Laboratory of Robust Identification and Control

Lecture notes

A. Ayanmanesh Motlaghmofrad

A.Y. 2024/2025

Contents

Chapter 1

Introduction

Disturbances and uncertainties

The key word *Robust* suggests that we are taking uncertainties into account. The model of the plant is as following:

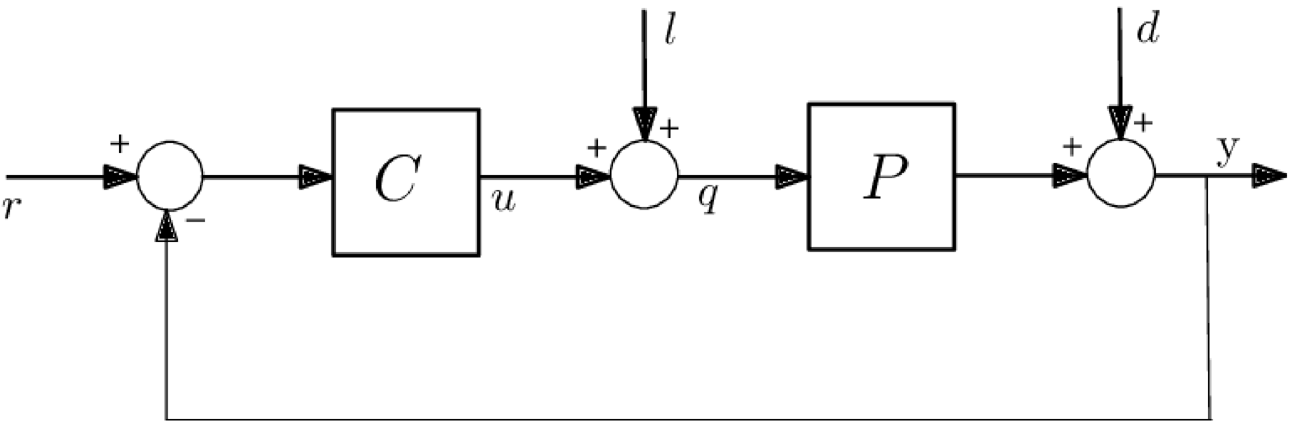


Figure 1.1: A general control plant with additive disturbances l and d

The general nonlinear state-space representation of the system is:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t), u(t)) \end{cases} \tag{1.1}$$

Where:

- $x(t)$ is the state vector,
- $u(t)$ is the input vector,
- $y(t)$ is the output vector,
- $f(\cdot)$ is a nonlinear function describing the system dynamics,
- $g(\cdot)$ is a nonlinear function describing the output equation.

The first step of any control problem is typically **derivation of mathematical model of the plant.** This step is the most crucial step, because if we drive the model by applying first principles of physics, we are likely to adopt approximated models, adopting simplifying assumptions, e.g. rigid body assumption etc. Further, the value of the physical parameters involved in the equations, such as friction coefficient, are not exactly known. Such approximations introduce errors and uncertainty in the mathematical description of the plant to be studied and controlled.

This fact is critical, since standard approaches to controller design are model-based; that is, the controller design has a strong dependency on the mathematical models used to describe the plant to be studied and controlled.

Neglecting some physical details* $\equiv$ *Neglecting some state variables

For example, for modelling a robotic arm, generally, rigidity is assumed for the joints. Nevertheless, in fast movements this assumption does not hold anymore, and the model does not predict the real performance of the robot, neglecting some state variables. Further, in some applications, we are not even aware of the phenomenon or phenomena that is being neglected.

Counter act for disturbances and uncertainties

- **First counter act** These uncertainties and disturbances directly affect the controller design in the time domain, since the feedback gain and observer are directly calculated by solving algebraic equations including physical parameters with uncertainties. Nontheless, In frequency domain, the design of the controller is less affected by these uncertainties.

  This does not mean that *Transfer Function* is not affected by uncertainties of the parameters, because not considering some phenomona leads to the transfer function having less poles or zeros and because the uncertainty of the parameters affect the coefficients of the complex variables, being s or z. In the frequency domain design, we design the controller based on the frequency response of the system, considering cutting frequencies that reject high-frequency and low-frequency disturbances. In addition, by considering phase-margin and gain-margin, some margin for disturbances and uncertainties are taken into accoutn.

  Question to be answered: the sensitivity function is high-pass filter, meaning that the high frequency phenomenon, on the other-hand T is a low-pass filter, if the plant is designed for having a fast rising time, high-frequency phenomena also passes T.

- **Second counter act**: Optimization problems in state-space where introduced to tackle this problem.

- **Third counter act**: Optimization problem, in state space, is combined with the concept of robustness, in frequency space, which is called H_∞.

What is going to be discussed in this course

In the first part of this course, we are going to learn how to learn from the mapping from the input to the output of the system, extracting a **mathematical model** for the plant. Further, it elaborates on this data to drive also a **discription for uncertainty**. These, together, can be used for **robust controller design** in a model-based approach.

In the second step, the aim is to design a controller directly from the data, without the intervention of the model.

Professor's Quote

In conclusion, even if the physical model is very precise, at some point, in order to measure the parameters used in the physical models, it is required to do some experimental measurements, subjected to noise, introducing uncertainties to our model.

If these uncertainties are not taken into account, the controller will not have a good performance when implemented physically, and it is going to work only in simulations.

There are many approaches to tackle this problem. Here, in the first part of the course, we will focus on **System Identification**, which is another modelling paradigm. In this paradigm, we learn the mathematical model of the system to be controlled by using experimentally collected data. Not only do we introduce what *System identification* is and what are the possible approached, but maintly to focus on ***Set-membership Identification Technique*** that allow us to learn the model of the system, and drive information about how the uncertainty is affecting out model; this is used to design a controller in a robust manner. Robust controller designed can be directly applied to the real physical plants.

In the second part of the course, we shift our paradigm again. In this part, assuming that the collected data represents the behavior of the mapping between the input and output, we try to design the controller directly from the data, called **Direct Data-Driver Controller Design**.

Chapter 2

System Identification

What system identificaiton is about?

System identification deals with the problem of building mathematical model of dynamical systems from sets of experimentally collected input, output data.

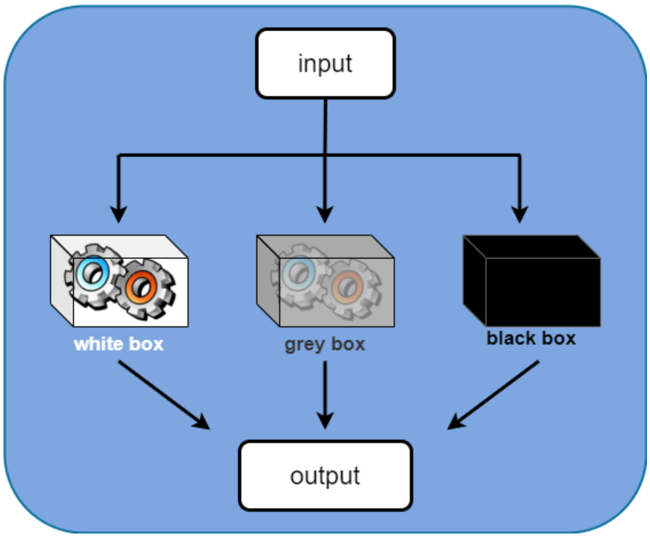


Figure 2.1: Classifications of System Identification Problems

There are three different approaches to mathematically model dynamical systems:

1. **White-box modelling**:
   Models, in this approach, are obtained by applying **first principles for physics**. All the physical phenomena and also all the physical parameters involved in the equiation are assumed to be exactly know. This approach was applied in the course of *Automatic Control* for modelling systems.

2. **Grey-box modelling**: Here, **models** are base on the equations obtained by applying first principles of **physics**, but **the parameters** entering the equations are not completely known, so they need **to be estimated** from experimental data.

3. **Black-box modelling**: In this case, the structure of the equations is selected by the user on the basis of some **"general" a-priori information**, e.g. linearity, on the system physics, or at any rate by the system properties.

Professor's Quote

In this method, the designer of the system model, has some degree of freedom in selecteing the structure of the model, provided that the model embeds the "general" a-priori information.

If we do not have any a-priori information, an artificial-neural-network may be selected as the structure of the model.

the parameters involved in the equations of the black-box models are **to be estimated**, or computed, by using experimentally collected data. In general, **the parameters of a black-box model do not have a clear physical meaning**.

In general, **white-box models are not very useful in practice**, at least for control applications, because they are based on the assumption that the physics involved in the system under study is well-known.

The comparison between the grey-box and Black-box models:

Similarity In both cases:

- physical insight is exploited to drive/select the structure of the equations.
- experimentally collected data is used to estimate/compute the parameters involved in the equations.

Difference

- In grey-box modelling, the structure of the equations is not selected by the user, since it is forced by the first principles of physics. This suggests that, in general, the equations of a grey-box model depends, in a possibly complex, non-linear ways, on the physical parameters to-be-estimated.

  for instance of a grey-box model: $\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$ where:

  $$A = \begin{bmatrix} \dfrac{m}{k^2} & \sqrt{\beta} \\ \dfrac{\alpha^2 . k^3}{\gamma} & 1 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}, \quad D = \begin{bmatrix} d \end{bmatrix}$$

  In this case, using the physical principles for modelling seems like a good idea; the system is simple, and its physics is well-known. The problem is that the parameters are to-be-estimated. This problem leads to an optimization problem. Now, A difficulty may arise, because if the equation we are going to write so that they relates input and output data obtained experimentally depends on parameters in a complex and non-linear fashion, the mathematical problem of driving what are the correct value of the system parameter satisfying the relation between the intput and output is going to be a complex problem. **This is the main limitation of grey-box model.**

- In the black-box version of the same problem, just some general information such as liniearity and time-invariantion is exploited. Now, we found the four matricies *A, B, C, D* in a form that is much simpller, just 4 parameters for A, and so on, for instance. In this case, the system output depends on the input in a way simpler manner.

  $$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 \end{bmatrix}, \quad D = \begin{bmatrix} d \end{bmatrix}$$

Professor's Quote

Therefore, in black-box model, we have more freedom to select the struture of the equations in a way that is computationally more convenient, by embedding/exploiting some general properties drived from our physical insight. The idea is to consider only the most important a-priori information and/or the information that we trust the most!

General procedure for building a grey/black-box modelling:

Let's compare grey-box and black-box models from the point of view of the parameter estimation problem. While building the mathematical model of the system, in general, we follow a procedure similar to the following one, it be a grey-box or black-box structure:

1. to exploint availabe a-priori information on the system under study to select **the structure of the mathematical equation** describing the input-output mapping.

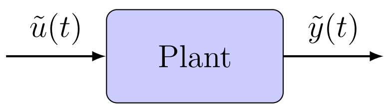
   Professor's Quote

   The model is always in the input/output form. Signals that we can apply to the system is called input, and signals that we can measure, is called output. If we are in the case where we can measure all **state variables** involved in the system, we can have a full description of the system. However, in a general case, the system involves input, output, and state variables, but we are able to measure only a subset of physical variables in the system - being the value to be monitored or the control output.
   Now, these parameters have physical meaning in grey-box case, or are merely mathematical parameters in black-bock case.

   In the end of this step, we have a mathematical model of the following form:

   $$y(t) = f\left(u(t), \theta\right) \tag{2.1}$$

2. To collect input-output data representing the behavior of the system under study by performing an experiment. $\tilde{u}$ and $\tilde{y}$ are noise-corrupted data, since, in general, noise can corrupt the output measurements as well as measurement of the input signal applied to the system.

3. To formulate a suitable mathematical problem to estimate/compute the values of the vector of parameters θ in such a way that our mathematical model is going to describe the behavior of real system as well as possible. for example:

   $$\hat{\theta} = \arg\min_{\theta \in S} f(\theta) = \arg\min_{\theta} \|\tilde{y} - f(\tilde{u}, \theta)\|_2$$

   J(.) here is a **cost function**. In this case, it is the euclidian norm of $\tilde{y}$ and $f(\tilde{u}, \theta)$.

At the third stage, the difference between grey-box and black-box models comes into play, since:

- In grey-box model: $f(\tilde{u}, \theta)$ will, in general depends in a complex, non-linear manner from θ, **leading to multi-minima convex cost function which is really hard to be minimize**

  While

- In black-box model: $f(\tilde{u}, \theta)$ will be selected by the user in order to depend linearly from θ,if possible, or in the simplest possible way, **leading to a single-minima convex funciton to be minimized**, which is much easier in comparison.

For example, consider the following multi-minima cost-function:

$$J(\theta_1, \theta_2) = \sin(\theta_1) \cdot \cos(\theta_2) + \frac{\theta_1^2 + \theta_2^2}{10}$$

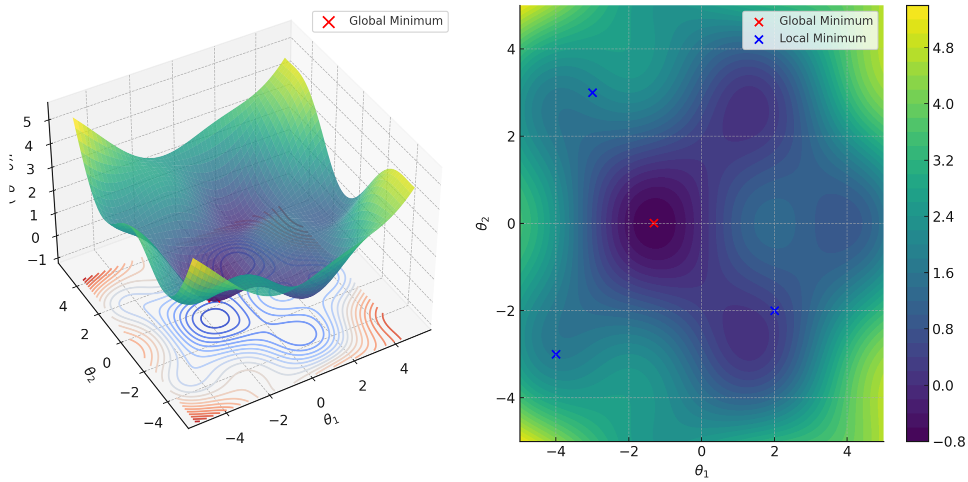


Figure 2.2: The 3D plot and countor plot of the afforementioned cost-function, which is a multi-minima cost-function

In this example, we look for the global minimum of the cost-function. Finding this point, however, is not an easy task, due to the fact that there is no guarantee that our optimization algorithm will not be trapped in one of the local minima, which may correspond to a "very bad" estimation of θ. Even if we find the global minimum by chance, there is no guarantee that output is indeed the global minima.

On the other hand, When a black-box model is considered, a parametrization is considered so that the function $f(u, \theta)$ is a convex function of θ, thereby having a single global minimum. In this case, no matter what optimization algorithm is used, reaching the global minimum is guaranteed.This is evident in the following cost-function:

$$J(\theta_1, \theta_2) = \theta_1^2 + \theta_2^2$$

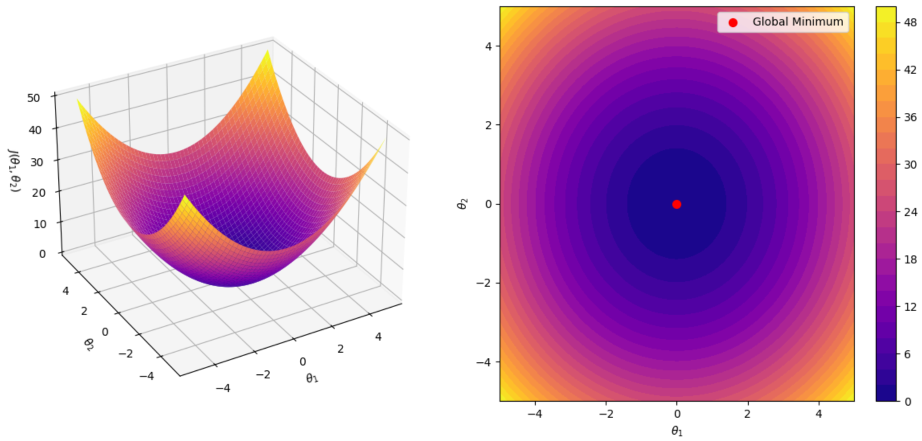


Figure 2.3: The 3D plot and countor plot of the afforementioned cost-function, which is a convex cost-function

Q and A

Question: we use grey-box model when we have an insight about the system and parameters. Hence, when we run the optimization problem, we can initialize the optimization problem with more suitable initial condition. Further, we can neglect some of the estimation, knowing that the estimated value does not correspond to the physical quantity the parameter is expected to have.

Professor's answer: This is true for simple systems. Nonetheless, in such compolex systems as chemical processes or optical lazer systems, it's hard to have an insight before hand about some parameters. In addition, since we are dealling with multi-dimensional problems, it might be the case that we confine the expected value for some parameters but still, for some parameters, we need to deal with the issue mentioned about the multi-minima cost functions. However, it might be the case that the estimation of those physical parameters are of interest, which is not the concern of this course. Here, we do system identification for control purposees.

To clarify the matter further, consider an ***LTI system*** such that the **transfer function** obtained from $H(s) = C(sI - A)^{-1}B + D$ where matrices A, B, C, D are teh state-space matrices obtained by applying the first principles of physics.

$$H(s) = \frac{\frac{P_1}{P_2}s + \frac{P_3}{\sqrt{P_4}}}{s^2 + \frac{P_1.P_2}{P_3^3} + 1}$$

Where P_1, P_2, P_3, P_4 are the physical parameters. Now, in terms of modelling the input-output behavior of the system, we don't miss anything by considering the following transfer function.

$$H(s) = \frac{\theta_1 s + \theta_2}{s^2 + \theta_3 s + \theta_4}$$

In this black-box model, we just take into consideration the "general" information that the system is an LTI system - hence being able to be modelled as a transfer function - and of order two. Therefore, we can use a transfer function model for describing the system behavior.

"Phylosophical Remark"

Pay attention that **we cannot build our model without any assumption about the system**. No matter how much input-output sample is available, we need to have an assuption about the system to be able to descibe it.

In conclusion: **black-box models** are thet best choice for the following purposes:

- Simulalting the input-output behavior of the system
- Modelling for the purpose of designing a controller

Grey-box models are the best choice when it is desired to:

- Simulalting the input output behavior of the system
- Modelling for the purpose of designing a controller