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H1 $y' = x/y \Rightarrow y\,dy = x\,dx \Rightarrow \begin{cases} x^2 - y^2 = C & \text{(implict)} \\ y = \pm\sqrt{C + x^2} & \text{(explict)} \end{cases}$

$y = -y/x \Rightarrow \frac{1}{y}dy = -\frac{1}{x}dx \Rightarrow \ln|y| = -\ln|x| + \ln|C|$

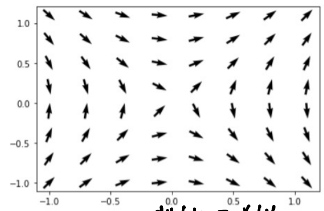
$\Rightarrow \ln|xy| = \ln|C|$

$\Rightarrow \begin{cases} xy = C & \text{(implict)} \\ y = \frac{C}{x} & \text{(explict)} \end{cases}$

$y = y/x \Rightarrow \frac{1}{y}dy = \frac{1}{x}dx \Rightarrow \ln|y| = \ln|x| + \ln C$

$\Rightarrow \begin{cases} y = Cx & \text{(explict)} \\ y + Cx = 0 & \text{(implict)} \end{cases}$

49]: <matplotlib.quiver.Quiver at 0x7ff1b25c2700>



$dy/dx = x/y$

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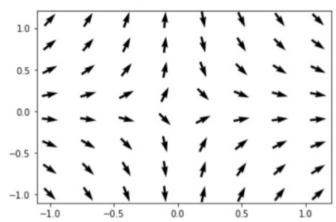
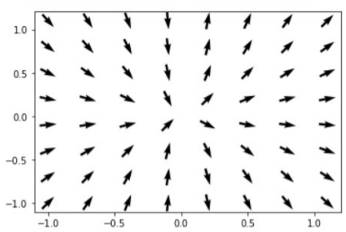


$dy/dx = -y/x$

<matplotlib.quiver.Quiver at 0x7ff1b2625e50>



$dy/dx = y/x$

H2 $y' = \sqrt{|y|} \Rightarrow \frac{1}{\sqrt{|y|}}dy = dx \Rightarrow \begin{cases} \text{① for } y>0 \text{ then } \frac{1}{\sqrt{y}}dy = dx \Rightarrow 2\sqrt{y} = x + C_1 \\ \quad \Rightarrow y = \frac{1}{4}(x+C_1)^2 \\ \text{② for } y=0 \text{ then } y=0 \\ \text{③ for } y<0 \text{ then } \frac{1}{\sqrt{-y}}dy = dx \Rightarrow y = -\frac{1}{4}(x+C_2)^2 \end{cases}$

C_1, C_2 are arbitrary random numbers

$\boxed{\text{Claim, Sols can be uniquely determined for all } t \geq t_0 \text{ iff } y_0 = y(t_0) > 0}$

① for $y_0 > 0$ then As $y \geq 0$ $y = \frac{1}{4}(x+C_1)^2$ C_1 can be determined.
so for $[t_0, \infty)$ all the function can be determined.

② if $y_0 < 0$, then the $y = -\frac{1}{4}(x+C_2)^2$ can be detemined function.
but as the C_1 cannot be calculated, so $[t_0, \infty)$ cannot be determined

③ if $y_0 = 0$, then $y=0$ determined neither C_1 nor C_2, then we can figure out $[t_0, \infty)$ cannot be determined.

H3 $y'' = -y \Rightarrow \frac{1}{y}\frac{dy}{dx}dy = -dx \Rightarrow \ln|y|\frac{dy}{dx} = -x + C_1 \quad \ln|y|dy = (-x + C_1)dx \Rightarrow y\ln|y| - y = -\frac{1}{2}x^2 + C_1 x + C_2$

$\Rightarrow \begin{cases} y(t_0) = y_0 \\ y'(t_0) = y_1 \end{cases} \Rightarrow \begin{cases} y_0\ln|y_0| - y_0 = \frac{1}{2}t_0^2 + C_1 t_0 + C_2 \\ y_1 \ln|y_1| = -t_0 + C_1 \end{cases}$

$\Rightarrow \begin{cases} C_1 = y_1\ln|y_1| + t_0 \\ C_2 = y_0\ln|y_0| - y_0 - \frac{1}{2}t_0^2 - t_0\cdot(y_1\ln|y_1| + t_0) \end{cases} \Rightarrow$ So, C_1, C_2 can be determined
$\Rightarrow$ IVP has a unique sol.
Q.E.D.

H4 (a) Assume $y = bx^\beta \Rightarrow y'' = b\beta(\beta-1)x^{\beta-2} = b^2x^{2\beta}$
$\Rightarrow b^2x^{2\beta} - b\beta(\beta-1)x^{\beta-2}$ $\Rightarrow$ let $\beta = -2$ $b = b$
$\boxed{\text{then there is } y = bx^{-2}}$

(b) Assume $y = ae^{\alpha x} \Rightarrow y'' - 5y' + 6y = \alpha^2 a e^{\alpha x} - 5\alpha a e^{\alpha x} + 6ae^{\alpha x}$
$= (\alpha^2 - 5\alpha + 6)ae^{\alpha x} \Rightarrow \boxed{y(x) = a_1 e^{2x} + a_2 e^{3x}}$

(c) let $y = ae^{\alpha x} \Rightarrow (a^2-5a+6)ae^{\alpha x} = e^x \Rightarrow a^3-5a^2+6a=1 \Rightarrow y = \frac{1}{2}e^x$

(d) let $y = bx^\beta \Rightarrow y'' - \frac{1}{2x}y' + \frac{1}{2x^2}y = \beta(\beta-1)x^{\beta-2} - \frac{\beta x^{\beta-1}}{2x} + \frac{x^\beta}{2x^2} = (\beta^2 - \frac{3}{2}\beta + \frac{1}{2})x^{\beta-2} = 0$

$\Rightarrow 2\beta^2 - 3\beta + 1 = 0 \Rightarrow \beta_1 = \frac{1}{2} \quad \beta_2 = 1 \quad \boxed{\Rightarrow y = x \text{ or } y = \sqrt{x}}$

(e) $y(x) = e^{\alpha x} \Rightarrow (2x)\alpha^2 e^{\alpha x} + (4x-2)\alpha - 8 = ((2x+1)\alpha^2 + (4x-2)\alpha - 8)e^{\alpha x} = 0$

$\Rightarrow (2x+1)\alpha^2 + (4x-2)\alpha - 8 = (2\alpha^2+4\alpha)x + \alpha^2 - 2\alpha + 8$

$\Rightarrow 2\alpha^2 + 4\alpha = \alpha^2 - 2\alpha - 8 = 0$

$\Rightarrow \alpha = -2 \Rightarrow \boxed{y(x) = e^{-2x}}$

(f) let $y = x^{\beta x} \Rightarrow x^2(1-x)\beta(\beta-1)x^{\beta-2} + 2x(2-x)\beta x^{\beta-1} + 2(1+x)x^\beta$

$= (x^2(1-x)\beta(\beta-1) + 2x(2-x)\beta x + 2(1+x)x^2)x^{\beta-2} = 0$

$\Rightarrow \begin{cases} -\beta^2 - \beta + 2 = 0 \\ \beta^2 + 3\beta + 2 = 0 \end{cases} \Rightarrow \beta = 0 \Rightarrow \boxed{y = x^{-2}}$

H5 Ex23 (a) Kinetic Energy $= \frac{1}{2}mv^2 = \frac{1}{2}m(L \cdot \frac{d\theta}{dt})^2 = \frac{1}{2}mL^2 \cdot (\frac{d\theta}{dt})^2$ Q.E.D

(b) Assume energy of rest position is 0

$\Rightarrow$ potential Energy $= mg \cdot \Delta H = mg(L - L\cos\theta)$

$= mgL(1-\cos\theta)$ Q.E.D.

(c) $\Rightarrow \sum E = \frac{1}{2}mL^2(\frac{d\theta}{dt})^2 + mg(L - L\cos\theta)$

$\Rightarrow \frac{dE}{dt} = \frac{1}{2}mL^2 \cdot 2 \cdot (\frac{d\theta}{dt}) \cdot \frac{d^2\theta}{dt^2} + mgL(\sin\theta) \cdot \frac{d\theta}{dt} = 0$

$\Rightarrow \frac{1}{2}mL^2 \cdot 2 \frac{d^2\theta}{dt^2} + mgL\sin\theta = 0$

$\Rightarrow \frac{d^2\theta}{dt^2} + \frac{mgL}{mL^2}\sin\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$

Ex24 (a) the angular momentum rate of change: $\frac{dM}{dt} = \sum F \times L$

$\frac{dM}{dt} = ma \cdot L = m\frac{dv}{dt} \cdot L = mL^2(\frac{d\theta}{dt})$

(b) $\frac{dM}{dt} = mL^2(\frac{d\theta}{dt}) = -mgL\sin\theta \Rightarrow mL^2(\frac{d\theta}{dt}) + mgL\sin\theta = 0$

$\Rightarrow \frac{d\theta}{dt} + \frac{g}{L}\sin\theta = 0$ Q.E.D.