DEPARTMENT OF MECHANICAL ENGINEERING

JADAVPUR UNIVERSITY

3RD YEAR 1ST SEMESTER 2021-22

MINOR PROJECT REPORT

NAME - AYANAVA BAGCHI

CLASS ROLL NUMBER - 001911201026

PROJECT TITLE: MODELLING AND CONTROL OF 1-DOF ACTIVE MAGNETIC BEARING

PROJECT ADVISOR - PROF. ANINDYA SUNDAR DAS

DATE-23-12-2021

Contents Page Number

ACTIVE MAGNETIC BEARING

Introduction

Active Magnetic Bearings have been increasingly in application over the years. The primary reason for their application in the high-speed turbo-machines over other kinds of bearings is their property of contactless rotation. Since, while rotation there is no surface contact between the rotor and the bearing walls, the rotor can be allowed to rotate at very high speeds without worrying about lubrication or wear and tear. Among the other benefits include lower maintenance costs, higher lifetime and reasonable cost. However, since they work on the principle of magnetic levitation, there is a significant challenge in keeping the rotor afloat in a particular position while running at high rpm. This is an interesting control problem and a control scheme has been discussed in this report which attempts to solve this problem.

Working Principle

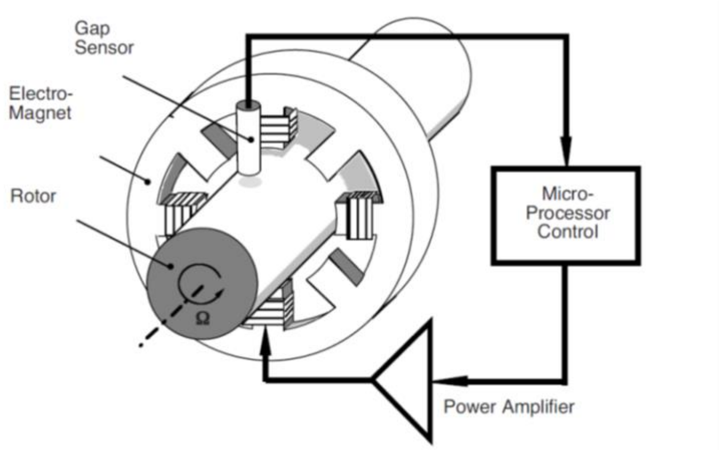


Fig.1

As shown in Fig.1 A sensor measures the displacement of the rotor from a reference position, a microprocessor as a controller generates a control signal from the measurement and feeds it into a power amplifier which then transforms this control signal into a control current. The control current flows through the electromagnets, which then change the magnetic field and thus changing the forces acting on the rotor, in such a way that the rotor remains in its hovering position.

There is a significant challenge in controlling the system because the rotor is always in an unstable equilibrium. The magnetic forces are attractive in nature and inversely proportional to the air gap which causes the rotor to move in the direction in which the gap between the electromagnet and itself decreases. As the gap decreases the attractive force also increases in that direction which makes the system unstable. There may be additional disturbance forces acting on the rotor which may further destabilize the system.

SETTING UP THE PROBLEM

A rotor in an actual magnetic bearing can have 6 degrees of freedom. However, to analyze the simplest model, a one degree of freedom model has been developed.

1-DOF Simplified Model

Physical Description of the Setup

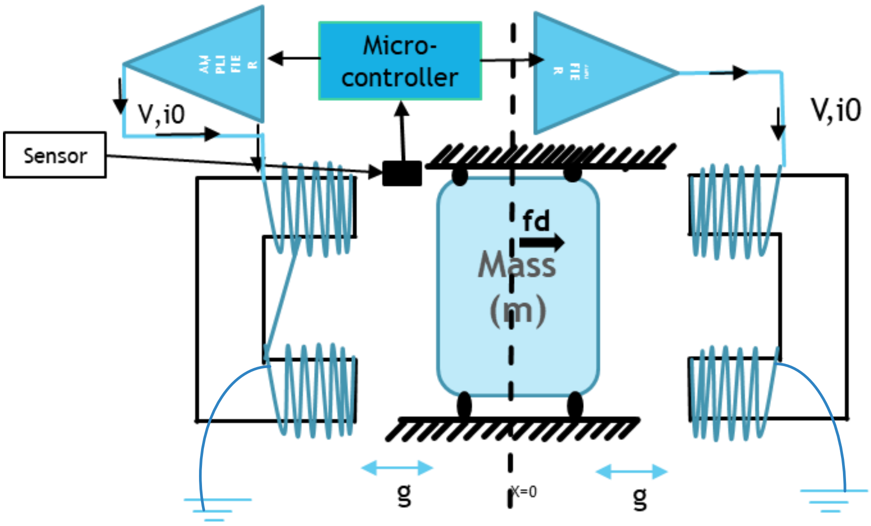


Fig.2 The Setup

A block of mass is equidistantly placed in between two identical electromagnets each having N coils.

The block can move freely along the x-axis. The air gap between the block and either electromagnet is g.

A sensor is placed in between the block and one of the electromagnets.

A time varying periodic disturbance force fd acts on the block.

The sensor is connected to a microcontroller which generates a control signal and passes it to the amplifier on either side.

The amplified signal changes the current (i1, i2) in respective electromagnet. At time t=0, the current and voltage across both the electromagnets are same (V-voltage, i0-bias current).

Control Problem

The goal is to keep the block of mass at the mean position that is x=0 at all times, despite the action of a time varying disturbance force on the block at all times. The controller must be robust to sudden changes in the disturbance force and stabilize the block, taking it back to the mean position in least amount of time.

Modelling of the system

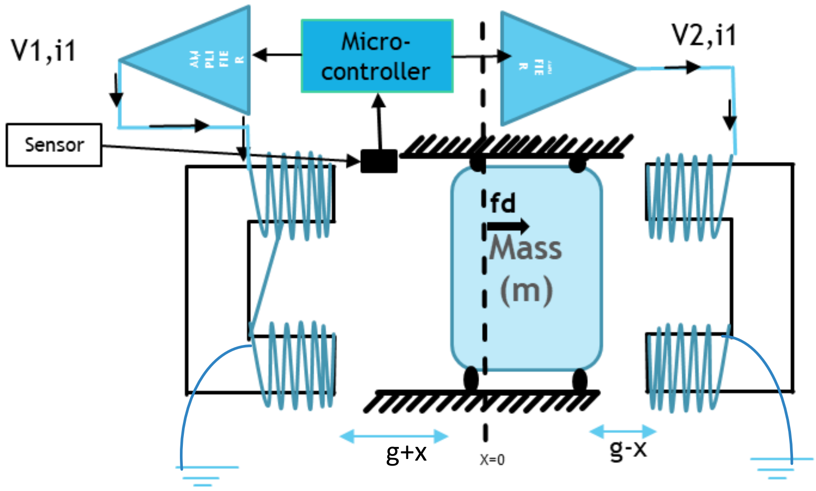


Fig. 3 Situation when the block is displaced by x units

Let m be the mass of the block. The block is initially at x=0 as shown in Fig.2. Bias current i0 flows through both the electromagnets. The nominal air gap is g on either side as shown in Fig 2.

A disturbance force starts acting at time t=0.

At time t=0, magnitude of the magnetic forces on the block by the two electromagnets are equal.

But, as the disturbance force acts and moves the block from x=0(shown in Fig.3), magnetic forces by the electromagnets become unequal in magnitude due to change in the effective magnetic path caused due to change in air gap.

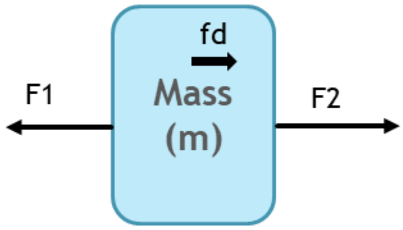


Fig. 4 FBD of block in Fig.3

Net force: F_{net}

F_{net} = F2-F1 +fd

Where fd is the disturbance force.

Magnetic force F1 = $\frac{B1^2A}{\mu 0} = \frac{(\mu 0Ni1)^2A}{4\mu_0(g+x)^2}$

Magnetic force F2 = $\frac{B2^2A}{\mu 0} = \frac{(\mu 0Ni2)^2A}{4\mu_0(g-x)^2}$

F2-F1 = $\frac{\mu 0N^2A}{4}\{\left(\frac{i2^2}{(g-x)^2}\right) - \left(\frac{i1^2}{(g+x)^2}\right)\}$

Let f = F2-F1

The net magnetic force is non-linear in current and air gap.

f $\propto$ i^2 *and* $\propto \frac{1}{g^2}$ *where i is the curent and g is the air gap.*

Linearizing f about x = 0, ix=0;

Let $i1 = i0 - ix, i2 = i0 + ix$

$f(x, ix) = ki\ ix - kg\ x$

Also,

$f(x, ix) = f|_{(0,0)} + \frac{\partial f}{\partial x}\Big|_{(0,0)}(x-0) + \frac{\partial f}{\partial ix}\Big|_{(0,0)}(ix-0)$ ----------(1)

On solving Eq (1) we get,

$\therefore ki = \frac{\mu_0N^2A\ i0}{g^2}$ and $kg = -\frac{\mu_0\ N^2A\ \ i0^2}{g^3}$

CONTROL LAW

PID Control

Fig.5

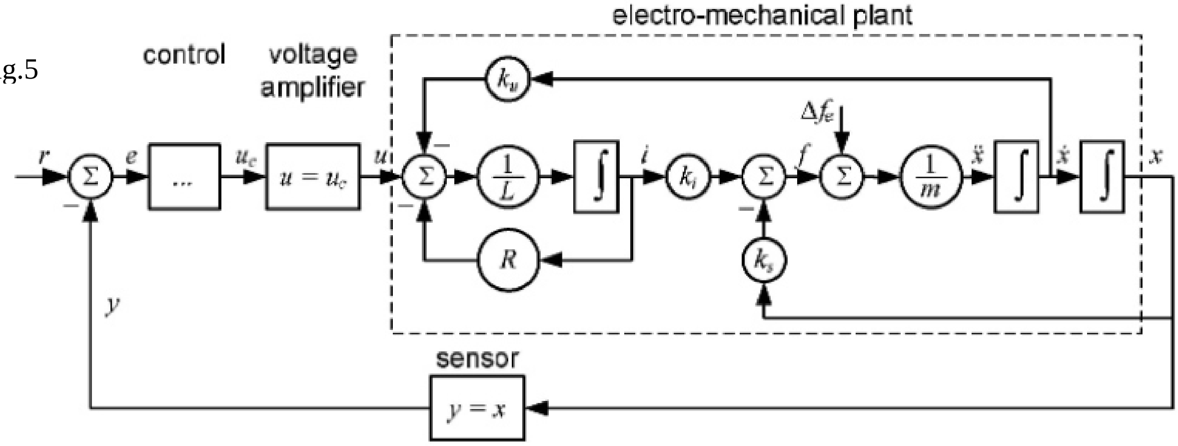


Fig 5 Voltage controlled linearized magnetic bearing system with voltage command signal uc coil inductance L, coil resistance R, disturbance force Δfe and induced voltage ku dx/dt where ku = ki.

The acceleration is computed from the net force, which is then integrated to obtain the velocity and further integrated to get the position. The position is then fed into the sensor which then outputs a signal y. This signal y is compared with a reference command and the error e is computed. The error e goes into the control block which in our case is a PID controller and out comes a control voltage u.

The voltage u is then amplified to obtain the actuating control voltage u1. This control voltage is responsible for changing the current in the electromagnets so as to stabilize the block. This current is also known as the control current with voltage command.

But there are two other factors which affect the overall voltage across the electromagnets which change the overall control current. They are the inductor with resistance R which produces a back emf whenever there is a change in emf, and the induced voltage produced due to the motion of the block which changes the fluxes in the magnetic circuits pertaining to the electromagnets.

The overall voltage is computed, from which the overall control current is obtained.

The net linearized magnetic force is computed which is a function of control current and position. And again, the loop continues.

If suitable PID constants are used the system will eventually stabilize and the position of the block will converge very close to the mean position (x = 0).

PID Control with Voltage Command

Linearization of the magnetic force with respect to current and position about x=0 assumes that current changes instantly as position changes. But this is not the case. The inductance of the coils will resist any immediate change in current. Also, there is an induced emf due to the movement of the block. Hence the actuating current does not take the above factors into consideration. Hence a voltage command would be a better actuating signal for developing the appropriate control current. The electrical have been considered and integrated into the model as discussed below.

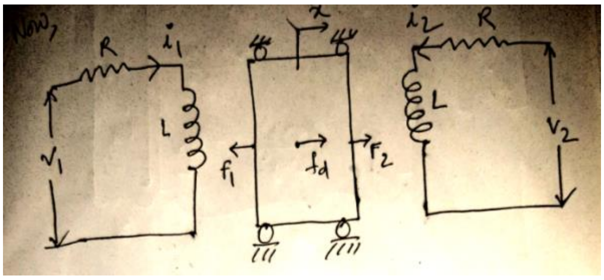


Fig. 6

In Fig.6 for a displacement x: $V1$ and $V2$ be the voltages across respective electromagnets.

Let $V1 = V - V_x$ and $V2 = V + V_x$

The equations of the R-L circuits:

$V1 = V - V_x = R\ i1 + L\frac{di_1}{dt} + ki\ \frac{dx}{dt}$------(2)

$V2 = V + V_x = R\ i2 + L\frac{di_2}{dt} - ki\ \frac{dx}{dt}$-------(3)

Also, $i1 = i0 - ix, i2 = i0 + ix$

On subtracting Eq (2) from Eq (3) we get,

$$V_2 - V_1 = 2v_x = R(i_2 - i_1) + L\left(\frac{di_2}{dt} - \frac{di_1}{dt}\right) - 2k_i\frac{dx}{dt}$$

$\boldsymbol{V_x = R(i_x) + L\left(\frac{di_x}{dt}\right) - k_i\frac{dx}{dt}}$ where R is the resistance of the coils,

L is the initial inductance (at t = 0).

$\therefore \frac{dix}{dt} = -\frac{R}{L}ix + \frac{k_i}{L}\frac{dx}{dt} + V_x$---------------------(4)

STATE SPACE DESCRIPTION of the AMB SYSTEM

The state space vector X consists of 4 variables or state parameters.

X = [x_1 x_2 x_3 i_x] where x_1 = $\boxed{x}$ $(displacement)$,

$$x2 = \dot{x1} = \dot{x}$$

$$\dot{x2} = \ddot{x},$$

$$x3 = \int_0^t x dt,$$

$$\dot{x3} = x$$

$$ix\ (control\ current)$$

From Eq (4) we get $\frac{dix}{dt} = -\frac{R}{L} ix + \frac{k_i}{L}\frac{dx}{dt} + V_x$

<u>Control Equation:</u>

$\dot{X} = AX + B1U1 + B2U2$ where A = $\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{kg}{m} & 0 & 0 & \frac{k_i}{m} \\ 1 & 0 & 0 & 0 \\ 0 & \frac{k_i}{L} & 0 & \frac{-R}{L} \end{bmatrix}$ governs the behavior of the block without any influence of disturbance force or control law.

B1 = $\begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \\ 0 \end{bmatrix}$ U1 = [fd] B2 = $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/L \end{bmatrix}$ U2 = $[V_x]$

Now in a PID Control:

V_x can be expressed as:

$$Vx = KP(0 - x) + KI\left(0 - \int_0^t xdt\right) + KD(0 - \frac{dx}{dt})$$

Now the modified A matrix is: A = $\begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{kg}{m} & 0 & 0 & \frac{k_i}{m} \\ 1 & 0 & 0 & 0 \\ \frac{-KP}{L} & \frac{k_i - KD}{L} & \frac{-KI}{L} & \frac{-R}{L} \end{bmatrix}$

The final state space equation is : $\begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \dot{ix} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \frac{kg}{m} & \mathbf{0} & \mathbf{0} & \frac{k_i}{m} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \frac{-KP}{L} & \frac{k_i-KD}{L} & \frac{-KI}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ ix \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \frac{1}{m} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$ **[fd]**--------(3)

CALCULATIONS

Table of Constants

Sl.No.	Item	Value
1	Permeability of free space(μ0)	1. 1.2 x 10^{-6} H/m
2	Number of coils(N)	600
3	Area of cross section of the block (A)	2.25 x 10^{-4} m^2
4	Air gap(g)	0.5 x 10^{-3} m
5	Bias current(i0)	2A
6	Inductance L = $(\mu0)^2 N^2 A/2g$	0.0972 H
7	ki = $(\mu0)\ N^2 A(i0)/g^2$	777.6 H^2 A/m
8	kg = $-(\mu0)\ N^2 A(i0)^2/g^3$	-3.11 x 10^5 H^2A^2/m^2
9	Mass of the block m	6 kg
PID CONTROLLER CONSTANTS		
10	KP	5000
11	KI	8000
12	KD	1000

The disturbance force is taken as: fd = 100 + 0.5sin(100πt)

Equation 3 is solved in python numerical software Numpy and the state variables are plotted with respect to time.

RESULTS AND REMARKS

Variation of displacement x (in m) with time

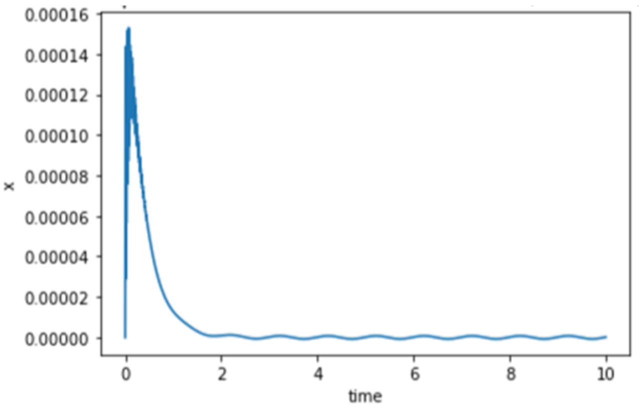


Fig. 7.1

Variation of velocity (in m/s) with time

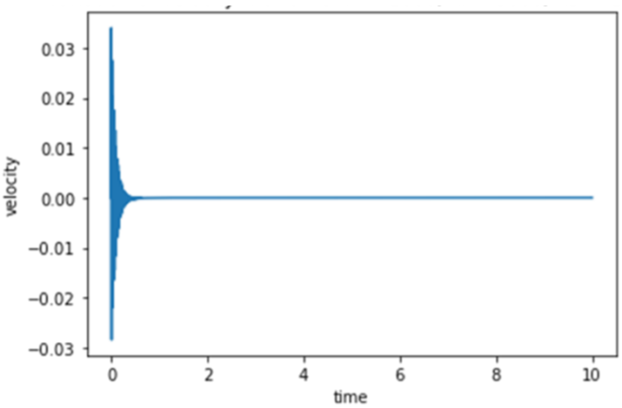


Fig. 7.2

In Fig.7.1 the displacement of the block converges to zero (becomes negligible) after 2 seconds. The maximum displacement reached is 0.16mm. For an air gap of 0.5mm this is a fairly small and acceptable.

In Fig.7.2 the block comes to rest (or negligible speed) within 1 second.

STABILITY OF THE SYSTEM

<u>Eigenvalues of matrix A **before** applying control law</u>

```
Pole


 0.00e+00
-1.06e+03
 1.02e+03
-1.96e+00
```

Matrix A before applying control law has an eigenvalue with positive real part. This signifies that the system is unstable.

<u>Eigenvalues of matrix A **after** applying control law</u>

```
Pole


-1.13e+01 + 4.94e+02i
-1.13e+01 - 4.94e+02i
-1.58e+01
-2.77e+00
```

The real part of all the eigenvalues of matrix A after applying control law are negative which signifies that the system has become stable. However, this is so for a particular set of KP, KI, KD values of the PID control.

RESULTS AND REMARKS

Variation of control current (in A) with time

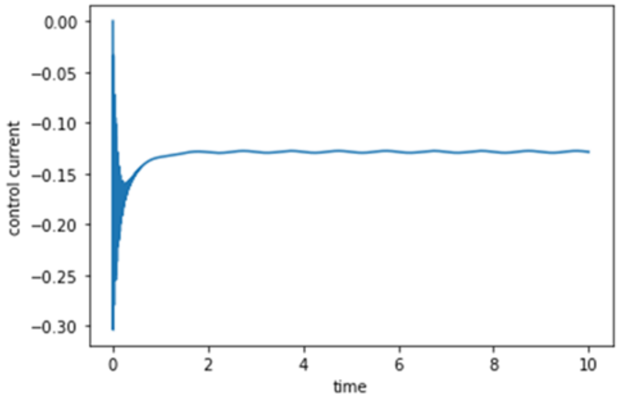


Fig. 8.1

Variation of control current (in A) with time

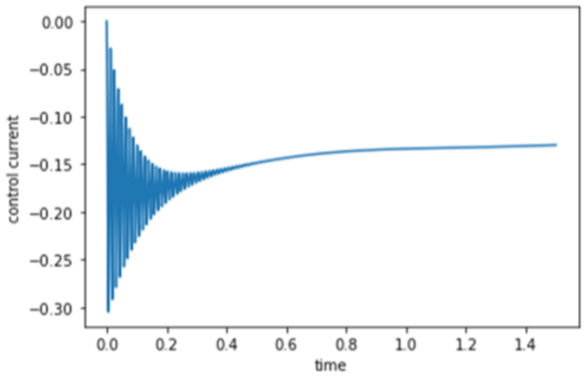


Fig. 8.2

For time t>=0.2seconds:

Control current ix remains below 10%(i0).

For time t<=0.2 seconds, control current ix remains within 10%-15% of the bias current ix.

We can observe that the control current converges and remains within an acceptable range after 0.2 seconds.

REFERENCES

1. Lichuan Li, T. Shinshi and A. Shimokohbe, "Asymptotically exact linearizations for active magnetic bearing actuators in voltage control configuration," in IEEE Transactions on Control Systems Technology, vol. 11, no. 2, pp. 185-195, March 2003, doi: 10.1109/TCST.2003.809241.

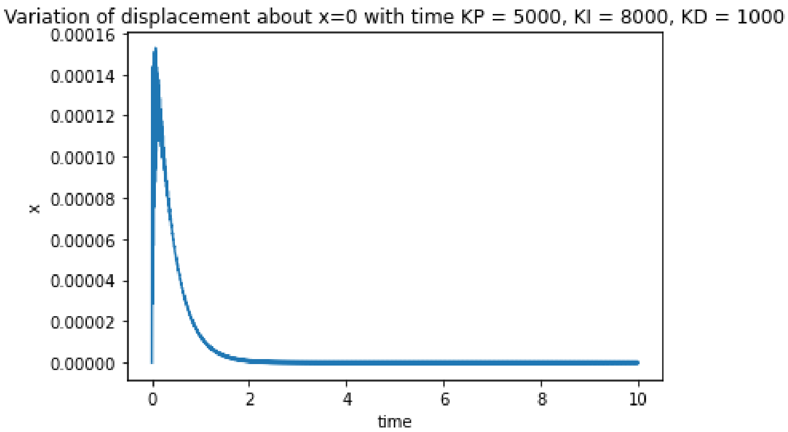
2. Magnetic Bearings-Theory, Design, and Application to Rotating Machinery Gerhard Scheitzer, Eric H. Maslen, Contributors: Hannes Bleuler, Matthew Cole, Patrick Keogh, Ren´e Larsonneur, Eric Maslen, Rainer Nordmann, Yohji Okada, Gerhard Schweitzer, Alfons Traxler

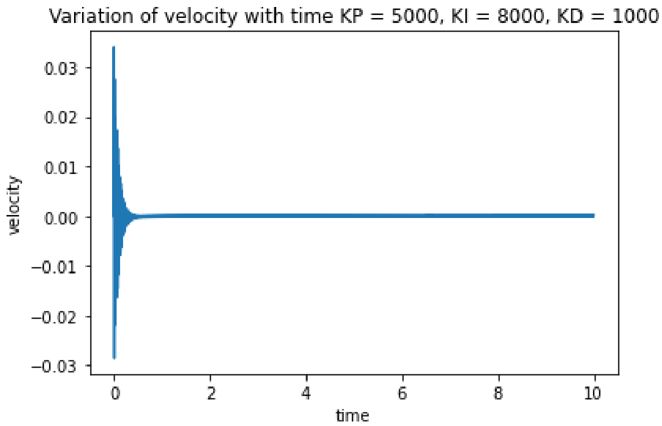
CODE

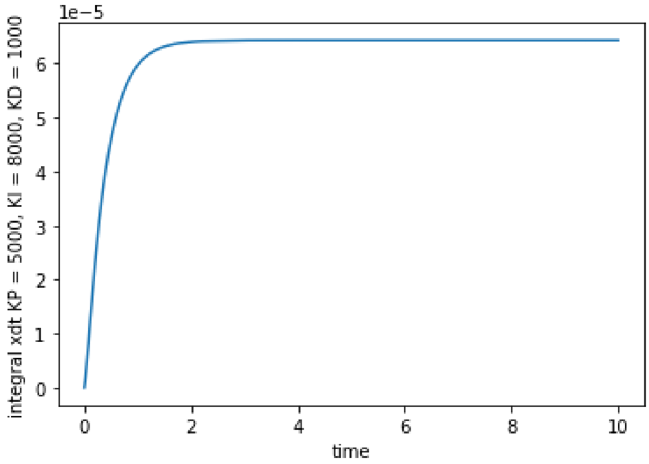
```
import matplotlib.pyplot as plt
import numpy as np
from scipy.integrate import odeint
from numpy import linalg as LA
%matplotlib inline

# Model Constants
Ki = 777.6
Kg = -3.11*100000
R = 4
L = 0.0972
M = 6
# Controller constants
kd = 1000
kp = 5000
ki = 8000

A = np.array([[0,1, 0, 0],
        [-Kg/M,0,0,Ki/M],
        [1,0,0,0],
        [-kp/L, (Ki-kd)/L ,-ki/L, -R/L]])
```

```
# ODE function
def amb_fun(x,t):
    input_force =100 + 0.5*np.sin(100*np.pi*t)

   dxdt = np.array([0,0,0,0])
    B = np.array([0, 1, 0, 0])
    U = np.array([input_force/M])
    dxdt = np.dot(A,x) + B*U
    return dxdt

# Initial Conditions
x0 = np.array([0,0,0,0]) #State vector at t=0
# Time span
t_span = np.linspace(0,10,3000)
# Solving the system of ODE
sol = odeint(amb_fun, x0, t_span)

#Plotting the results
plt.xlabel("time")
plt.ylabel("x")
plt.title("Variation of displacement about x=0 with time KP = 5000, KI = 8000, KD = 1000")
plt.plot(t_span,sol[:,0])
plt.show()
plt.title("Variation of velocity with time KP = 5000, KI = 8000, KD = 1000")
plt.xlabel("time")
plt.ylabel("velocity")
plt.plot(t_span,sol[:,1])
plt.show()
plt.xlabel("time")
```

```
plt.ylabel("integral xdt KP = 5000, KI = 8000, KD = 1000")

plt.plot(t_span,sol[:,2])

plt.show()

plt.title("Variation of control current with time KP = 5000, KI = 8000, KD = 1000 Bias
current(i0) = 2A")

plt.xlabel("time")

plt.ylabel("control current")

plt.plot(t_span,sol[:,3])

plt.show()
```







Variation of control current with time KP = 5000, KI = 8000, KD = 1000 Bias current(i0) = 2A