人工智能基础 HW8

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14.12

Two astronomers in different parts of the world make measurements M_1 and M_2 of the number of stars N in some small region of the sky, using their telescopes. Normally, there is a small possibility e of error by up to one star in each direction. Each telescope can also (with a much smaller probability f) be badly out of focus (events F_1 and F_2), in which case the scientist will undercount by three or more stars (or if N is less than 3, fail to detect any star at all). Consider the three networks shown in Figure 14.22

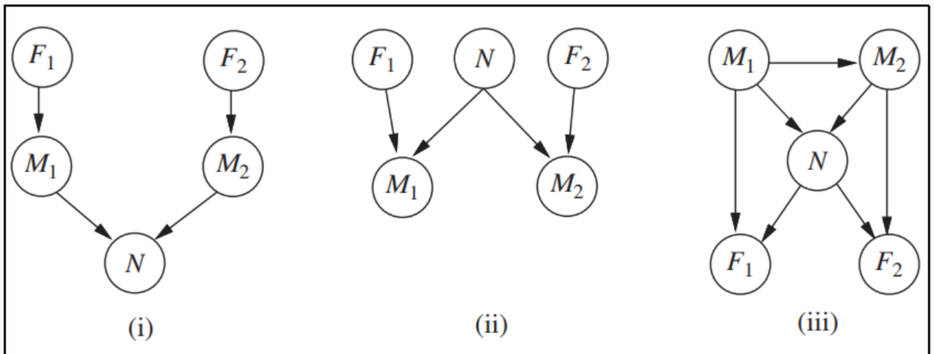

Figure 14.22 Three possible networks for the telescope problem.

a.

Which of these Bayesian networks are correct (but not necessarily efficient) representations of the preceding information?

(ii) 是最佳的, 因为 M_1 确实由星星数量 N 和 F_1 所影响, 而 M_2 确实由星星数量 N 和 F_2 所影响. (i) 显然是有问题的, 当给定 M_1 和 M_2, 不可能 F_1 和 F_2 就对 N 无影响. (iii) 是按照 M_1, M_2, N, F_1, F_2 的顺序排出的, 没有问题.

b.

Which is the best network? Explain.

(ii) 是最佳的. 它结构相对简单, 且准确描述了问题.

c.

Write out a conditional distribution for $P(M_1|N)$, for the case where $N \in 1, 2, 3$, and $M_1 \in 0, 1, 2, 3, 4$. Each entry in the conditional distribution should be expressed as a function of the parameters e and/or f.

依题意知:

$$P(M_1|N) = P(M_1|N, F_1)P(F_1) + P(M_1|N, \neg F_1)P(\neg F_1)$$

假设少数和多数一颗概率是 e, 不多不少是 $1-2e$. 那么可以列表如下:

M_1 \ N	1	2	3
0	$f + e(1-f)$	f	f
1	$(1-2e)(1-f)$	$e(1-f)$	0
2	$e(1-f)$	$(1-2e)(1-f)$	$e(1-f)$
3	0	$e(1-f)$	$(1-2e)(1-f)$
4	0	0	$e(1-f)$

d.

Suppose $M_1 = 1$ and $M_2 = 3$. What are the possible numbers of stars if you assume no prior constraint on the values of N ?

可能因失焦少了 3 颗, 或因数错多了 ± 1颗. 因此由 $M_1 = 1$ 知道 $N \in \{0, 1, 2, 4, 5, \cdots\}$; 由 $M_2 = 3$ 知道 $N \in \{2, 3, 4, 6, 7, \cdots\}$. N 的可能值必然是二者交集. 故 N 可能为 $\{2, 4\} \cup \{6, 7, 8, 9, \cdots\}$.

e.

What is the most *likely* number of stars, given these observations? Explain how to compute this, or if it is not possible to compute, explain what additional information is needed and how it would affect the result.

假设 $p_2 = P(N = 2)$, $p_4 = P(N = 4)$, $p = P(N >= 6)$, 那么

- $P(N = 2|M_1 = 1, M_2 = 3) = \frac{P(N=2,M_1=1,M_2=3)}{P(M_1=1,M_2=3)} = \frac{p_2 e(1-f)e(1-f)}{P(M_1=1,M_2=3)} = \frac{P(N=2,M_1=1,M_2=3)}{P(M_1=1,M_2=3)} = \frac{p_2 e^2(1-f)^2}{P(M_1=1,M_2=3)}$
- $P(N = 4|M_1 = 1, M_2 = 3) = \frac{P(N=4,M_1=1,M_2=3)}{P(M_1=1,M_2=3)} = \frac{p_4(1-2e)fe(1-f)}{P(M_1=1,M_2=3)} = \frac{p_4 e(1-2e)f(1-f)}{P(M_1=1,M_2=3)}$
- $\forall k \geq 6$, $P(N = k|M_1 = 1, M_2 = 3) = \frac{P(N=k,M_1=1,M_2=3)}{P(M_1=1,M_2=3)} = \frac{pf^2}{P(M_1=1,M_2=3)}$

由于 $f \ll e$, 显然上述三个概率最大的是 $P(N = 2|M_1 = 1, M_2 = 3)$. 因此 $N = 2$ 是最有可能的.

14.13

Consider the network shown in Figure 14.22(ii), and assume that the two telescopes work identically. $N \in 1,2,3$, and $M_1, M_2 \in 0,1,2,3,4$, with the symbolic CPTs as described in Exercise 14.12. Using the enumeration algorithm (Figure 14.9 on page 525), calculate the probability distributio $P(N|M_1 = 2, M_2 = 2)$

```
function ENUMERATION-ASK(X, e, bn) returns a distribution over X
  inputs: X, the query variable
          e, observed values for variables E
          bn, a Bayes net with variables {X} ∪ E ∪ Y   /* Y = hidden variables */

  Q(X) ← a distribution over X, initially empty
  for each value x_i of X do
      Q(x_i) ← ENUMERATE-ALL(bn.VARS, e_{x_i})
          where e_{x_i} is e extended with X = x_i
  return NORMALIZE(Q(X))
---------------------------------------------------------------
function ENUMERATE-ALL(vars, e) returns a real number
  if EMPTY?(vars) then return 1.0
  Y ← FIRST(vars)
  if Y has value y in e
      then return P(y | parents(Y)) × ENUMERATE-ALL(REST(vars), e)
      else return ∑_y P(y | parents(Y)) × ENUMERATE-ALL(REST(vars), e_y)
          where e_y is e extended with Y = y
```

Figure 14.9 The enumeration algorithm for answering queries on Bayesian networks.

有变量 $\{N, M_1, M_2, F_1, F_2\}$, 可计算如下:

$$
\begin{aligned}
P(N|M_1 = 2, M_2 = 2) &= \alpha \sum_{f_1} \sum_{f_2} P(N, M_1 = 2, M_2 = 2, f_1, f_2) \\
&= \alpha \sum_{f_1} \sum_{f_2} P(N)P(f_1)P(f_2)P(M_1 = 2|f_1, N)P(M_2 = 2|f_2, N)
\end{aligned}
$$

由于若 $f_1 = true$ 或 $f_2 = true$ 时对应项 M_i 都不可能达到 2 个星星, 因此只要考虑 $f_1 = f_2 = false$, 故

$$
P(N|M_1 = 2, M_2 = 2) = \alpha P(N)(1-f)^2 P(M_1 = 2|false, N)P(M_2 = 2|false, N)
$$

故有

$$
P(N = k|M_1 = 2, M_2 = 2) = \begin{cases} \alpha P(N = 1)(1-f)^2 e^2 & k = 1 \\ \alpha P(N = 2)(1-f)^2 (1-2e)^2 & k = 2 \\ \alpha P(N = 3)(1-f)^2 e^2 & k = 3 \end{cases}
$$