Student ID	3200110970	Pre-lab	/25
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Semester/Section	SP22	Total	/83

Lab 3: Frequency Response and Fourier Series

In this lab you will build an active bandpass filter circuit with two capacitors and an op-amp, and examine the response of the circuit to periodic inputs over a range of frequencies. The same circuit will be used in Lab 4 in your AM radio receiver system as an intermediate frequency (IF) filter, but in this lab our main focus will be on the frequency response $H(\omega)$ of the filter circuit and the Fourier series of its periodic input and output signals. In particular we want to examine and gain experience about the response of linear time-invariant circuits to periodic inputs.

1 Prelab

1. Determine the compact-form Fourier series of the periodic square wave signal, $f(t)$ shown in Figure 1, with a period T and amplitude A. That is, find c_n and θ_n such that

$$f(t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n), \text{ where } \omega_o = \frac{2\pi}{T}.$$

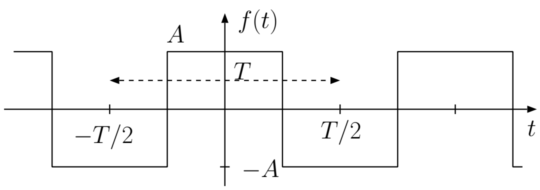


Figure 1: Square wave signal for prelab.

Notice $\frac{C_0}{2} = 0$. How could you have determined that without any calculation?

if $C_0/2 \neq 0$ cannot be symmetric with respect to t-axis.

(____/2)

(____/11)

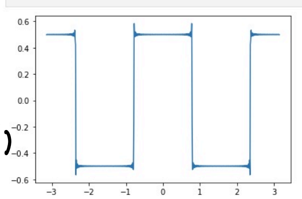
Show your work.(____/3)

$$f(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n w_0 t) + b_n \cos(n w_0 t)]$$

$$a_0 = \frac{1}{T_0}\int_{t_0}^{t_0+T} f(t)dt \qquad a_n = \frac{\int_{t_0}^{t_0+T_0} f(t)\cos(n w_0 t)dt}{\int_{t_0}^{t_0+T_0} \cos^2(n w_0 t)dt} = \frac{2}{T_0}\int_{t_0}^{t_0+T} f(t)\cos(n w_0 t)dt$$

$$b_n = \frac{2}{T_0}\int_{t_0}^{t_0+T} f(t)\sin(n w_0 t)dt$$

```
import matplotlib.pyplot as plt
import numpy as np
#define mathmetical fomulas
def FSeries(n,A,T):
    list2 = np.linspace(1,2*n,2*n)
    list3 = np.linspace(-T,T,int(100*2*T))
    func = np.zeros(int(100*2*T))
    for i in range(int(100*2*T)):
        for j in range(n):
            idx = 2*j+1
            factor = 2*A*(-1)**j/(idx*np.pi)
            func[i] += factor * np.cos(idx*2*np.pi/T*list3[i])
    plt.plot(list3,func)
FSeries(100,1,np.pi)
```



$$a_n = \frac{2}{T}\left(A\int_{-\frac{T}{4}}^{\frac{T}{4}} \cos(n\frac{2\pi}{T}t)dt - A\int_{\frac{T}{4}}^{\frac{3T}{4}} \cos(n\frac{2\pi}{T}t)dt\right)$$

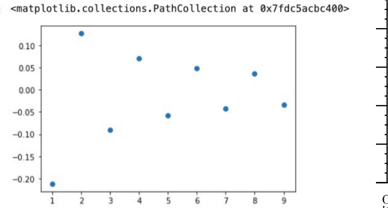
$$= \frac{A}{n\pi}\left(\sin(n\frac{2\pi}{T}t)\Big|_{-\frac{T}{4}}^{\frac{T}{4}} - \sin(n\frac{2\pi}{T}t)\Big|_{\frac{T}{4}}^{\frac{3T}{4}}\right)$$

$$= \frac{A}{n\pi} 2\cdot(\sin(\frac{n\pi}{2}))$$

$$\Rightarrow \begin{cases} a_{2k} = \dfrac{2\cdot(-1)^k}{(2k+1)\pi} \\ b_n = 0 \end{cases} \Rightarrow F_n = \sum_{k=1}^{\infty} \frac{(-1)^k\cdot 2A}{\pi(2k+1)} \cos[(2k+1)w_0 t] \quad (w_0 = \frac{2\pi}{T})$$

```
[48]: def FourierCoeff(n,A):
          idx = 2*n+1
          factor = 2*A*(-1)**n/(idx*np.pi)
          return factor
      list1 = np.linspace(1,9,9)
      plt.scatter(list1,FourierCoeff(list1,1))

[48]: <matplotlib.collections.PathCollection at 0x7fdc5acbc400>
```



$F_n = \sum_{n=1}^{\infty} \frac{(-1)^n 2A}{\pi(2n+1)} \cos((2n+1)w_0 t) \quad (w_0 = \frac{2\pi}{T})$ (____/2)

$c_n = \frac{(-1)^n 2A}{\pi(2n+1)}$ (____/2)

$\theta_n = 0$ (____/2)

With $A = 1$, plot c_n over n ($n \in [1, 9]$) (____/2)

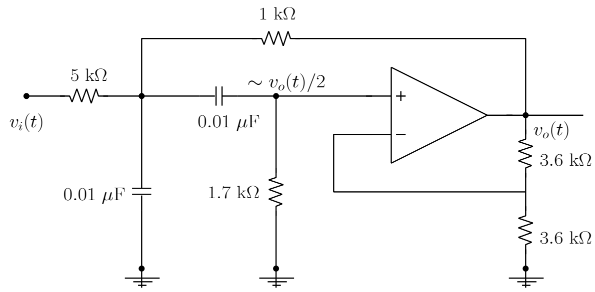


Figure 2: Circuit for analysis in prelab and lab.

2. Consider the circuit in Figure 2 where $v_i(t)$ is a co-sinusoidal input with some radian frequency ω.

   (a) What is the phasor gain $\frac{V_o}{V_i}$ in the circuit as $\omega \to 0$? (Hint: How does one model a capacitor at DC — open or short?)

   Show your work
   the gain is almost zero as the capacitor is the same as openckt.

   (____/3)

   (b) What is the gain $\frac{V_o}{V_i}$ as $\omega \to \infty$? (Hint: think of capacitor behavior in $\omega \to \infty$ limit)

   Show your work
   the capacitor is like to be short, so, the Vo is 0. so the gain will be zero

   (____/3)

   (c) In view of the answers to part (a) and (b), and the fact that the circuit is 2nd order (it contains two energy storage elements), try to guess what kind of a filter the system frequency response $H(\omega) \equiv \frac{V_o}{V_i}$ implements — lowpass, highpass, or bandpass? The amplitude response $|H(\omega)|$ of the circuit will be measured in the lab.

   Give your answer and explain your reasoning.
   band pass. , as both too high & too low frequency cannot pass the circuit

   (____/2)

3. Decibels (dB) is a unit of measurement widely used in science and engineering to compare power or intensity quantities. A decibel (dB) is one-tenth of a Bel (B), which is the name given to $\log_{10}\left(\frac{P_1}{P_0}\right)$, where $\log_{10}$ is the base 10 logarithm, and $\frac{P_1}{P_0}$ is the ratio of two power quantities. The formula for calculating decibels is : $10\log_{10}\left(\frac{P_1}{P_0}\right)$. and for comparing voltages we can use: $20\log_{10}\left(\frac{V_1}{V_0}\right)$, which is derived from $10\log_{10}\left(\frac{V_1^2/R}{V_0^2/R}\right)$. Complete the following table of useful ratios: (use accuracy of 1 decimal in dB row)

P_1/P_0	1000	100	10	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$
V_1/V_0	$\sqrt{1000}$	10	$\sqrt{10}$	$2\sqrt{2}$	2	$\sqrt{2}$	1	$\frac{\sqrt{2}}{2}$	2	$\frac{\sqrt{10}}{10}$	$\frac{1}{10}$	$\frac{1}{\sqrt{1000}}$
Decibels (dB)	30	20	10	9.03	6.02	3	0	-3.01	-6.02	-10	-20	-30

(____/2)

4. In the case of the Fourier analysis, the oscilloscope compares the signals with a reference of $V_{\text{rms}} = 1\,\text{V}$. Recall that a sine wave with rms (root mean square) amplitude of $V_{\text{rms}} = 1\,\text{V}$ corresponds to a sine wave with a peak amplitude of $\sqrt{2}\,\text{V}$, which is to say a peak-to-peak amplitude of $2\sqrt{2}\,\text{V}$. To specify that the reference is $V_{\text{rms}} = 1\,\text{V}$, the decibel symbol is modified with the suffix "V" becoming "dBV". Convert the voltages in the table below to dBV units (use accuracy of 1 decimal).
   $\rightarrow A = \sqrt{2}V \quad V_{pp} = 2\sqrt{2}V$
   $\Leftrightarrow 20\log_{10}(V_1/V_0) = 20\log_{10}(V_{rms})$

v	$V_{\text{rms}} = 1\,\text{V}$	$V_{\text{rms}} = 2\,\text{V}$	$V_{\text{rms}} = \sqrt{10}\,\text{V}$	$V_{\text{rms}} = \sqrt{2}\,\text{V}$	$V_{\text{rms}} = 1/10\,\text{V}$	$2\sqrt{2}\,\text{V}$peak-to-peak	$4\sqrt{5}\,\text{V}$peak-to-peak	$2\,\text{V}$peak-to-peak	$\frac{2\sqrt{2}}{\sqrt{10}}\,\text{V}$peak-to-peak
v (dBV)	0.0	6.02	10	3.01	-20	0.0	10	-3.01	-10

(____/2)

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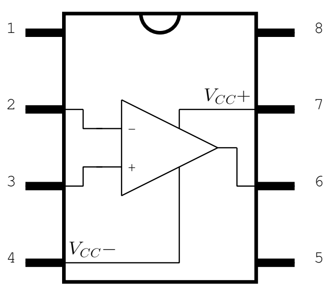
2 Laboratory exercise

- Equipment: Function generator, oscilloscope, protoboard, and wires.
- Components: 741 op-amp, two 0.01 μF capacitors, one 1 kΩ resistor, one 1.7 kΩ resistor, two 3.6 kΩ resistors, and one 5 kΩ resistor.

2.1 Frequency Response $H(\omega)$

The **frequency response** $H(\omega)$ of a linear and dissipative time-invariant circuit contains all the key information about the circuit which is needed to predict the circuit response to arbitrary inputs. Its magnitude $|H(\omega)|$ is known as **amplitude response** and $\angle H(\omega)$ is usually referred to as **phase response**. In this section, you will construct an active bandpass filter circuit and measure its amplitude response over the frequency range 1-20 kHz.

1. Construct the circuit shown in Figure 4 on your protoboard. For now, do not connect it to the three-stage circuit from Lab 2. Remember the rules for wiring and using the 741 op-amp, which are repeated in Figure 3.



Pins 1,5	offset null correction — not used
2	inverting input (-)
3	non-inverting input (+)
4	$V_{CC}-$ supply set to -15 V (DC)
7	$V_{CC}+$ supply set to 15 V (DC)
6	output voltage v_o
8	not connected

Figure 3: Pin-out diagram for the 741 op-am.

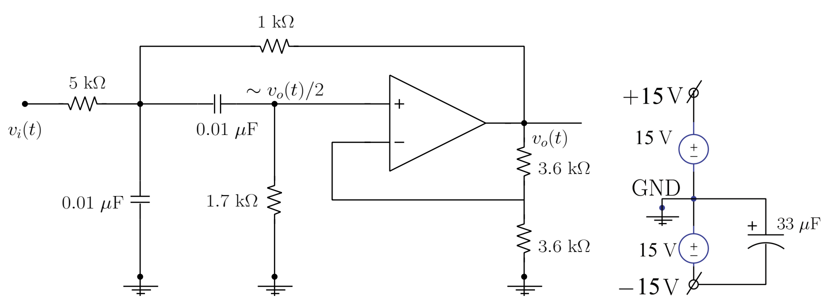


Figure 4: Circuit for analysis in prelab and lab.

2. Turn on the DC supplies, then connect a 1 kHz sine wave with amplitude 1 V peak-to-peak as the AC input $v_i(t)$ (Don't forget to set the function generator in High Z mode). Display $v_i(t)$ on channel 1 of the oscilloscope and $v_o(t)$ on channel 2. The Trigger should be set to the rising edge of Channel 1, and the time/div around 500 μs. You should obtain an output of approximately of 30 to 50 mV peak-to-peak. To obtain a clear measurement please follow the instructions below:

   - Be sure to have the 33μF electrolytic capacitor in your protoboard, between -15V and ground as shown in Figure 4. This is done to filter out voltage fluctuations produced by the power-supply. Recall that electrolytic capacitors have a voltage polarity requirement. The correct polarity is indicated on the packaging with minus signs and possible arrowheads, denoting the negative terminal. Also the negative terminal lead of radial electrolytic capacitors are shorter. (a reverse-bias voltage above 1 to 1.5 V will destroy the capacitor).
   - Select, in the Oscilloscope, the "High Resolution" acquisition mode. This is done by pressing the "Acquire" button and selecting the "High Res" mode under the "Acq Mode" menu on the display.

- Use the "Meas" button, and select the peak-to-peak measurement "Pk-Pk" applied to the Source 2(Channel 2 $= v_0(t)$). **Recall, that in any measurement performed by the oscilloscope, the entire signal has to fit inside the display, to have a valid value.**

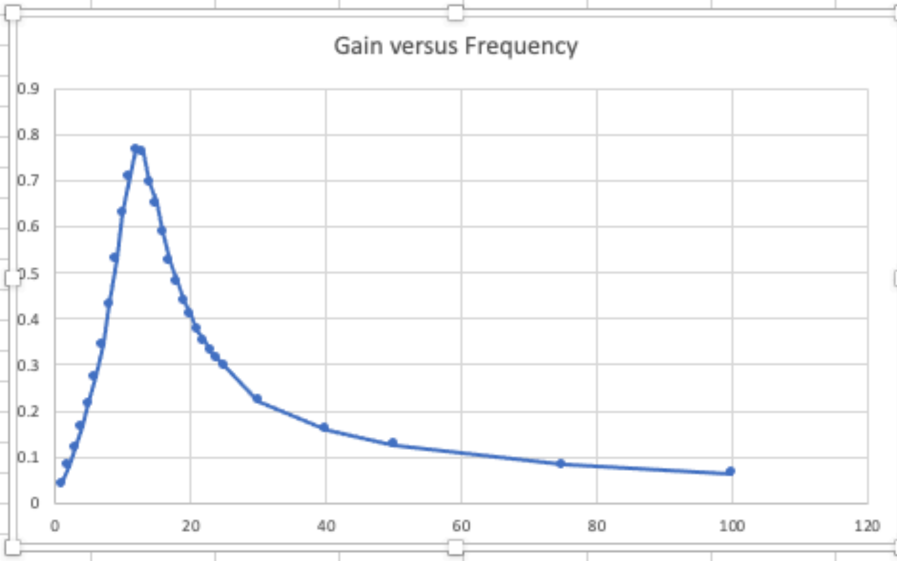
3. Increase the function generator frequency from 1 kHz to 100 kHz according to the table below. At each frequency record the magnitude of the phasor voltage gain $\frac{V_o}{V_i}$ and plot the values in the semi-log axis shown below only from 1 kHz to 20 kHz — $\frac{V_o}{V_i}$ is the system frequency response $H(\omega)$ and its magnitude $\frac{|V_o|}{|V_i|}$ is the system amplitude response $|H(\omega)|$.

f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert}$ =	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert}$ =	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert}$ =	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert}$ =	f(kHz)	$\frac{\lvert V_o\rvert}{\lvert V_i\rvert}$ =
1		7		13		19		25	
2		8		14		20		30	
3		9		15		21		40	
4		10		16		22		50	
5		11		17		23		75	
6		12		18		24		100	

(____/3)

Vout	Vin	freq	gain
42	1020	1	0.04117647
82	1020	2	0.08039216
121.6	1020	3	0.11921569
170	1020	4	0.16666667
220	1020	5	0.21568627
280	1020	6	0.2745098
352	1020	7	0.34509804
440	1020	8	0.43137255
540	1020	9	0.52941176
644	1020	10	0.63137255
724	1020	11	0.70980392
784	1020	12	0.76862745
776	1020	13	0.76078431
712	1020	14	0.69803922
664	1020	15	0.65098039
600	1020	16	0.58823529
536	1020	17	0.5254902
488	1020	18	0.47843137
448	1020	19	0.43921569
416	1020	20	0.40784314
384	1020	21	0.37647059
360	1020	22	0.35294118
336	1020	23	0.32941176
320	1020	24	0.31372549
304	1020	25	0.29803922
226	1020	30	0.22156863
162	1020	40	0.15882353
130	1020	50	0.12745098
84	1020	75	0.08235294
65	1020	100	0.06372549

$\left|\frac{V_o}{V_i}\right|$



Frequency (kHz)

(____/5)

4. The **center frequency** $\omega_o = 2\pi f_o$ of a bandpass $H(\omega)$ is defined as the frequency at which the amplitude response $|H(\omega)|$ is maximized. What is the center frequency f_o in kHz units and what is the maximum amplitude response $|H(\omega_o)|$ of the circuit? Estimate f_o and $|H(\omega_o)|$ from your graph as accurately as you can.

$f_0 =$ 12KHz	(____/2)	$\lvert H(\omega_0)\rvert =$ 0.769	(____/2)

5. The **3 dB cutoff frequencies** $\omega_u = 2\pi f_u$ and $\omega_l = 2\pi f_l$ are the frequencies above and below $\omega_o = 2\pi f_o$ at which the amplitude response $|H(\omega)|$ is $\frac{1}{\sqrt{2}} \approx 0.707$ times its maximum value $|H(\omega_o)|$. The same frequencies are also known as **half-power** cutoff frequencies since at frequencies ω_u and ω_l the output signal power is one half the value at ω_o, assuming equal input powers at all three frequencies.

$\frac{1}{\sqrt{2}}\|H(\omega_0)\|=$ 0.544	(____/2)	$f_l =$ 10 kHz	(____/2)	$f_u =$ 16 kHz	(____/2)

6. Determine the **3 dB bandwidth** $B \equiv f_u - f_l$ of the bandpass filter in kHz units and calculate the **quality factor** of the circuit defined as $Q \equiv \frac{\omega_o}{2\pi B} = \frac{f_o}{B}$.

$B =$ 6 kHz	(____/2)	$Q =$ 2	(____/2)

2.2 Displaying Fourier coefficients

In order to display the Fourier coefficients of a periodic signal on the oscilloscope we can use the built in FFT function[1]. On the "FFT screen" of your scope the horizontal axis will represent frequency ω (like in the frequency response plot of the last section) normalized by 2π, and you will see narrow spikes positioned at values $f = \frac{\omega}{2\pi}$ equal to harmonic frequencies $n\frac{\omega_o}{2\pi}$ of the periodic input signal; spike amplitudes will be proportional to compact Fourier coefficients $c_n = 2|F_n|$ in dB. The next set of instructions tells you how to view the single Fourier coefficient (namely c_1) of a co-sinusoidal signal:

1. Connect Channel 1 of your oscilloscope to the input terminal of your circuit from the previous section. Connect the output of the circuit to Channel 2.
2. Set the function generator to a 13 kHz sinusoid with amplitude 500 mV peak-to-peak. In the scope, press the "Auto-Scale" button and adjust the volts/div "Amplitude" knobs, so that the peak-to-peak amplitude of the input and the output are of the order of 1 division in the display (i.e. the signals should occupy approximately 1 division). For better results use the acquisition mode "High Res".
3. Set the oscilloscope to compute the Fourier transform:
   (a) Press "Math"
   (b) Select "FFT" under the "Operator" item menu on the display.
   (c) Set the Operand to Source 2 (The output).
   (d) Press "More FFT". The default "Window" setting "Hanning" should be used.
   (e) Then set the time/div to 2 ms, so that the frequency span(second item in the menu) is 25.0 kHz and the Center frequency is 12.5 kHz.
4. Observe the output signal's Fourier coefficient . How does the FFT display change as you sweep the frequency of the input from 1 kHz to 20 kHz? Describe how the signal changes in frequency domain and explain why the signal changes as a function of the frequency. If necessary turn off the time-domain signals by pressing "1" and/or "2" until the signal disappears.

Describe how the signal changes in the frequency domain and explain why. (____/4)

The peak of the signal moves from the left end to the right end of the oscilloscope display screen with the increase of frequency.

[1] *FFT* stands for *fast Fourier transform* and it is a method for calculating Fourier transforms with sampled signal data — see Example 9.26 in Section 9.3 of Chapter 9 to understand the relation of windowed Fourier transforms to Fourier coefficients.

2.3 Fourier coefficients of a square wave

Now you will introduce a periodic signal with a more interesting set of Fourier coefficients — a square-wave:

1. Change the function generator setting to create a 15 kHz square wave with amplitude 0.5 V peak-to peak as the input to your circuit.

2. Display the FFT of the square wave at the filter input by setting the Operand to Channel 1, and set the time/div to 100μs in order to have a frequency span of 500kHz and centered at 250kHz. Fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power. Keeping in mind the result for Problem 1 in the Prelab, describe the signal in the frequency domain:

(____/4)

freq.	Ampl.(dBV)	Ampl.(V)
14.5K	-12.74dBV	217.20mV
45.0K	-24.03dBV	67.3 mV
75.4K	-27.70 dBV	42.20mV
104.3K	-30.2 dBV	31.8mV

Vrms graph like many peaks with some interval.
dBV graph is also peaks but the buttom is not
a straight line

3. Display the FFT of the filter's output, by setting the FFT Operand to Channel 2. In addition, display on the scope both the input and output signals in the time domain. Describe the output in time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

(____/4)

freq.	Ampl.(dBV)	Ampl.(V)
14.1k	-21.4 dBV	86.2mV
44.2k	-34.4 dBV	17.4dBV
90K	-41.7dBV	8.23mV
61K	-42.2 dBV	7.60mV

Describe the output signal in the time domain.

like input, diagram has peaks. but the peak does
not arrange from high to low this time.
Although the highest one is the first one, the height of the rest peaks
are not sorted.

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain:

(____/4)

It looks very similar to the sinoid wave, with frequency around 14 kHz
as input.

Its FFT only has one major peak, the rest are minor and at large frequency.
Thus the image is dominated by the majority sinod wave.

4. Repeat for a 10 kHz square wave. Describe the output signal in the time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

(____/4)

freq.	Ampl.(dBV)	Ampl.(V)
10K	-21.8dBV	76.27mV
20K	-36.4dBV	17.2mV
40K	-37.6dBV	14.3mV
30K	-43.2dBV	7.24mV

Describe the output signal in the time domain.

The output wave looks kind of like triangular wave
but seems more round.

Based on the measured amplitudes of the harmonics of the output signal calculate the total harmonic distortion (THD), and explain the shape of the output signal seen in time domain:

(____/4)

The output wave is a superposition of many sine-waves. some compound sine waves have frequency only twice or three time as the major one that makes the output wave seems similar to triangular wave.

5. Repeat for a 5 kHz square wave. Describe the output signal in time domain. For the frequency domain, fill in the table specifying the frequency and the amplitude of the 4 harmonics with the largest power.

(____/4)

freq.	Ampl.(dBV)	Ampl.(V)
4.8K	-29.3dBV	28.8mV
14.6k	-30.0dBV	27.6mV
9.8K	-35.7 dBV	14.2mV
24.2k	-40.3dBV	8.9mV

Describe the output signal in the time domain.

The output wave is very complicated, with many rises and falls in one period.

Based on the amplitude of the harmonics of the output signal, explain the shape of the output signal seen in time domain:

(____/3)

The component waves have similar amplitude and close frequency, which makes the output signal to have complicated waveform.

6. In terms of the amplitude response of the system ($|H(\omega)|$) explain the change in amplitude of the harmonics from the input to the output (Hint: what frequencies are been attenuated, and what frequencies are inside the bandwidth of the filter):

(____/5)

The amplitude of major harmonic wave drops, but the sub harmonic waves' amplitude rises.

Important!

Leave your active filter assembled on your protoboard! You will need it in the next lab session.

The Next Step

The active filter is the last component you will build for the AM radio receiver. In Lab 4, you will combine your components from Labs 1 through 3 to create a working AM radio receiver. The frequency-domain techniques you learned this week will be essential to following the AM signal through each stage of the receiver system. Please make sure you have your circuits ready (i.e. working lab 2 and lab 3) for lab 4 if you want to get **bonus**!