Indian Institute Of Technology
MSc Economics Dept.
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MACROECONOMICS Assignment
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Replicating "The Contributions to the Empirics of Economic Growth by Mankiw Romer Weil , 1992"

Basic Theory of the Standard Solow model

The model starts with a Cobb-Douglas production function:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

The evolution of capital is determined by the following equation:

$$\dot{k(t)} = sk(t)^{\alpha} - (n+g+\delta)k(t)$$

From this we can get the steady state level of $k(t)$ by equating $\dot{k(t)}$ to 0. We then get a simple formula to determine the steady state $k(t)^*$

$$k(t)^* = \left(\frac{s}{n+g+\delta}\right)^{1/(1-\alpha)}$$

We can take logs of the production function and substitute in $\dot{k(t)}$ to find the equation we want to estimate, which is the steady state level of output. We also add an initial $A(0)$ which we take as equal to $a+\epsilon$ where a is a constant and ϵ is a country specific shock

$$log(Y/L) = a + \frac{\alpha}{(1-\alpha)}log(s) - \frac{\alpha}{(1-\alpha)}log(n+g+\delta) + \epsilon$$

where: a represents the constant in the assumed $log(A)$ equation
ϵ represents the country shock for $log(a)$
α represents the share of capital
s represents the exogenous savings rate (measured with average share of investment in GDP)
n represents the exogenous population growth rate
g represents the exogenous tfp growth rate
δ represents the depreciation rate of capital

MAKING TABLE 1 FOR 1990-2018 FROM PENN WORLD TABLE

REGRESSION RESULTS

Unrestricted Regressions

	Non-Oil	Intermediate	OECD
const	-1221.92***	-928.66***	172.04
	(161.03)	(176.22)	(178.56)
ln(s)	0.48**	0.42*	0.41**
	(0.23)	(0.25)	(0.16)
ln(n+g+d)	-412.73***	-314.68***	52.82
	(53.83)	(58.93)	(59.69)
R-squared	0.49	0.36	0.31
R-squared Adj.	0.47	0.34	0.24
R^2	0.4697	0.3386	0.2351
N	68.0000	55.0000	21.0000
s.e.e.	0.8050	0.7029	0.2115
Implied	0.32	0.30	0.29

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Restricted Regressions

	Non-Oil	Intermediate	OECD
const	11.50***	11.48***	12.74***
	(0.94)	(0.88)	(0.45)
ln(s)-ln(n+g+d)	0.38	0.27	0.43**
	(0.32)	(0.30)	(0.16)
R-squared	0.02	0.01	0.28
R-squared Adj.	0.01	-0.00	0.24
R^2	0.0063	-0.0041	0.2433
N	68.0000	55.0000	21.0000
s.e.e.	1.1019	0.8661	0.2103
Implied	0.28	0.21	0.30

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Here, (g+δ) = 0.05 as assumed in the paper, for the purpose of regression.
As the result suggests let us consider, $\frac{\alpha}{(1-\alpha)} = 0.38$, Solving yields,$\alpha \cong 0.27 \neq \frac{1}{3}$
$So, the\ implied\ \alpha$ level is quite closer but it isn't consistent with the empirical value of capital share, according to which the capital share should've been around 1/3.

The value of R^2 in the Un-Restricted model is 0.49 which is quite good , considering the original result obtained by Mankiw-Romer-Weil in the paper and without incorporating Human capital in the model

Now, we have to consider the Augmented Solow model by introducing the effect of Human Capital in the Model.

AUGMENTED SOLOW MODEL

Considering the impact of Human Capital, the Cobb-Douglas production function becomes:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}$$

The change in the stock of physical human capital can be defined as:

$$\dot{k(t)} = s_k y(t) - (n+g+\delta)k(t)$$
$$\dot{h(t)} = s_h y(t) - (n+g+\delta)h(t)$$

The steady state values for k and h can be calculated by setting $\dot{k(t)}$ and $\dot{h(t)}$ to 0. Using this we can derive the equation:

$$log(Y(t)/L(t)) = log(A(0)) + gt + \frac{\alpha}{1-\alpha-\beta} log(s_k) - \frac{\alpha+\beta}{1-\alpha-\beta} log(n+g+\delta) + \frac{\beta}{1-\alpha-\beta} log(s_h)$$

REGRESSING THE PER-CA-PITA INCOME WITH THE LOG VARIABLES AS MENTIONED ,YIELDS THE FOLLOWING TABLES:-

MAKING TABLE 2

REGRESSION RESULTS

Unrestricted Regressions

	Non-Oil	Intermediate	OECD
const	-393.31***	-404.47***	110.69
	(141.56)	(125.39)	(117.01)
ln(s)	0.38**	0.47***	0.67***
	(0.16)	(0.16)	(0.12)
ln(n+g+d)	-134.81***	-138.58***	32.40
	(47.39)	(41.97)	(39.11)
ln(school)	2.62***	2.99***	1.48***
	(0.29)	(0.33)	(0.29)
R-squared	0.77	0.75	0.72
R-squared Adj.	0.76	0.74	0.68
R^2	0.7628	0.7380	0.6751
N	68.0000	55.0000	21.0000
s.e.e.	0.5383	0.4424	0.1378
Implied	0.09	0.10	0.21
Implied	0.66	0.67	0.47

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Restricted Regressions

	Non-Oil	Intermediate	OECD
const	-1.05	-2.21	7.19***
	(1.04)	(1.34)	(1.11)
ln(s)-ln(n+g+d)	0.34**	0.42**	0.69***
	(0.16)	(0.17)	(0.11)
ln(school)-ln(n+g+d)	3.15***	3.50***	1.50***
	(0.23)	(0.32)	(0.29)
R-squared	0.75	0.70	0.71
R-squared Adj.	0.74	0.69	0.68
R^2	0.7385	0.6912	0.679
N	68.00	55.00	21.0
s.e.e.	0.5653	0.4803	0.137
Implied	0.08	0.09	0.22
Implied	0.70	0.71	0.47

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

It is clearly evident that by introducing the Human Capital in our model the value of R^2 has increased by around 50 percent.The α and β implied from the regressions are much closer to what we would expect.The α and β implied from the regressions are much closer to what we would expect. So adding human capital seems to have improved the Solow model's performance. R^2 also seems to be much higher.

TEST FOR CONVERGENCE

SOLOW RATE OF CONVERGENCE

The basic equation of Convergence is given as:

$$\ln(y(t)) - \ln(y(0)) = (1-e^{-\lambda t})\left(\frac{\alpha}{1-\alpha-\beta}\right)\ln(s_k) + (1-e^{-\lambda t})\left(\frac{\beta}{1-\alpha-\beta}\right)\ln(s_h) - (1-e^{-\lambda t})\left(\frac{\alpha+\beta}{1-\alpha+\beta}\right)\ln(n+g+\delta) - (1-e^{-\lambda t})\ln(y(0)) \tag{1}$$

First we will go through a much more basic regression:

$$\ln(y(t)) - \ln(y(0)) = \beta_0 + \beta_1 \ln(y(0)) + \epsilon \tag{2}$$

REGRESSION RESULTS

Tests for Unconditional Convergence

	Non-Oil	Intermediate	OECD
const	-0.43013	-0.46833	-4.49062
	(0.45464)	(0.67589)	(3.33935)
ly18	0.10154**	0.10317	0.44409
	(0.04351)	(0.06290)	(0.28981)
R-squared	0.07624	0.04830	0.10999
R-squared Adj.	0.06224	0.03034	0.06315
R^2	0.0622	0.0303	0.0631
N	68.0000	55.0000	21.0000
s.e.e.	0.3937	0.3995	0.3134
Implied	-0.00387	-0.00393	-0.01470

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Tests for Conditional Convergence

	Non-Oil	Intermediate	OECD
const	-124.22978	-147.73274	108.07097
	(106.98651)	(112.59975)	(161.57487)
ly90	-0.08962	-0.15903**	-0.69602***
	(0.06148)	(0.07063)	(0.12827)
ln(s)	0.33559***	0.41837***	0.43607***
	(0.11186)	(0.12904)	(0.14433)
ln(n+g+d)	-42.67624	-50.92778	32.50030
	(35.91151)	(37.78338)	(53.93434)
R-squared	0.14050	0.23530	0.71141
R-squared Adj.	0.10021	0.19031	0.66048
R^2	0.1002	0.1903	0.6605
N	68.0000	55.0000	21.000
s.e.e.	0.3856	0.3651	0.1887
Implied	0.00376	0.00693	0.04763

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Tests for Conditional Convergence

	Non-Oil	Intermediate	OECD
const	-70.82611	-138.98108	88.96801
	(102.01263)	(99.77411)	(115.15009)
ly90	-0.27073***	-0.40233***	-0.86294***
	(0.08243)	(0.08868)	(0.09967)
ln(s)	0.33147***	0.43994***	0.65264***
	(0.10511)	(0.11444)	(0.11506)
ln(n+g+d)	-25.13920	-48.39472	25.61343
	(34.22073)	(33.47747)	(38.44258)
ln(school)	0.85992***	1.33763***	1.30439***
	(0.27918)	(0.34551)	(0.31160)
R-squared	0.25299	0.41166	0.86226
R-squared Adj.	0.20556	0.36459	0.82783
R^2	0.2056	0.3646	0.8278
N	68.0000	55.0000	21.0000
s.e.e.	0.3623	0.3234	0.1344
Implied	0.01263	0.02059	0.07949

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

Rate of Convergence - Restricted

	Non-Oil	Intermediate	OECD
const	0.48403	-0.04364	6.31858***
	(0.68100)	(0.95179)	(1.22090)
ly90	-0.25022***	-0.35388***	-0.85349***
	(0.07673)	(0.08232)	(0.09738)
ln(s)-ln(n+g+d)	0.32420***	0.42535***	0.66730***
	(0.10418)	(0.11501)	(0.11161)
ln(school)-ln(n+g+d)	0.89394***	1.35208***	1.31186***
	(0.27381)	(0.34852)	(0.30695)
R-squared	0.24720	0.38884	0.85783
R-squared Adj.	0.21191	0.35289	0.83274
R^2	0.2119	0.3529	0.8327
N	68.0000	55.0000	21.0000
s.e.e.	0.3609	0.3264	0.1324
Implied	0.01152	0.01747	0.07683

Standard errors in parentheses.
* p<.1, ** p<.05, ***p<.01

REPLICATING PLOTS