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General Physics Report

*Moment of Inertia, Angular Velocity & Angular
Acceleration*

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Purpose

The purpose of the experiment is to measure the angular velocity and angular acceleration cause by the torque, and to verify the applicability of the Newton's second law of motion to describe the rotation of a rigid body around a fixed axis. Also to verify law of conservation of energy.

Introduction

Consider figure 1. A fixed object is rotating about an axis situated at a point O perpendicular to the plane of the paper. Let θ_1 be the position of the object at time t_1 and θ_2 is the position of the object at time t_2 , made with x-axis. Therefore the average angular velocity is given as the ratio of the angular displacement of a rigid object in the time interval Δt during which the displacement occurs,

$$\bar{\omega} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

If an object travels a small distance $\Delta\theta$ in very small interval of time Δt then instantaneous angular velocity is given as

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

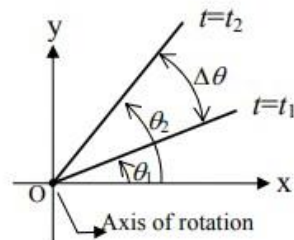


Figure 1. An object rotating about an axis situated at point O

If the angular velocity of an object changes with time, then average angular acceleration is given by,

$$\bar{\alpha} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Therefore, instantaneous angular acceleration is obtained at very small interval of time and it is given as,

$$\alpha = \lim_{\Delta t \rightarrow 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Equation (1) and (2) defines the angular velocity and angular acceleration of an object which not only describe only rotation of a rigid object in fact, it describes the object at each of the turning point. The dimensions for these two physical quantities

is angular velocity (rad/s) or (Rev./s); angular acceleration (rad/s²) or (Rev./s²). If we have many points in the rigid bodies then the kinetic energy of the whole rigid body is sum of kinetic energies of all points and is expressed as,

$$K = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2 + \dots = \sum_i \frac{1}{2}m_i v_i^2$$

Where m_i and v_i are the mass and velocity of an object where $i = 1, 2, 3, \dots$. If a point object is rotating with radius of rotation r_i then the equation (3) is simplified by using formula $v=r\omega$ as,

$$K = \sum \frac{1}{2}m_i(\omega r_i)^2 = \frac{1}{2}(\sum m_i r_i^2)\omega^2$$

We defined a new physical quantity in equation (3) which is known as moment of inertia (I) and it is rewritten as

$$I = \sum m_i r_i^2$$

Where, r_i is the distance of an object from the axis of rotation and m_i is the mass of an object. The unit of moment of inertia in SI system is given as kg.m². If the rigid body consists of many non-discrete points then moment of inertia is given as,

$$I = \int r^2 dm = \int \rho r^2 dV$$

Where ρ is the density of an object and dV is small volume. If an object is rotating around a fixed axis of rotation then the torque of an object is given by Newton's 2nd law of motion and it is expressed as shown in equation (7), where m is mass of an object

$$\tau = F_r = m(\alpha r)r = (mr^2)\alpha = I\alpha$$

From equation (7) it is observed that, the angular acceleration of an object is directly proportional to the torque for a fixed moment of inertia

Materials/methods

The materials that were used for the experiment were disks ($M_{\text{disk}} = 1.5 \text{ kg}$; $M_{\text{Ring}} = 1.42 \text{ kg}$), a box of weights, String, photoelectric Timer, photo gate system and electronic vernier.

The following procedure was followed when carrying out the experiment:

❑ Measure the angular acceleration at fixed torque:

Experimental setup is shown in figure 2. Take a string whose length is longer than the distance between disks to the floor. Tie one end of the string to the mass hanger ($\sim 20\text{g}$) and the other end to the pedestal of the disk. (Attention: When winding string you should avoid overlap of string) Then add mass to the mass hanger, at the same time it will drop due to the gravity

1. Place the photo gate system on the path when the Mass Hanger loading. Attention : Do not hit the photo gate sensor while loading of mass hanger
2. Switch on the power of the photo gate system and set the function to "Function 4". Release the mass and measure the time when it dropping to the location of the photo gate system. And redo it again but changing the winding direction from clockwise to counterclockwise. Then repeat for three times. Also calculate the average value \bar{t} and the average standard deviation. (The instructions of the photogate system you can read experiment 2).
3. According to the distance s and the time \bar{t} , calculate the linear acceleration and get the angular acceleration α of the disk. Where r is the radius of the pedestal. You need to calculate the average standard deviation of α and \bar{t} .
4. Change the weight of the mass and repeat the step 1-4. You need calculate them and record in a list of table.
5. Place the ring on the disk to change the rotor inertia I and repeat 1-4 steps

Results

Experiment 1

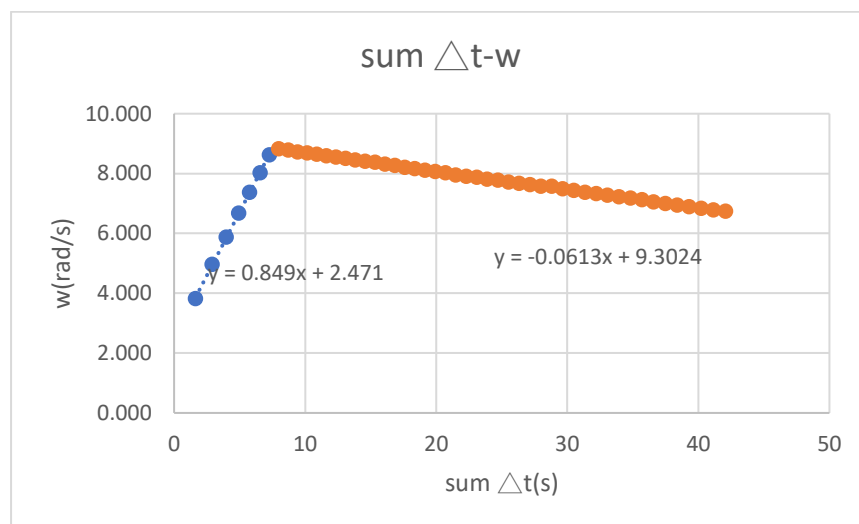
Expl	m(g)	m(kg)	h(cm)	h(m)	t(s)	Taverage	t^2(s^2)	a(m/s^2)	r(mm)	r(m)	R(m)	M(kg)	Iexp(kg*m^2)	Itho(kg*m^2)	Error(%)
1	53.31	0.05331	65.3	0.653	9.795				15.36	0.01536	0.114	1.378	0.009341	0.008954	4.32%
2	53.31	0.05331	65.3	0.653	9.847	9.955	99.109	0.0132							
3	53.31	0.05331	65.3	0.653	10.224										
1	102.39	0.10239	65.3	0.653	7.299										
2	102.39	0.10239	65.3	0.653	7.071	7.170	51.414	0.0254	15.36	0.01536	0.114	1.378	0.009084	0.008954	1.45%
3	102.39	0.10239	65.3	0.653	7.141										
1	152.37	0.15237	65.3	0.653	5.756										
2	152.37	0.15237	65.3	0.653	5.27	5.648	31.904	0.0409	15.36	0.01536	0.114	1.378	0.008570	0.008954	4.29%
3	152.37	0.15237	65.3	0.653	5.919										

Experiment 2

Mass : 0.5331kg

Exp2	$\Delta t(s)$	sum $\Delta t(s)$	$w(\text{rad/s})$
1	1.644	1.644	3.822
2	1.265	2.909	4.967
3	1.07	3.979	5.872
4	0.941	4.92	6.677
5	0.852	5.772	7.375
6	0.783	6.555	8.025
7	0.728	7.283	8.631
8	0.712	7.995	8.825
9	0.715	8.71	8.788
10	0.72	9.43	8.727
11	0.723	10.153	8.690
12	0.727	10.88	8.643
13	0.731	11.611	8.595
14	0.735	12.346	8.549
15	0.738	13.084	8.514
16	0.743	13.827	8.457
17	0.747	14.574	8.411
18	0.75	15.324	8.378
19	0.756	16.08	8.311
20	0.759	16.839	8.278
21	0.765	17.604	8.213
22	0.769	18.373	8.171
23	0.774	19.147	8.118
24	0.779	19.926	8.066
25	0.783	20.709	8.025
26	0.79	21.499	7.953
27	0.794	22.293	7.913
28	0.798	23.091	7.874
29	0.804	23.895	7.815
30	0.808	24.703	7.776
31	0.814	25.517	7.719
32	0.819	26.336	7.672
33	0.823	27.159	7.634
34	0.829	27.988	7.579
35	0.829	28.817	7.579
36	0.839	29.656	7.489
37	0.845	30.501	7.436
38	0.852	31.353	7.375
39	0.857	32.21	7.332
40	0.863	33.073	7.281
41	0.87	33.943	7.222
42	0.875	34.818	7.181
43	0.882	35.7	7.124
44	0.891	36.591	7.052
45	0.897	37.488	7.005
46	0.905	38.393	6.943
47	0.912	39.305	6.889
48	0.918	40.223	6.844
49	0.926	41.149	6.785
50	0.932	42.081	6.742

h(cm)	α 1(rad/s)	0.849
75	α 2(rad/s)	-0.0613
h(m)	wmax(rad/s)	8.842
0.75		
	Iexp	0.009439
	Itho	0.008954
	error(%)	5%
	Iexp	0.01017
	Itho	0.00895
	error(%)	14%



Above is a graph of sum against angular velocity

Experiment 3

exp3				
E potential	0.391829		(without f)error(%)	5.57%
I(without friction)	0.009439			
I(with friction)	0.010174		(with f)error(%)	6.14%
v(rad/s)	0.135819			
Zf(N*m)	-0.00062			
θ (rad)	48.82813			
E RHS(without f)	0.370001			
E RHS(with f)	0.367775			

Discussion

Experiment 1 delved into measuring angular acceleration under fixed torque conditions, yielding results with relatively small percentage errors. However, it's crucial to acknowledge potential sources of imprecision, such as air resistance and variations in experimental setups. Despite these challenges, the consistency in observed trends suggests a reasonable alignment with theoretical expectations.

Experiment 2, exploring angular acceleration resulting from changes in angular velocity, revealed notable errors. Potential contributors to these discrepancies include the influence of air resistance and limitations in the experimental setup. Refining the apparatus and accounting for additional environmental factors in future experiments could enhance the accuracy of the findings.

Experiment 3, investigating the law of conservation of energy, provided insightful results despite encountering friction-related challenges. The observed differences

between kinetic and potential energy underscore the impact of friction on the overall energy balance. A more comprehensive analysis of frictional effects and their quantification could deepen our understanding of energy transformations in rotational systems.

In summary, while the experiments showcased fundamental principles governing rotational motion, ongoing refinement and consideration of external factors are necessary for a more accurate representation of these intricate physical phenomena. The exploration of potential improvements in experimental design and acknowledgment of influencing factors will contribute to the continued advancement of our understanding of rotational dynamics.

Conclusion

In conclusion, this experiment provided valuable insights into angular acceleration, torque, and the conservation of energy in rotational motion. The results, while showing some discrepancies, align with theoretical expectations. The observed errors, though present, can be attributed to factors such as experimental limitations, environmental conditions, and potential inaccuracies in measurements.

To improve the experiment's precision, a larger space and enhanced apparatus could be considered in future iterations. Addressing these factors may contribute to a reduction in percentage errors and a more accurate representation of the physical principles involved.

Moreover, the findings support the application of Newton's second law of motion to rotational dynamics, confirming the relationship between torque and angular acceleration. The exploration of the law of conservation of energy, despite encountering friction-related challenges, sheds light on the complex interplay between kinetic and potential energy in rotating systems.

Reference

- Angular velocity and centripetal acceleration relationship : Kaps, T Splith, F Stallmach. (2021) Implementation of smartphone-based experimental exercises for physics courses at universities. Physics Education 56:3, 035004. Online publication date: 11-Feb-2021
- <https://study.com/learn/lesson/friction-function-types-motion.html>
- <https://physics.stackexchange.com/questions/371707/how-does-friction-increase-energy-of-a-system>
- Barker, B. M., & O'Connell, R. F. 1975, Phys. Rev. D, 12, 329

- Cottam, J., Paerels, F., & Mendez, M. 2002, *Nature*, 420, 50