ANALOG SIGNAL PROCESSING

ECE 210 & 211

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Objectives

- LTI System Response to Energy Signals
- Amplitude modulation
- Coherent demodulation of AM signal

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- LTI System Response to Energy Signals
- Amplitude modulation
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LTI Circuit Response

- Previously, we obtained the input – Output relation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

LTI

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) F(\omega) e^{j\omega t} d\omega$$

LTI Circuit Response

- Alternatively, we can write this relation as,

$$y(t) \Longleftrightarrow Y(\omega) = H(\omega)F(\omega)$$

- The system output is the IFT (Inverse Fourier Transform) of the product of the system frequency response and Fourier transform of the input
- The relation is simply drawn from,

$$e^{j\omega t} \rightarrow \boxed{\text{LTI}} \rightarrow H(\omega)e^{j\omega t}$$

LTI Circuit Response

$$y(t) \Longleftrightarrow Y(\omega) = H(\omega)F(\omega)$$

- **The relation describes just steady state response *$y(t)$* to a dissipative LTI system response to an input**

$$f(t) \Longleftrightarrow F(\omega)$$

- **In dissipative systems, transient part of zero-state response to input $cos(\omega t)$ and $sin(\omega t)$ applied at $t = -\infty$ must be vanished for finite times**

$$e^{j\omega t} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(\omega)e^{j\omega t}$$

zero-state response **for all finite times**

LTI Circuit Response

- **The Inverse Fourier Transform of the**

$$y(t) \Longleftrightarrow Y(\omega) = H(\omega)F(\omega)$$

Represents , for all finite t, *the entire* (steady state + transient) zero state response of the system $H(\omega)$ **to the input**

$$f(t) \Longleftrightarrow F(\omega)$$

Dissipative LTI Response – Example 1

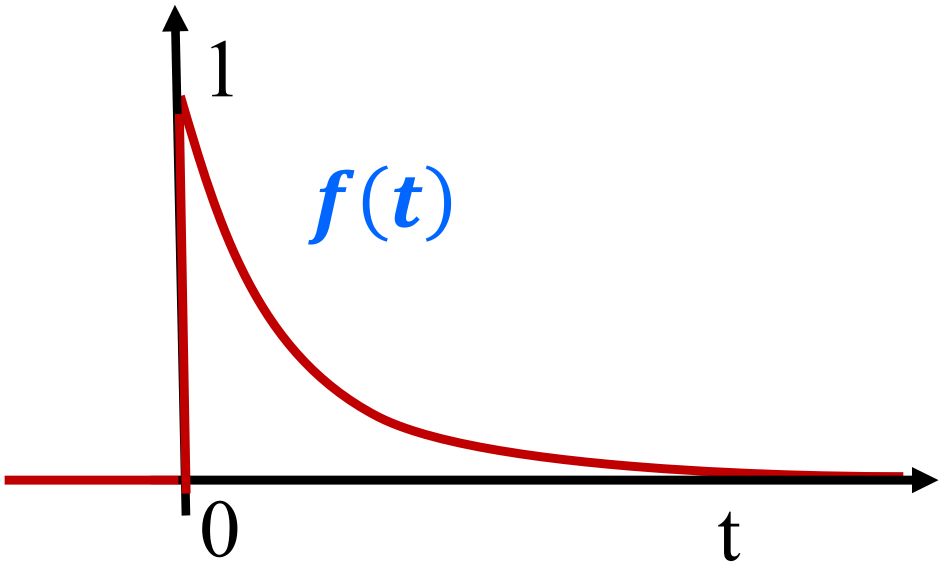
Question: The input of an LTI system,

$$H(\omega) = \frac{1}{1 + j\omega}$$

is

$$f(t) = e^{-t}u(t)$$

Determine the *zero-state response* *y(t)*?



Dissipative LTI Response – Example 1

Solution: Since,

$$f(t) = e^{-t}u(t) \Longleftrightarrow F(\omega) = \frac{1}{1 + j\omega}$$

The Fourier transform of *y(t)*,

$$Y(\omega) = H(\omega)F(\omega) = \frac{1}{1 + j\omega}\frac{1}{1 + j\omega} = \frac{1}{(1 + j\omega)^2}$$

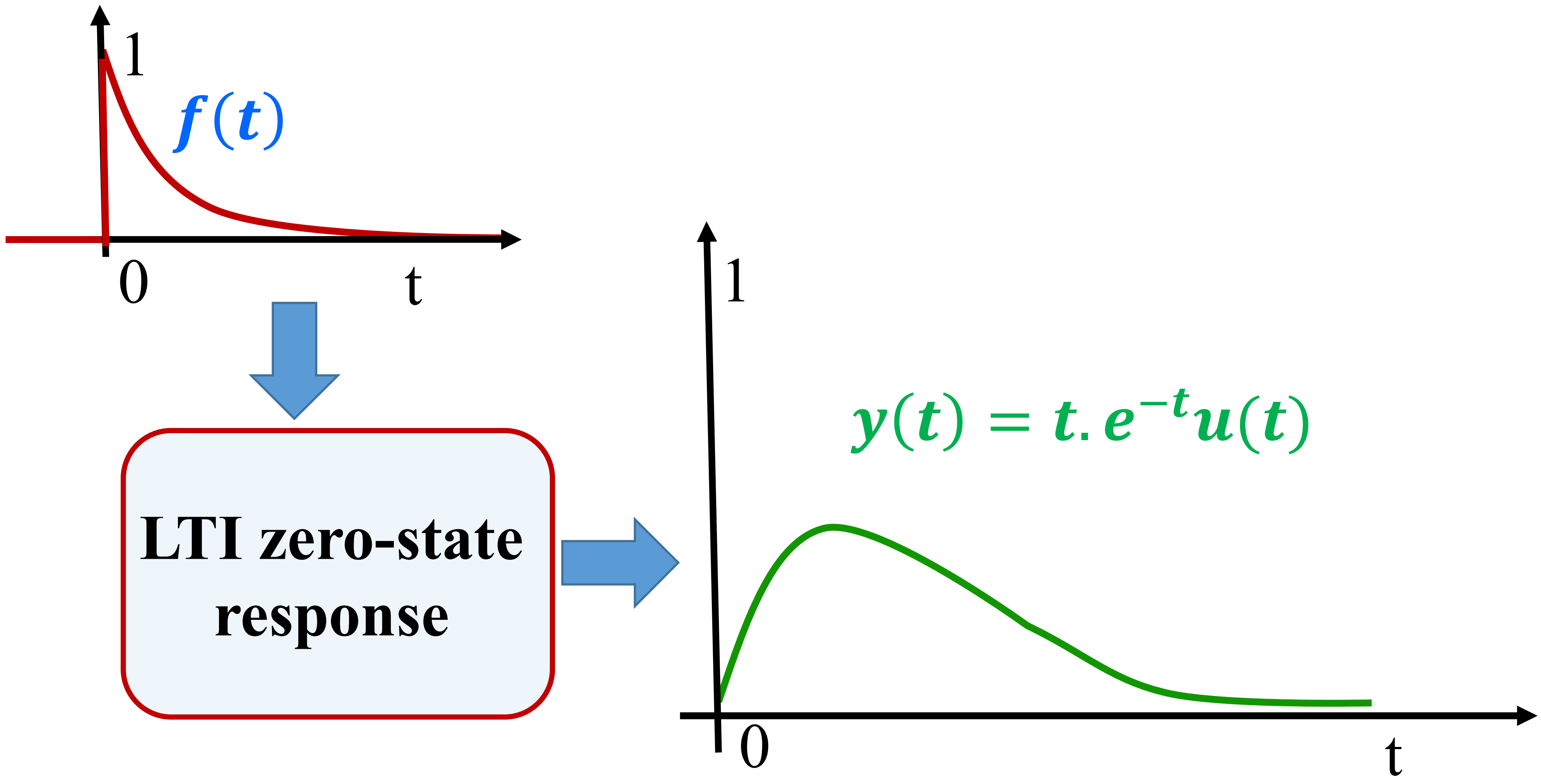
From Transform pairs,

$$t.e^{-t}u(t) \Longleftrightarrow \frac{1}{(1 + j\omega)^2}$$

Thus,

$$y(t) = t.e^{-t}u(t)$$

Dissipative LTI Response – Example 1



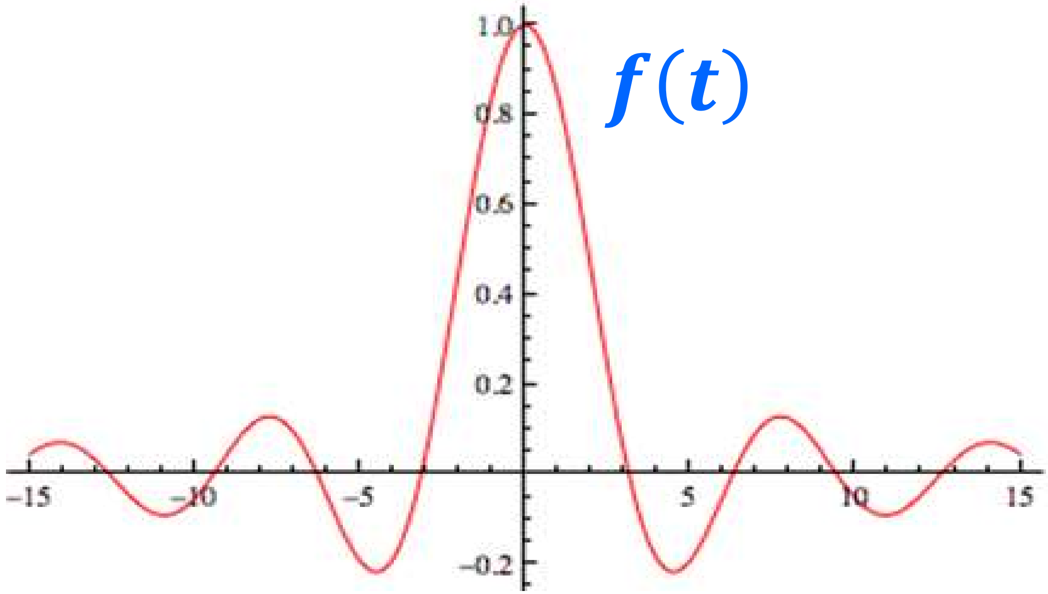
Dissipative LTI Response – Example 2

Question: **The input of an LTI system,**

$$f(t) = sinc(t) \Longleftrightarrow F(\omega) = \pi rect\left(\frac{\omega}{2}\right)$$

having the frequency response, $H(\omega) = rect(\omega)$

Determine the ***zero-state response y(t)*****?**



Dissipative LTI Response – Example 2

Solution: Given that,

$$F(\omega) = \pi rect\left(\frac{\omega}{2}\right) \quad \text{and} \quad H(\omega) = rect(\omega)$$

We have, $Y(\omega) = H(\omega)F(\omega) = rect(\omega).\pi rect\left(\frac{\omega}{2}\right)$

$$= \pi rect(\omega)$$

taking the inverse Fourier transform of $Y(\omega)$,

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(\omega)\, e^{j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \pi rect(\omega) e^{j\omega t} d\omega$$

Dissipative LTI Response – Example 2

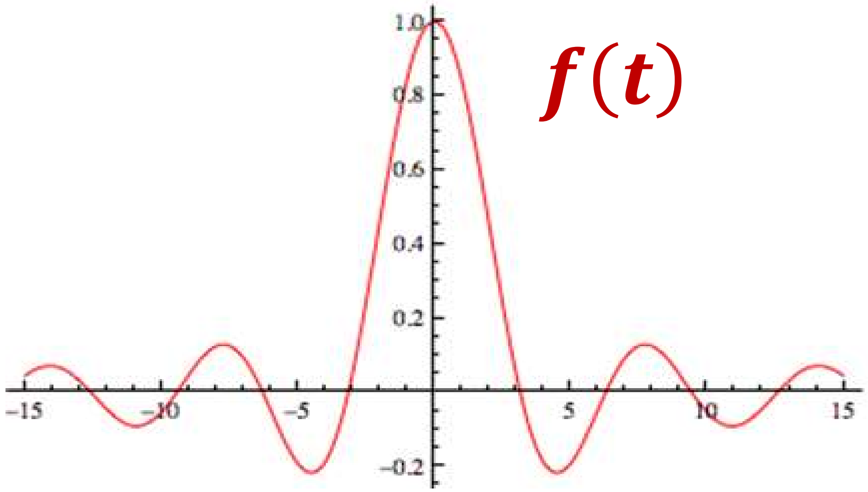
$$y(t) = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{j\omega t}\,d\omega = \frac{e^{j\frac{t}{2}} - e^{-j\frac{t}{2}}}{2jt}$$

the result simplifies to,

$$y(t) = \frac{1}{2} sinc\left(\frac{t}{2}\right)$$

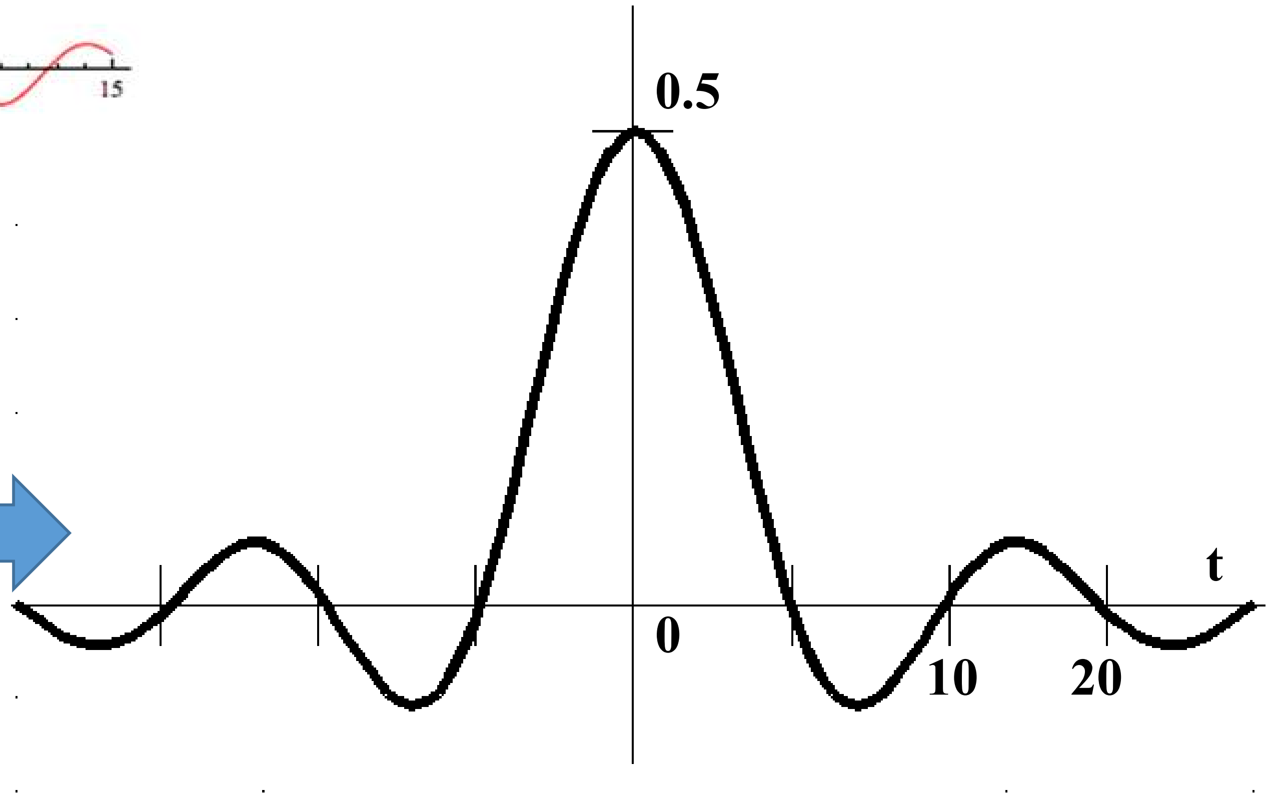
- **The system broadens the input *f(t)* by a factor of 2 by having its bandwidth**

Dissipative LTI Response – Example 2



LTI zero-state response

$$y(t) = \frac{1}{2} sinc\left(\frac{t}{2}\right)$$



Dissipative LTI Response – Example 3

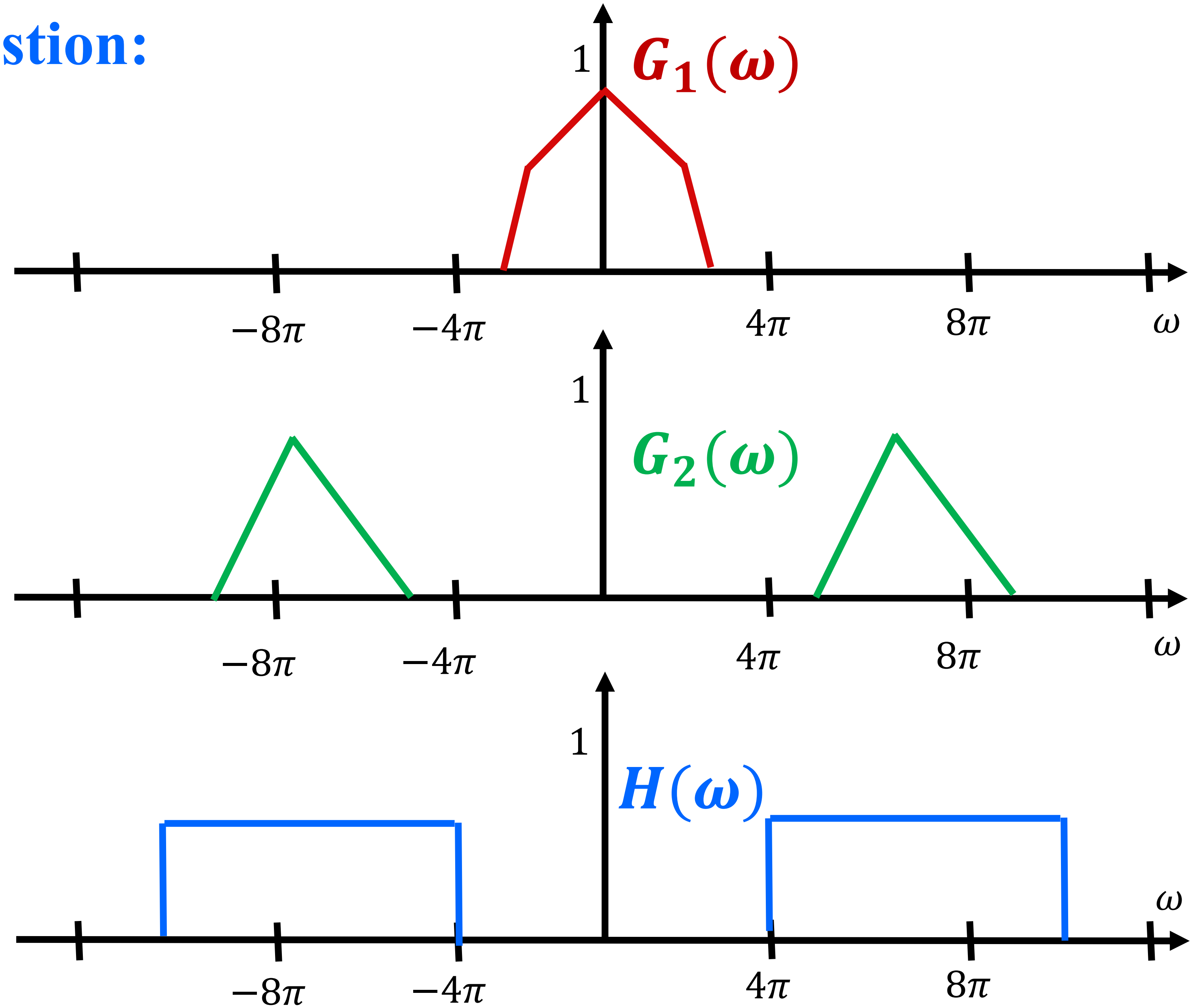
Question: The signal

$$f(t) = g_1(t) + g_2(t)$$

is passed through a bandpass filter having frequency response $H(\omega)$ shown.
Determine the zero-state response $y(t)$ in terms of $g_1(t)$ and $g_2(t)$?
$G_1(\omega)$ and $G_2(\omega)$ are Fourier transform of $g_1(t)$ and $g_2(t)$, respectively.

Dissipative LTI Response – Example 3

Question:



Dissipative LTI Response – Example 3

Solution: Since,

$$f(t) = g_1(t) + g_2(t)$$

$$f(t) \Longleftrightarrow F(\omega) = G_1(\omega) + G_2(\omega)$$

Therefore, the Fourier transform of the output *y(t)*

$$Y(\omega) = H(\omega)F(\omega) = H(\omega)G_1(\omega) + H(\omega)G_2(\omega)$$

from the figure, we can observe that,

$$H(\omega)G_1(\omega) = 0$$

Dissipative LTI Response – Example 3

and,

$$\boldsymbol{H(\omega)G_2(\omega) = \frac{1}{2}G_2(\omega)}$$

Hence,

$$\boldsymbol{Y(\omega) = H(\omega)F(\omega) = \frac{1}{2}G_2(\omega)}$$

So, the inverse Fourier transform will be,

$$\boldsymbol{y(t) = \frac{1}{2}g_2(t)}$$

The system filters out the component *$g_1(t)$* from the input and delivers a scaled-down replica of *$g_2(t)$* as the output

Dissipative LTI Response – Example 4

Question: **The input $f(t)$ is passed through a system having frequency response of**

$$H(\omega) = e^{-jt_o\omega}$$

Determine the zero-state response *y(t)*?

Solution:

$$Y(\omega) = H(\omega)F(\omega) = F(\omega)e^{-jt_o\omega}$$

Using the time sifting property of the Fourier transforms, we found that output *y(t)* is,

$$y(t) = f(t - t_o)$$

which is a delayed copy of the input $f(t)$

Dissipative LTI Response – Example 5

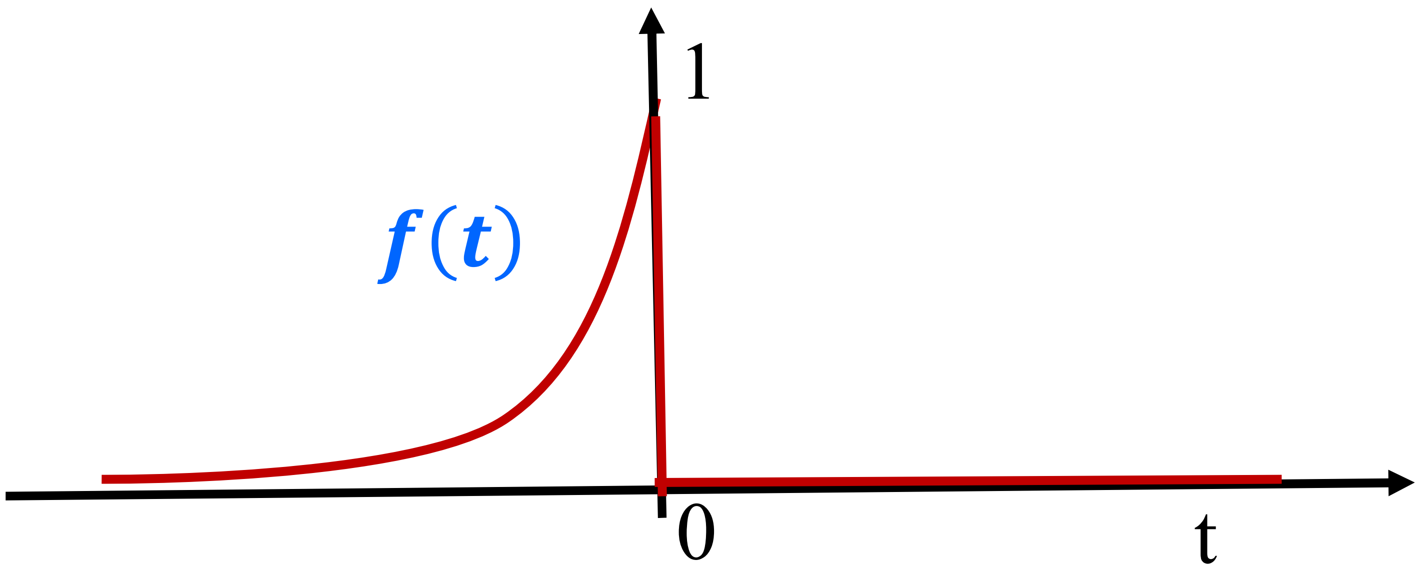
Question: **The input of an LTI system,**

$$H(\omega) = \frac{1}{1 + j\omega}$$

is

$$f(t) = e^{t}u(-t)$$

Determine the ***zero-state response y(t)?***



Dissipative LTI Response – Example 5

Solution: Since,

$$f(t) = e^{-t}u(-t) \Longleftrightarrow F(\omega) = \frac{1}{1 - j\omega}$$

The Fourier transform of *y(t)*,

$$Y(\omega) = H(\omega)F(\omega) = \frac{1}{1 + j\omega}\frac{1}{1 - j\omega} = \frac{1}{1 + \omega^2}$$

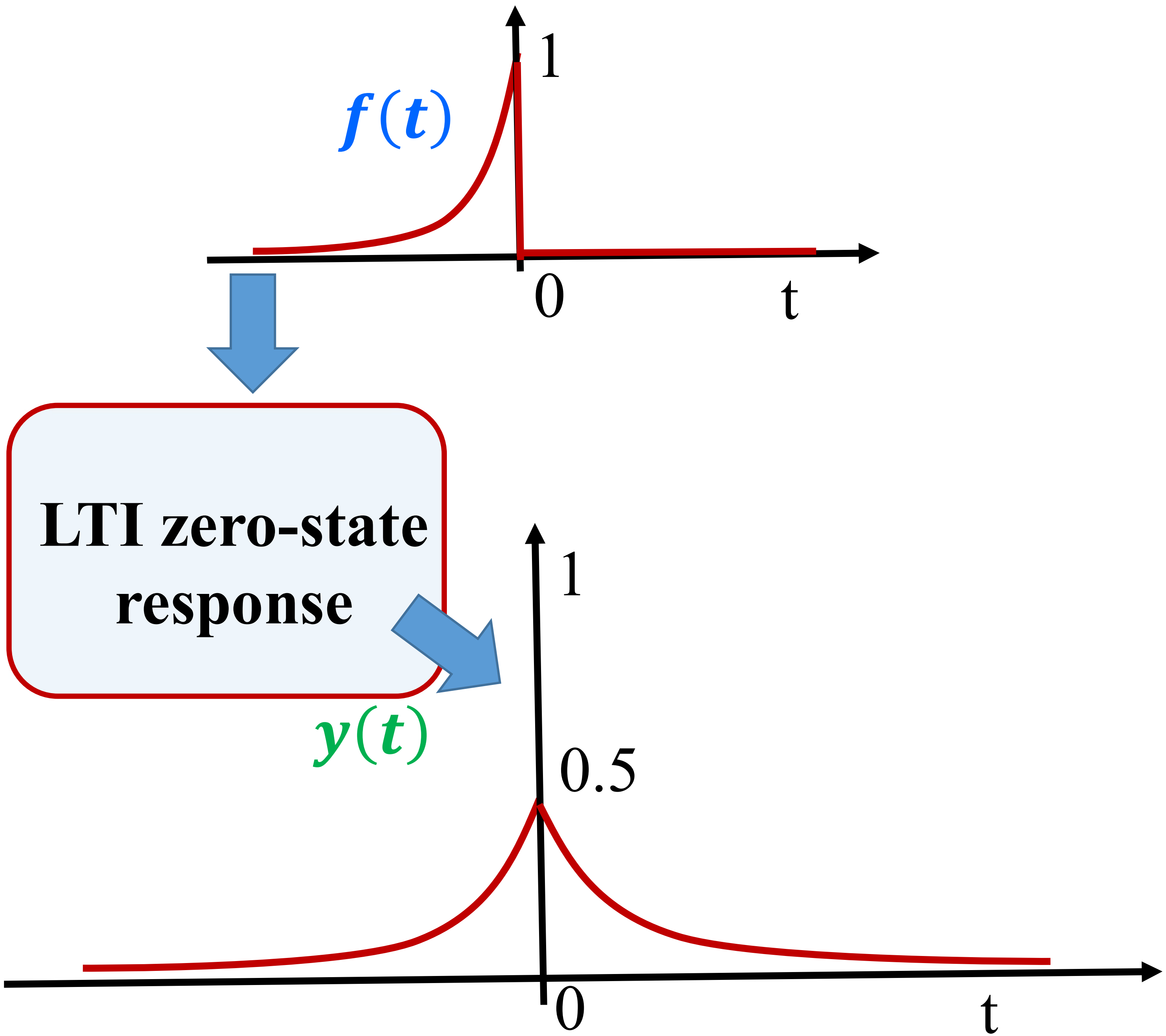
From Transform pairs,

$$e^{-|t|} \Longleftrightarrow \frac{2}{(1 + j\omega)^2}$$

Thus output will be,

$$y(t) = \frac{1}{2}e^{-|t|}$$

Dissipative LTI Response – Example 5



Dissipative LTI Response – Example 6

Question: **The input of an LTI system,**

$$\boldsymbol{H(\omega) = \frac{1}{1 + j\omega}}$$

is

$$\boldsymbol{f(t) = rect(t)}$$

What is the system output *y(t)*?

Dissipative LTI Response – Example 6

Solution: Since,

$$f(t) = rect(t) \Longleftrightarrow F(\omega) = sinc\left(\frac{\omega}{2}\right)$$

The Fourier transform of $y(t)$,

$$Y(\omega) = H(\omega)F(\omega) = \frac{1}{1 + j\omega} sinc\left(\frac{\omega}{2}\right)$$

taking the inverse Fourier transform,

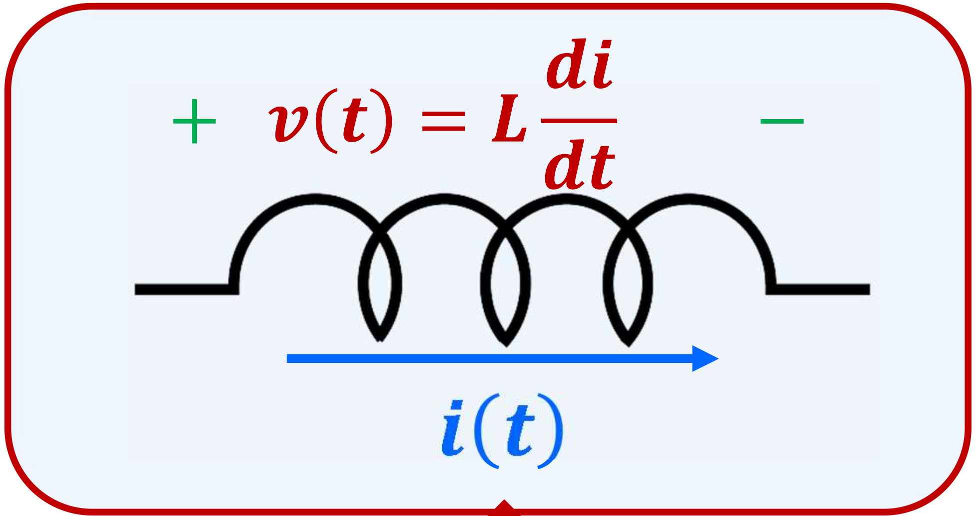
$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{1 + j\omega} sinc\left(\frac{\omega}{2}\right) e^{j\omega t} d\omega$$

Dissipative LTI Response for Circuits

We know that for inductor, *i-v* relation looks like,

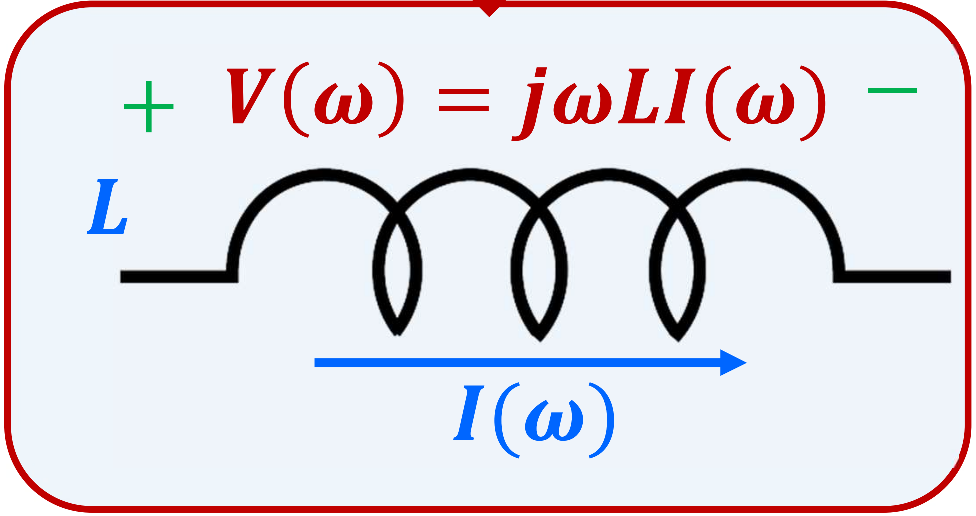
Time Domain

$$v(t) = L\frac{di}{dt}$$



Frequency Domain

$$V(\omega) = j\omega L I(\omega)$$



Dissipative LTI Response for circuits

From the correspondence $\frac{di}{dt} \Leftrightarrow j\omega I(\omega)$

Amplitude-scaling property implies that,

$$V(\omega) = j\omega L I(\omega) \Rightarrow V(\omega) = Z I(\omega)$$

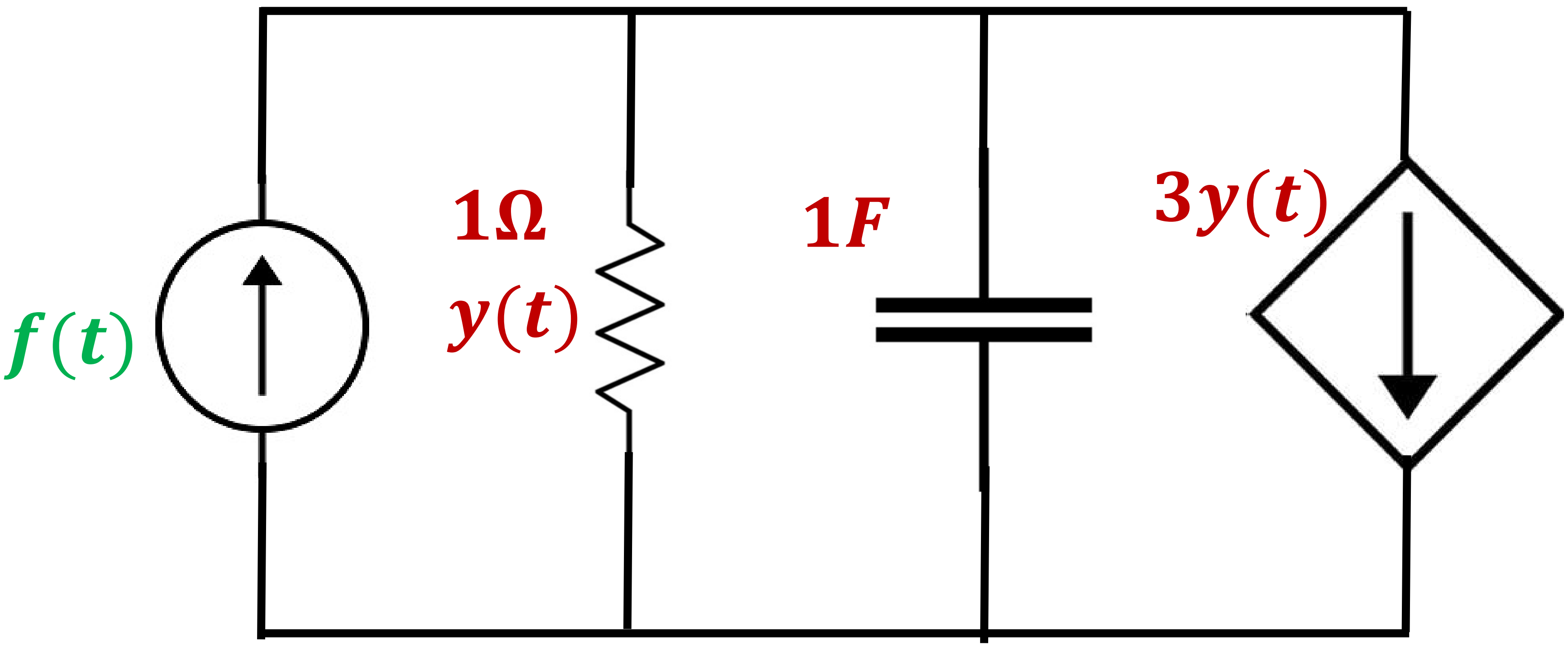
$$Z = j\omega L$$

For capacitor case, $Z = \frac{1}{j\omega C}$

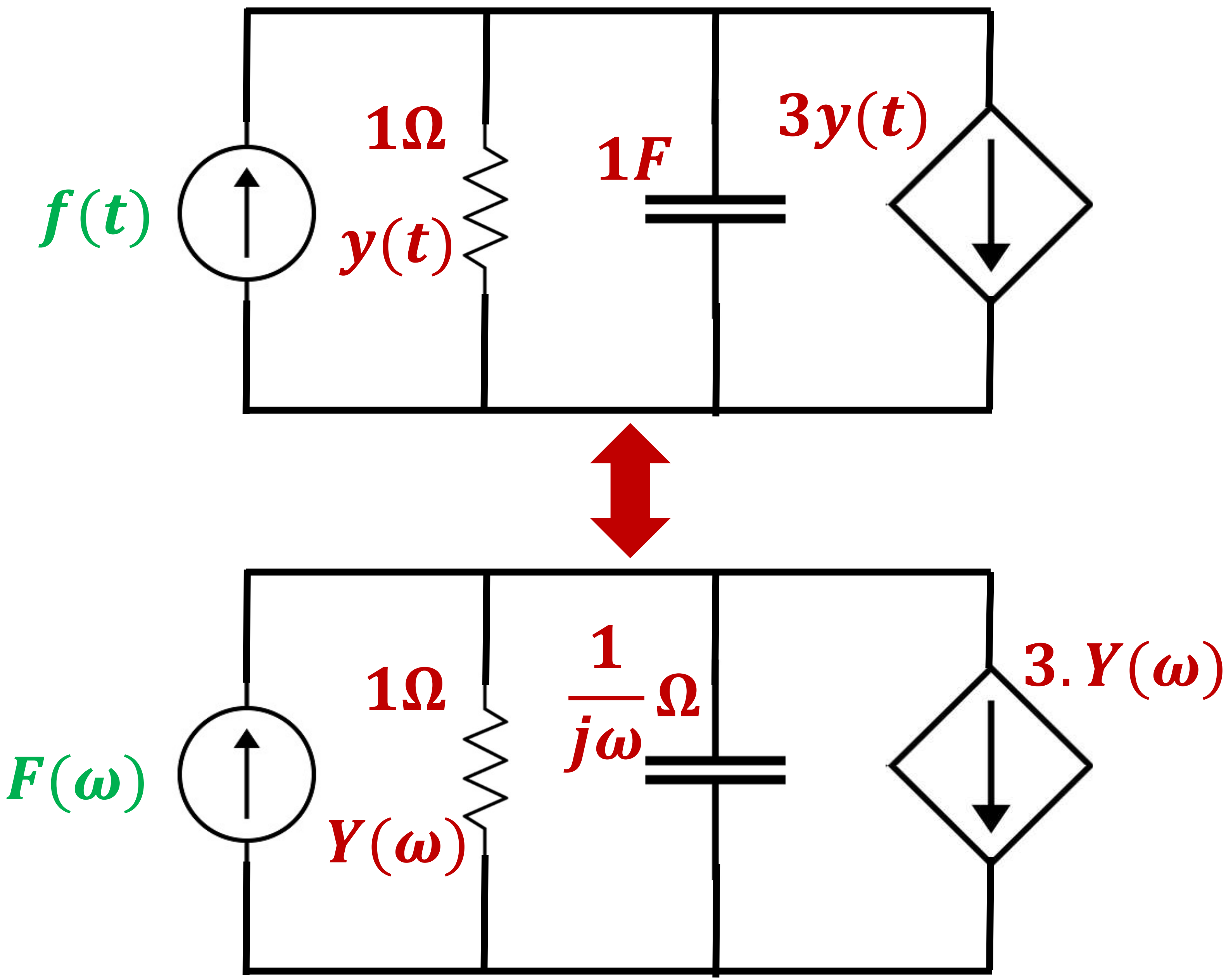
By using above, we will apply Fourier transform method to directly analyze circuits

Dissipative LTI Response – Example 7

Question: **Determine the $y(t) \leftrightarrow Y(\omega)$ for an arbitrary input $f(t) \leftrightarrow F(\omega)$ for the circuit given below?**



Dissipative LTI Response – Example 7



Dissipative LTI Response – Example 7

Applying node voltage method, the KCL equation for the top node is,

$$\boldsymbol{F(\omega) = \frac{Y(\omega)}{1} + \frac{Y(\omega)}{\frac{1}{j\omega}} + 3Y(\omega) = (4 + j\omega)Y(\omega)}$$

Thus,

$$\boldsymbol{Y(\omega) = \frac{1}{(4+j\omega)}F(\omega)}$$

which is a Fourier transform of the zero-state response *y(t)*

Dissipative LTI Response – Example 7

Hence, the system frequency response is,

$$H(\omega) = \frac{1}{(4 + j\omega)}$$

Using the inverse Fourier transform,

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{4 + j\omega} F(\omega) e^{j\omega t} d\omega$$

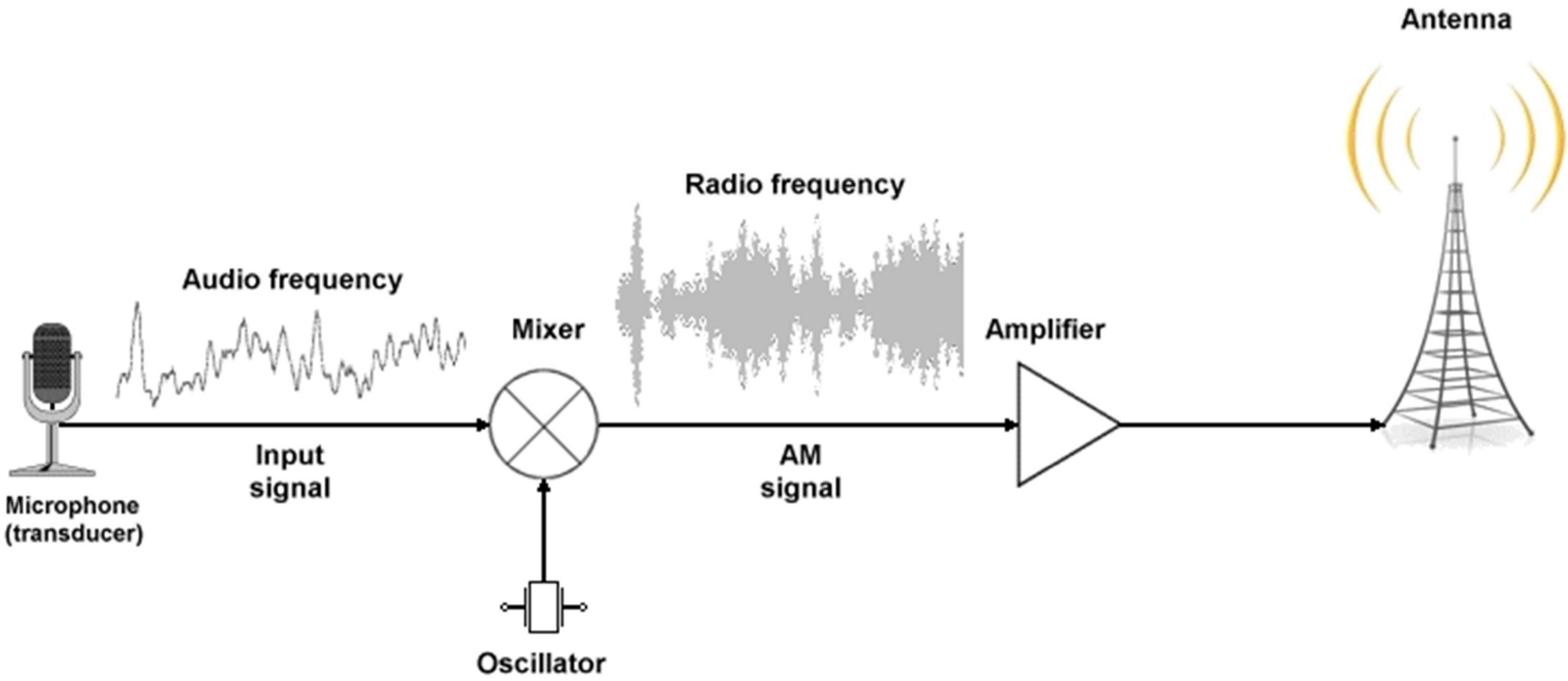
This response is valid for all lab generated input having finite energy spectrum and Fourier transformable signals

Objectives

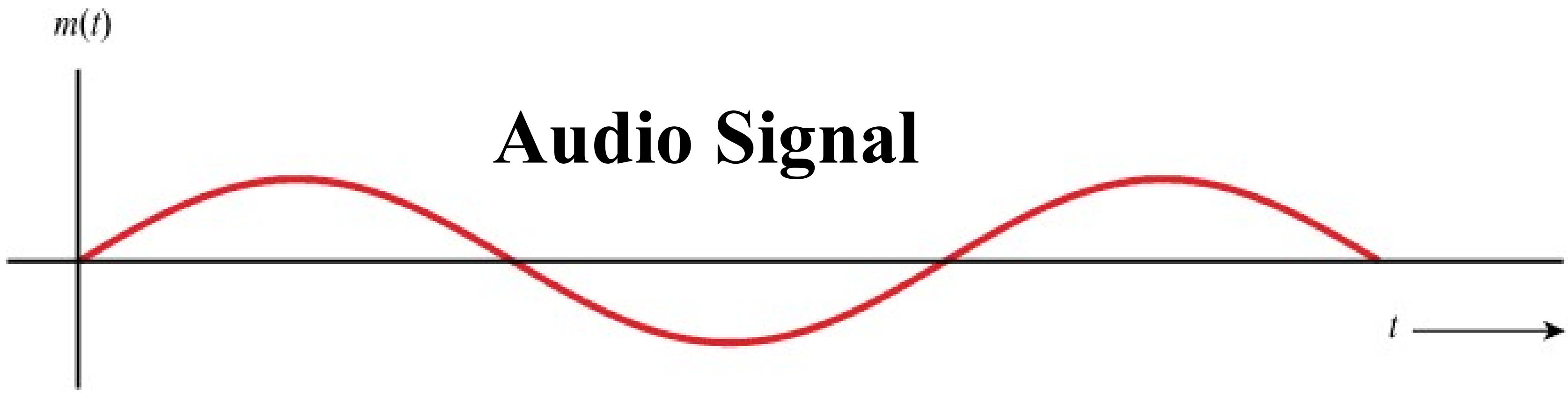
- LTI System Response to Energy Signals
- **Amplitude modulation**
- Coherent demodulation of AM signal

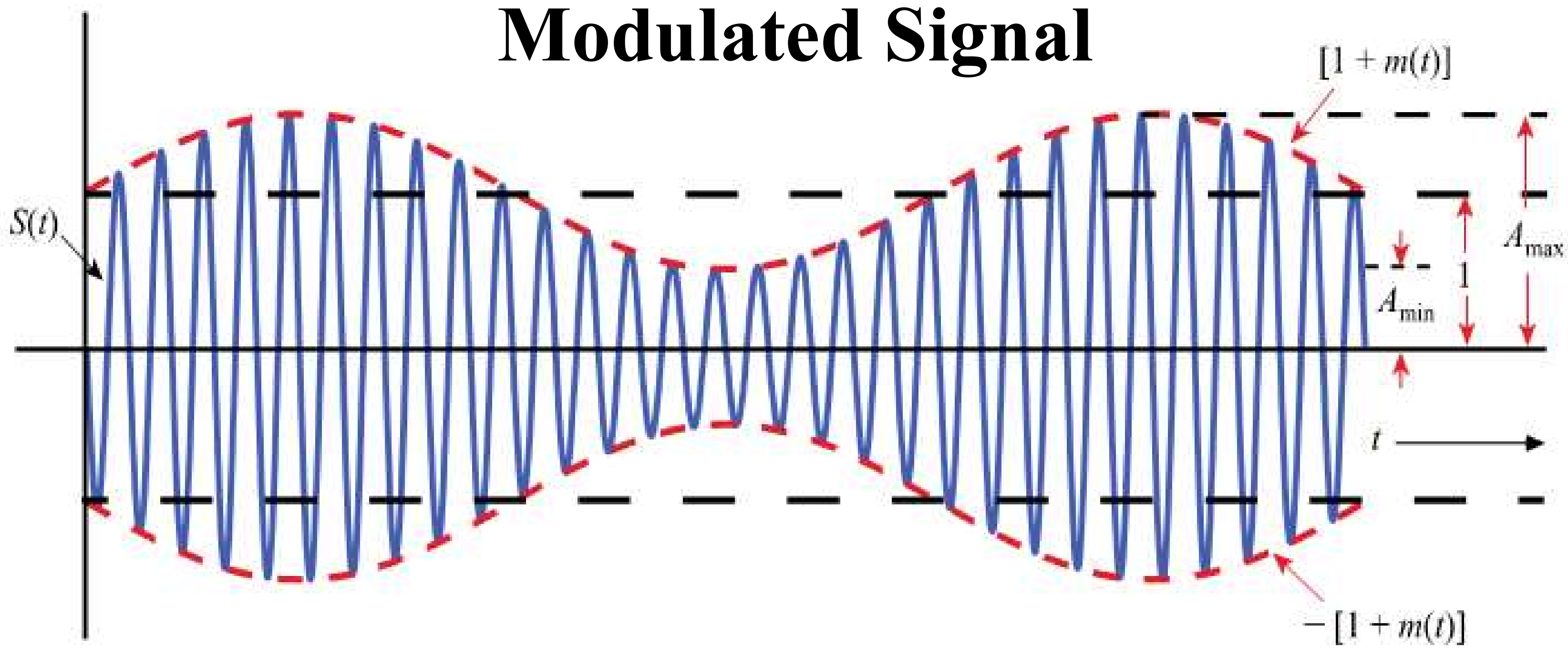
Amplitude Modulation

- The power limitations for voice communication at long distances evolved modulation techniques



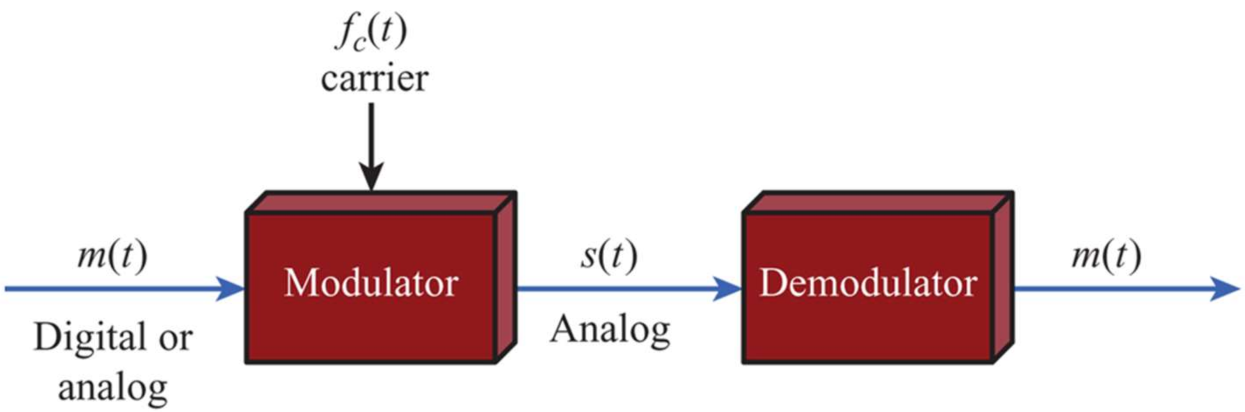
Amplitude Modulation

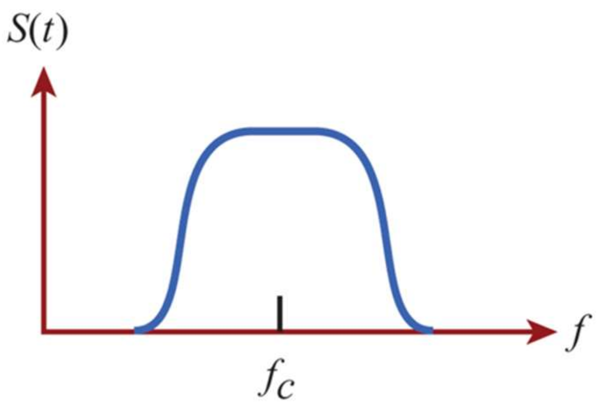




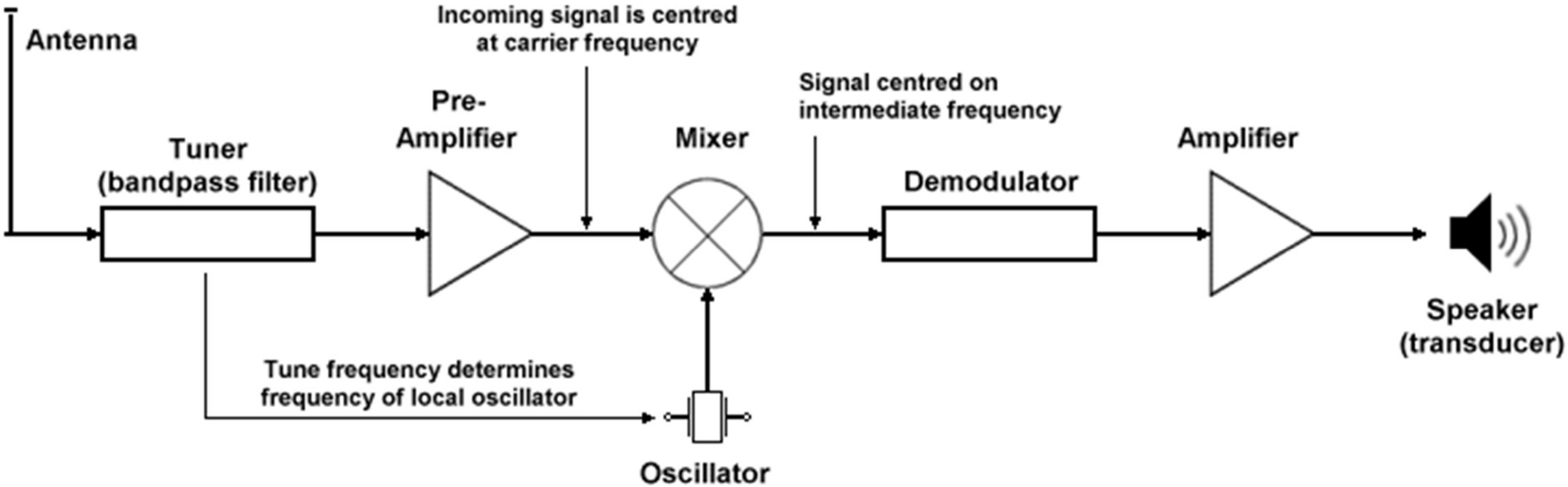
Amplitude demodulation

- **The prime responsibilities of a demodulation system:**
  - ✓ **Receive signal power through antenna**
  - ✓ **Extraction of message signal from modulated signal**
  - ✓ **Selection of specific band spectrum (BP filter)**
  - ✓ **Conversion of signal into voltage/current signal**
  - ✓ **Passing the signal through a transducer (Headphone, speaker)**





Demodulation of AM signal



Fourier properties for AM

- **Time – Shifting Property**

The time shifting property states that,

$$f(t - t_o) \Longleftrightarrow F(\omega)e^{-j\omega t_o}$$

where $F(\omega)$ is the Fourier transform of $f(t)$

- **Frequency – Shifting Property**

The frequency shifting property states that,

$$F(\omega - \omega_o) \Longleftrightarrow f(t)e^{jt\omega_o}$$

Fourier properties for AM – Heterodyne

- **We will make use of these two properties to better understand heterodyning phenomena**

- ***Heterodyning*** **refers to translation of signal spectrum to another frequency**

- **The demodulating circuit needs to translate high frequency signals broadcasted by AM station to *detectable* low frequency range**

Fourier properties for AM

Let's consider both frequency shifting as,

$$F(\omega - \omega_c) \Longleftrightarrow f(t)e^{jt\omega_c}$$

$$F(\omega + \omega_c) \Longleftrightarrow f(t)e^{-jt\omega_c}$$

summing of these will lead us to,

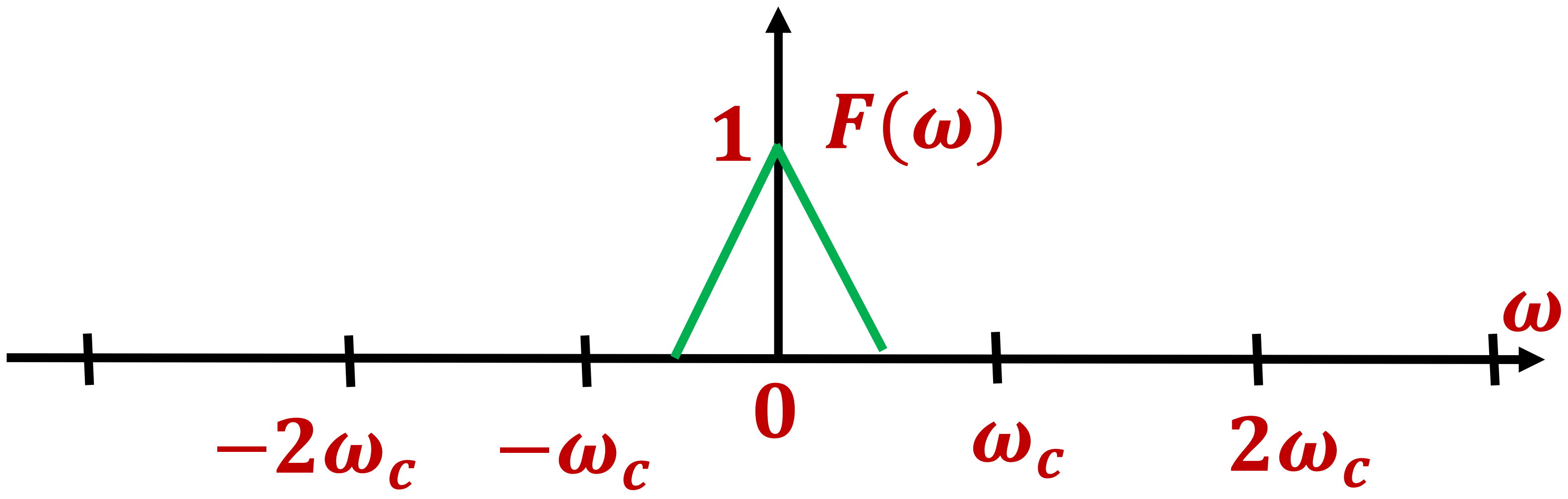
$$\boxed{f(t)\left(e^{jt\omega_c} + e^{-jt\omega_c}\right) = 2\,f(t)\cos(\omega_c t)}$$

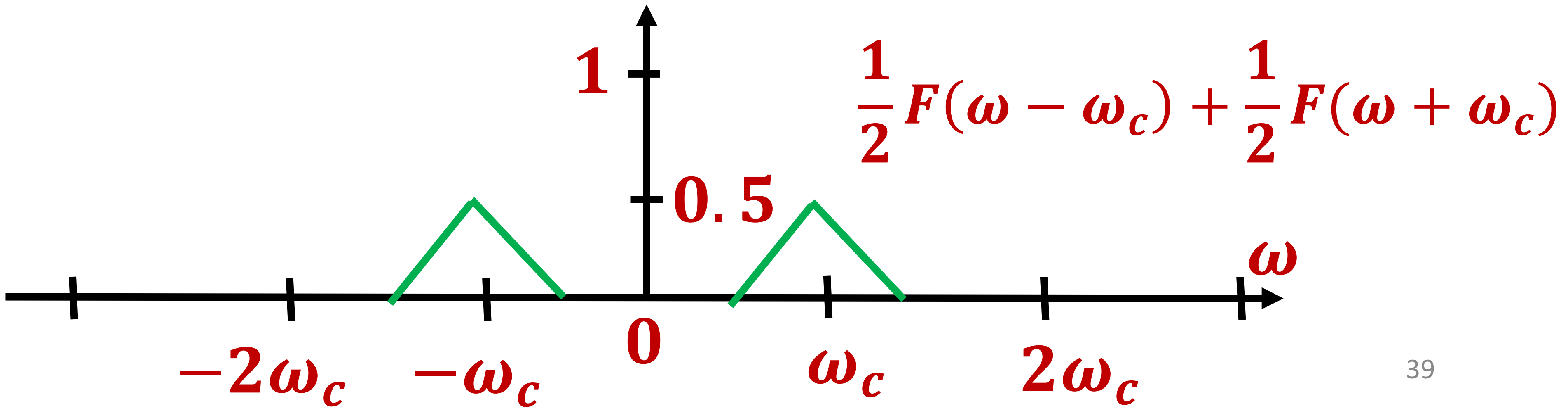
$$\Updownarrow$$

$$\boxed{F(\omega - \omega_c) + F(\omega + \omega_c)}$$

Fourier properties for AM

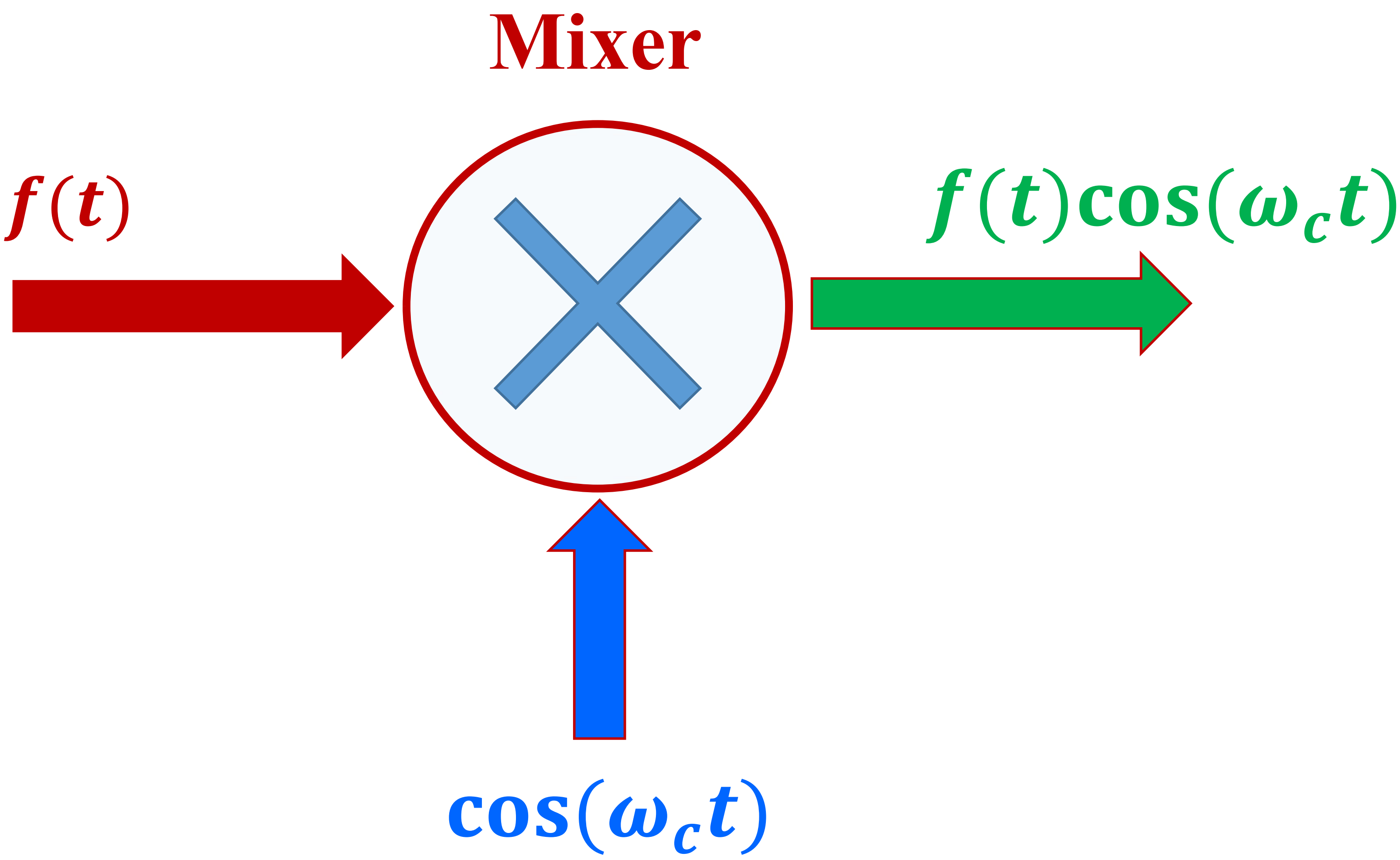
$$f(t)\cos(\omega_c t) \Longleftrightarrow \frac{1}{2}F(\omega-\omega_c)+\frac{1}{2}F(\omega+\omega_c)$$





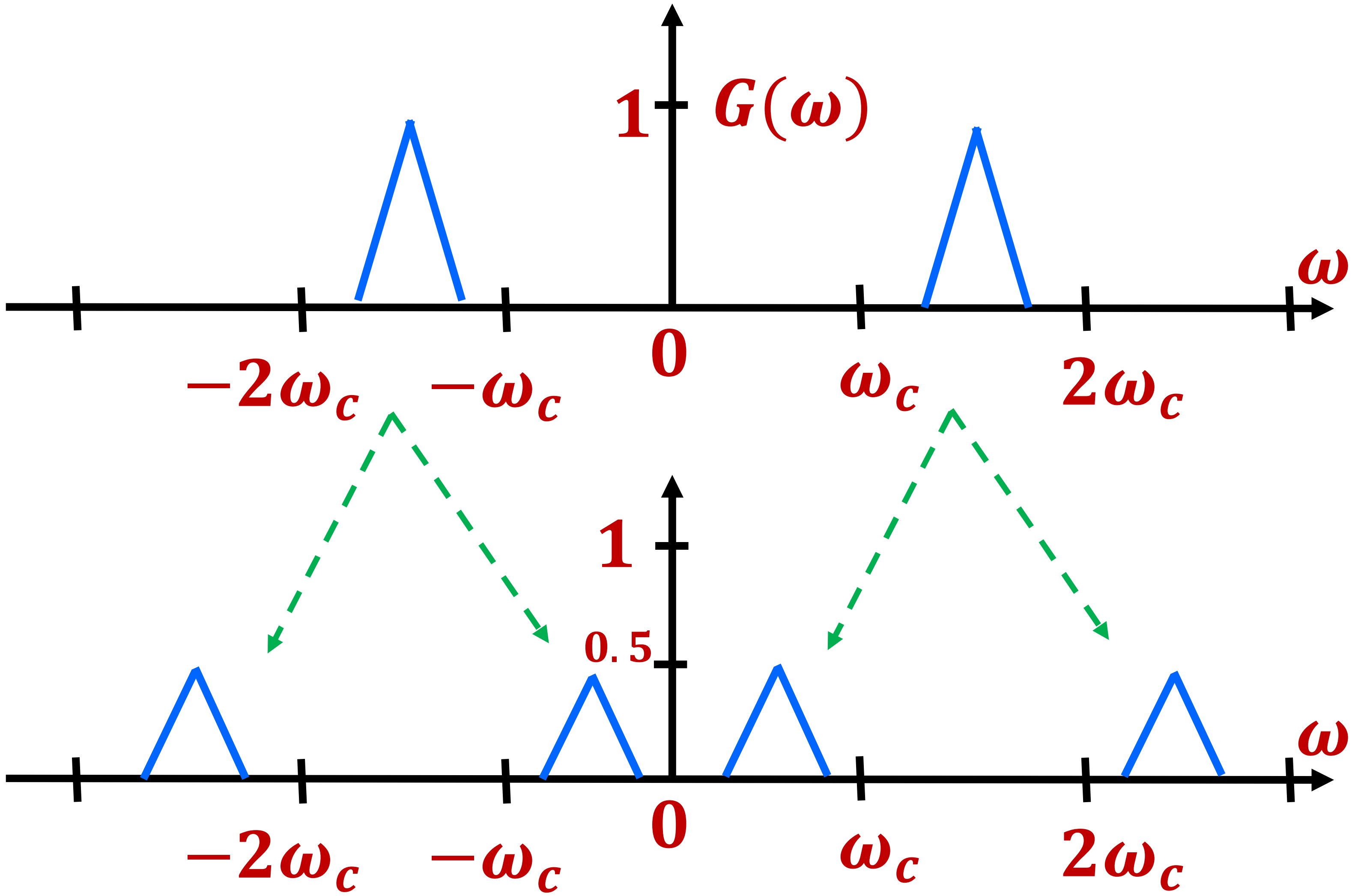
Heterodyning – Mixer

- **Shifting frequency content to a new location in** $\omega - space$ **is called heterodyning**



Another Example of modulation

$$g(t)\cos(\omega_c t) \Longleftrightarrow \frac{1}{2}G(\omega-\omega_c)+\frac{1}{2}G(\omega+\omega_c)$$



Motivation for AM

- The antenna converts radio waves into electrical signal (and vice versa)
- The frequency response of antenna allows us to transmit signal at higher frequencies,

$$|H_{ant}(\omega)| = \frac{\pi c}{2L}$$

- For human voice, can you calculate antenna size ***L*** when ***c*** is the speed of light?

- Using ***bandpass filtration*** advantage, we can generate multiple carriers and receive the same without channel interfering problem

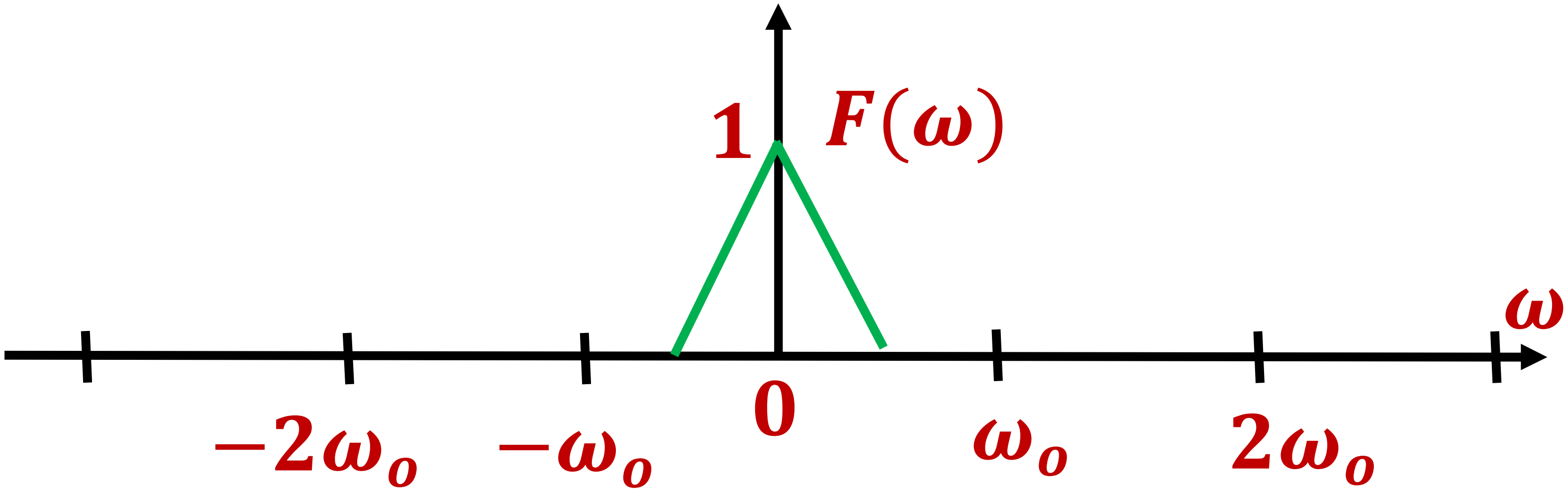
Fourier properties for AM – Example 8

Question: A mixer is used to multiply

$$1+\sum_{n=1}^{\infty}\frac{1}{n}\cos(n\omega_o t)$$ with a low pass signal $f(t)$

plot the Fourier transform $M(\omega)$ as output of mixer?

$$m(t)=f(t)\left\{1+\sum_{n=1}^{\infty}\frac{1}{n}\cos(n\omega_o t)\right\}$$



Fourier properties for AM – Example 8

Solution: **We will start by expanding the expression,**

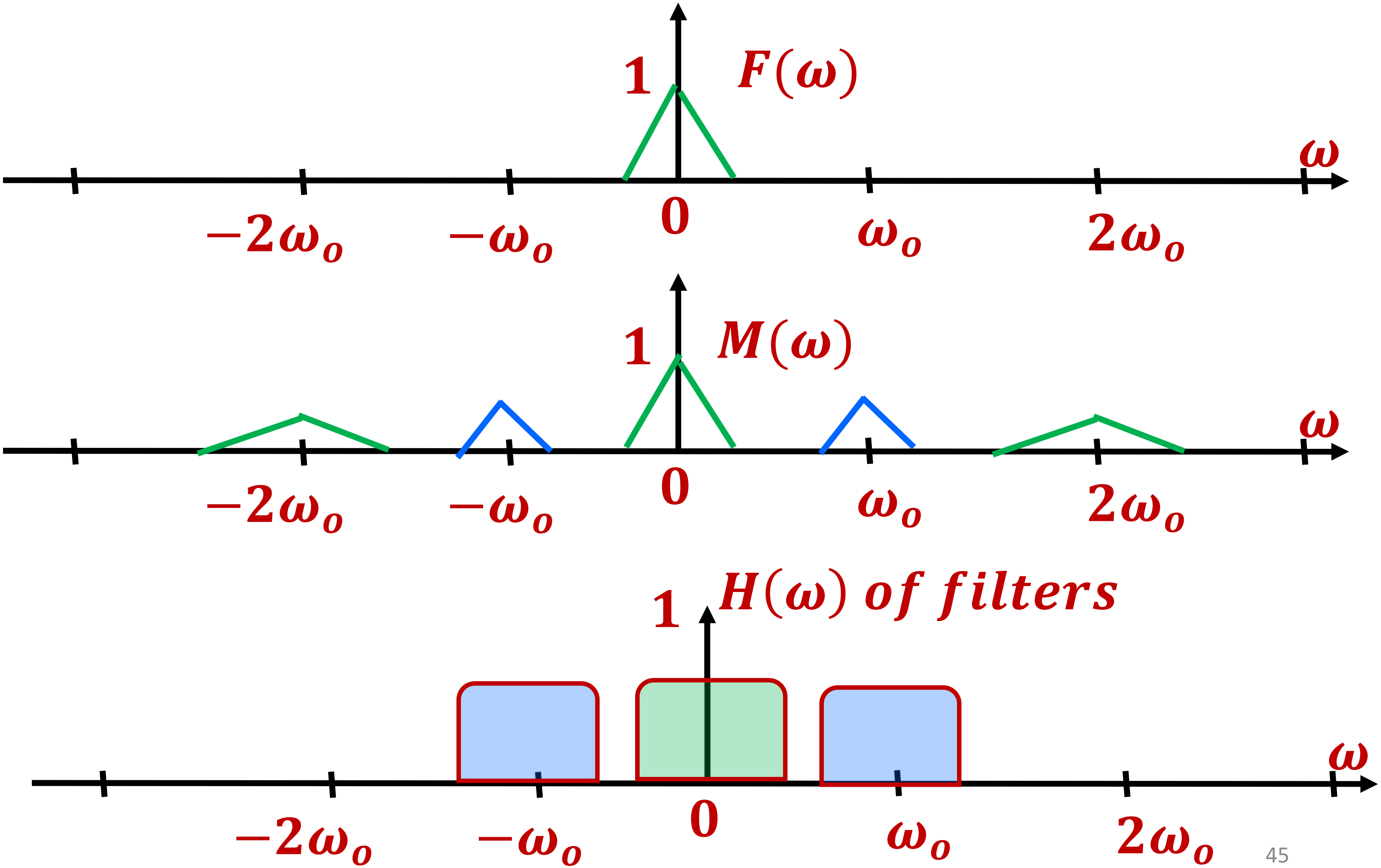
$$m(t) = f(t) + \sum_{n=1}^{\infty} \frac{1}{n} f(t)\cos(n\omega_o t)$$

taking Fourier transforms and applying Fourier transform properties gives,

$$M(\omega) = F(\omega) + \sum_{n=1}^{\infty} \frac{1}{2n} \{F(\omega - n\omega_o) + F(\omega + n\omega_o)\}$$

Let's draw the signal spectrum and frequency responses for HPF and BPF

Fourier properties for AM –Example 8



Objectives

- LTI System Response to Energy Signals
- Amplitude modulation
- **Coherent demodulation of AM signal**

Coherent demodulation

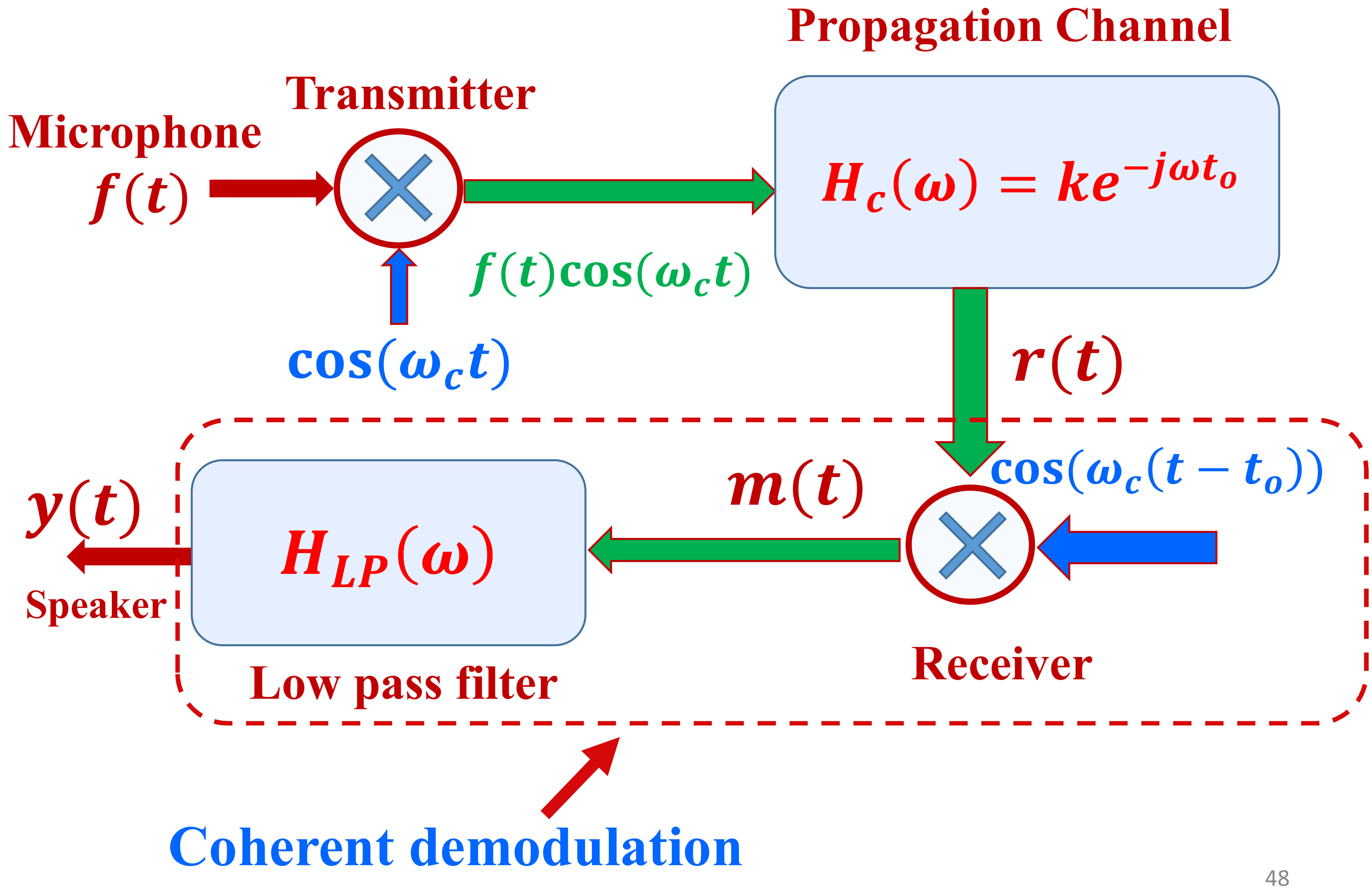
- **As the transmitter and receiver are situated hundreds of kilometers away**

- **Any good detection should contain *rejection* component for channel delay and unwanted scaling (simply noise)**

- **The delay and scaling can be modelled in Fourier domain and Bandpass filter are good enough to reject unwanted frequency components in the signals**

- **Let's see this step by step…**

Coherent demodulation –Overall picture



Coherent demodulation

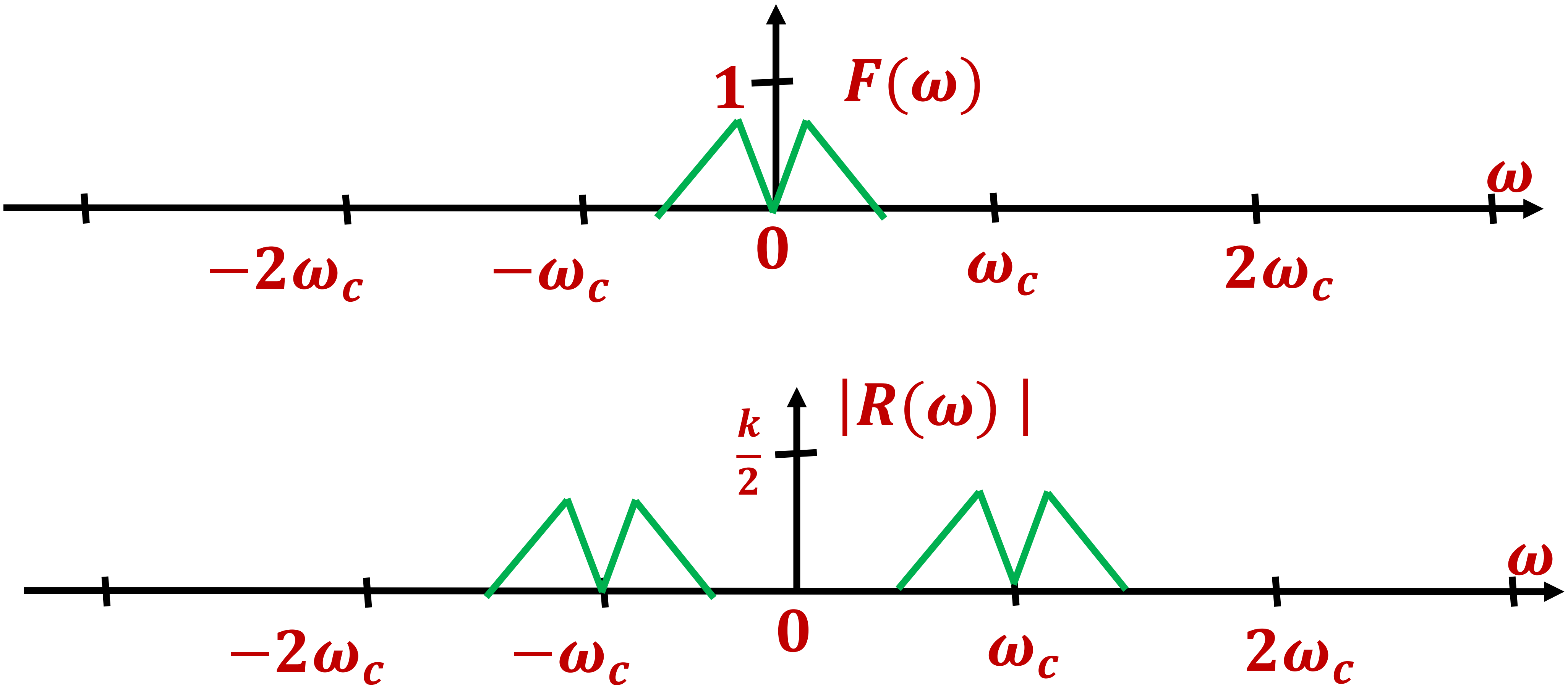
- **The ideal propagation channel (delayed by t_o and scaled by k) has frequency response of,**

$$H_c(\omega) = ke^{-jt_o\omega}$$

- **Due to radio channel propagation losses, the signal arrived at the receiver end is delayed by t_o and scaled by k, can be written as,**

$$r(t) = kf(t - t_o)\cos(\omega_c(t - t_o))$$

Coherent demodulation



Coherent demodulation

- The mixer output in the receiver is

$$m(t) = r(t)cos(\omega - \omega_o))$$

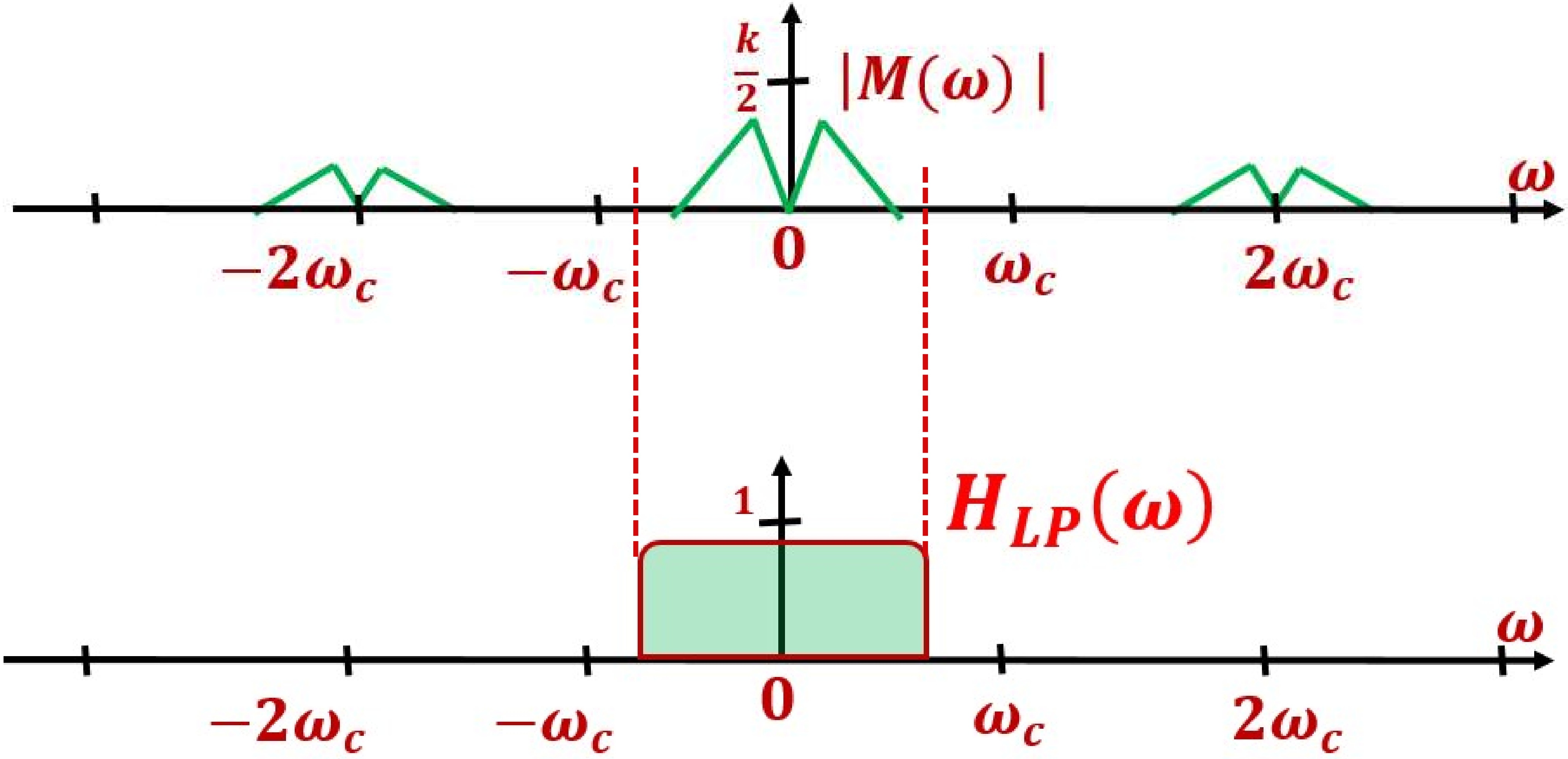
$$m(t) = f(t - t_o)\frac{k}{2}\{1 + cos(2\omega_c(t - t_o))\}$$

- Using addition, modulation and shifting properties of Fourier transform, we can calculate $M(\omega)$ as,

$$M(\omega) = \frac{k}{2}F(\omega)e^{-j\omega t_0} \; +$$

$$\frac{k}{4}\{F(\omega - 2\omega_c) + F(\omega + 2\omega_c)\}e^{-j\omega t_0}$$

Coherent demodulation



Coherent demodulation

- It can be seen that , the first term of $m(t)$,

$$\frac{k}{2}f(t-t_0)$$

is delayed audio signal we want to recover. To extract this, we pass it to a LPF ,

$$Y(\omega)=H_{LP}(\omega)M(\omega)=\frac{k}{2}F(\omega)e^{-j\omega t_0}$$

taking the inverse Fourier transform will lead us to the signal that can be input to the speaker

$$y(t)=\frac{k}{2}f(t-t_0)$$

Trouble with coherent demodulation

- The mixing of *r(t)* and $cos(\omega_c(t - t_o))$ at the mixer stage creates complexity as we need to generate $cos(\omega_c t + \theta)$locally with the right frequency ω_c

- An arbitrary phase shift $\theta \neq -\omega_c t_0$ will not work for us , as a small shift in t will generate a large *phase shift* and *y(t)* will always fluctuate

- *Coherent,* thus, refers to the requirement that phase shift θ of incoming signal $cos(\omega_c t + \theta)$ be coherent with phase shift $\omega_c t_0$ of incoming carrier

$$\theta \equiv -\omega_c t_0$$

How to deal with it…

- The receiver complexity can be minimized if the incoming signal is of the form,

$$r(t) = k(f(t - t_0) + \alpha)cos(\omega_c(t - t_o))$$

- It contains a constant cosine term $k\alpha cos(\omega_c(t - t_o))$ plus primary signal term $k\left(f(t - t_0)\right) cos\left(\omega_c(t - t_o)\right)$ carrying the voice signal *f(t)*

- So, the loss can be adjusted by α and delay can be settled by t_o *locally*

Summary

- **The system output is the IFT (Inverse Fourier Transform) of the product of the system frequency response and Fourier transform of the input**

- **In dissipative systems, transient part of zero-state response to input $cos(\omega t)$ and $sin(\omega t)$ applied at $t=-\infty$ must be vanished for finite times**

- **Once we know the system frequency response, we can estimate system output for any practical input**

- **The complex circuits can be directly analyzed by taking Fourier transform and evaluating frequency response**

Summary

- **Any energy signal can be transmitted far away by modulating it with high frequency signal**

- **When the modulated signal is propagated in radio channel transmitted by antenna *mixed* with channel losses and cause attenuation losses and delay in the signal**

- **The receiver includes demodulation circuit and filter circuits to extract message signal**

- ***Coherent* refers to the requirement that phase shift of incoming signal matches with the local oscillator signal**

Further reading

1. Ch. 7 (page 248-258), Ch. 8 (page 261-268), E. Kudeki and D. C. Munson, ***Analog Signals and Systems,*** Prentice Hall, 2008.
2. Ch. 8 (page 582-593), C. K. Allan V. Oppehnheim, ***Signals and Systems*** , 5th ed., Prentice hall,1996.
3. Ch. 14 (page 650-670), C. K. Alexander and M. Sadiku, ***Fundamentals of Electric Circuits***, 5th ed., McGraw-Hill, 2013.
4. Ch. 15 (page 755-765, page 770-785), J. D. Irwin, and R. M. Nelms. ***Basic Engineering Circuit Analysis,*** 10th ed. Wiley, 2011.

Preview:

1. Ch. 8 (page 268-275), E. Kudeki and D. C. Munson, ***Analog Signals and Systems,*** Prentice Hall, 2008.

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Deadline: 10:00 PM, 27th April, 2022

Thank you!