



ANALOG SIGNAL PROCESSING



ECE 210 & 211
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Objectives

- **Orthogonal Projection and Fourier series**
- **Exponential Fourier series**
- **Periodic and non-periodic sums**
- **Shifting Property of signals**
- **Differentiation of signals**

Objectives

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Orthogonal Projections and Fourier Series

➤ There is a strong mathematical correlation between Fourier series and vector of ***n -dimensional space***

➤ Suppose a 3-D vector as,

$$\vec{v} = (3, -2, 5)$$

can be re-written as a weighted sum of three mutually orthogonal vectors,

$$\vec{u}_1 = (1, 0, 0), \quad \vec{u}_2 = (0, 1, 0), \quad \vec{u}_3 = (0, 0, 1)$$

as,

$$\vec{v} = 3\vec{u}_1 - 2\vec{u}_2 + 5\vec{u}_3$$

Orthogonal Projections and Fourier Series

In general, any 3D vector can be expressed as,

$$\vec{v} = \sum_{n=1}^3 V_n \vec{u}_n \quad \text{where,} \quad V_n = \vec{v} \cdot \vec{u}_n$$

- The coefficient V_n of the \vec{v} can be regarded as projections of \vec{v} along the basis function \vec{u}_n
- By this analogy, a convergent Fourier series as

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$$

Orthogonal Projections and Fourier Series

can be treated as an infinite weighted sum of orthogonal basis function,

$$e^{jn\omega_0 t}, -\infty \leq n \leq \infty$$

satisfying an orthogonality condition,

$$\int_T (e^{jn\omega_0 t})(e^{jm\omega_0 t}) dt = 0 \text{ for } m \neq n$$

A Fourier coefficient F_m of $f(t)$ is then the projection of $f(t)$ *along basis function* $e^{jm\omega_0 t}$.

Orthogonal Projections and Fourier Series

Inner product property for \vec{a} with \vec{b} , is

$$\langle a, b \rangle = a^T b = [a_0, a_1, \dots, a_n] \begin{bmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{bmatrix} = \sum_{i=1}^n a_i b_i$$

Inner product property for $f[n]$ with $g[n]$, is

$$\langle f[n], g[n] \rangle = \sum_1^n f[n] g^*[n]$$

Inner product property for $f(t)$ with $g(t)$, is

$$\langle f(t), g(t) \rangle = \int_a^b f(t) g^*(t) dt$$

Orthogonal Projections and Fourier Series

Using inner product property for $f(t)$ with $e^{jm\omega_o t}$, is

$$\begin{aligned}\int_T f(t) (e^{-jm\omega_o t}) dt &= \int_T \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t} e^{-jm\omega_o t} dt \\ &= \int_T F_n \sum_{n=-\infty}^{\infty} e^{jn\omega_o t} (e^{jm\omega_o t})^* dt = F_m\end{aligned}$$

Only interchanging m with n , we may have,

$$F_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_o t} dt$$

which can be used for any periodic and convergent signal (satisfying Dirichlet's conditions))

Orthogonal Projections and Fourier Series

Trigonometric Fourier series as Orthogonal Projection

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \underbrace{\cos(n\omega_o t)} + b_n \underbrace{\sin(n\omega_o t)}$$

Orthogonal Basis functions

$$F_n = a_n - jb_n$$

$$a_n = F_n + F_{-n}$$

$$b_n = j(F_n - F_{-n})$$

Objectives

- Orthogonal Projection and Fourier series
- **Exponential Fourier series**
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Exponential Fourier series

- A compact way to represent Fourier series is to put in to exponential form
- We need to use Euler's identity for sine and cosine functions,

$$\cos n\omega_0 t = \frac{1}{2}[e^{jn\omega_0 t} + e^{-jn\omega_0 t}]$$

$$\sin n\omega_0 t = \frac{1}{2j}[e^{jn\omega_0 t} - e^{-jn\omega_0 t}]$$

Exponential Fourier series

$$f(t) = \underbrace{a_0}_{\text{dc}} + \underbrace{\sum_{n=1}^{\infty} (a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)}_{\text{ac}}$$

Using the values of $\cos n\omega_0 t$ and $\sin n\omega_0 t$,

$$f(t) = a_0 + \frac{1}{2} \sum_{n=1}^{\infty} [(a_n - jb_n)e^{jn\omega_0 t} + (a_n + jb_n)e^{-jn\omega_0 t}]$$

If we define a new coefficient c_n so that,

$$c_0 = a_0, \quad c_n = \frac{(a_n - jb_n)}{2}, \quad c_{-n} = c_n^* = \frac{(a_n + jb_n)}{2}$$

Exponential Fourier series

then $f(t)$ becomes,

$$f(t) = c_0 + \sum_{n=1}^{\infty} (c_n e^{jn\omega_0 t} + c_{-n} e^{-jn\omega_0 t})$$

or,

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

Exponential /complex form of Fourier transform

Exponential Fourier series

- The plots of the magnitude and phase of c_n versus $n\omega_0$ are called the *complex amplitude spectrum* and *complex phase spectrum* of $f(t)$, respectively
- The exponential Fourier series of a periodic function $f(t)$ describes the spectrum of $f(t)$ in terms of the amplitude and phase angle of ac components at positive and negative harmonic frequencies

$$A_n \angle \phi_n = a_n - jb_n = 2c_n$$

Exponential Fourier series

Alternatively,

$$c_n = |c_n| \angle \theta_n = \frac{\sqrt{a_n^2 + b_n^2}}{2} \angle -\tan^{-1} b_n/a_n \quad \text{if positive } a_n$$

The rms value can be calculated as,

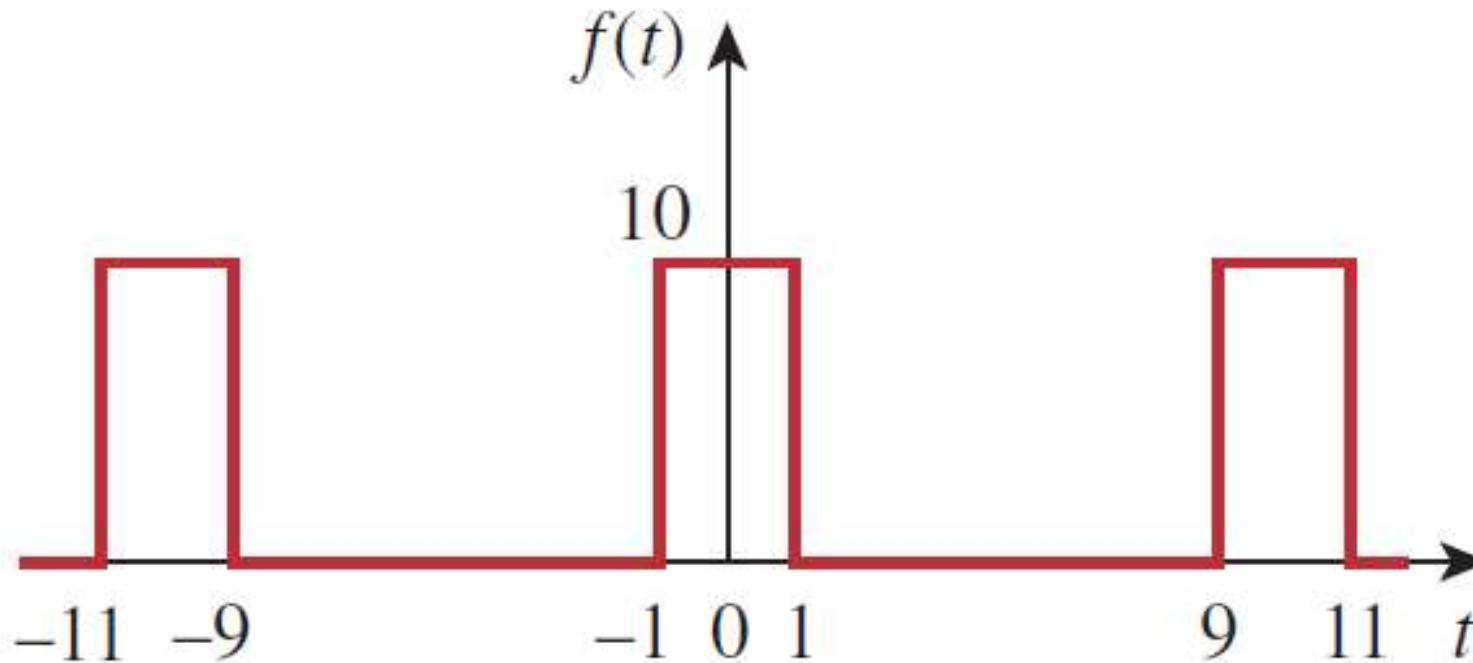
$$F_{\text{rms}} = \sqrt{\sum_{n=-\infty}^{\infty} |c_n|^2}$$

For a periodic input with period ω_o can be expressed,

$$F_{\text{rms}}^2 = |c_0|^2 + 2 \sum_{n=1}^{\infty} |c_n|^2$$

Exponential Fourier series

Question: Find the amplitude and phase spectra of the pulse train shown below?



Exponential Fourier series

Solution: The period of the pulse train is $T = 10 \text{ s}$, Thus the fundamental frequency will be $\omega_o = \frac{\pi}{5} \text{ rad/s}$

$$\begin{aligned} c_n &= \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_o t} dt = \frac{1}{10} \int_{-1}^1 10 e^{-jn\omega_o t} dt \\ &= \frac{1}{-jn\omega_o} e^{-jn\omega_o t} \Big|_{-1}^1 = \frac{1}{-jn\omega_o} (e^{-jn\omega_o} - e^{jn\omega_o}) \\ &= \frac{2}{n\omega_o} \frac{e^{jn\omega_o} - e^{-jn\omega_o}}{2j} = 2 \frac{\sin n\omega_o}{n\omega_o}, \quad \omega_o = \frac{\pi}{5} \\ &= 2 \frac{\sin n\pi/5}{n\pi/5} \end{aligned}$$

Exponential Fourier series

$$f(t) = 2 \sum_{n=-\infty}^{\infty} \frac{\sin n\pi/5}{n\pi/5} e^{jn\pi t/5}$$

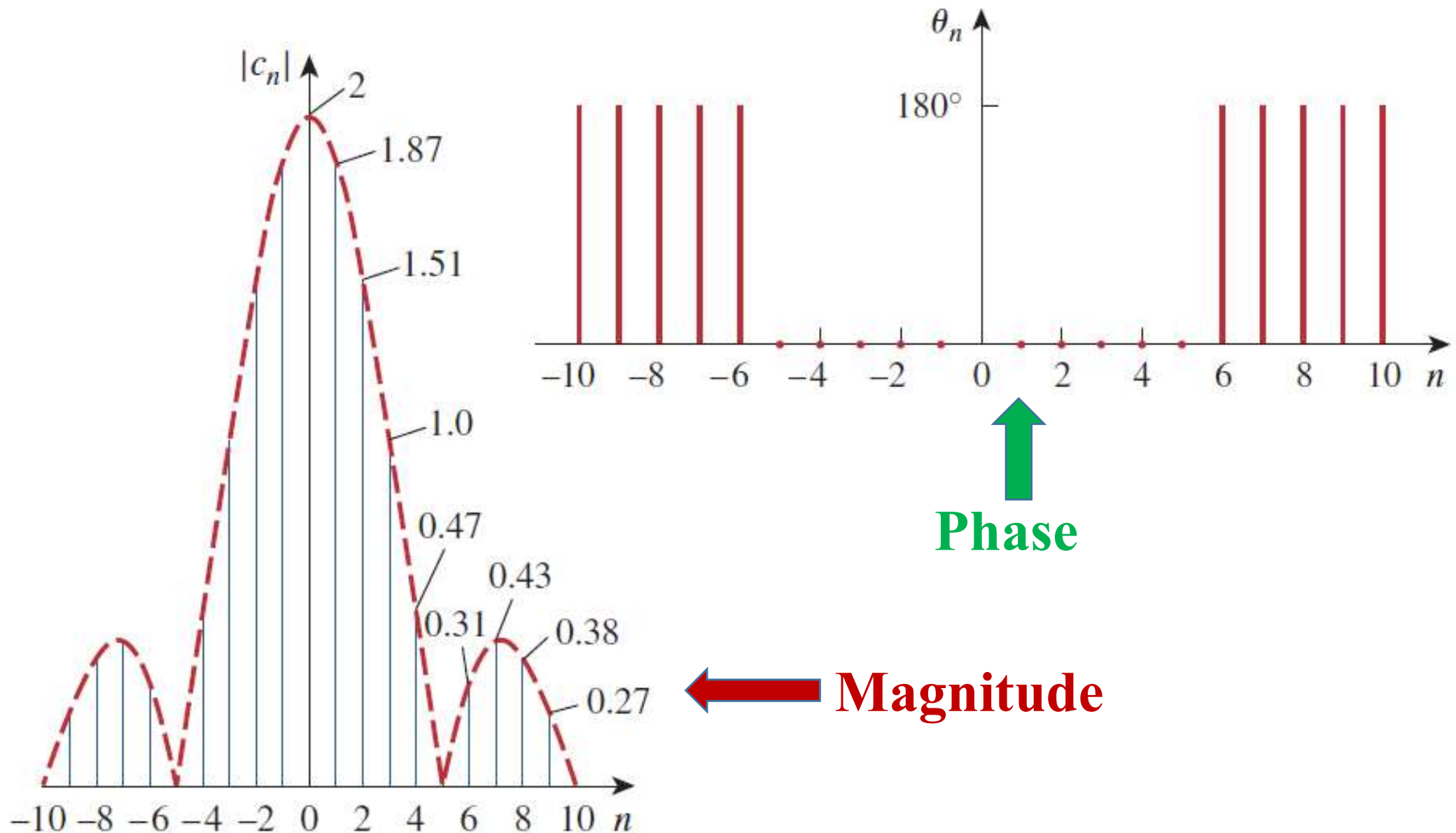
It is a ***Sinc function***. Amplitude of the $f(t)$ is,

$$|c_n| = 2 \left| \frac{\sin n\pi/5}{n\pi/5} \right|$$

The phase of the $f(t)$ is,

$$\theta_n = \begin{cases} 0^\circ, & \sin \frac{n\pi}{5} > 0 \\ 180^\circ, & \sin \frac{n\pi}{5} < 0 \end{cases}$$

Exponential Fourier series



Exponential Fourier series

Question: Find the exponential Fourier expansion of the periodic function $f(t) = e^t$, $0 < t < 2\pi$, with period 2π ?

Solution:

As $\omega_o = 1$ rad/s, Hence,

$$\begin{aligned} c_n &= \frac{1}{T} \int_0^T f(t) e^{-jn\omega_o t} dt = \frac{1}{2\pi} \int_0^{2\pi} e^t e^{-jnt} dt \\ &= \frac{1}{2\pi} \frac{1}{1 - jn} e^{(1-jn)t} \Big|_0^{2\pi} = \frac{1}{2\pi(1 - jn)} [e^{2\pi} e^{-j2\pi n} - 1] \end{aligned}$$

Exponential Fourier series

By using Euler's identity,

$$e^{-j2\pi n} = \cos 2\pi n - j \sin 2\pi n = 1 - j0 = 1$$

Thus,

$$c_n = \frac{1}{2\pi(1 - jn)} [e^{2\pi} - 1] = \frac{85}{1 - jn}$$

The complex Fourier series will be,

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{85}{1 - jn} e^{jnt}$$

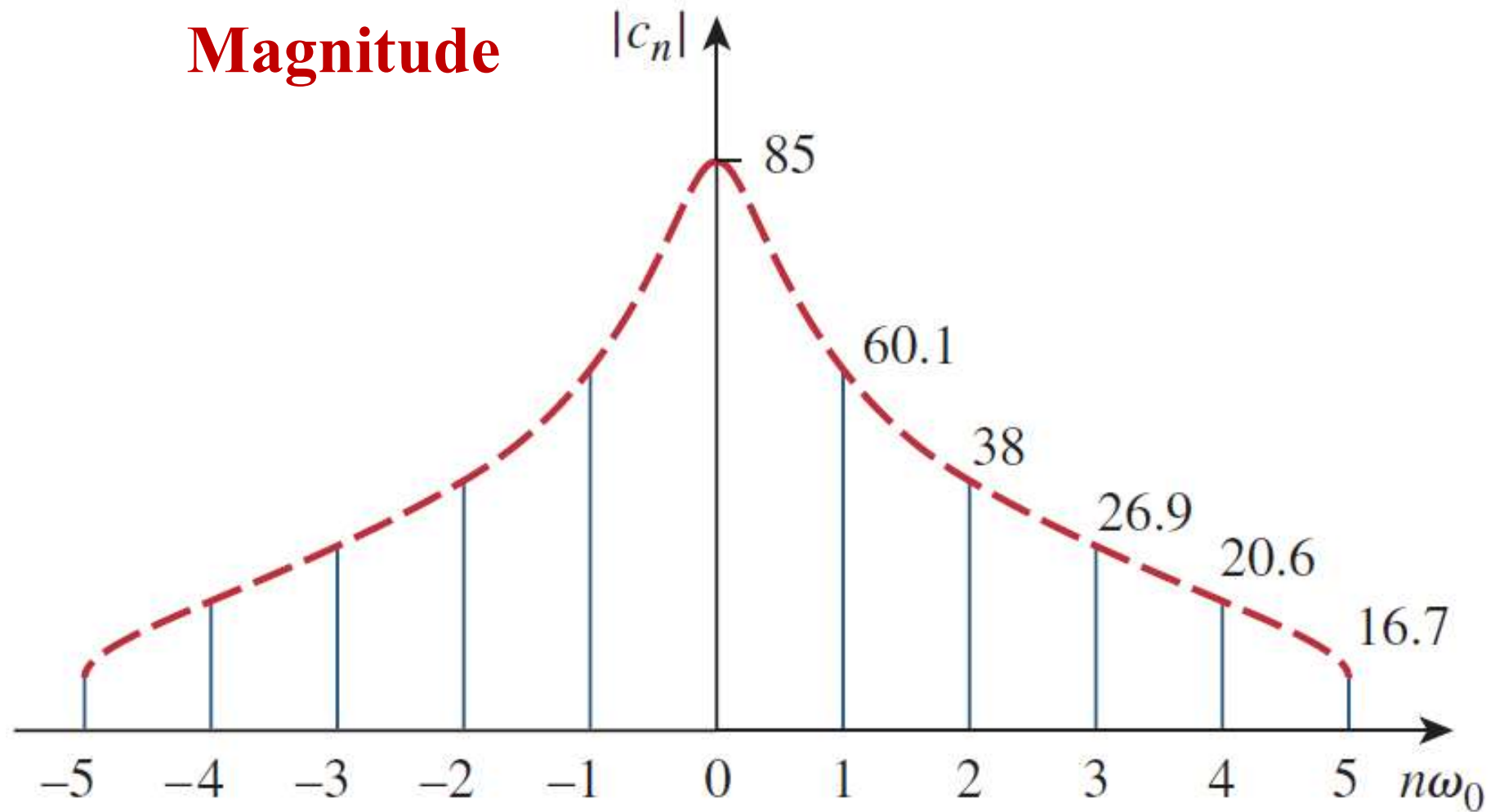
Exponential Fourier series

We may want to plot the complex frequency spectrum of $f(t)$. If we let $c_n = |c_n| \angle \theta_n$, then,

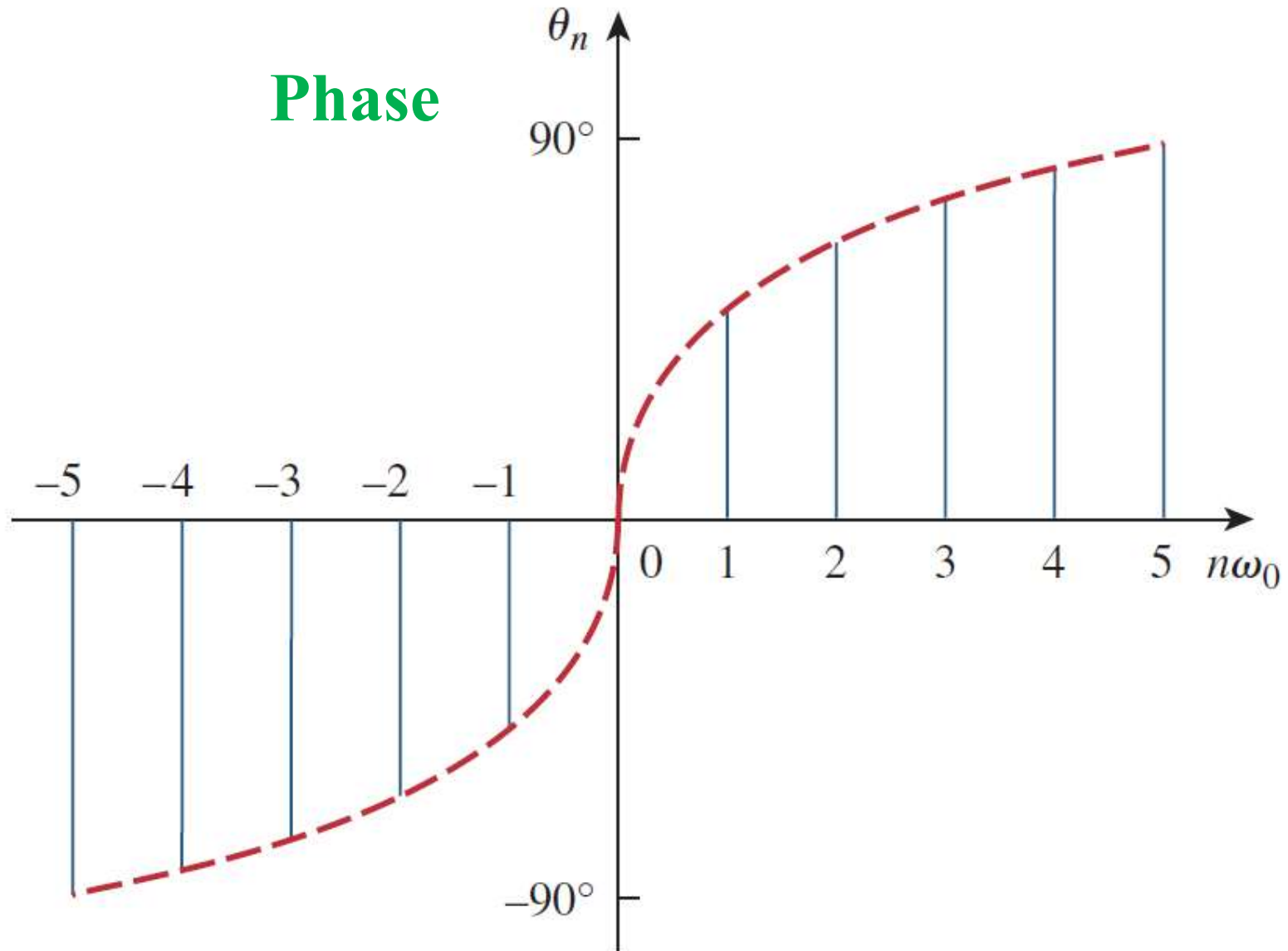
$$|c_n| = \frac{85}{\sqrt{1 + n^2}}, \quad \theta_n = \tan^{-1} n$$

By inserting the positive and negative values of n , we can obtain the magnitude and phase of c_n versus $n\omega_o = n$

Exponential Fourier series

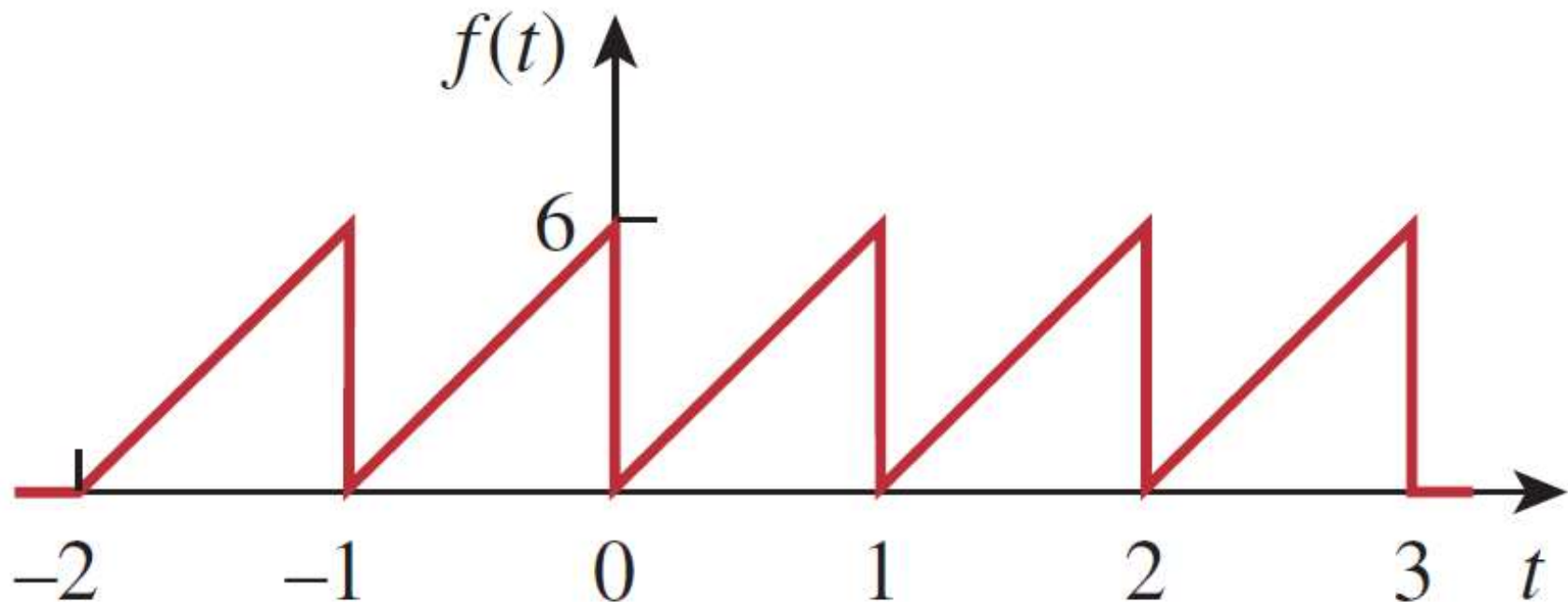


Exponential Fourier series



Exponential Fourier series

Question: Find the exponential Fourier expansion of the sawtooth wave. Also plot amplitude and phase spectra?



Exponential Fourier series

Solution: As $\omega_o = 2\pi$ rad/s, Hence,

$$c_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_o t} dt = \frac{1}{1} \int_0^1 t e^{-j2n\pi t} dt$$

Applying integration,

$$\begin{aligned} c_n &= \frac{e^{-j2n\pi t}}{(-j2n\pi)^2} (-j2n\pi t - 1) \Big|_0^1 \\ &= \frac{e^{-j2n\pi} (-j2n\pi - 1) + 1}{-4n^2\pi^2} = \frac{-j2n\pi}{-4n^2\pi^2} = \frac{j}{2n\pi} \end{aligned}$$

Exponential Fourier series

This does not include the case when $n = 0$, So, considering this condition gives

$$c_0 = \frac{1}{T} \int_0^T f(t) dt = \frac{1}{1} \int_0^1 t dt = \left. \frac{t^2}{2} \right|_0^1 = 0.5$$

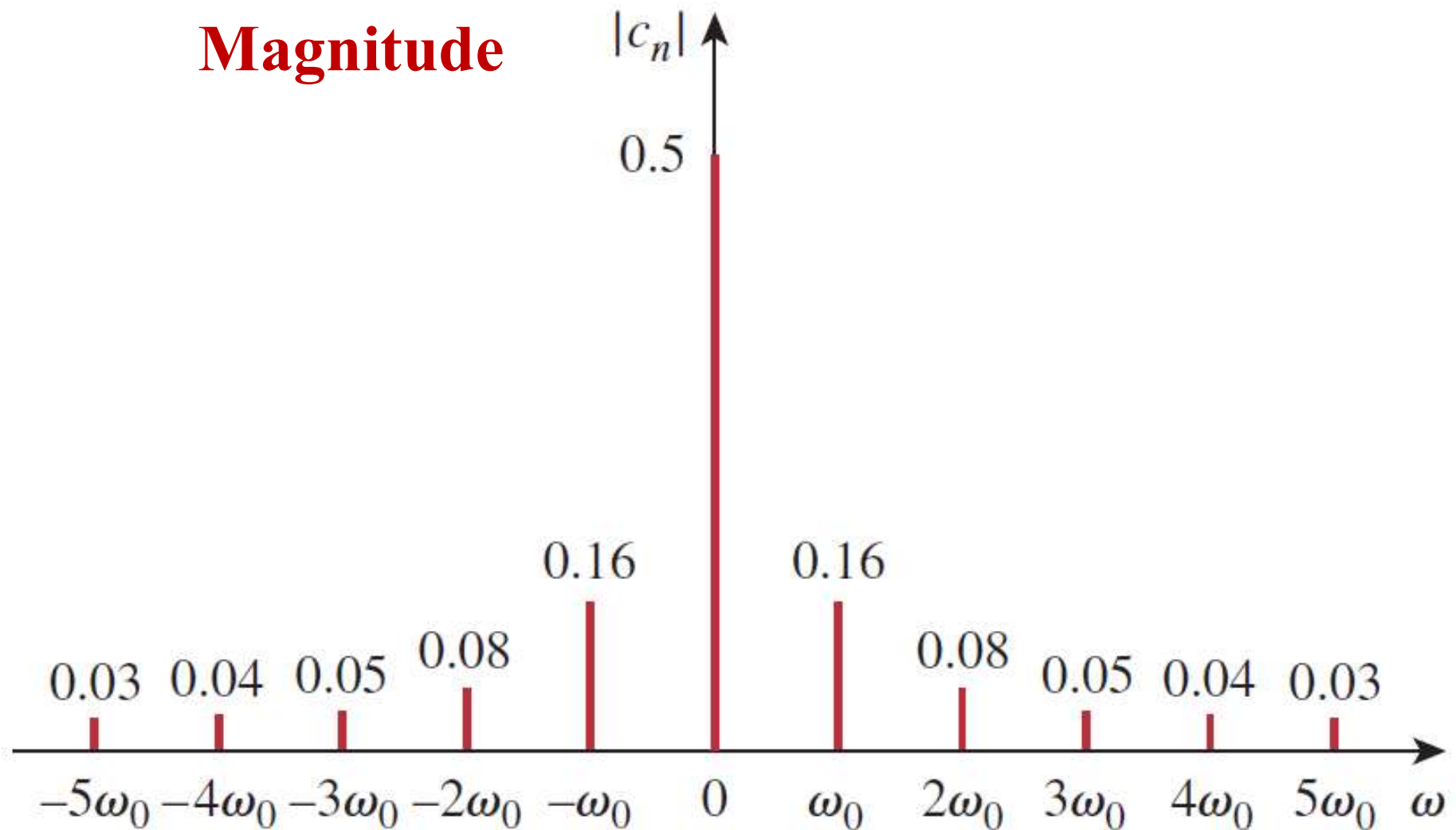
Hence,

$$f(t) = 0.5 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{j}{2n\pi} e^{j2n\pi t}$$

So, the **amplitude** and **phase** is,

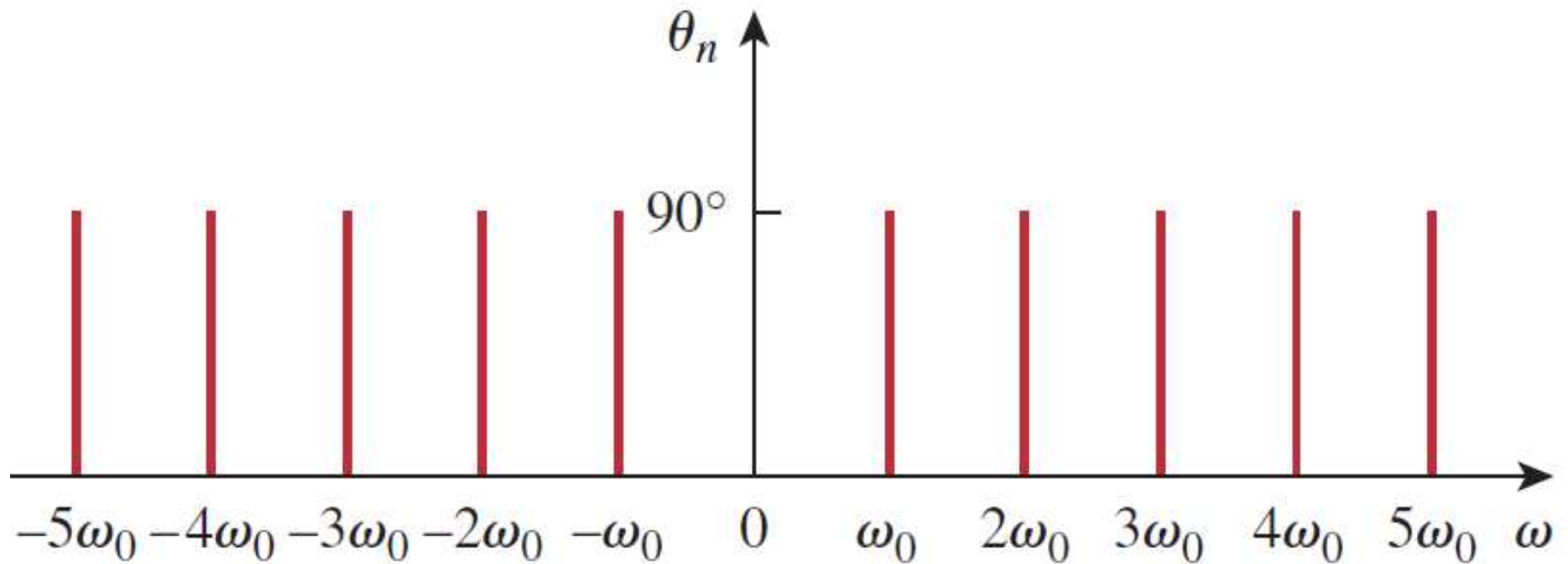
$$|c_n| = \begin{cases} \frac{1}{2|n|\pi}, & n \neq 0 \\ 0.5, & n = 0 \end{cases}, \quad \theta_n = 90^\circ, \quad n \neq 0$$

Exponential Fourier series



Exponential Fourier series

Phase



Objectives

- Orthogonal Projection and Fourier series
- Exponential Fourier series
- **Periodic and non-periodic sums**
- Shifting Property of signals
- Differentiation of signals

Periodic and non-periodic sums

- It is seen that *not* all the co-sinusoids or exponentials are periodic, as mentioned below,

$$g(t) = \sum_{k=1}^{\infty} c_k \cos(\omega_k t + \theta_k)$$

- It is periodic only for any number ω_o that is all frequencies ω_k are **integer multiple of ω_o**
- If the sum is periodic then all possible ratios of ω_k are *rational*
- ω_o : Largest number whose integer multiples *matches* each and every ω_k

Non-periodic sums – Example 1

Consider a signal,

$$p(t) = 2 \cos(\pi t) + 4 \cos(2t)$$

Taking the ratio of two frequencies,

$$\frac{\pi t}{2t} = \frac{\pi}{2}$$

- It is not a *rational number* so not a periodic signal

Periodic sums – Example 2

Consider another signal,

$$q(t) = \cos(4t) + 5\sin(6t) + 2\cos(7t - \frac{\pi}{7})$$

is a periodic signal as frequencies **4,6** and **7 rad/s** are integral multiples of **1 rad/s**

- Fundamental frequency of signal is $\omega_o = 1 \frac{\text{rad}}{\text{s}}$
- The period is 2π seconds

Periodic sums – Example 3

For the signal,

$$f(t) = 1 + \cos(8\pi t) + 7.6\sin(10\pi t)$$

It can be seen that 2π is the largest number whose integer multiples ($\times 4$ and $\times 5$) matches frequencies 8π and 10π

- Fundamental frequency of signal is $\omega_o = 2\pi \frac{\text{rad}}{\text{s}}$
- The period will be *1 second*
- What are exponential Fourier series of $f(t)$?

Exponential Fourier Series

we can re-write $f(t)$ as,

$$f(t) = 1e^{j(0) \cdot 2\pi t} + \frac{e^{j(4)2\pi t} + e^{j(-4)2\pi t}}{2} + 7.6 \frac{e^{j(5)2\pi t} - e^{j(-5)2\pi t}}{2j}$$

All the Fourier series coefficients F_n are zero except :

➤ $F_0 = 1$

➤ $F_{4,-4} = \frac{1}{2}$

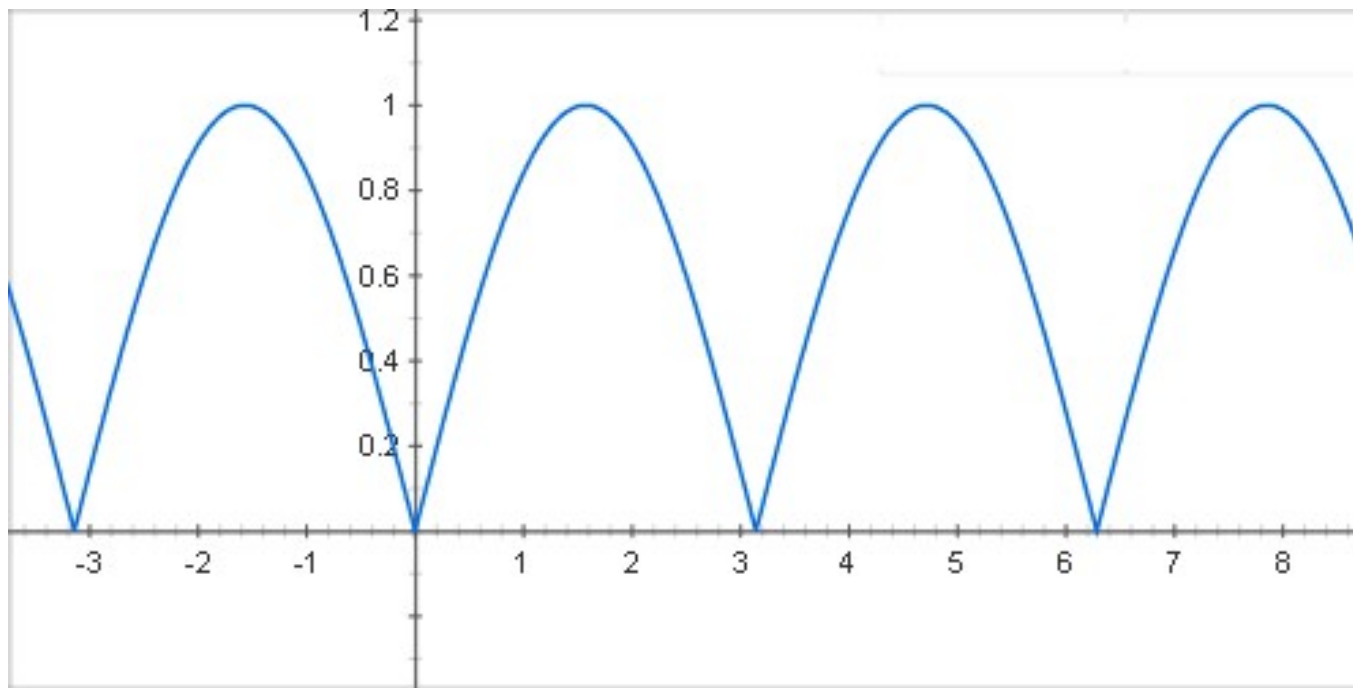
➤ $F_5 = \frac{7.6}{2j} = -j3.8$

➤ $F_{-5} = \frac{-7.6}{2j} = j3.8$

Periodic sums – Example 4

A sine modulus signal is shown below,

$$f(t) = |\sin(t)|$$



Find the exponential and compact Fourier coefficients for $f(t)$?

Periodic sums – Example 4

From the plot of $|\sin(t)|$, it can be seen that $\omega_o = 2 \text{ rad/sec}$ and $T = \pi \text{ seconds}$, and it will be integrated over $[0, \pi]$, for exponential form:

$$F_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_o t} dt = \frac{1}{\pi} \int_0^{\pi} \sin(t) e^{-jn2t} dt$$

Solving the integral over the period gives,

$$F_n = -\frac{1}{2\pi} \left(\frac{e^{j(1-2n)\pi} - 1}{1 - 2n} + \frac{e^{-j(1+2n)\pi} - 1}{1 + 2n} \right)$$

Exponential Fourier series

$$F_n = \frac{1}{\pi} \left(\frac{1}{1-2n} + \frac{1}{1+2n} \right) = \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right)$$

The exponential Fourier series for $|\sin(t)|$ is,

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) e^{jn2t}$$

The coefficients for compact form (**for $n > 1$**) are,

$$c_n = 2|F| = 2 \sqrt{\left[\frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) \right]^2 + 0^2} = \frac{1}{\pi} \frac{1}{n^2 - \frac{1}{4}}$$

$$\theta_n = \angle F_n = \pi \text{ rad}$$

Compact form of Fourier series

Now, the compact form,

$$c_0 = F_0 = \frac{2}{\pi}$$

$$f(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{1}{\pi} \frac{1}{n^2 - \frac{1}{4}} \cos(n2t + \pi)$$

As we know that $f(t)$ is *continuous*, so the Fourier series *converges* to $f(t)$ for all values of t

Objectives

- Orthogonal Projection and Fourier series
- Exponential Fourier series
- Periodic and non-periodic sums
- **Shifting Property of signals**
- Differentiation of signals

Shifting property

The shifting property states that,

$$f(t) \longleftrightarrow F_n \xrightarrow{\quad} f(t - t_o) \longleftrightarrow F_n e^{-jn\omega_o t_o}$$

Expressing $f(t)$ in Fourier series,

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$$

replacing t with $t - t_o$ both sides,

$$f(t - t_o) = \sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o(t-t_o)} = \sum_{n=-\infty}^{\infty} (F_n e^{-jn\omega_o t_o}) e^{jn\omega_o t}$$

The expression in the parenthesis is n -th Fourier coefficient of $f(t - t_o)$, proving shifting property

Time scaling property


Previously, we have seen that for $|\sin(t)|$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) e^{jn2t}$$

to see *time scaling property*, we introduce

$$g(t) = \left| \sin \left(\frac{t}{2} \right) \right|$$

note that $g(t) = f\left(\frac{t}{2}\right)$, replacing t with $\frac{t}{2}$ gives

$$g(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi} \left(\frac{1}{1-4n^2} \right) e^{jnt}$$


Time scaling property

and for compact form,

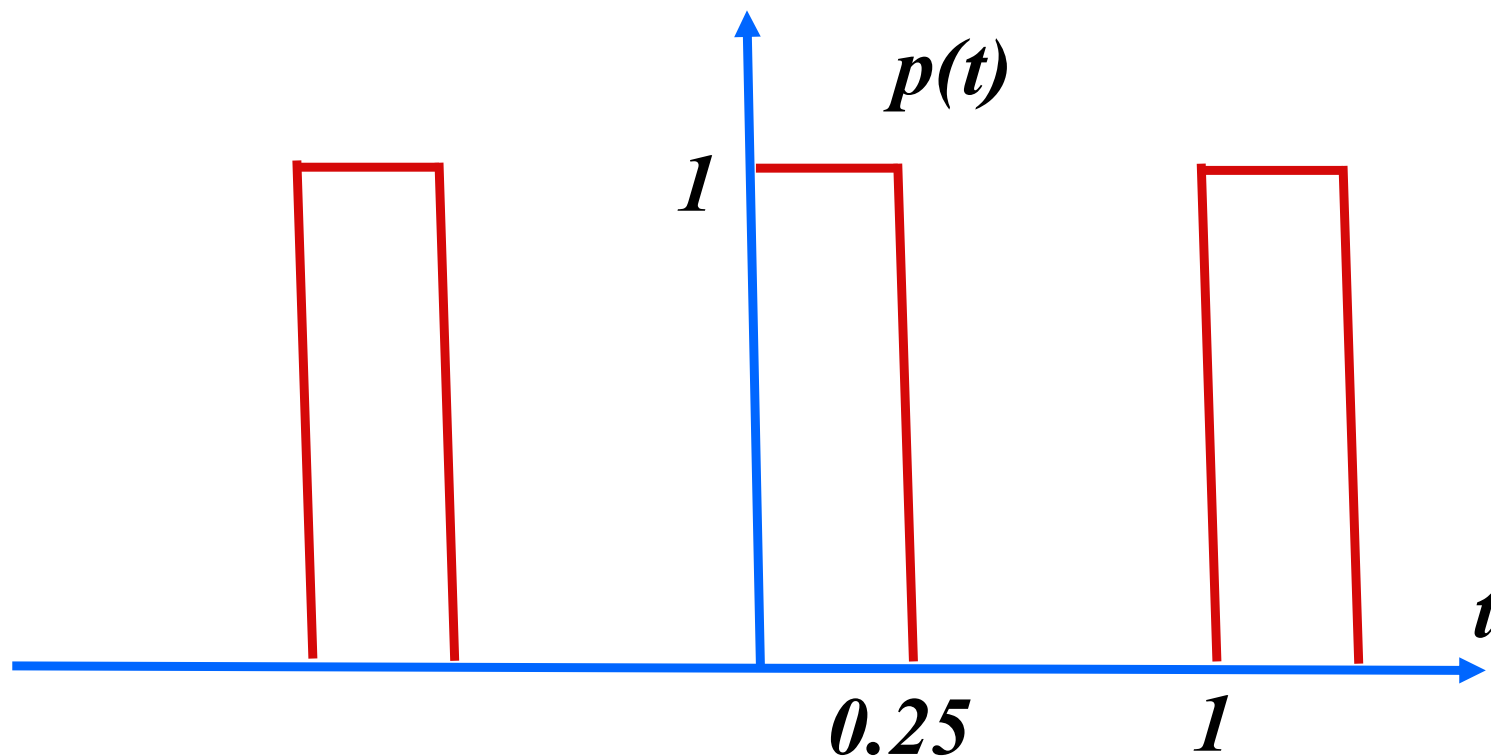
$$g(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{1}{\pi} \frac{1}{n^2 - \frac{1}{4}} \cos(nt + \pi)$$



- We may observe a *stretching* or *squashing* of a periodic waveform corresponds to a change in *period* and *fundamental frequency*
- $g(t)$ is stretched by factor of 2

Periodic sums – Example 5

Question: Find the exponential and compact form of Fourier series of given signal having $T=1$ s and *duty cycle 25%* and amplitude 1 unit?



Periodic sums – Example 5

Solution: We can express the signal as,

$$p(t) = \begin{cases} 1 & 0 < t < D \\ 0 & D < t < 1 \end{cases} \quad \text{for } D = 0.25$$

The signal has period of 1 s so fundamental frequency is 2π , evaluating Fourier coefficients,

$$P_n = \frac{1}{T} \int_T p(t) e^{-jn\omega_o t} dt = \frac{1}{1} \int_0^D e^{-jn2\pi t} dt$$

For the case $n = 0$,

$$P_0 = D$$

Periodic sums – Example 5

For the case $n \neq 0$,

$$P_n = \left. \frac{e^{-jn2\pi t}}{-jn2\pi t} \right|_0^D = \frac{e^{-jn2\pi D} - 1}{-jn2\pi t} = \frac{\sin n\pi D}{n\pi} e^{-jn\pi D}$$

So, the exponential form will be,

$$p(t) = \sum_{n=-\infty}^{\infty} \frac{\sin n\pi D}{n\pi} e^{j(n2\pi t - n\pi D)}$$

and compact form will be,

$$p(t) = D + \sum_{n=1}^{\infty} \frac{2\sin n\pi D}{n\pi} \cos(n2\pi t - n\pi D)$$

Periodic sums – Example 6

Question: Find the compact trigonometric Fourier series for $q(t)$ having time period $T = 4\text{ s}$?

$$q(t) = \begin{cases} 2t & 0 < t < 2s \\ 0 & 2 < t < 4s \end{cases}$$

Solution: The fundamental frequency is $\omega_o = \frac{\pi}{2} \text{ rad/s}$

First, for $n = 0$,

$$Q_o = \frac{1}{4} \int_0^2 2t dt = \frac{1}{4} t^2 \Big|_0^2 = 1$$

Periodic sums – Example 6

secondly, for all other values of n ,

$$Q_n = \frac{1}{4} \int_0^2 2te^{-jn\frac{\pi}{2}t} dt = \frac{1}{2} \int_0^2 t \frac{d}{dt} \cdot \left[\frac{e^{-jn\frac{\pi}{2}t}}{-jn\frac{\pi}{2}} \right] dt$$

using integration by part will result into,

$$Q_n = \begin{cases} \frac{j2}{\pi n} & \text{for even } n > 0 \\ -\frac{4 + j2\pi n}{\pi^2 n^2} & \text{for odd } n \end{cases}$$

Compact Form – Example 6

after finding the magnitude and phase of Q_n , for even and odd terms, we can write compact form as,

$$q(t) = 1 + \sum_{n=2(\text{even})}^{\infty} \frac{4}{\pi n} \cos\left(n \frac{\pi}{2} t + \frac{\pi}{2}\right) + \sum_{n=1(\text{odd})}^{\infty} \frac{4\sqrt{4 + \pi^2 n^2}}{\pi^2 n^2} \\ \times \cos\left(n \frac{\pi}{2} t + \pi + \tan^{-1} \frac{\pi n}{2}\right)$$

Objectives

- Orthogonal Projection and Fourier series
- Exponential Fourier series
- Periodic and non-periodic sums
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- **Differentiation of signals**

Differentiation of Signal – Example 7

Question: Determine the exponential Fourier series and compact trigonometric Fourier series of $g(t)$?

$$g(t) = \frac{df}{dt}$$

where,

$$f(t) = |\sin(t)|$$

Differentiation of Signal – Example 7

Solution:

From Example 4, $\omega_o = 2 \frac{\text{rad}}{\text{s}}$

For continuous $f(t)$, differentiation property states,

$$\text{For Continuous } f(t) : \quad \frac{df}{dt} \longleftrightarrow j\omega F_n = jn\omega_o F_n$$

can be used, so putting the value of ω_o , and Fourier coefficient,

$$G_n = jn\omega_o F_n = jn2F_n \quad \text{Eq. 1}$$

Compact form – Example 7

as we seen in example 4, the Fourier coefficient of $|\sin(t)|$ is,

$$F_n = \frac{2}{\pi} \frac{1}{1 - 4n^2}$$

Putting the value of F_n in Eq. 1 gives the Fourier coefficient for $g(t)$,

$$G_n = j \frac{n^4}{\pi} \frac{1}{1 - 4n^2}$$

Compact form – Example 7

To obtain compact form, we differentiate $f(t)$ term by term,

$$g(t) = \frac{df}{dt} = \frac{d}{dt} \left(\frac{2}{\pi} \right) + \sum_{n=1}^{\infty} \frac{1}{\pi} \frac{1}{n^2 - \frac{1}{4}} \frac{d}{dt} (\cos(2nt + \pi))$$

Solving differentiation gives,

$$g(t) = \sum_{n=1}^{\infty} \frac{1}{\pi} \frac{2n}{n^2 - \frac{1}{4}} \cos(n2t + \frac{3\pi}{2})$$

Summary

- A periodic signal can be represented in orthogonal projections forms having a finite time period and satisfying Dirichlet's conditions
- The Dirichlet conditions are sufficient to satisfy Fourier series
- A compact way to represent Fourier series is to put in to exponential form
- The plots of the magnitude and phase of c_n versus $n\omega_0$ are called the *complex amplitude spectrum* and *complex phase spectrum* of $f(t)$, respectively

Summary

- **Periodic and non-periodic signals can be used for Fourier series taking their integral multiple of fundamental frequency**
- **It is periodic only for any number ω_o that is all frequencies ω_k are integer multiple of ω_o**
- **A stretching or squashing of a periodic waveform corresponds to a change in period and fundamental frequency**
- **The time scaling property holds for compact and exponential form of Fourier Transform**

Further reading

1. Ch. 6 (page 190-208), E. Kudeki and D. C. Munson, *Analog Signals and Systems*, Prentice Hall, 2008.
2. Ch. 17 (page 760-780, page 787-791), C. K. Alexander and M. Sadiku, *Fundamentals of Electric Circuits*, 5th ed., McGraw-Hill, 2013.
3. Ch. 15 (page 751-773), J. D. Irwin, and R. M. Nelms. *Basic Engineering Circuit Analysis*, 10th ed. Wiley, 2011.

Preview:

1. Ch. 6 (page 208-218), E. Kudeki and D. C. Munson, *Analog Signals and Systems*, Prentice Hall, 2008.

Homework 8

Deadline: 10:00 PM, 20th April, 2022

Thank you!