Math 286 Differential Equations Plus (Prof. Honold) **Midterm Exam** 2021/04/01

Name: _____ Student No.: _____ Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2 \colon I \to \mathbb{R}$ satisfying $y_1(t_0) = y_2(t_0)$ for some $t_0 \in I$?
   ☐ $y' = \sin(ty^2)$ ☐ $yy' = 0$ ☐ $y' = |ty|$ ☐ $y' = y\sqrt{t}$ ■ $y' = \sqrt{|ty|}$

2. $(x^4y^2 - y)\,dx + (x^2y^4 - x)\,dy = 0$ has the integrating factor
   ☐ $1/(xy)$ ☐ 1 ■ $1/(xy)^2$ ☐ $1/(xy^2)$ ☐ $1/(x^2y)$

   这个也是纯粹的带进去试试看。。。。

3. The family of curves $y = c/x^2$, $c \in \mathbb{R}$ satisfies the ODE
   ☐ $dy = x^{-2}\,dx$ ☐ $dy = 2x^{-3}\,dx$ ■ $2xy\,dx + x^2\,dy = 0$ ☐ $dx = dy$
   ☐ $2yx^{-3}\,dx - x^{-2}\,dy = 0$

4. For the solution $y(t)$ of the IVP $y' = y^3 - 7y + 6,\ y(0) = 0$ the limit $\lim\limits_{t\to+\infty} y(t)$ equals
   ☐ -3 ☐ -2 ☐ -1 ■ 1 ☐ 2

5. For the solution $y(t)$ of the IVP $y' = (y/t) - 1,\ y(1) = \ln 2$ the value $y(2)$ is equal to
   ■ 0 ☐ 1 ☐ 2 ☐ $\ln 2$ ☐ $2\ln 2$

6. For the solution $y(t)$ of the IVP $y' = y^2 e^{-t},\ y(0) = -1$ the value $y(-1)$ is equal to
   ☐ e ■ $1/(e-2)$ ☐ $e+2$ ☐ $1/(e+2)$ ☐ $e-2$

7. For the solution $y \colon (0,+\infty) \to \mathbb{R}$ of the IVP $t^2y'' + 2ty' - 2y = 1$, $y(1) = 0$, $y'(1) = 1$ the value $y(2)$ is equal to
   ☐ $\frac{13}{8}$ ☐ 1 ☐ $\frac{13}{24}$ ☐ $\frac{19}{8}$ ■ $\frac{19}{24}$

8. The power series $\sum\limits_{k=1}^{\infty} 2^k z^{k^2}$ has radius of convergence
   ☐ 0 ☐ $\frac{1}{2}$ ■ 1 ☐ 2 ☐ ∞

9. The smallest integer s such that $f_s(x) = \sum\limits_{k=1}^{\infty} \dfrac{\cos(k^2x)}{k^s}$ is differentiable on $\mathbb{R}$ is equal to
   ☐ 0 ☐ 1 ☐ 2 ☐ 3 ■ 4

10. If $y(t)$ solves $y' = t^2y + ty^2$ then $z = 1/y(t)$ solves
    ☐ $z' = -t^2z$ ☐ $z' = -z^2 + t$ ☐ $z' = t^2/z + t/z^2$ ☐ $z' = -z^2$
    ■ $z' = -t^2z - t$

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11. The sequence $\phi_0, \phi_1, \phi_2, \ldots$ of Picard-Lindelöf iterates for the IVP $y' = y + 2t,\ y(0) = -2$ has $\phi_2(t)$ equal to

☐ $-2 + 2t + \frac{1}{3}t^3$ ☐ $-t^2 + \frac{1}{3}t^3$ ■ $-2 - 2t + \frac{1}{3}t^3$ ☐ $t^2 + \frac{1}{3}t^3$

☐ $1 + t + \frac{3}{2}t^2 + \frac{1}{3}t^3$

12. $y'' - 4y' + 4y = 2t + \mathrm{e}^{2t}$ has a particular solution $y_p(t)$ of the form

☐ $c_0 + c_1 t + c_2 \mathrm{e}^{2t}$ ■ $c_0 + c_1 t + c_2 t^2 \mathrm{e}^{2t}$ ☐ $c_0 t + c_1 t^2 \mathrm{e}^{2t}$ ☐ $c_0 + c_1 t$

☐ $c_0 t + c_1 \mathrm{e}^{2t}$

13. Maximal solutions of $y' = y^2 + y$ satisfying $y(0) > 0$ are defined on an interval of the form

☐ (a, b) ☐ $[a, b]$ ☐ $(a, +\infty)$ ■ $(-\infty, b)$ ☐ $(-\infty, +\infty)$

with $a, b \in \mathbb{R}$.

14. For $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$, the matrix $\mathrm{e}^{\mathbf{A}t}$ is equal to

☐ $\begin{pmatrix} \mathrm{e}^{t} & 0 \\ 0 & \mathrm{e}^{-t} \end{pmatrix}$ ☐ $\begin{pmatrix} \mathrm{e}^{t} & \mathrm{e}^{-t} \\ \mathrm{e}^{t} & \mathrm{e}^{-t} \end{pmatrix}$ ☐ $\begin{pmatrix} \mathrm{e}^{t} & \mathrm{e}^{t} \\ \mathrm{e}^{-t} & \mathrm{e}^{-t} \end{pmatrix}$

☐ $\begin{pmatrix} 1+t & t \\ -t & 1-t \end{pmatrix}$ ■ $\begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix}$

15. The matrix norm of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (subordinate to the Euclidean length on $\mathbb{R}^2$) is equal to

☐ 0 ☐ 1 ■ $\sqrt{2}$ ☐ 2 ☐ 4

Time allowed: 60 min CLOSED BOOK ***Good luck!***