Lect 16 Freq Domain Discription of Signals.

Signal energy & Parseval's Theorem. → Extension from Fourier Series. 帕塞瓦尔定理

$$W = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(w)|^2 dw$$

finite energy signals

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notion of energy spectrum, useful. for signal classification parameter definitions.

① Signal class based on $|F(w)|^2$

Low pass signal $|F(w)| \to 0$ as $w \to$ high freq.

high pass signal $|F(w)| \to 0$ as $w \to 0$

band pass signal $|F(w)| \to 0$ as w deviates from immediate center freq.

② Signal bandwidth.

low pass signal bandwidth.

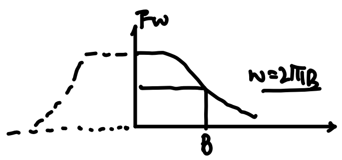
$W = \Omega = 2\pi B$ beyond which energy spectrum $|F(w)|^2$ is very small.

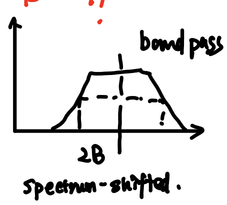
eg. 3-dB bandwidth. → most commonly used by default.

$$\frac{|F(\Omega)|^2}{|F(0)|^2} = \frac{1}{2} \qquad 10\log\frac{|F(w)|^2}{|F(0)|^2} = -3dB$$

$$\frac{1}{2\pi}\int_{-\Omega}^{\Omega} |F(w)|^2 dw = rW$$

$\hookrightarrow 0.95 - 0.99$





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example. $f(t) = \text{rect}(\frac{t}{\tau})$

$\hookrightarrow F(w) = \tau\,\text{sinc}(\frac{w\tau}{2})$

$$|F(w)|^2 = \tau^2 \cdot \text{sinc}^2(\frac{w\tau}{2})$$

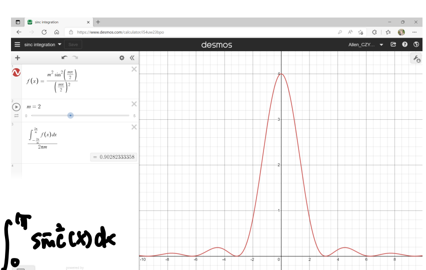
$w = \frac{2\pi}{\tau}$

$\frac{w\tau}{2} = \frac{2\pi}{\tau} \cdot \frac{\tau}{2} = \pi$

$\text{Sinc}\,\pi = 0$

$\text{Sinc}(-\pi) = 0 \quad w = -\frac{2\pi}{\tau}$

90% within Bandwith.



$f(t) = \text{rect}(\frac{t}{\tau}) \quad W = \int_{-\infty}^{\infty} |\text{Rect}(\frac{t}{\tau})|^2 dt = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} dt = \tau$

$\Omega = \frac{2\pi}{\tau}$

$$\frac{1}{2\pi}\int_{-\frac{2\pi}{\tau}}^{\frac{2\pi}{\tau}} \tau^2 \text{sinc}^2(\frac{w\tau}{2})\, dw = r\tau$$

$x = \frac{w\tau}{2} \quad w = \frac{2}{\tau}x$

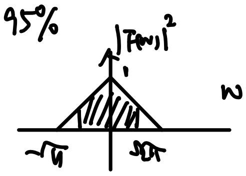
$r = \frac{2}{\pi}\int_0^{\pi} \text{sinc}^2(x)\,dx$

$\simeq 0.9028213$

$r = \frac{2}{\pi}$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \tau^2 \text{sinc}^2(x)\, d\frac{2}{\tau}x = r\tau$$

95%



$w = \frac{1}{2}$

$\tan\theta = \frac{1}{\pi}$

$(\pi - \Omega)\frac{1}{\pi}\cdot(\pi-\Omega)/2 = 0.025$

$\Rightarrow \Omega = \pi(1-\sqrt{0.05}) = 0.776\pi$

bandwidth is $\Omega = 1.55\pi$

LTI CKT & system Response to energy sgnls.

$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(w)e^{jwt}dw \rightarrow \boxed{LTI} \rightarrow y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(w)F(w)e^{jwt}dw$ → zero state → Not just steadystate part.

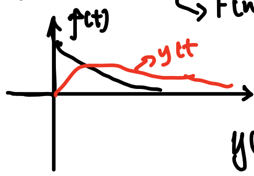
$y(t) \Rightarrow Y(w) = H(w)F(w)$

$f(t) \Leftrightarrow F(w)$

LTI system $H(w) = \frac{1}{1+jw}$

Ex1 $f(t) = e^{-t}u(t)$

$\hookrightarrow F(w) = \frac{1}{1+jw}$



$Y(w) = H(w)\cdot F(w) = \frac{1}{(1+jw)^2}$ → make inside table.

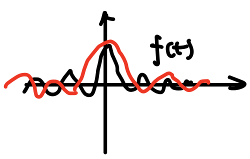
zero-state response

$\Updownarrow$

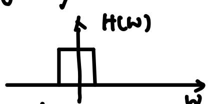
$te^{-t}u(t)$

$y(t) = te^{-t}u(t)$

Ex2: $f(t) = \text{sinc}(t) \Leftrightarrow F(w) = \pi\,\text{rect}(\frac{w}{2})$



$y(t)$ of $H(w) = \text{rect}(w)$



$Y(w) = H(w)F(w) = \underline{\text{rect}(w)\,\pi\,\text{rect}(\frac{w}{2})}$

$= \underline{\text{rect}(w)\,\pi}$

$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(w)e^{jwt}dw$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi\,\text{rect}(w)e^{jwt}dw$

$= \frac{e^{jwt}}{jt}\Big|_{-\frac{1}{2}}^{\frac{1}{2}}$

$= \frac{1}{2}\text{sinc}(\frac{t}{2})$

Ex3. $f(t) \rightarrow \boxed{LTI}$ $H(w) = e^{-jwt_0}$ → zero-state $y(t)$

$Y(w) = H(w)F(w) = e^{-jwt_0}\cdot F(w)$

time-shift properties.

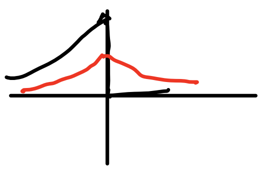
$\Rightarrow y(t) = f(t-t_0)$ delayed copy of input.

Ex4. $f(t)\ e^{t}u(-t)$

$\hookrightarrow \text{LTI} \rightarrow y(t)$

$H(\omega) = \frac{1}{1+j\omega} \rightarrow \underline{y(t)}$

zero state.



$f(t) = e^{t}u(-t)$

$\hookrightarrow F(\omega) = \frac{1}{(1-j\omega)}$

$Y(\omega) = H(\omega)\cdot F(\omega) = \frac{1}{1+j\omega}\cdot\frac{1}{1-j\omega}$

$= \frac{1}{1+\omega^2}$ 逆变换 $\Rightarrow \frac{1}{2}e^{-|t|}$

$y(t) = \frac{1}{2}e^{-|t|}$