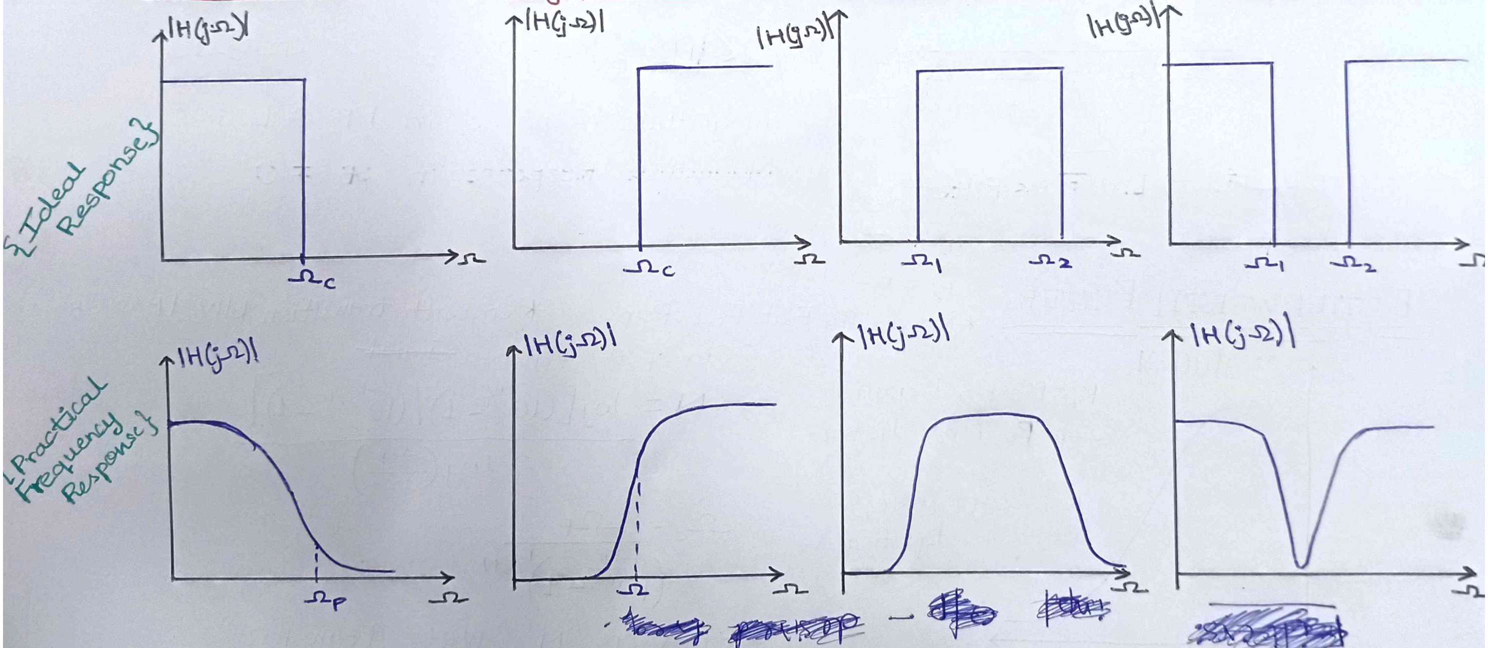
Filters : are Algorithms / Systems that modify / manipulate signals to remove unwanted Components or enhance Certain Features.

Applicⁿ: Noise Reductions, Signal enhancement, Feature extraction.

※ Types of Filters :

i) Low Pass Filter : ii) Highpass Filter iii) Band-Pass Filter iv) Bandstop Filter.



ANALOG FILTER SPECIFICATION :

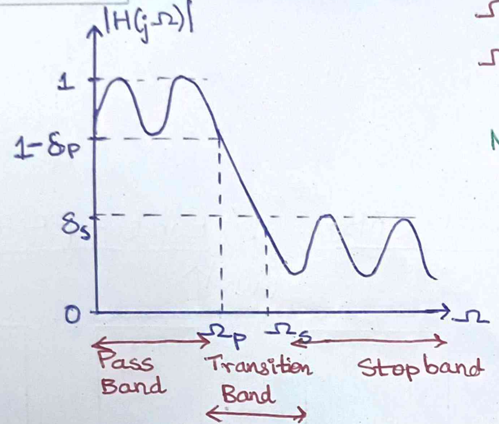


Fig: Specif^n of Low Pass Filter.

Ω_p - Passband edge freq.

Ω_s - Stopband edge freq.

Mathematical descrip^n of Freq. Response:

$$(1-\delta_p) \leq |H(j\Omega)| \leq 1 \quad ; \quad 0 \leq \Omega \leq \Omega_p$$

$$0 \leq |H(j\Omega)| \leq \delta_s \quad ; \quad \Omega \geq \Omega_s$$

Magnitude Response in PB = 1

Magnitude Response in SB = 0

a) Butterworth
b) Chebyshev.

BUTTERWORTH FILTER :

* Maximally Flat Freq. Resp. * Smooth transition b/w PB & SB

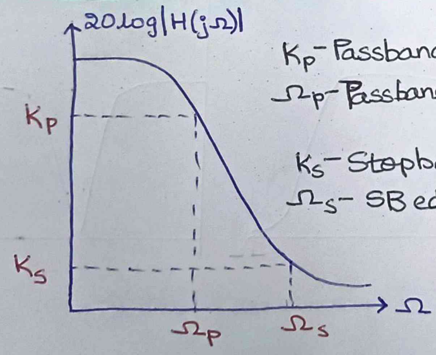


Fig: LowPass Butterworth Filter.

K_p - Passband gain

Ω_p - Passband edge freq.

K_s - Stopband gain

Ω_s - SB edge freq.

Order of the filter, ~~Smoot~~

$$N = \frac{\log\left[\left(10^{-K_p/10} - 1\right) / \left(10^{-K_s/10} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}.$$

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}} - 1\right)^{1/2N}}$$

* Based on 'N', Write Transfer f^n.

* Apply appropriate Transformation.

* $H(s)\Big|_{@ s=j\Omega} \longrightarrow H(j\Omega)$; Evaluate at Ω_p; Evaluate at Ω_s.

N	Butterworth Polynomial $B_N(s)$
1	$(s+1)$
2	$(s^2+\sqrt{2}s+1)$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.76545s+1)(s^2+1.8478s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

$$H_N(s) = \frac{1}{B_N(s)}$$

* Normalised Analog LPF : $\Omega_p = 1$ rad/sec.

Backward design eq^n :

LP : $\Omega_s = \dfrac{\Omega'_s}{\Omega_u}$

HP : $\Omega_s = \dfrac{\Omega_u}{\Omega'_s}$

BP : $A = \dfrac{\Omega_1^2 + \Omega_l\Omega_u}{\Omega_1(\Omega_u - \Omega_l)}$, $B = \dfrac{\Omega_2^2 - \Omega_l\Omega_u}{\Omega_2(\Omega_u - \Omega_l)}$

$\Omega_s = \min\{|A|, |B|\}$

BS : $A = \dfrac{\Omega_1(\Omega_u - \Omega_l)}{-\Omega_1^2 + \Omega_u\Omega_l}$, $B = \dfrac{\Omega_2(\Omega_u - \Omega_l)}{-\Omega_2^2 + (\Omega_u\Omega_l)}$

IIR Filter Transformation :

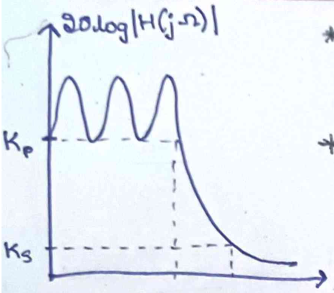
LP–LP : $s \to \dfrac{s}{\Omega_u}$

LP–HP : $s \to \dfrac{\Omega_u}{s}$

LP–BP : $s \to \dfrac{s^2 + \Omega_u\Omega_l}{s(\Omega_u - \Omega_l)}$

LP–BS : $s \to \dfrac{s(\Omega_u - \Omega_l)}{s^2 + \Omega_u\Omega_l}$

Chebyshev Filters:



* $K_p = 20\log\left[\dfrac{1}{\sqrt{1+\varepsilon^2}}\right]$

* $\delta_p = 1 - \dfrac{1}{\sqrt{1+\varepsilon^2}}$

* $K_s = 20\log\delta_s$

* Selectivity factor: $K = \dfrac{\Omega_p}{\Omega_s}$

* Discrimination factor: $d = \sqrt{\dfrac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$

* $N = \dfrac{\cosh^{-1}\left(\frac{1}{d}\right)}{\cosh^{-1}\left(\frac{1}{K}\right)}$

$$a = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{\frac{1}{N}} - \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-\frac{1}{N}}$$

$$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/N} + \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-1/N}$$

k	$\sigma_k = -a\sin(2k-1)\frac{\pi}{2N}$	$\Omega_k = b\cos(2k-1)\frac{\pi}{2N}$
1		
⋮		
N		

$S_k = \sigma_k + j\Omega_k$

$$H_N(s) = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)\cdots(s-s_N)}$$

If N is odd,
$K_N = b_0$

If N is even,
$K_N = \dfrac{b_0}{\sqrt{1+\varepsilon^2}}$

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Q:1 A ButterWorth Lowpass Filter (LPF) has to meet the following specifications.

a) Passband gain, $K_p = -1dB$, $\Omega_p = 4 rad/sec$

b) Stopband attenuation $\geq 20dB$ @ $\Omega_s = 8 rad/sec$.

Determine the transfer func^n. $H_a(s)$ of Lowest order BW filter to meet the given specific^n.

Sol^n:

$$N = \frac{\log\left[\left(10^{\frac{-K_p}{10}} - 1\right) / \left(10^{\frac{-K_s}{10}} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\left(10^{\frac{-(-1)}{10}} - 1\right) / \left(10^{\frac{20}{10}} - 1\right)\right]}{2\log\left(\frac{4}{8}\right)}$$

$$= \frac{\log\left[\left(10^{0.1} - 1\right) / \left(10^{2} - 1\right)\right]}{2\log\left(\frac{1}{2}\right)}$$

$$= \frac{\log\left[(1.25893 - 1) / (100 - 1)\right]}{2\log(0.5)}$$

$$= \frac{\log\left[0.2589/99\right]}{2\log(0.5)}$$

$$= \frac{-2.5825}{-0.60205}$$

$$N = 4.2894$$

$$\boxed{N = 5}$$

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N	$B_N(s)$
$\cancel{0}$	
1	$s+1$
2	$s^2+\sqrt{2}s+1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7653s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

Step 1:

$$H_5(s) = \frac{1}{(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)}$$

$$H_5(s) = \frac{1}{s^5+3.236s^4+5.236s^3+5.236s^2+3.236s+1}$$

Step 2:

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}}-1\right)^{1/2N}} = \frac{4}{\left(10^{\frac{-(-1)}{10}}-1\right)^{1/2(5)}}$$

$$= \frac{4}{\left(10^{0.1}-1\right)^{1/10}}$$

$$= \frac{4}{(0.2589)^{1/10}}$$

$$= \frac{4}{0.8736}$$

$$\boxed{\Omega_c = 4.5787 \text{ rad/sec}}$$

Step 3:

$$H_a(s) = H_5(s)\Big|_{s \to \frac{s}{\Omega_c}}$$

$$= H_5(s)\Big|_{s \to \frac{s}{4.5787}}$$

$$\therefore H_5(s) = \frac{1}{\left(\frac{s}{4.5787}\right)^5 + 3.236\left(\frac{s}{4.5787}\right)^4 + 5.236\left(\frac{s}{4.5787}\right)^3 + 5.236\left(\frac{s}{4.5787}\right)^2 + 3.236\left(\frac{s}{4.5787}\right) + 1}$$

$$= \frac{1}{4.9692\times10^{-4}s^5 + 3.236(2.2753\times10^{-3})s^4 + 5.236(0.01042)s^3 + 5.236(0.047699)s^2 + 0.2184(3.236)s + 1}$$

Step 4:

Divide Numerator & Denominator by (4.9692×10^{-4})

$$= \frac{1/(4.9692\times10^{-4})}{s^5+14.817s^4+109.675s^3+502.4953s^2+1422.260s+2012.3963}$$

$$\therefore H_5(s) = \frac{2012.3963}{s^5+14.817s^4+109.675s^3+502.4953s^2+1422.260s+2012.3963}$$

Step 5:

Evaluate at $\Omega_p = 4$ rad/s

$$H_5(s)\Big|_{s=j\Omega_p=4j} = \frac{2012.3963}{(4j)^5+14.817(4j)^4+109.675(4j)^3+502.4953(4j)^2+1422.260(4j)+2012.3963}$$

$$= \frac{2012.3963}{1024j+3793+(-j)7019.2-8039.9248+5689.04j+2012.396}$$

$$20\log|H_5(s)| = 20\log\frac{2012.3963}{\sqrt{(\cancel{-13845.3211}\ -2234.5285)^2+(-306.16)^2}}$$

$$= 20\log\left(\frac{2012.3963}{\cancel{13848.70573}\ 2255.4}\right)$$

$$= 20\log(\cancel{0.1453124}\ 0.892256)$$

$$\boxed{20\log|H_5(s)| = 0.99 \text{ dB}}$$

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Q. 2) A Butterworth LPF with following Specific^n.

a) $K_p = -1.5dB$ at $\Omega_p = 10$ rad/s

b) $K_s \geq 30dB$ at $\Omega_s = 30$ rad/s

$H_a(s) = ?$

Sol^n:

$$N = \frac{\log\left[\left(10^{-K_p/10} - 1\right)/\left(10^{-K_s/10} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\left(10^{0.15} - 1\right)/\left(10^{3} - 1\right)\right]}{2\log\left(\frac{10}{30}\right)}$$

$$= \frac{\log\left[0.412/999\right]}{2\log(1/3)}$$

$$= \frac{-3.3846}{-0.95424}$$

$N = 3.54689$ $\boxed{N = 4}$

N	$B_N(s)$
1	$s+1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7658s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

step 1:

$$H_4(s) = \frac{1}{(s^2+0.7653s+1)(s^2+1.8477s+1)}$$

$$= \frac{1}{s^4+1.84776s^3+s^2+0.7653s^3+1.4118s^2+0.7653s+s^2+1.8477s+1}$$

$$H_4(s) = \frac{1}{s^4+2.613s^3+3.4118s^2+2.6107s+1}$$

step 2:

$$\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}}-1\right)^{1/2N}} = \frac{10}{\left(10^{0.15}-1\right)^{1/2(4)}}$$

$$= \frac{10}{(0.41254)^{1/8}}$$

$\therefore \boxed{\Omega_c = 11.17 \text{ rad/sec}}$

step 3: $H_a(s) = H_4(s)\Big|_{s = \frac{s}{11.17}}$

$$H_4(s) = \frac{1}{\left(\frac{s}{11.17}\right)^4 + 2.613\left(\frac{s}{11.17}\right)^3 + 3.4118\left(\frac{s}{11.17}\right)^2 + 2.6107\left(\frac{s}{11.17}\right) + 1}$$

$$= \frac{1}{6.42\times10^{-5}s^4 + 2.613(7.175\times10^{-4})s^3 + 3.4118(8.0148\times10^{-3})s^2 + 2.6107(0.08952)s + 1}$$

step 4:

Divide Numerator & denominator by (6.42×10^{-5});

$$H_4(s) = \frac{1/6.42\times10^{-5}}{s^4+29.2029s^3+426.806s^2+3640.340s+15576.3239}$$

at $\Omega_p = 10$ rad/s,

$$H_4(s) = \frac{15576.3239}{(10j)^4+29.2029(10j)^3+426.806(10j)^2+3640.340(10j)+15576.3239}$$

step 5:

$$20\log H_4(s) = 20\log\left(\frac{15576.3239}{\sqrt{(17103.6761)^2+(7200.5)^2}}\right)$$

$$= 20\log(0.83935)$$

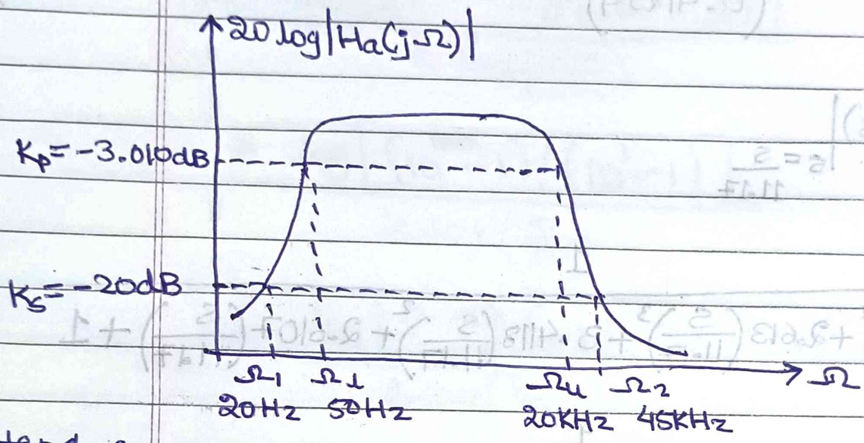
$$\boxed{20\log H_4(s) = -1.52 \text{ dB}}$$

Q. 3) Design an analog Bandpass Filter to meet freq. domain specif^n.

a) 3.0103 dB with upper & lower cutoff freq. 50 Hz & 20 K

b) Stopband attenuation of 20 dB @ 20 Hz & 45 KHz.

c) Monotonic freq. responce.



Step 1 :

$$\Omega_1 = 2\pi f_1 = 2\pi(20) = 125.663 \text{ rad/sec.}$$
$$\Omega_2 = 2\pi f_2 = 2\pi(45K) = 2.827 \times 10^5 \text{ rad/sec.}$$
$$\Omega_l = 2\pi f_l = 2\pi(50) = 314.159 \text{ rad/sec}$$
$$\Omega_u = 2\pi f_u = 2\pi(20K) = 1.257 \times 10^5 \text{ rad/sec.}$$

Backward design eq^n :

$$A = \frac{-\Omega_1^2 + \Omega_l \Omega_u}{\Omega_1(\Omega_u - \Omega_l)} = 2.5$$

$$B = \frac{\Omega_2^2 - \Omega_l \Omega_u}{\Omega_2(\Omega_u - \Omega_l)} = 2.25$$

$$\Omega_s = \min\{|A|, |B|\}$$

Step 2(a) :

Normalized Lowpass ButterWorth filter $\Omega_p = 1$

$$N = \frac{\log\left[\left(10^{\frac{3.010}{10}} - 1\right) / \left(10^{20/10} - 1\right)\right]}{2\log\left(\frac{1}{2.25}\right)}$$

$$= \frac{\log\left[(1.0717 - 1)/99\right]}{2\log(1/2.25)} = \frac{-\cancel{3.1397}\ 1.9956}{-0.704} = 2.833$$

$$\boxed{N = 3}$$

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p 2B :

N	$B_N(s)$
1	$s+1$
2	$s^2+\sqrt{2}s+1$
~~3~~	$(s+1)(s^2+s+1)$
4	$(s^2+0.7653s+1)(s^2+1.84776s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

Step 3 : $H_3(s) = \dfrac{1}{s^3+2s^2+2s+1}$ //

Step 4: Lowpass to Band pass Transformation :

$$H_a(s) = H_3(s)\Big|_{s\to\frac{s^2+\Omega_u\Omega_l}{s(\Omega_u-\Omega_l)}} = H_3(s)\Big|_{s\to\frac{s^2+(1.257\times10^5)(314.159)}{s(1.257\times10^5-314.159)}}$$

$$H_3(s)\Big|_{s\to\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}}$$ //

$$H_3(s) = \frac{1}{\left(\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right)^3 + 2\left(\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right)^2 + 2\left(\frac{s^2+3.949\times10^7}{s(1.2538\times10^5)}\right) + 1}$$

$$= \frac{1}{\frac{s^6+6.1583\times10^{22}+11847\times10^4s^4+4.6784\times10^{15}s^2}{s^3(1.97099\times10^{45})} + \frac{2(s^4+7898\times10^4s^2+1.5595\times10^{15})}{s^2(3.1843\times10^{25})} + \frac{1.5951\times10^{-5}}{s}(s^2+3.949\times10^7)+1}$$

$$H_3(s) = \frac{1}{\frac{5.6736\times10^{-46}}{s^3}(s^6+6.1583\times10^{22}+11847\times10^4s^4+4.678\times10^{15}s^2)+\frac{6.455\times10^{-26}}{s^2}(s^4+7898\times10^4s^2+1.5595\times10^{15})+\frac{1.5951\times10^{-5}}{s}(s^2+3.949\times10^7)+1}.$$

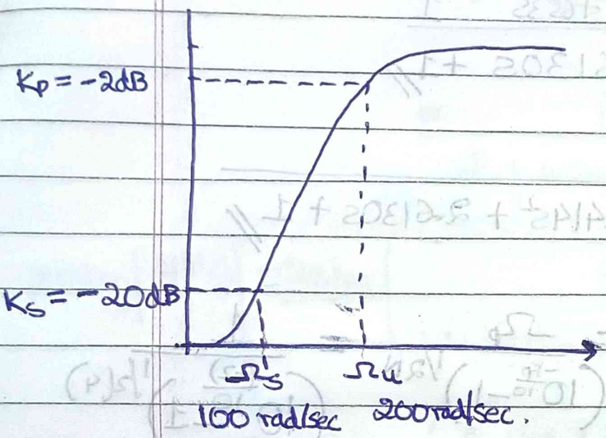
Q.4.) Design analog BW HPF that meets specific^n.

a) Max^m. Passband attenuation 2dB

b) Passband edge freq. 200 rad/sec.

c) Min. stopband attenuation 20dB

d) Stopband edge freq. 100 rad/s.



Step 1: From Backward design eq^n,

$$\Omega_s = \frac{\Omega_u}{\Omega'_s} = \frac{200}{100} = 2 \text{ rad/sec}$$

Step 2: LP normalized BW filter:

$\Omega_p = 1$ rad/sec, $K_p = -2dB$

$\Omega_s = 2$ rad/sec, $K_s = -20dB$

$$N = \frac{\log\left[\left(10^{-K_p/10} - 1\right) / \left(10^{-K_s/10} - 1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\left(10^{\frac{-(-2)}{10}} - 1\right) / \left(10^{20/10} - 1\right)\right]}{2\log\left(\frac{1}{2}\right)}$$

$$= \frac{\log\left[(1.5848 - 1)/(100 - 1)\right]}{2\log(0.5)}$$

$$= \frac{-2.2286}{-0.6021}$$

$N = 3.701$ $\boxed{N = 4}$

Step 3:

N	$B_N(s)$
1	$s+1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s+1)(s^2+s+1)$
4	$(s^2+0.7653s+1)(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.6180s+1)(s^2+1.6180s+1)$

$$H_4(s) = \frac{1}{(s^2+0.765s+1)(s^2+1.84775s+1)}$$

	s^4	s^3	s^2	s	1
			s^2	$0.7653s$	1
s^2	s^4	$0.7653s^3$	s^2		
1.84775		$1.84775s^3$	$s^2(1.414)$	$1.84775s$	
1			s^2	$0.7653s$	1

$$s^4+2.6130s^3+3.414s^2+2.6130s+1$$

$$\therefore H_4(s) = \frac{1}{s^4+2.6130s^3+3.414s^2+2.6130s+1}$$

Step 4: Cutoff frequency, $\Omega_c = \dfrac{\Omega_p}{\left(10^{\frac{-k_p}{10}}-1\right)^{1/2N}} = \dfrac{1}{\left(10^{\frac{-(-2)}{10}}-1\right)^{1/2(4)}}$

$$\Omega_c = \frac{1}{(0.5848)^{1/8}} \quad \therefore \boxed{\Omega_c = 1.0693 \text{ rad/sec}}$$

Step 5: LP prototype, $H_p(s) = H_4(s)\Big|_{\frac{s}{\Omega_c} \to \frac{s}{1.0693}}$

Highpass Butterworth Filter,

$$H_a(s) = H_{LP}(s)\Big|_{s \to \frac{200}{s} \Rightarrow \frac{200}{1.0693s} \to \frac{187.031}{s}}$$

$$H_4(s) = \frac{1}{\left(\frac{187.031}{s}\right)^4 + 2.6130\left(\frac{187.031}{s}\right)^3 + 3.414\left(\frac{187.031}{s}\right)^2 + 2.6130\left(\frac{187.031}{s}\right) + 1}$$

$$= \frac{1}{\frac{1223642024}{s^4} + \frac{17095436.63}{s^3} + \frac{119423}{s^2} + \frac{488}{s} + 1}$$

$$= \frac{1}{\left(\frac{1223642024 + 17095436.63s + 119423s^2 + 488s^3 + s^4}{s^4}\right)}$$

$$\therefore H_4(s) = \frac{s^4}{s^4 + 488s^3 + 119423s^2 + 17095436.63\,s + 1223642024}$$

$$H_4(s)\Big|_{s \to j\Omega = 100j} = \frac{(100j)^4}{(100j)^4 + 488(100j)^3 + 119423(100j)^2 + 11095436(100j) + 1223642024}$$

$$= \frac{10^8}{10^8 + 448\times10^6(-j) - 119423\times10^4 + 1109543600j + 1223642024}$$

$$20\log|H_4(s)| = 20\log\left(\frac{10^8}{\sqrt{129412024^2 + 661543600^2}}\right)$$

$$= 20\log(0.1483497)$$

$$\boxed{20\log|H_4(j\Omega)| = -16.574 \text{ dB}}$$

Q.5) Design a Lowpass Chebyshev Filter which has

a) Passband ripple ≤ 2dB (K_p)

b) Passband edge 1 rad/sec (Ω_p)

c) Stop band attenuation ≥ 20dB (K_s)

d) Stopband edge 1.3 rad/sec, (Ω_s)

Solⁿ:

Step 1 (a):

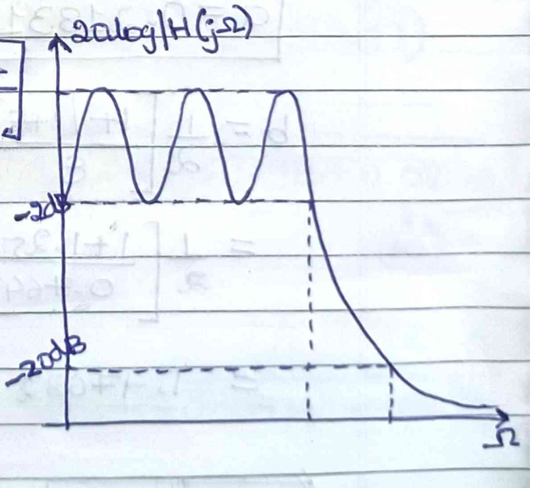
$$K_p = 20\log\left[\frac{1}{\sqrt{1+\varepsilon^2}}\right]$$

$$-2 = 20\log\left[\frac{1}{\sqrt{1+\varepsilon^2}}\right]$$

$$-0.1 = \log\left[\frac{1}{\sqrt{1+\varepsilon^2}}\right]$$

ripple factor,

$$\boxed{\varepsilon = 0.76478}$$



Step 1.(b):

$$\delta_p = 1 - \frac{1}{\sqrt{1+\varepsilon^2}}$$

$$= 1 - \frac{1}{\sqrt{1+0.76478^2}}$$

passband ripple

$$\boxed{\delta_p = 0.20567}$$

$$K_s = 20\log\delta_s$$

$$-20 = 20\log\delta_s$$

$$\boxed{\delta_s = 0.1}$$

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Selectivity factor, $K = \frac{\Omega_p}{\Omega_s} = \frac{1}{1.3} = 0.769$

Discrimination factor, $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = \sqrt{\frac{(1-0.20567)^{-2}-1}{0.1^{-2}-1}}$

$d = 0.07686$

$\boxed{d = 0.077}$

Step 2: $N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = \frac{\cosh^{-1}(12.987)}{\cosh^{-1}(1.3004)}$

$= \frac{3.2556}{0.756} = 4.3$

$\boxed{N=5}$

Step 3: $a = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{\frac{1}{N}} - \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-\frac{1}{N}}$

$= \frac{1}{2}\left[\frac{1+1.2589}{0.76478}\right]^{\frac{1}{5}} - \frac{1}{2}\left[\frac{1+1.2589}{0.76478}\right]^{-\frac{1}{5}}$

$= \frac{1}{2}(1.24186) - \frac{1}{2}(0.80524)$

$= 0.62093 - 0.40262$

$\boxed{a = 0.21831}$

$b = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right] + \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-\frac{1}{N}}$

$= \frac{1}{2}\left[\frac{1+1.2589}{0.76478}\right] + \frac{1}{2}\left[\frac{1+12589}{0.76478}\right]^{-\frac{1}{5}}$

$= 1.47682 +$

$\boxed{b = 1.02355}$

K	$\sigma_K = -a\sin(2k-1)\frac{\pi}{2N}$ $\sigma_K = -0.2183\sin(0.3142(2k-1))$	$\Omega_K = b\cos(2k-1)\frac{\pi}{2N}$ $\Omega_K = 1.02355(0.3142(2k-1))$
1	-0.067467	0.97344
2	-0.176624	0.601526
3	-0.218299	0
4	-0.176572	-0.6016287
5	-0.06738	-0.97344557

step 5: $H_5(s) = \dfrac{K_N}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)}$

$$= \frac{K_N}{(s-(0.06746+j0.97345))(s+0.176615-j0.6016287)(s+0.2183)(s+0.176615+j0.601628)(s+0.06746+j0.973455)}$$

$$= \frac{K_N}{((s-0.06746)^2-(j0.97345)^2)((s+0.1766)^2-(j0.60162)^2)(s+0.2183)}$$

$$= \frac{K_N}{(s^2-1.3492s+1.402699)(s^2+0.35323s+0.39315)(s+0.2183)}$$

$$H_5(s) = \frac{K_N}{s^5+0.7064s^4+4.995s^3+0.6934s^2+0.459349s+0.08172}$$

$0.08172 \to (b_0)$

$N=5$ (odd) $\Rightarrow K_N = b_0$

$$\therefore H_5(s) = \frac{0.08172}{s^5+0.7064s^4+4.995s^3+0.6934s^2+0.45934s+0.08172}$$

at $\Omega=1$, $s=j\Omega$, $s=j$

$$H_5(j\Omega) = \frac{0.08172}{j^5+0.7064(j)^4+4.995(j^3)+0.6934(j^2)+0.45934j+0.08172}$$

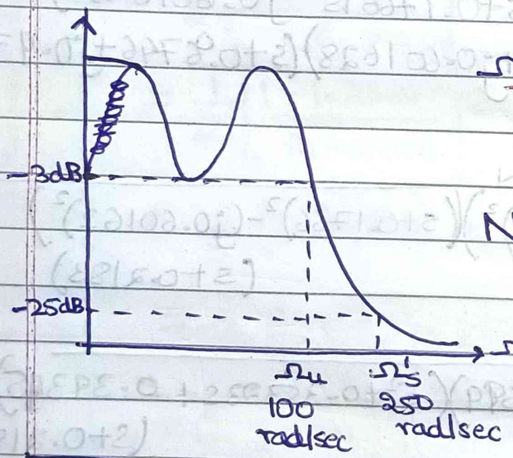
$$\therefore 20\log|H_5(j\Omega)| = \frac{0.08172}{\sqrt{(0.09472)^2+(-3.53566)^2}}$$

$$= 20\log\left(\frac{0.08172}{3.5669178}\right)$$

$$= 20 \times \log\left(\frac{1.20154}{\cancel{0.02291}}\right)$$

$$\boxed{20\log|H(j\Omega)| = 1.594765}$$

Q.6) Design a Chebyshev Analog LowPass filter that has: -3dB, cutoff freq of 100 rad/sec. Stopband attenuation 25 dB for all radian freq. past 250 rad/sec. Find the transfer func^n.



$$\Omega_s = \frac{\Omega_s'}{\Omega_u} = \frac{250}{100} = 2.5 \text{ rad/sec}$$

Normalized LPF,

$\Omega_p = 1$ rad/sec, $\Omega_s = 2.5$ rad/sec

$K_p = -3$dB, $K_s = -25$ dB.

$$K_p = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$-3 = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$-0.15 = \log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$$

$$0.5012 = \frac{1}{1+\varepsilon^2}$$

$$\boxed{\varepsilon = 0.99762}$$

$$\delta_p = 1 - \frac{1}{\sqrt{1+\varepsilon^2}}$$

$$= 1 - \frac{1}{\sqrt{1+0.99762^2}}$$

$$= 1 - 0.707$$

$$\boxed{\delta_p = 0.29205}$$

$(K_s = -25$ dB ;$)$

$$K_s = 20\log\delta_s$$

$$\frac{-25}{20} = \log\delta_s$$

$$\boxed{\delta_s = 0.0562}$$

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(15)

Selectivity factor, $K = \frac{\Omega_P}{\Omega_S} = \frac{1}{2.5} = 0.4$ $\boxed{K = 0.4}$

Discrimination factor, $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$

$= \sqrt{\frac{(1-0.29205)^{-2}-1}{0.0582^{-2}-1}}$ $\therefore \boxed{d = 0.05615}$

$N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = \frac{\cosh^{-1}(17.8079)}{\cosh^{-1}(2.5)}$ $\therefore N = 2.2798$

$\boxed{N=3}$

$$a = \frac{1}{2}\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{\frac{1}{N}} - \frac{1}{2}\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{-\frac{1}{N}}$$

$$= \frac{1}{2}\left(\frac{1+\sqrt{1.99524}}{0.99762}\right)^{\frac{1}{3}} - \frac{1}{2}\left(\frac{1+\sqrt{1.99525}}{0.99762}\right)^{-\frac{1}{3}}$$

$$= \frac{1}{2}(2.4183)^{1/3} - \frac{1}{2}(2.41828)^{-1/3}$$

$$= 0.671128 - 0.37251$$

$$\boxed{a = 0.298618}$$

$$b = \frac{1}{2}\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right) + \frac{1}{2}\left(\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right)^{-\frac{1}{N}}$$

$$= \frac{1}{2}(2.4183) + \frac{1}{2}(2.41828)^{-1/3}$$

$$= 0.671128 + 0.37251$$

$$\boxed{b = 1.04363}$$

K	$\sigma_k = -a\sin(2k-1)\frac{\pi}{2N}$ $\sigma_k = -0.29868\sin[0.52359(2k-1)]$	$\Omega_k = b\cos(2k-1)\frac{\pi}{2N}$ $\Omega_k = 1.04363\cos[0.52359(2k-1)]$
1.	-0.14931	0.90381
2.	-0.29862	0
3.	-0.14931	-0.90381

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$$H_3(s) = \frac{K_N}{(s-s_1)(s-s_2)(s-s_3)} = \frac{K_N}{(s-(-0.14931+j0.90381))(s-0.29862)(s-(-0.14931-j0.96831))}$$

$$\begin{array}{c|lll}
 & s & 0.14931 & -j0.90381 \\
s & s^2 & 0.14931s & -j0.90381s \\
0.14931 & & 0.14931s & -j\cancel{0.13495} & 0.222934 \\
+j0.9381 & & & j\cancel{0.90381s} & 0.816873 \\
 & & & j\cancel{0.13495} & \\
\end{array}$$

$$(s^2 + 0.29862s + 1.039807)(s-0.29862)$$

$$= s^3 + 0.29862s^2 + 1.039807s - 0.29862s^2 - 0.08917s - 0.31051$$

$$= s^3 + 0.59724s^2 + 0.928s - 0.280943$$

$$\therefore H_3(s) = \frac{0.2505943}{\left(\frac{s}{100}\right)^3 + 0.59724\left(\frac{s^2}{100}\right) + 0.928\left(\frac{s}{100}\right) - 0.2505943}$$

$$H_3(s)\Big|_{s=\frac{100j}{100}} = \frac{2505943}{(100j^3) + 0.59724(100j)^2 + 0.928(100j) - 0.2505943}$$

$$= \frac{2505943}{-j10^6 - 5972.4 + 92.8j - 0.2505943}$$

$$\cancel{20\log|H_3(j\Omega)| = 20\log\left(\frac{0.2505943}{\sqrt{(-5972.65)^2 + (-999907.2)^2}}\right)}$$

$$20\log|H_3(j\Omega)| = 20\log\left(\frac{250594.3}{-j100^3 - 59.72404(100^2) + 0.834(100j) + 2505943.3}\right)$$

~~999925.0378~~ $\div$ ~~2.506×10^{-7}~~

$$= 20\log\left(\frac{2505943.3}{\sqrt{(3103183)^2 + (-999906.6)^2}}\right)$$

$$\boxed{20\log|H_3(j\Omega)| = -2.76\ dB} \approx -3dB$$

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Bilinear Transformation :

Q: A digital LPF is req. to meet following specificⁿ.
a) Monotonic passband & stopband.
b) −3.01dB, cutoff freq. of 0.5π rad.
c) SB attenuation of 15 dB at 0.75π rad.
Find Register Funcⁿ. H(z) & the differential equⁿ. realizⁿ. Use Bilinear Transformation.

→

i) $\Omega'_p = \frac{2}{T}\tan\left(\frac{w_p}{2}\right) = 2\tan\left(\frac{0.5\pi}{2}\right) = 2$ rad/sec.

$\Omega'_s = \frac{2}{T}\tan\left(\frac{W_s}{2}\right) = 2\tan\left(\frac{0.75\pi}{2}\right) = 4.828$ rad/sec.

$K_p = -3.01\text{dB}, \quad \Omega'_p = 2\text{rad/sec}$
$K_s = 15\text{dB}, \quad \Omega'_s = 4.828 \text{ rad/sec}.$

ii) $$N = \frac{\log\left[\left(10^{\frac{-k_p}{10}}-1\right)/\left(10^{\frac{-k_s}{10}}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}$$

$$= \frac{\log\left[\left(10^{0.301}-1\right)/\left(10^{1.5}-1\right)\right]}{2\log\left(2/4.828\right)} = \frac{\log[0.03265]}{2(-0.3827)}$$

$$N = 1.9416$$

$$\boxed{N=2}$$

iii) $$\Omega_c = \frac{\Omega'_p}{\left(10^{\frac{-k_p}{10}}-1\right)^{1/2N}} = \frac{2}{\left(10^{0.301}-1\right)^{1/2(2)}} = 2 \text{ rad/sec}$$

{Write Butterworth Polynomial table}

$$H(s) = \frac{1}{(s^2+\sqrt{2}s+1)}$$

LP → LP transformation,
$s \to s/2 \to s/\Omega_c$

$$= \frac{1}{\frac{s^2}{4}+\sqrt{2}\frac{s}{2}+1}$$

$$\boxed{H(s) = \frac{4}{s^2+2\sqrt{2}s+4}}$$

iv) $H(z) = H_2(s)\Big|_{s \to \frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})}}$ {Bilinear Transformation}

$$H(z) = \frac{4}{\left(\frac{2(1-z^{-1})}{(1+z^{-1})}\right)^2 + 2.828\left(\frac{2(1-z^{-1})}{(1+z^{-1})}\right) + 4}$$

$$= \frac{4}{\frac{4(1-2z^{-1}+z^{-2})}{(1+z^{-1})^2} + 5.6568\frac{(1-z^{-1})}{(1+z^{-1})} + 4}$$

$$= \frac{4(1+z^{-1})^2}{4-8z^{-1}+4z^{-2}+5.6568-5.6568z^{-2}+4+4z^{-2}+8z^{-1}}$$

$$= \frac{(1+z^{-1})^2}{1-2z^{-1}+z^{-2}+1.4142-1.4142z^{-2}+1+z^{-2}+2z^{-1}}$$

$$H(z) = \frac{(1+z^{-1})^2}{3.4142+0.5858z^{-2}}$$

v) $z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{1+2e^{-j\omega}+e^{-j2\omega}}{3.4142+0.5858e^{-j2\omega}}$$

$$= \frac{1+2(\cos\omega - j\sin\omega)+(\cos 2\omega - j\sin 2\omega)}{3.4142+0.5858(\cos 2\omega - j\sin 2\omega)}$$

(a) $\omega_p = 0.5\pi$

$$20\log|H(\omega)|\Big|_{\omega=0.5\pi} = \frac{1+2(0-j)+(-1-j(0))}{3.4142+0.5858(-1-j0)}$$

$$= 20\log\left|\frac{0-2j}{2.828}\right| \qquad a+jb = \sqrt{a^2+b^2} = \sqrt{0^2+2^2} = 2$$

$$= -3.01\ dB$$

(b) $\omega_s = 0.75\pi$: $\frac{1+2(-0.707-j0.707)+(0-j(-1))}{3.4142+0.5858(0-j(-1))}$

$$20\log|H(\omega)| = 20\log(0.\cancel{5854}169)$$

$$= -15.43\ dB$$

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Differential eqⁿ: $H(z) = \frac{Y(z)}{X(z)} = \frac{1+2z^{-1}+z^{-2}}{3.4142 + 0.5858z^{-2}}$

$\Rightarrow 3.4142Y(z) + 0.5858\,Y(z)\,z^{-2} = X(z) + 2X(z)z^{-1} + X(z)z^{-2}.$

$\Rightarrow 3.4142\,y(n) + 0.5858\,y(n-2) = x(n) + 2x(n-1) + x(n-2)$

$$y(n) = \frac{x(n) + 2x(n-1) + x(n-2) - 0.5857\,y(n-2)}{3.4142}$$

$$y(n) = 0.29289\,x(n) + 0.58578\,x(n-1) + 0.2928\,x(n-2) - 0.\cancel{01472}\,171577\,y(n-2)$$

Q: Determine H(z) of lowest order Chebyshev Filter to meet the following specificⁿ:

a) 3dB ripple in PB $0 \leq |\omega| \leq 0.3\pi$.

b) atleast 20dB attenuation in SB $0.6\pi \leq |\omega| \leq \pi$. Use Biliear Transformⁿ.

→ $K_p = -3dB$, $\omega_p = 0.3\pi$

$K_s = -20dB$, $\omega_s = 0.6\pi$.

i) $\Omega'_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = 1.019$ rad/sec

$\Omega'_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = 2.75$ rad/sec

ii) a) Selectivity factor: $K = \frac{\Omega'_p}{\Omega'_s} = 0.37$

Discrimⁿ. Factor: $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}}$

$d = \sqrt{\frac{(1-0.292)^{-2}-1}{0.1^{-2}-1}}$

$\boxed{d = 0.1}$

$N = \frac{\cosh^{-1}(1/0.1)}{\cosh^{-1}(1/0.37)} = 1.81$

$\boxed{N = 2}$

$K_p = 20\log(1+\delta_p)$

$-3 = 20\log(1+\delta_p)$

$\boxed{\delta_p = -0.292}$

$20\log\delta_s = K_s$

$20\log\delta_s = -20$

$\boxed{\delta_s = 0.1}$

$$a=\frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/N}-\frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-1/N}$$

$$=\frac{1}{2}(2.422)^{1/2}-\frac{1}{2}(2.422)^{-1/2}$$

$$\boxed{\begin{aligned}a&=0.45691\\ b&=0.65925\end{aligned}}$$

iii) $K_p=20\log\left[\frac{1}{\sqrt{1+\varepsilon^2}}\right]$

$-3=20\log\left[\frac{1}{\sqrt{1+\varepsilon^2}}\right]$

$\boxed{\varepsilon=0.995263}$

K	$\sigma_k=-a\sin(2k-1)\frac{\pi}{2N}$ $=-0.45691\sin(2k-1)\pi/2(2)$	$\Omega_k=b\cos(2k-1)\pi/2N$ $=1.099\cos(0.78539(2k-1))$
1	-0.32308	0.7771
2	-0.323	-0.7771

$$H_2(s)=\frac{K_N}{(s-s_1)(s-s_2)}$$

$$=\frac{K_N}{(s+0.32308-j0.7771)(s+0.323+j0.7771)}$$

$$=\frac{K_N}{s^2+0.644899s+\underbrace{0.707838}_{b_0}}$$

$N\rightarrow$ even $=2$.

$$K_n=\frac{b_0}{\sqrt{1+\varepsilon^2}}=\frac{0.707838}{\sqrt{1+0.995263^2}}=0.5017$$

$$\therefore H_2(s)=\frac{0.5017}{s^2+0.644899s+0.707838}$$

- Apply LP-LP Transform^n. $s\rightarrow s/\Omega_p=s/1.019 \rightarrow \frac{0.5017}{0.963s^2+0.6327s+0.70783}$

- $$H_2(s)=\frac{0.52}{s^2+0.657185s+0.73502}$$

$s\rightarrow\frac{2}{T}\frac{(1-z^{-1})}{(1+z^{-1})} \longrightarrow H_2(z)=\frac{(1+z^{-1})^2}{5.18519z^{-2}-15.3846z^{-1}+11.6129}$.

$z\rightarrow e^{j\omega}=\cos\omega+j\sin\omega$.

- $20\log|H(\omega)|_{at\ \Omega_p}=?$ $\approx -3dB$
- $20\log|H(\omega)|_{at\ \Omega_s}=?$ $\approx -20dB$.

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Impulse variant transforⁿ Technique:

1) A 3^{rd} Order Butterworth LPF has $H(s) = \frac{1}{(s+1)(s^2+s+1)}$. Design $H(z)$ using impulse variant technique.

→

$$H(s) = \frac{1}{(s+1)(s^2+s+1)} \quad \left\{ s^2+s+1 = (s+0.5-j0.866)(s+0.5+j0.866) \right.$$

$$= \frac{1}{(s+1)(s+0.5-j0.866)(s+0.5+j0.866)}$$

$$H(s) = \frac{C_1}{s+1} + \frac{C_2}{(s+0.5-j0.866)} + \frac{C_2^*}{(s+0.5+j0.866)}$$

$$C_1 = 1, \quad C_2 = 0.577\, e^{-j2.62}, \quad C_2^* = 0.557\, e^{j2.62}$$

$$s_1 = -1, \quad s_2 = -0.5+j0.866, \quad s_3 = 0.5-j0.866$$

Impulse invariant technique,

$$H(z) = \sum_{i=1}^{3} \frac{C_i}{1-e^{s_i T} z^{-1}}$$

$$= \frac{C_1}{1-e^{s_1 T} z^{-1}} + \frac{C_2}{1-e^{s_2 T} z^{-1}} + \frac{C_2^*}{1-e^{s_3 T} z^{-1}}$$

$$H(z) = \frac{1}{1-e^{-T} z^{-1}} + \frac{0.577\, e^{-j2.62}}{1-e^{(-0.5+j0.866)T} z^{-1}} + \frac{0.577\, e^{j2.62}}{1-e^{(-0.5-j0.868)T} z^{-1}}$$

$$e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$= \frac{1}{1+e^{-T} z^{-1}} + \frac{0.577\, e^{-j2.62}\left(1-e^{(-0.5-j0.866)T} z^{-1}\right) + 0.577\, e^{j2.62}\left(1-e^{(0.5-j0.8\ldots)} z^{-1}\right)}{1-e^{(-0.5+j0.866)T} z^{-1} + 1-e^{(-0.5-j0.866)T} z^{-1}}$$

$$H(z) = \frac{1}{1+e^{-T} z^{-1}} + \frac{2(0.577)\cos(2.67) - 2.055\, e^{-0.5T}\cos(2.62+0.2467)}{1-2e^{-0.5T}\cos(0.866T)e^{-T} + e^{-T} z^{-2}}$$

2.) Let $H_a(s) = \frac{1}{(s+a)^2+b^2}$. $H(s)$ - 2^{nd} order Analog T. Func^n.

ST. digital TF $H(z)$ is given by a) $H(z) = \frac{e^{-aT}\sin bT z^{-1}}{1-2e^{-aT}\cos bT z^{-1} + e^{3aT} z^{-2}}$

b) $H(z)$ when $H(s) = \frac{1}{s^2+2s+2}$.

$\rightarrow$ $(s+a)^2+b^2 = (s+a-jb)(s+a+jb)$.

$$H_a(s) = \frac{1}{(s+a-jb)(s+a+jb)} = \frac{C_1}{(s+a-jb)} + \frac{C_1^*}{(s+a+jb)}$$

$$b = C_1(s+a+jb) + C_1^*(s+a-jb).$$

* $s = -a-jb$;

$$b = C_1^*(-2jb)$$

$$C_1^* = \frac{-1}{2j}$$

* $s = -a+jb$,

$$b = C_1(2jb)$$

$$C_1 = \frac{1}{2j}$$

Impulse invariance Technique: $H(z) = \sum_{i=1}^{N} \frac{C_i}{1-e^{S_iT}z^{-1}}$; $s_1 = -a+jb$, $s_2 = -a-jb$.

$$H(z) = \frac{1/2j}{1-e^{(-a+jb)T}z^{-1}} + \frac{-1/2j}{1-e^{(-a-jb)T}z^{-1}}$$

$$= \frac{\frac{1}{2j}(1-e^{(-a-jb)T}z^{-1}) - \frac{1}{2j}(1-e^{(-a+jb)T}z^{-1})}{1-2e^{-aT}\cos bT z^{-1} + e^{-2aT}z^{-2}}$$

$$H(z) = \frac{e^{-aT}\sin bT z^{-1}}{1-2e^{-aT}\cos bT z^{-1} + e^{-2aT}z^{-2}}$$

Denominator

$$(1-e^{(-a+jb)T}z^{-1})(1-e^{(-a-jb)T}z^{-1})$$

$$= 1-e^{(-a-jb)T}z^{-1} - e^{(-a+jb)T}z^{-1} + e^{(-a+jb)T}e^{(-a-jb)T}z^{-2}$$

$$= 1-e^{-aT}e^{-jbT}z^{-1} - e^{-aT}e^{+jbT}z^{-1} + e^{-2aT}e^{+jbT-jbT}z^{-2}$$

$$= 1-e^{-aT}z^{-1}(e^{-jbT}+e^{jbT})z^{-1} + e^{-2aT}z^{-2} = 1-2e^{-2a}$$

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3.) $H_a(s) = \frac{s+a}{(s+a)^2+b^2}$. ($2^{nd}$ order Analog T.F. ST. 2^{nd} order digital TF $H(s)$ is $H(z) = \frac{1-e^{-aT}\cos bT\, z^{-1}}{1-2\cos bT\, e^{-aT}z^{-1}+e^{-2aT}z^{-1}}$.

$\rightarrow$ $(s+a)^2+b^2 = (s+a-jb)(s+a-jb)$

$s_1 = -a+jb$

$s_2 = -a-jb$. $\quad H_a(s) = \frac{s+a}{(s+a-jb)(s+a+jb)}$

$= \frac{C_1}{s+a-jb} + \frac{C_1^*}{(s+a+jb)}$

Let $s = -a+jb$

$\frac{s+a}{\cancel{(s+a)}} = C_1(s+a-jb) + C_1^*(s+a+jb)$

$-a+jb+a = C_1(-a+jb+a-jb) + C_1^*(-a+jb+a+jb)$

$jb = C_1^* 2jb$

$\boxed{C_1^* = 1/2}$ ||| ^{ly} $C_1 = 1/2$

$H(s) = \frac{1/2}{s+a+jb}$ {Impulse invariance technique}

$H(z) = \sum_{i=1}^{N} \frac{C_i}{1-e^{s_iT}z^{-1}} = \frac{1/2}{1-e^{(-a+jb)T}z^{-1}} + \frac{1/2}{1-e^{(-a-jb)T}z^{-1}}$

$= \frac{\frac{1}{2}(1-e^{(-a-jb)T}z^{-1}) + \frac{1}{2}(1-e^{(-a+jb)T}z^{-1})}{(1-e^{(-a+jb)T}z^{-1})(1-e^{(-a-jb)T}z^{-1})}$

$= \frac{\frac{1}{2}(1-z^{-1}e^{-aT}z^{-1}e^{-jbT}) + \frac{1}{2}(1-z^{-1}e^{-aT}.e^{jbT}z^{-1})}{1-e^{(-(a-jb)T}z^{-1}-e^{(a+jb)T}z^{-1}+e^{-2aT}z^{-2}}$

$= \frac{1-e^{-aT}\cos bT z^{-1}}{1-2\cos bT e^{-aT}z^{-1}+e^{-2aT}z^{-2}}$ //

$z^{-1}\left(\frac{e^{-jbT}+e^{jbT}}{2}\right) + e^{-2aT}z^{-2}$

$= 1-2e^{-aT}\cos bT z^{-1}+e^{2aT}z^{2}$

4.) Transform Analog filter $H_a(s) = \frac{(s+1)}{s^2+5s+6}$.
$T = 0.1$ sec.

$\rightarrow$ $s^2+5s+6 \rightarrow (s+2)(s+3)$

$H_a(s) = \frac{s+1}{(s+2)(s+3)} \Rightarrow \frac{C_1}{s+2} + \frac{C_2}{s+3}$.—(1)

$= \frac{C_1(s+3)+C_2(s+2)}{(s+2)(s+3)}$

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$e^{j\theta} = \cos\theta + j\sin\theta$

$S+1 = C_1(S+3) + C_2(S+2)$ — (2)

$S = -3$; $\boxed{C_2 = 2}$

$S = -2$, $\boxed{C_1 = 1}$

$$H_a(S) = \frac{-1}{S+2} + \frac{2}{S+3} \quad \text{— (3)}$$

Comparing (1) & (3),

$P_1 = -2$, $P_2 = -3$, $C_1 = -1$, $C_2 = 2$.

WKT, $$H(z) = \sum_{k=1}^{2} \frac{C_k}{1 - z^{-1} e^{P_k T}} = \frac{C_1}{1 - z^{-1} e^{P_1 T}} + \frac{C_2}{1 - z^{-1} e^{P_2 T}}$$

$$H(z) = \frac{-1}{1 - z^{-1} e^{-2(0.1)}} + \frac{2}{1 - z^{-1} e^{-3(0.1)}}$$

$$= \frac{-1}{1 - z^{-1}(0.8187)} + \frac{2}{1 - z^{-1}(0.7408)}$$

$$= \frac{-1}{1 - \frac{0.8187}{z}} + \frac{2}{1 - \frac{0.7408}{z}}$$

$$= \frac{-z}{z - 0.8187} + \frac{2z}{z - 0.7408}$$

$$H(z) = \frac{-z(z - 0.7408) + 2z(z - 0.8187)}{(z - 0.8187)(z - 0.7408)}$$

$$= \frac{z^2 - 0.8966z}{z^2 - 1.5592z + 0.6065}$$

$\therefore (\div z^2)$

$$= \frac{1 - 0.8966z^{-1}}{1 - 1.5592z^{-1} + 0.6065z^{-2}}$$

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Impulse Invariante Transformation :

1.) A digital LPF is req. to meet the following specif^n.

$$20\log|H(\omega)|_{\omega=0.2\pi} \geq -1.9328\text{ dB}.$$

$$20\log|H(\omega)|_{\omega=0.6\pi} <= -13.9794\text{ dB}.$$

It has Maximally Flat Freq. response.
Find H(z) to meet the specif^n. using Impulse inv r. transfo

→ We have to design BW LP filter.

$K_p = -1.9328$ dB , $\omega_p = 0.2\pi$

$K_s = -13.9794$ dB , $\omega_s = 0.6\pi$ $T = 1$ sec

- $\Omega_p = \frac{\omega_p}{T} = 0.2\pi$ • $\Omega_s = \frac{\omega_s}{T} = 0.6\pi$ rad/sec

$$\boxed{N = \frac{\log\left[\left(10^{\frac{-K_p}{10}}-1\right)/\left(10^{\frac{-K_s}{10}}-1\right)\right]}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)}}$$

$$= \frac{\log\left[(0.56056/23.999)\right]}{2\log 0.333} = 1.7096 \quad \boxed{N=2}$$

$$B_N(s) = s^2+\sqrt{2}s+1.$$

$$H_2(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

$$\boxed{\Omega_c = \frac{\Omega_p}{\left(10^{\frac{-K_p}{10}}-1\right)^{1/2N}}} = \frac{0.2\pi}{(0.56056)^{1/4}} = 0.726146\text{ rad/sec}.$$

LP – LP Transformation. $s \to s/\Omega_c$

$$H_2(s)\Big|_{s\to\frac{s}{\Omega_c}} = \frac{1}{\left(\frac{s}{0.726146}\right)^2+\sqrt{2}\left(\frac{s}{0.726146}\right)+1}$$

$$= \frac{1}{1.896528s^2+1.94456s+1}.$$

$$H_2(s) = \frac{0.52728}{s^2 + 1.02690\,s + 0.52728}$$

$$s^2 + 2a\,s + \cancel{a^2} + b^2 = (s+a)^2 + b^2$$

$2as = 1.02690\,s$ $\qquad$ $a^2 + b^2 = 0.52728$

$\boxed{a = 0.512929}$ $\qquad$ $b^2 = 0.52728 - (0.512929)^2$

$\boxed{b = 0.51398}$

$$H_2(s) = \frac{0.52728}{(s + 0.152929)^2 + (0.51398)^2}$$

Impulse Invariance technique: $(T = 1\,\text{sec})$

$$\boxed{\frac{b}{(s+a)^2 + b^2} = \frac{e^{-aT}\sin bT\, z^{-1}}{1 - 2e^{-aT}\cos bT\, z^{-1} + e^{-2aT} z^{-2}}}$$

$$= \frac{e^{-0.512929}\sin(0.52635)\,z^{-1}}{1 - 2e^{-0.512929T}\cos(0.52635)\,z^{-1} + e^{-2(0.512929T)}\,z^{-2}}$$

$$= \frac{0.301\,z^{-1}}{1 - 1.0433\,z^{-1} + 0.3585\,z^{-2}}$$

$z = e^{j\omega}$

$z^{-1} = e^{-j\omega}$

$$H_2(\omega) = \frac{0.301\,e^{-j\omega}}{1 - 1.0433\,e^{-j\omega} + 0.3585\,e^{-2j\omega}}$$

$$H_2(\omega)\Big|_{\text{at } \omega_p = 0.2\pi} = \frac{0.301(0.8090 - j\,0.5877)}{(1 - 1.0433(0.8090 - j0.5877) + 0.3585(0.3090 - j0.951))}$$

$$20\log|H_2(\omega)|_{\omega_p = 0.2\pi} = \frac{0.30085}{\sqrt{0.266742^2 + 0.272178^2}} = 20\log(0.7894419)$$

$$= -2.0535\ \text{dB}$$

$$H_2(\omega)\Big|_{\omega_p = 0.6\pi} = \frac{0.301(-0.309 - j\,0.951)}{(1 - 1.0433(-0.309 - j0.5877) + 0.3585(-0.809 + j0.5877))}$$

$$= 20\log\left(\frac{0.3}{\sqrt{1.03^2 + 0.82384^2}}\right)$$

$$= -12.9\ \text{dB}$$

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Q: Design a Chebyshev LP filter With :

$0.8 \leq |H(\omega)| \leq 1$, $0 \leq \omega \leq 0.2\pi$

$|H(\omega)| \leq 0.2$, $0.6\pi \leq \omega \leq \pi$

→

$\delta_p = 1-0.8 = 0.2$, $\omega_p = 0.2\pi$

$\delta_s = 0.2$, $\omega_s = 0.6\pi$

$\Omega_p = \frac{\omega_p}{T} = 0.2\pi$ rad/sec.

$\Omega_s = \frac{\omega_s}{T} = 0.6\pi$ rad/sec.

$K_p = 20\log(1-\delta_p)$
$= 20\log(1-0.2)$
$\boxed{K_p = -1.9382}$

$K_s = 20\log\delta_s$
$= 20\log 0.2$
$\boxed{K_s = -14\,dB}$

★ $K = \frac{\Omega_p}{\Omega_s} = \frac{0.2\pi}{0.6\pi} = 0.333$ (Selectivity factor)

★ $d = \sqrt{\frac{(1-\delta_p)^{-2}-1}{\delta_s^{-2}-1}} = 0.15309$ (Discrimination factor)

★ $N = \frac{\cosh^{-1}(1/d)}{\cosh^{-1}(1/k)} = \frac{2.5645}{1.76380} = 1.45395$

$\boxed{N=2}$

★ $K_p = 20\log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$ or ★ $(1-\delta_p) = \frac{1}{\sqrt{1+\varepsilon^2}}$

$\frac{-1.9382}{20} = \log\left(\frac{1}{\sqrt{1+\varepsilon^2}}\right)$

$10^{-0.09664} = \frac{1}{\sqrt{1+\varepsilon^2}}$

(Squaring both sides)

$0.6407 + 0.6407\varepsilon^2 = 1$

$\boxed{\varepsilon = 0.7504}$

$$a = \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{1/N} - \frac{1}{2}\left[\frac{1+\sqrt{1+\varepsilon^2}}{\varepsilon}\right]^{-1/N}$$

$$a = \frac{1}{2}(1.732) - \frac{1}{2}(0.57735)$$

$$\boxed{a = 0.57735}$$

$$\boxed{b = 1.547}$$

K	$\sigma_K = -0.5775 \sin(2k-1)(0.785398)$	$\Omega_K = 1.1547 \cos(2k-1) 0.785398$
	$\sigma_K = -a \sin(2k-1)\pi/2N$	$\Omega_K = b \cos(2k-1)\pi/2N$
1	-0.4083	0.81649
2	-0.40835	-0.81649

$$H_2(s) = \frac{K_N}{(s-s_1)(s-s_2)}$$

$$= \frac{K_N}{(s-(0.4083+j0.81649))(s-(-0.4835-j0.81649))}$$

$$= \frac{K_N}{s^2+0.8166s+\underbrace{0.83335}_{b_0}}$$

$$N \to \text{Odd}, \quad K_N = \frac{b_0}{\sqrt{1+\varepsilon^2}} = \frac{0.83335}{1.25} = 0.66653$$

$$H_2(s) = \frac{0.6665}{s^2+0.8166s+0.8335}$$

$$H_2(s)\Big|_{\frac{s}{\omega_p} = \frac{s}{0.2\pi}} = \frac{0.66653}{\left(\frac{s^2}{0.2\pi^2}\right) + \frac{0.8166s}{0.2\pi} + 0.8335}$$

$$= \frac{0.66653}{s^2 + 0.513095 + 0.3290564}$$

$$\{s^2 + 2as + a^2 + b^2\}$$

$$2as = 0.513095$$

$$\boxed{a = 0.2565}$$

$$a^2 + b^2 = 0.3290564.$$

$$b = \sqrt{0.3290564 - 0.2565^2}$$

$$\boxed{b = 0.51329}$$