c}{8} = 3$

$\begin{cases}\frac{dV_c}{dt} + \frac{3V_c}{8} = 3 \\ V_c = V_R\end{cases}$

$\Rightarrow V_c = V_R = (\int 3e^{\frac{3t}{8}}dt + c)e^{-\frac{3t}{8}}$
$= (8e^{\frac{3t}{8}} + c)e^{-\frac{3t}{8}}$
$= 8 + ce^{-\frac{3t}{8}}\,V$

$$\boxed{\begin{aligned} &\text{as } V_c(0) = 12V \Rightarrow C = 4 \\ &V_R = V_c = 8 + 4e^{-\frac{3t}{8}}\ (V)\end{aligned}}$$

4. Assume linear operation and $V_c(0) = -1V$, determine $V_0(t)$ in the following circuit:



$V_0(t)|_{t=0} = -4V$

$\begin{cases}\frac{4 - V_-}{2k\Omega} = \frac{V_- - V_0}{2k\Omega} \\ V_+ \approx V_- \approx 0\end{cases}$

$\Rightarrow V_0 = -4V$

$\Leftrightarrow \frac{4 - V_c(t)}{2k\Omega} = \frac{V_c(t) - V_0(t)}{2k\Omega} \Rightarrow V_0(t) = 2V_c(t) - 4$

$-C\frac{dV_c(t)}{dt} = \frac{V_c(t)}{R} \Rightarrow \frac{dV_c(t)}{dt} + \frac{1}{RC}V_c(t) = 0$

$\Rightarrow V_c(t) = Ae^{-\frac{1}{RC}t}$

$V_c(0) = A = -1V$

$\therefore V_c = -e^{-\frac{t}{RC}}$

$$\boxed{\begin{aligned} \Rightarrow V_0(t) &= -2e^{-\frac{t}{RC}} - 4\ V \\ &= -2e^{-1000t} - 4\,(V)\end{aligned}}$$