

ECE 210 Analog Signal Processing

Formula Sheet

I. Properties of the frequency response $H(\omega)$ of LTI systems

	Description	Property
1	Conjugate symmetry	$H(-\omega) = H^*(\omega)$
2	Even amplitude response	$ H(-\omega) = H(\omega) $
3	Odd phase response	$\angle H(-\omega) = -\angle H(\omega)$
4	Real DC response	$H(0) = H^*(0)$ is real valued
5	Steady-state response to $e^{j\omega t}$	$e^{j\omega t} \rightarrow \boxed{\text{LTI}} \rightarrow H(\omega)e^{j\omega t}$

II. Fourier Series & Different forms

$f(t)$, period $T = \frac{2\pi}{\omega_o}$	Form	Coefficients
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_o t}$	Exponential	$F_n = \frac{1}{T} \int_T f(t) e^{-jn\omega_o t} dt$
$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)$	Trigonometric	$a_n = F_n + F_{-n}$ $b_n = j(F_n - F_{-n})$
$\frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_o t + \theta_n)$	Compact for real $f(t)$	$c_n = 2 F_n $ $\theta_n = \angle F_n$

III. Properties of Fourier series

	Name:	Condition:	Property:
1	Scaling	Constant K	$Kf(t) \leftrightarrow KF_n$
2	Addition	$f(t) \leftrightarrow F_n, g(t) \leftrightarrow G_n, \dots$	$f(t) + g(t) + \dots \leftrightarrow F_n + G_n + \dots$
3	Time shift	Delay t_o	$f(t - t_o) \leftrightarrow F_n e^{-jn\omega_o t_o}$
4	Derivative	Continuous $f(t)$	$\frac{df}{dt} \leftrightarrow jn\omega_o F_n$
5	Hermitian	Real $f(t)$	$F_{-n} = F_n^*$
6	Even function	$f(-t) = f(t)$	$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega_o t)$
7	Odd function	$f(-t) = -f(t)$	$f(t) = \sum_{n=1}^{\infty} b_n \sin(n\omega_o t)$
8	Average power		$P \equiv \frac{1}{T} \int_T f(t) ^2 dt = \sum_{n=-\infty}^{\infty} F_n ^2$

IV. Important Fourier transform pairs

	$f(t) \leftrightarrow F(\omega)$		
1	$e^{-at}u(t) \leftrightarrow \frac{1}{a+j\omega}, a > 0$	14	$\delta(t) \leftrightarrow 1$
2	$e^{at}u(-t) \leftrightarrow \frac{1}{a-j\omega}, a > 0$	15	$1 \leftrightarrow 2\pi\delta(\omega)$
3	$e^{-a t } \leftrightarrow \frac{2a}{a^2+\omega^2}, a > 0$	16	$\delta(t-t_0) \leftrightarrow e^{-j\omega t_0}$
4	$\frac{a^2}{a^2+t^2} \leftrightarrow \pi a e^{-a \omega }, a > 0$	17	$e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$
5	$te^{-at}u(t) \leftrightarrow \frac{1}{(a+j\omega)^2}, a > 0$	18	$\cos(\omega_0 t) \leftrightarrow \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
6	$t^n e^{-at}u(t) \leftrightarrow \frac{n!}{(a+j\omega)^{n+1}}, a > 0$	19	$\sin(\omega_0 t) \leftrightarrow j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
7	$\text{rect}(\frac{t}{\tau}) \leftrightarrow \tau \text{sinc}(\frac{\omega\tau}{2})$	20	$\cos(\omega_0 t)u(t) \leftrightarrow \frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
8	$\text{sinc}(Wt) \leftrightarrow \frac{\pi}{W} \text{rect}(\frac{\omega}{2W})$	21	$\sin(\omega_0 t)u(t) \leftrightarrow j\frac{\pi}{2}[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$
9	$\Delta(\frac{t}{\tau}) \leftrightarrow \frac{\tau}{2} \text{sinc}^2(\frac{\omega\tau}{4})$	22	$\text{sgn}(t) \leftrightarrow \frac{2}{j\omega}$
10	$\text{sinc}^2(\frac{Wt}{2}) \leftrightarrow \frac{2\pi}{W} \Delta(\frac{\omega}{2W})$	23	$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$
11	$\frac{e^{-at} \sin(\omega_0 t)u(t)}{(a+j\omega)^2 + \omega_0^2}, a > 0$	24	$\sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\omega - n\frac{2\pi}{T})$
12	$\frac{e^{-at} \cos(\omega_0 t)u(t)}{(a+j\omega)^2 + \omega_0^2}, a > 0$	25	$\sum_{n=-\infty}^{\infty} f(t)\delta(t - nT) \leftrightarrow \sum_{n=-\infty}^{\infty} \frac{1}{T} F(\omega - n\frac{2\pi}{T})$
13	$e^{-\frac{t^2}{2\sigma^2}} \leftrightarrow \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$		

V. Important properties of Fourier Transform.

	Name:	Condition:	Property:
1	Amplitude scaling	$f(t) \leftrightarrow F(\omega)$, constant K	$Kf(t) \leftrightarrow KF(\omega)$
2	Addition	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$, ...	$f(t) + g(t) + \dots \leftrightarrow F(\omega) + G(\omega) + \dots$
3	Hermitian	Real $f(t) \leftrightarrow F(\omega)$	$F(-\omega) = F^*(\omega)$
4	Even	Real and even $f(t)$	Real and even $F(\omega)$
5	Odd	Real and odd $f(t)$	Imaginary and odd $F(\omega)$
6	Symmetry	$f(t) \leftrightarrow F(\omega)$	$F(t) \leftrightarrow 2\pi f(-\omega)$
7	Time scaling	$f(t) \leftrightarrow F(\omega)$, real c	$f(ct) \leftrightarrow \frac{1}{ c } F(\frac{\omega}{c})$
8	Time shift	$f(t) \leftrightarrow F(\omega)$	$f(t - t_0) \leftrightarrow F(\omega)e^{-j\omega t_0}$
9	Frequency shift	$f(t) \leftrightarrow F(\omega)$	$f(t)e^{j\omega_0 t} \leftrightarrow F(\omega - \omega_0)$
10	Modulation	$f(t) \leftrightarrow F(\omega)$	$f(t) \cos(\omega_0 t) \leftrightarrow \frac{1}{2} F(\omega - \omega_0) + \frac{1}{2} F(\omega + \omega_0)$
11	Time derivative	Differentiable $f(t) \leftrightarrow F(\omega)$	$\frac{df}{dt} \leftrightarrow j\omega F(\omega)$
12	Freq derivative	$f(t) \leftrightarrow F(\omega)$	$-jtf(t) \leftrightarrow \frac{d}{d\omega} F(\omega)$
13	Time convolution	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$	$f(t) * g(t) \leftrightarrow F(\omega)G(\omega)$
14	Freq convolution	$f(t) \leftrightarrow F(\omega)$, $g(t) \leftrightarrow G(\omega)$	$f(t)g(t) \leftrightarrow \frac{1}{2\pi} F(\omega) * G(\omega)$
15	Compact form	Real $f(t)$	$f(t) = \frac{1}{2\pi} \int_0^\infty 2 F(\omega) \cos(\omega t + \angle F(\omega)) d\omega$
16	Parseval, Energy W	$f(t) \leftrightarrow F(\omega)$	$W \equiv \int_{-\infty}^\infty f(t) ^2 dt = \frac{1}{2\pi} \int_{-\infty}^\infty F(\omega) ^2 d\omega$