Lab Assignment

Purpose

Obtain basic knowledge about numerical methods for solving ODEs.

Reading Material

[BDM17], Chapters 2.7, 8.1–8.3, 8.6

Format

Work in groups of 5 students.
Scores for presentations (max. 5) and written reports (max. 10) will be assigned group-wise and account for 15 % of the total score.
Work groups will be formed during the discussion sessions (no later than Fri May 6).

Presentations/Reports Deadline

The presentations (length ca. 15 min) will be held during the last week of the Spring semester (May 18–24). Every group has to hand in a written lab report of 6–8 text pages (computer printouts exclusive) before their presentation.

Lab Problem

Consider the two initial value problems

$$y' = y^2 + t^2 - t, \quad y(0) = 0, \tag{IVP1}$$

$$y' = (y - t)(y^2 - t^2), \quad y(0) = 0. \tag{IVP2}$$

Using a combination of numerical and analytical methods, determine the maximal solutions of (IVP1), (IVP2) as accurately as possible, including an accurate computation of their domains (a_1, b_1) and (a_2, b_2), respectively.

Further Remarks

- Information on how to obtain the vertical asymptotes $t = a_i$, $t = b_i$, $i = 1, 2$, of the solution graphs can be found in the example on p. 378 of [BDM17] (p. 380 in the 11th edition). This particular example was also discussed in the lecture from a different point of view; cf. `lecture32-36_handout.pdf`, Slide 6 ff.
- When computing the solutions of (IVP1), (IVP2), you are urged to use different numerical methods (e.g., Euler, Improved Euler, Runge-Kutta) and step sizes, and compare the results. Please pay particular attention to the values near the vertical asymptotes.
- While not mandatory for a full score, you are invited to explore other routes to the solution, e.g., symmetry considerations, power series representations of the solutions, or reduction to simpler ODE's by means of appropriate substitutions. (But don't blame me when some of these suggestions lead nowhere!) For excellent contributions in this voluntary part work groups can earn up to 2 extra credits.

Further Remarks (cont'd)

- For the presentation make an appropriate selection of the topics discussed in your report. Don't overload your presentation! In order to obtain a presentation score, all work group members must participate in the presentation with a fair share (but not necessarily the same amount of time). The time limit for the presentations will be enforced strictly (cutting your speech off, if necessary).