ZJU-UIUC INSTITUTE

Zhejiang University – University of Illinois at Urbana-Champaign Institute

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ECE-210 Analog Signal Processing Spring 2022
Homework #13: Submission Deadline 25th May (10:00 PM)

1. Consider the following zero-state input-output relations for a variety of systems. In each case, determine whether the system is zero-state linear, time invariant, and causal.

   (a) $y(t) = f(t-1) + f(t+1)$.
   (b) $y(t) = 5f(t) * u(t)$.
   (c) $y(t) = \int_{-\infty}^{t-2} f(\tau)d\tau$.
   (d) $y(t) = \delta(t-4) * f(t) - \int_{-\infty}^{t+2} f(\tau)d\tau$.
   (e) $y(t) = f(t^2)$.

2. Find the impulse responses $h(t)$ of the LTI systems having the following unit-step responses.

   (a) $g(t) = 3u(t-3)$.
   (b) $g(t) = t^2u(t)$.
   (c) $g(t) = (2 - e^{-t})u(t-3)$.

3. If the unit-step response of an LTI system is $g(t) = 3\text{rect}\left(\frac{t-3}{2}\right)$, find the system zero-state responses to the following inputs.

   (a) $f(t) = \text{rect}(t)$.
   (b) $f(t) = e^{-2t}u(t)$.
   (c) $f(t) = 2\delta(t)$.

4. For each one of the 5 signals $f(t)$ in parts (a), (b), (c), (d), and (e), do the following

   i. Obtain its Laplace transform $\hat{F}(s)$.
   ii. Indicate the poles of $\hat{F}(s)$.
   iii. Indicate the ROC of $\hat{F}(s)$.

   (a) $f(t) = u(t) - u(t-6)$
   (b) $f(t) = te^{2(t-1)}u(t)$
   (c) $f(t) = (t-1)\,e^{-4t} + \delta(t)$
   (d) $f(t) = e^{2t}\cos(t)u(t+1)$.

5. Determine whether the LTIC systems with the following transfer functions are BIBO stable and explain why or why not.

   (a) $\hat{H}_2(s) = 2 + \frac{s}{(s+1)(s-2)}$
   (b) $\hat{H}_3(s) = \frac{s^2+4s+6}{(s+1+j6)(s+1-j6)}$
   (c) $\hat{H}_1(s) = \frac{s^3+1}{(s+2)(s+4)}$
   (d) $\hat{H}_4(s) = \frac{1}{s^2+16}$
   (e) $\hat{H}_5(s) = \frac{s-2}{s^2-4}$.

6. For each of the following Laplace transforms $\hat{F}(s)$, determine the inverse Laplace transform $f(t)$.

   (a) $\hat{F}(s) = \frac{s+3}{(s+2)(s+4)}$
   (b) $\hat{F}(s) = \frac{1}{s(s-5)^2}$
   (c) $\hat{F}(s) = \frac{s^2+2s+1}{(s+1)(s+2)}$

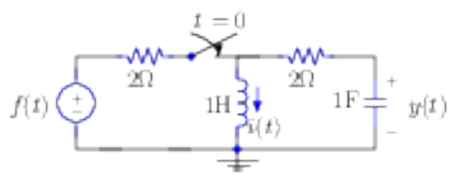
8. Determine the transfer functions $\hat{H}(s)$ and the zero-state response for the LTIC system described by the following ODE:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = e^{3t}u(t) \qquad \text{input signal f(t)=e^(3t)u(t)}$$

9. Take the Laplace transform of the following ODE to determine $\hat{Y}(s)$ assuming $f(t) = u(t)$, $y(0^-) = 1$, and $y'(0^-) = 0$. Determine $y(t)$ for $t > 0$ by taking the inverse Laplace transform of $\hat{Y}(s)$.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y(t) = \frac{df}{dt} + 2f(t).$$

10. Consider the circuit:

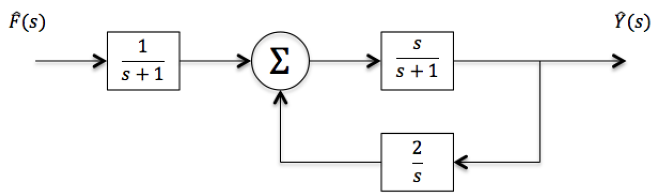


(a) Show that the transfer function of the circuit for $t > 0$ is $\hat{H}(s) = \frac{\hat{Y}(s)}{\hat{F}(s)} = \frac{s}{4s^2+5s+2}$.

(b) What are the characteristic modes of the circuit?

(c) Determine $y(t)$ for $t > 0$ if $f(t) = 1$ V, $y(0^-) = 1$ V, and $i(0^-) = 0$.

11. Determine the transfer function $\hat{H}(s)$ of the system shown below. Also determin whether the system is BIBO stable.



1. Consider the following zero-state input-output relations for a variety of systems. In each case, determine whether the system is zero-state linear, time invariant, and causal.

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   (b) $y(t) = 5f(t) * u(t)$.
   (c) $y(t) = \int_{-\infty}^{t-2} f(\tau)d\tau$.
   (d) $y(t) = \delta(t-4) * f(t) - \int_{-\infty}^{t+2} f(\tau)d\tau$.
   (e) $y(t) = f(t^2)$.

(a) linear & time invariant: let $y_1 = f_1(t-1)+f_1(t+1)$ $y_2 = f_2(t-1)+f_2(t+1)$

$y(t) = f(t-1)+f(t+1) = af_1(t-1)+bf_2(t-1)+af_1(t+1)+bf_2(t+1)$

$= a(f_1(t-1)+f_1(t+1)) + b(f_2(t-1)+f_2(t+1)) = ay_1(t)+by_2(t)$

$\Rightarrow$ linear Q.E.D.

time-invariant: let input be $f_1(t) = f(t-t_0)$

$\Rightarrow y(t) = f_1(t-1)+f_1(t+1) = f(t-1-t_0)+f(t+1-t_0) = y(t-t_0)$ Q.E.D.

(b) LTI (linear time-invariant) as convolution.
as $y(t) = 5f(t)*u(t) \Rightarrow$ system causal.

(c) $\int_{-\infty}^{t-2} f(\tau)d\tau = \int_{-\infty}^{\infty} f(\tau)\cdot u(t-2-\tau)d\tau \Leftrightarrow f(t)*u(t+2)$

$\Rightarrow$ LTI & causal.

(d) $y(t) = \delta(t-4)*f(t) - f(t)*u(t-2)$

let $f(t) = af_1(t)+bf_2(t)$ $y(t) = a\delta(t-4)*f_1(t) + b\delta(t-4)*f_2(t)$

$- bf(t)*u(t-2) - bf(t)*u(t-2) = ay_1(t)+by_2(t)$

$= ay_1(t)+by_2(t) \Rightarrow$ linear Q.E.D.

let $f_1(t) = f(t-t_0) \Rightarrow y_1(t) = \delta(t-4)*f(t-t_0) - f(t-t_0)*u(t)$

$= y(t-t_0) \Rightarrow$ Q.E.D. time-invariant

not causal as $\delta(t-4)*f(t) = f(t-4)$ depends on further value.

(e) linear, let $y(t) = af_1(t)+bf_2(t)$

$y(t) = af_1(t^2)+bf_2(t^2)$

$= ay_1+by_2 \Rightarrow$ Q.E.D.

not timeinvariant as $f((t+t_0)^2) \neq f(t^2+t_0)$

not causal as depending on further value.

2. Find the impulse responses $h(t)$ of the LTI $\frac{d}{dt}$ systems having the following unit-step responses.

   (a) $g(t) = 3u(t-3)$.
   (b) $g(t) = t^2u(t)$.
   (c) $g(t) = (2-e^{-t})u(t-3)$.

* unit step response $\rightarrow$ Impulse response $\Leftrightarrow$ zero-state response.

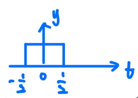
(a) $\frac{d}{dt}g(t) = 3\delta(t-3)$

(b) $\frac{d}{dt}g(t) = t^2\delta(t)+u(t)\cdot 2t = 2tu(t)$

(c) $\frac{d}{dt}g(t) = 2\delta(t-3) - e^{-t}\delta(t) + e^{-t}u(t-3)$

$= (2-e^{-t})\delta(t) + e^{-t}u(t-3)$

方形脉冲



3. If the unit-step response of an LTI system is $g(t) = 3\text{rect}\left(\frac{t-3}{2}\right)$, find the system zero-state responses to the following inputs.

   (a) $f(t) = \text{rect}(t)$.
   (b) $f(t) = e^{-2t}u(t)$.
   (c) $f(t) = 2\delta(t)$.

$\frac{d}{dt}g(t) = \frac{d}{dt}3(u(t-2)-u(t-4)) = 3\delta(t-2)-3\delta(t-4) = h(t) \Rightarrow H(\omega) = 3e^{-j\omega 2} - 3e^{-j\omega 4}$

$\Rightarrow$ (a) $y(t) = 3\text{rect}(t-2) - 3\text{rect}(t-4)$

(b) $y(t) = 3e^{-2(t-2)}u(t-2) - 3e^{-2(t-4)}u(t-4)$

(c) $y(t) = 6\delta(t-2) - 6\delta(t-4)$

4. For each one of the 5 signals $f(t)$ in parts (a), (b), (c), (d), and (e), do the following

   i. Obtain its Laplace transform $\hat{F}(s)$.
   ii. Indicate the poles of $\hat{F}(s)$.
   iii. Indicate the ROC of $\hat{F}(s)$.

   (a) $f(t) = u(t) - u(t-6)$
   (b) $f(t) = te^{2(t-1)}u(t)$
   (c) $f(t) = (t-1)e^{-4t} + \delta(t)$
   (d) $f(t) = e^{2t}\cos(t)u(t+1)$.

(a) $\mathcal{L}\{f(t)\} = \frac{1}{s}(1-e^{-6s})$ poles $\{-\infty+j\omega\}$ zeros: $\{\infty+j\omega, \frac{j\pi n}{3}(n\in N^+)\}$ ROC $\sigma = Re\{s\} > -\infty$

(b) $\mathcal{L}\{f(t)\} = \frac{e^{-2}}{(s-2)^2}$ poles: $s_{1,2} = 2$ (double pole) zeros: $\pm\infty+j\omega$ ROC $\sigma = Re\{s\} > 4$

(c) $\mathcal{L}\{(t-1)e^{-4t}+\delta(t)\} = 1 + \mathcal{L}\{te^{-4t}u(t)\} + \mathcal{L}\{-e^{-4t}u(t)\}$ $\Rightarrow$ poles: $\{-4(\text{double})\}$, list of zeros: $z_{1,2} = \frac{-7\pm j\sqrt{7}}{2}$

$= 1 + \frac{1}{(s+4)^2} - \frac{1}{s+4}$

(d) $\mathcal{L}\{e^{2t}\cos(t)u(t+1)\} = \frac{s-2}{(s-2)^2+1} = \frac{s-2}{(s-(2+j))(s-(2-j))}$ poles $= \{2+j, 2-j\}$ zeros: $\{2, \pm\infty+j\omega\}$

ROC $= \sigma = Re\{s\} > 2$

* LHP: left half plane Iff transfer func. has all of poles in the LHP.

5. Determine whether the LTIC systems with the following transfer functions are BIBO stable and explain why or why not.

   (a) $\hat{H}_2(s) = 2 + \frac{s}{(s+1)(s-2)}$
   (b) $\hat{H}_3(s) = \frac{s^2+4s+6}{(s+1+j6)(s+1-j6)}$
   (c) $\hat{H}_1(s) = \frac{s^3+1}{(s+2)(s+4)}$
   (d) $\hat{H}_4(s) = \frac{1}{s^2+16}$
   (e) $\hat{H}_5(s) = \frac{s-2}{s^2-4}$.

(a) as pole $s=2$ one RHP $\Rightarrow$ not BIBO

(b) as poles all in LHP $\Rightarrow$ BIBO

(c) $s=-2$, $s=-4$ $s=+\infty$ as ∞ is not reachable $\Rightarrow$ BIBO

(d) marginally stable but not BIBO stable.

(e) BIBO stable as $\hat{H}_5(s) = \frac{1}{s+2}$ $\Rightarrow$ BIBO stable.

6. For each of the following Laplace transforms $\hat{F}(s)$, determine the inverse Laplace transform $f(t)$.

   (a) $\hat{F}(s) = \frac{s+3}{(s+2)(s+4)}$
   (b) $\hat{F}(s) = \frac{1}{s(s-5)^2}$
   (c) $\hat{F}(s) = \frac{s^2+2s+1}{(s+1)(s+2)}$

(a) $\hat{F}(s) = \frac{k_1}{s+2} + \frac{k_2}{s+4}$

$k_1 = \frac{1}{2}$ $k_2 = \frac{1}{2}$ $\Rightarrow f(t) = \frac{1}{2}e^{-2t}u(t) + \frac{1}{2}e^{-4t}u(t)$

$\Rightarrow \hat{F}(s) = \frac{1}{2}\frac{1}{s+2} + \frac{1}{2}\frac{1}{s+4}$

(b) $\hat{F}(s) = \frac{k_1}{s} + \frac{k_2}{s-5} + \frac{k_3}{(s-5)^2}$ $k_1 = \frac{1}{25}$ $k_2 = -\frac{1}{25}$ $k_3 = \frac{1}{5}$

$\Rightarrow \hat{F}(s) = \frac{1}{25}\frac{1}{s} - \frac{1}{25}\frac{1}{s-5} + \frac{1}{5}\frac{1}{(s-5)^2} \Rightarrow f(t) = \frac{1}{25}u(t) - \frac{1}{25}e^{5t}u(t) + \frac{1}{5}te^{5t}u(t)$

(c) $\hat{F}(s) = \frac{(s+1)^2}{(s+1)(s+2)} = 1 - \frac{1}{s+2}$ $\Rightarrow f(t) = \delta(t) - e^{-2t}u(t)$

8. Determine the transfer functions $\hat{H}(s)$ and the zero-state response for the LTIC system described by the following ODE:

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = e^{3t}u(t)$$

input signal f(t)=e^(3t)u(t)

$s^2 \cdot Y - sf(0^-) - f'(0^-) + 3sY - 3f(0^-) + 2Y = F$

$\hat{H}(s) = \frac{1}{s^2+3s+2}$

$\Rightarrow Y = \frac{1}{(s^2+3s+2)(s-3)} \Rightarrow Y = \frac{1}{(s+2)(s+1)(s-3)} = \frac{k_1}{(s+2)} + \frac{k_2}{(s+1)} + \frac{k_3}{(s-3)}.$

$k_1 = \frac{1}{5}$ $k_2 = -\frac{1}{4}$ $k_3 = \frac{1}{20}$ $\Rightarrow y(t) = \frac{1}{5}e^{-2t}u(t) - \frac{1}{4}e^{-t}u(t) + \frac{1}{20}e^{3t}u(t)$

9. Take the Laplace transform of the following ODE to determine $\hat{Y}(s)$ assuming $f(t) = u(t)$, $y(0^-) = 1$, and $y'(0^-) = 0$. Determine $y(t)$ for $t > 0$ by taking the inverse Laplace transform of $\hat{Y}(s)$.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y(t) = \frac{df}{dt} + 2f(t).$$

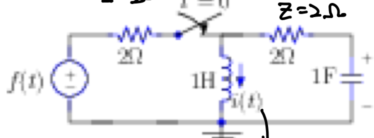
$s^2Y - y(0^-)\cdot s - y'(0^-) + 5sY - y(0^-) + 4Y = 1 + \frac{2}{s}$

$s^2Y - s + 5sY - 1 + 4Y = 1 + \frac{2}{s} \Rightarrow Y = \frac{2+s+\frac{2}{s}}{(s+4)(s+1)} = \frac{2}{s(s+4)(s+1)} + \frac{2+s}{(s+4)(s+1)} = \frac{1}{2}\frac{1}{s} - \frac{2}{3}\frac{1}{s+1} + \frac{1}{6}\frac{1}{s+4} + \frac{2}{3}\frac{1}{s+4} + \frac{1}{3}\frac{1}{s+1}$

$y(t) = \frac{1}{2}u(t) - \frac{1}{3}e^{-t}u(t) + \frac{5}{6}e^{-4t}u(t)$

10. Consider the circuit:

$Z = 2\Omega$ $Z = 2\Omega$



$\rightarrow Z = \frac{1}{sC} = \frac{1}{s} \Rightarrow$ $Y = F \cdot \frac{\frac{1}{s}}{2+\frac{1}{s}} \cdot \frac{\frac{2s+1}{s+2+\frac{1}{s}}}{2+\frac{2s+1}{s+2+\frac{1}{s}}}$ $\Rightarrow H = \frac{\frac{2s^2+s}{s^2+2s+1}}{2+\frac{2s^2+s}{s^2+2s+1}} \cdot \frac{1}{2s+1}$

$Z = sL = s$ $\frac{2s+1}{s+2+\frac{1}{s}}$ $= \frac{2s^2+s}{4s^2+5s+2} \cdot \frac{1}{2s+1}$

(a) Show that the transfer function of the circuit for $t > 0$ is $\hat{H}(s) = \frac{\hat{Y}(s)}{\hat{F}(s)} = \frac{s}{4s^2+5s+2}$. $= \frac{s}{4s^2+5s+2}$ Q.E.D.

(b) What are the characteristic modes of the circuit?

(c) Determine $y(t)$ for $t > 0$ if $f(t) = 1$ V, $y(0^-) = 1$ V, and $i(0^-) = 0$.

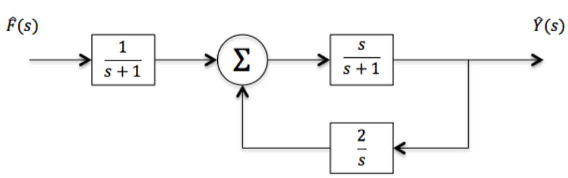
(c) $H = \frac{s}{4(s+\frac{5}{8}+\frac{i\sqrt{7}}{8})(s+\frac{5}{8}-\frac{i\sqrt{7}}{8})}$

$= \frac{1}{4}\left(\frac{1}{s+\frac{5}{8}+\frac{i\sqrt{7}}{8}} \cdot \frac{-j\sqrt{7}}{4} + \frac{1}{s+\frac{5}{8}-\frac{i\sqrt{7}}{8}} \cdot \frac{j\sqrt{7}}{4}\right)$

$= \frac{-j\sqrt{7}}{16}e^{(-\frac{5}{8}-\frac{j\sqrt{7}}{8})t}u(t) + \frac{j\sqrt{7}}{16}e^{(-\frac{5}{8}+\frac{j\sqrt{7}}{8})t}u(t)$

(b) band-pass filter ~~ckt~~.

11. Determine the transfer function $\hat{H}(s)$ of the system shown below. Also determin whether the system is BIBO stable.



$\hat{Y}(s) = \sum_{k=1}^{\infty} \left(\frac{s}{s+1}\right)^k \left(\frac{2}{s}\right)^{k-1} \cdot \frac{1}{s+1} = \sum_{k=1}^{\infty} \frac{2^{k-1}s}{(s+1)^k}$

$\Rightarrow$ BIBO as all poles on LHP.