

第一章 几何光学

1. 基本规律

光程 $\boxed{l = ns, \left(V = \frac{c}{n} \right)}$

费马原理

$$\boxed{\delta = 0, l \text{取极大、极小或常数} \Rightarrow i = -i', n_1 \sin \theta_1 = n_2 \sin \theta_2.}$$

2. 成像

① 单球面折射

$$\frac{s^2}{n^2(s+r)^2} - \frac{s'^2}{n'^2(s'-r')^2} = -4r \sin^2\left(\frac{\varphi}{2}\right) \left[\frac{1}{n^2(s+r)} + \frac{1}{n'^2(s'-r')} \right]$$

保持同心性物像点：

$$\frac{s^2}{n^2(s+r)^2} - \frac{s'^2}{n'^2(s'-r')^2} = 0, \frac{1}{n^2(s+r)} + \frac{1}{n'^2(s'-r')} = 0.$$

傍轴条件：

$$\boxed{\frac{n'}{s'} + \frac{n}{s} = \frac{n' - n}{r}}.$$

$$\boxed{f = \frac{nr}{n' - n}, f' = \frac{n'r}{n' - n}}$$

$$\boxed{\frac{f'}{s'} + \frac{f}{s} = 1}.$$

$$\boxed{V = \frac{y'}{y} = -\frac{ns'}{n's}}.$$

② 球面镜成像 $\boxed{n' = -n}$

$$\boxed{\frac{1}{s'} + \frac{1}{s} = -\frac{2}{r}}, \boxed{f = f' = -\frac{r}{2}}, \boxed{V = -\frac{s'}{s}}$$

③ 薄透镜

$$\frac{f'}{s'} + \frac{f}{s} = 1.$$

$$f = \frac{n}{\frac{n_L - n}{r_1} + \frac{n' - n_L}{r_2}}, f' = \frac{n'}{\frac{n_L - n}{r_1} + \frac{n' - n_L}{r_2}}.$$

$$n' = n, f' = f, \frac{1}{s'} + \frac{1}{s} = \frac{1}{f}.$$

$$s = x + f, s' = x' + f',$$

$$xx' = ff', V = -\frac{f}{x} = -\frac{x'}{f'}.$$

④密接透镜组

$$s_2 = -s'_1, \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

$$P = \frac{1}{f}, P = P_1 + P_2.$$

⑤望远镜

$$M \equiv -\frac{f_o}{f_E}$$

第二章 光的干涉

一.光波基本描述

$$1. \boxed{v = \frac{c}{n}} \quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.0 \times 10^8 \text{ m/s}, \quad n = \sqrt{\epsilon_r \mu_r}.$$

$$\boxed{\lambda = \frac{v}{\nu} = \frac{c}{n \nu} = \frac{\lambda_0}{n}, \lambda_0 = \frac{c}{\nu}}$$

2.单色波

①电场

$$\vec{E} = \vec{E}_0(p) \cos(\omega t - \varphi(p))$$

磁场

$$\vec{B} = \vec{B}_0(p) \cos(\omega t - \varphi(p))$$

②单色平面波

$$\vec{E} = \vec{E}_0 \cos[\omega t - \vec{k} \cdot \vec{r} + \varphi_0], \quad \tilde{E}(p, t) = E_0 \exp[-i(\omega t - \vec{k} \cdot \vec{r} + \varphi_0)]$$

波数 $k = \frac{2\pi}{\lambda}$, 波矢 $\vec{k} = \frac{2\pi}{\lambda} = k\vec{k}_0$, \vec{k}_0 为传播方向的单位方向矢量。

波的相位 $\omega t - \vec{k} \cdot \vec{r} + \varphi_0 = \omega t - R \frac{2\pi}{\lambda} + \varphi_0$, 其中 R 为 \vec{r} 在 \vec{k}_0 方向上的投影。

$$\text{复振幅} \quad \tilde{E}(p) = E_0 \exp(i\varphi(p)).$$

③单色球面波

$$E = \frac{A_0}{r} \cos[\omega t - kr + \varphi_0]$$

$$\tilde{E}(p, t) = \frac{A_0}{r} \exp[-i(\omega t - kr + \varphi_0)]$$

$$\text{复振幅为} \quad \tilde{E}(p) = \frac{A_0}{r} \exp[i(kr - \varphi_0)].$$

3.光强度

光强 $I = \langle \vec{s} \rangle = E_0^2 = \tilde{E}^*(p)\tilde{E}(p)$. \vec{s} 是电磁波能流密度。

谱密度 $i(\lambda) = \frac{dI_\lambda}{d\lambda}$, (dI_λ 是 $\lambda \sim \lambda + d\lambda$ 之间光强)。

$$I = \int_0^\infty dI_\lambda = \int_0^\infty i(\lambda) d\lambda$$

$$4. \text{反衬度} \quad \gamma \equiv \frac{I_M - I_m}{I_M + I_m}$$

二、线性叠加原理(弱光情况下成立):

$$1. \quad \vec{E}(p, t) = \vec{E}_1(p, t) + \vec{E}_2(p, t) + \cdots$$

同方向光振动叠加:

$$E(p, t) = E_1(p, t) + E_2(p, t) + \dots$$

2. 同频率、同振向波的叠加

$$E_1(p, t) = E_{10}(p) \cos(\omega t - \varphi_1(p)),$$

$$E_2(p, t) = E_{20}(p) \cos(\omega t - \varphi_2(p))$$

$$E(p, t) = E_0(p) \cos(\omega t - \varphi).$$

$$E_0^2(p) = E_{01}^2(p) + 2E_{10}(p)E_{02}(p)\cos[\varphi_1(p) - \varphi_2(p)] + E_{02}^2(p)$$

$$\tan \varphi(p) = \frac{E_{10}(p) \sin \varphi_1(p) + E_{20}(p) \sin \varphi_2(p)}{E_{10}(p) \cos \varphi_1(p) + E_{20}(p) \cos \varphi_2(p)}.$$

$$I(p) = I_1(p) + I_2(p) + 2\sqrt{I_1(p)I_2(p)}\cos \delta.$$

$$I = I_0(1 + \gamma \cos \delta), I_0 = I_1 + I_2.$$

三、光的干涉和相干条件

1. 相干条件

① 位相差判据

当 $\delta = 2\pi m, (m = 0, \pm 1, \pm 2, \dots)$ (同位相),

$I_M = (E_{01} + E_{02})^2$, 称为干涉极大, 对应亮纹;

当 $\delta = (2m + 1)\pi, (m = 0, \pm 1, \pm 2, \dots)$ (反位相),

$I_m = (E_{01} - E_{02})^2$, 称为干涉极小, 对应暗纹.

② 光程差判据

位相差 $\delta(p) = k(r_2 - r_1) = \frac{2\pi}{\lambda_0} \Delta l(p),$

其中 $\Delta l(p) = n_1 r_1 - n_2 r_2$.

干涉极大 $\Delta l(p) = m \lambda_0$.

干涉极小 $\Delta l(p) = \left(m + \frac{1}{2}\right) \lambda_0$.

四、杨氏实验

1. 光程差 $\Delta l = r_1 - r_2 \approx d \sin \theta$. $\Delta l \approx d \frac{x}{D}$.

2. 极大位置 $x = \frac{m \lambda_0 D}{d} \quad (m = 0, \pm 1, \pm 2, \dots)$.

极小位置 $x = \frac{(m + 1/2) \lambda_0 D}{d} \quad (m = 0, \pm 1, \pm 2, \dots)$.

3. 条纹宽度 $\Delta x = \frac{\lambda_0 D}{d}$.

4. 光强分布

$$\delta(p) = \frac{2\pi}{\lambda_0} \Delta l(p) = \frac{2\pi}{\lambda_0} d \frac{x}{D}.$$

实验中, $I_1 \approx I_2 = I_0$, $I = 2I_0 \left(1 + \cos \frac{2\pi}{\lambda_0} d \frac{x}{D}\right) = 4I_0 \cos^2 \left(\frac{\pi d}{D \lambda_0} x\right)$.

5. 最大光程差 $\Delta l_M = m' \lambda_0 = \frac{\lambda_0^2}{\Delta \lambda_0}$.

6. 光源 S 沿 x 方向移动 δs , 干涉条纹的移动 $\delta x \approx -\frac{D}{l} \delta s$.

7. 扩展光源

● 临界宽度 $b_c = \frac{l \lambda}{d}$.

干涉口径角 $\beta \equiv \frac{d}{l}$, 扩展光源干涉条件为 $b < \frac{\lambda}{\beta}$.

● 横向相干宽度 $d_c \equiv \frac{l\lambda}{b}$.

● 光场的空间相干性: $d < d_c$, 即 $\beta < \beta_c = \frac{d_c}{l}$ 内两点源都是相干点源.

● $b\beta_c = \lambda$.

五、薄膜干涉

1. 光程差

$$\Delta L = 2nt \cos \theta_r + \frac{\lambda}{2} = 2nt \sqrt{n^2 - n_1^2 \sin^2 \theta_i} + \frac{\lambda}{2}.$$

2. 等倾干涉

从中心向外数第 N 个亮环附近相邻两亮环间的角距离为

$$(\Delta N = 1) \Delta \theta_N = \frac{1}{n'} \sqrt{\frac{n\lambda}{t}} \frac{\Delta N}{2\sqrt{N}}.$$

$$\text{第 } N \text{ 个亮环半径 } r_N \approx \theta_N f = \frac{f}{n'} \sqrt{\frac{nN\lambda}{t}}.$$

$$\text{相邻两亮环间的径向距离为 } \Delta r_N \approx \Delta \theta_N f = \frac{n\lambda f}{2n'^2 t \theta_N}.$$

3. 等厚干涉

① 楔形

$$\text{相邻条纹的高度差 } \Delta t = t_{m+1} - t_m = \frac{\lambda}{2n}.$$

相邻条纹的间隔 $\Delta l = \frac{\Delta t}{\sin \alpha} = \frac{\lambda}{2n \sin \alpha}.$

②牛顿环

光程差 $\theta_i = 0, \Delta L = 2t - \frac{\lambda}{2}.$

m 级亮纹半径为 $r_m = \sqrt{\left(m + \frac{1}{2}\right)\lambda R}.$

m 级暗纹半径为: $r'_m = \sqrt{m\lambda R}.$

$$R = \frac{r_{m+N}'^2 - r_m'^2}{N\lambda}.$$

4.透射光

$$\Delta L = 2nt \cos \theta_r. \quad (I_0 = I_r + I_t)$$

5.薄膜厚度要求

$$\Delta L = 2nt \cos \theta < \Delta L_M = m' \lambda = \frac{\lambda^2}{\Delta \lambda}.$$

6.增透膜 $2nt = \left(m + \frac{1}{2}\right)\lambda, m = 0, 1, 2, \dots$

7.迈克尔逊干涉仪 $\Delta t = \pm N \frac{\lambda}{2}.$

六、光场的时间相干性: $t < \tau_0.$

光波的相干长度 $L_c = \Delta L_{max} = \frac{\bar{\lambda}^2}{\Delta \lambda},$ 相干时间 $\tau_0 \equiv \frac{L_c}{c}.$

第三章 光的衍射

一、惠更斯-菲涅耳原理

$$\tilde{E}(P) = k \iint_{(\Sigma)} \tilde{E}_0(Q) F(\theta_0, \theta) \frac{e^{ikr}}{r} d\Sigma.$$

$$\text{基尔霍夫公式 } \tilde{E}(P) = \frac{-i}{\lambda} \iint_{(\Sigma_0)} \frac{(\cos \theta_0 + \cos \theta)}{2} \tilde{E}_0(Q) \frac{e^{ikr}}{r} d\Sigma.$$

傍轴条件下, 即 $\theta_0 \approx \theta \approx 0, r \approx r_0$

$$\tilde{E}(P) = \frac{-i}{\lambda r_0} \iint_{(\Sigma_0)} \tilde{E}_0(Q) e^{ikr} d\Sigma.$$

二、巴俾涅原理

几何像点之外,

$$\because \tilde{E}_a(P) + \tilde{E}_b(P) = \tilde{E}_0(P) = 0,$$

$$\therefore |\tilde{E}_a(P)| = |\tilde{E}_b(P)|, \Rightarrow I_a(P) = I_b(P).$$

三、菲涅耳圆孔衍射和圆屏衍射

$$1. E_0(P) = \frac{1}{2} \Delta E_{10} + (-1)^{n+1} \frac{1}{2} \Delta E_{n0}.$$

$$2. k = \frac{\rho^2}{\lambda} \left(\frac{1}{R} + \frac{1}{r} \right).$$

平行光入射圆孔, 则 $R \rightarrow \infty$, $k = \frac{\rho^2}{\lambda R}.$

$$3. \text{自由传播 } E_0(P) = \frac{1}{2} \Delta E_{10}.$$

$$4. \text{圆屏衍射 } E_0(P) = \frac{1}{2} \Delta E_{k+10}(P) \neq 0$$

5. 波带片

• 遮住偶数带, 轴上 P 点的振幅为

$$E_0(P) = \Delta E_{10}(P) + \Delta E_{30}(P) + \Delta E_{50}(P) \cdots + \Delta E_{2n+10}(P).$$

• 遮住奇数带, 轴上 P 点的振幅为

$$E_0(P) = -(\Delta E_{20}(P) + \Delta E_{40}(P) + \Delta E_{60}(P) \cdots + \Delta E_{2n0}(P)).$$

• 半波带半径

$$\rho = \sqrt{k} \rho_1, \rho_1 = \sqrt{\frac{Rb\lambda}{R+b}}.$$

• 透镜作用: $\left(\frac{1}{R} + \frac{1}{b} \right) = \frac{k\lambda}{\rho_k^2}.$

四、夫琅禾费衍射

1. 单缝

① 光强

$$\tilde{E}_0(P_\theta) = \tilde{E}_0(P_0) \frac{\sin(\alpha)}{\alpha}, I_\theta = I_0 \left(\frac{\sin(\alpha)}{\alpha} \right)^2,$$

其中 I_0 为衍射场中心光强度,

$\left(\frac{\sin(\alpha)}{\alpha} \right)^2$ 为单缝衍射因子.

② 次极强 $\sin \theta = \pm 1.43 \frac{\lambda}{a}, \pm 2.46 \frac{\lambda}{a}, \pm 3.67 \frac{\lambda}{a}, \dots$

③ 暗纹位置 $\sin \theta = m \frac{\lambda}{a}, (m = \pm 1, \pm 2, \pm 3, \dots)$

④ 零级亮斑的半角宽度 $\Delta \theta \approx \frac{\lambda}{a}.$

2. 圆孔

中心角半径: $\theta = 0.610 \frac{\lambda}{a} \approx 1.22 \frac{\lambda}{D}, D = 2a.$

最小分辨角 $\delta \theta_m = \Delta \theta = 1.22 \frac{\lambda}{D}.$

3.光栅

①光强

$$\bullet I(P_\theta) = A_0^2(P_0) \left(\frac{\sin(\alpha)}{\alpha} \right)^2 \left(\frac{\sin(N\delta/2)}{\sin(\delta/2)} \right)^2.$$

$$\bullet \text{主极大: } d \sin \theta = k\lambda, k = 0, \pm 1, \pm 2, \pm 3, \dots$$

$$I_{MAX} = N^2 A_0^2 \left(\frac{\sin(\alpha)}{\alpha} \right)^2 \cdot k_{MAX} = \frac{d}{\lambda}.$$

$$\bullet \text{极小: } \sin \theta = \left(k + \frac{m}{N} \right) \frac{\lambda}{d}, m = 1, 2, \dots, N-1. (m \neq 0, N)$$

$$\bullet \text{主极大的半角宽度 } \Delta \theta = \frac{\lambda}{Nd \cos \theta_k}.$$

●主极大缺级:

$$\text{主极大 } d \sin \theta = k\lambda, k = 0, \pm 1, \pm 2, \pm 3, \dots$$

$$\text{单缝极小 } a \sin \theta = n\lambda, n = \pm 1, \pm 2, \pm 3, \dots$$

$$\text{当 } \sin \theta = \frac{k\lambda}{d} = \frac{n\lambda}{a} \text{ 时, 即 } k = \frac{dn}{a} \text{ 缺级.}$$

②光谱

$$\bullet \text{色散本领定义为 } D_\theta = \frac{\delta \theta_k}{\delta \lambda} = \frac{k}{d \cos \theta_k}.$$

●瑞利判据 : 最小分辨角 $\delta \theta'$ 等于光谱线的半角宽度, 即 $\delta \theta' = \Delta \theta$.

$$\bullet \text{色分辨本领 } R = \frac{\lambda}{\delta \lambda} = kN.$$

$$\text{③闪耀光栅 } d \sin 2\theta_B = k\lambda_B, k \text{ 级最亮.}$$

同时, $a \approx d$, $a \sin(2\theta_B) = k\lambda_B^k$ 也成立, 即其它干涉级均成为缺级.

④布拉格条件 $2d \sin \theta = k\lambda$.

第五章 光的电磁性

一、偏振

1. 偏振度: $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, 0 \leq P \leq 1.$

2. 马吕斯定律 $I_2 = I_1 \cos^2 \alpha.$

3. 布儒斯特角 θ_B $\tan \theta_B = \frac{n_2}{n_1}.$

4. e 光主折射率 $n_e = c / V_e = \frac{\sin \theta_i}{\sin \theta_r^e}.$

5. 波晶片

若 $(n_0 - n_e)d = \pm \frac{\lambda}{4} + m\lambda, m$ 为整数, $\varphi_{o-e} = \pm \frac{\pi}{2}$, 则称波晶片为 $\frac{\lambda}{4}$ 片.

若 $(n_0 - n_e)d = \pm \frac{\lambda}{2} + m\lambda, \varphi_{o-e} = \pm \pi, 2\pi$, 则称波晶片为 $\frac{\lambda}{2}$ 片.

6. 椭圆偏振光 $\vec{E} = \vec{E}_x + \vec{E}_y.$

其中 $E_x = E_{x0} \cos\left(\omega t - \frac{2\pi}{\lambda} z\right), E_y = E_{y0} \cos\left(\omega t - \frac{2\pi}{\lambda} z - \delta\right).$

二、光的吸收、色散和散射

1. 光的吸收 $I = I_0 e^{-\alpha d}$.

2. 色散

• 柯西公式: $n = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$.

• 色散率 $\frac{dn}{d\lambda} < 0$, 正常色散; $\frac{dn}{d\lambda} > 0$, 反常色散.

3. 瑞利散射: $I_s(\lambda) \propto \frac{f(\lambda)}{\lambda^4}, a < \lambda$.

第六章 光的量子性

1. 普朗克能量子 $\varepsilon_0 = h\nu$

2. 光电效应

$$\bullet \frac{1}{2} m v_m^2 = e V_0 = e K (\nu - \nu_0).$$

$$\bullet E = h\nu.$$

$$\bullet \frac{1}{2} m v_m^2 = h\nu - A,$$

$$\bullet \nu_0 = \frac{A}{h}.$$

3. 康普顿散射

$$\Delta\lambda = 2\lambda_c \sin^2 \frac{\theta}{2}, \lambda_c = \frac{h}{m_0 c} = 0.0243 \text{ \AA}$$

4. 光的波粒二象性 $E = h\nu, p = \frac{h\nu}{c} = \frac{h}{\lambda}$.

第七章 激光

自发辐射: $\frac{dN_{21}}{dt} = A_{21} N_2,$

受激辐射: $\frac{dN'_{21}}{dt} = B_{21}\rho(\nu)N_2,$

受激吸收: $\frac{dN_{12}}{dt} = B_{12}\rho(\nu)N_1,$

谐振腔纵模 $\nu_j = j \frac{c}{2nL}, j = 1, 2, \dots$ 间隔 $\Delta\nu = \frac{c}{2nL}.$

说明：打黑框的公式必须理解，并且记住！不打黑框的公式要求理解其物理意义。