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Subject:

Section **2**

Date / /

A)  $T(n) = 7T(\frac{n}{2}) + C(n^2)$  for  $n > 1$  and  $T(1) = 1$

$$T(\frac{n}{2}) = 7T(\frac{n}{4}) + C(\frac{n^2}{4}) \rightarrow T(n) = 7(7T(\frac{n}{4}) + C(\frac{n^2}{4})) + C(n^2)$$

$$T(n) = 7^2 T(\frac{n}{2^2}) + 7C(\frac{n^2}{2^2}) + C(n^2)$$

$$T(\frac{n}{4}) = 7T(\frac{n}{2^3}) + C(\frac{n^2}{2^4}) \rightarrow T(n) = 7^2(7T(\frac{n}{2^3}) + C(\frac{n^2}{2^4})) + 7C(\frac{n^2}{2^2}) + C(n^2)$$

$$T(\frac{n}{8}) = 7T(\frac{n}{2^4}) + C(\frac{n^2}{2^6}) \rightarrow T(n) = 7^3(7T(\frac{n}{2^4}) + C(\frac{n^2}{2^6})) + 7^2 C(\frac{n^2}{2^4}) + 7C(\frac{n^2}{2^2}) + C(n^2)$$

$$7^4 T(\frac{n}{2^4}) + 7^3 C(\frac{n^2}{2^6}) + 7^2 C(\frac{n^2}{2^4}) + 7C(\frac{n^2}{2^2}) + C(n^2)$$

$$7^k T(\frac{n}{2^k}) + 7^{k-1} C(\frac{n^2}{4^{k-1}}) + 7^{k-2} C(\frac{n^2}{4^{k-2}}) + 7C(\frac{n^2}{4}) + C(n^2)$$

$$\frac{n}{2^k} = 1 \rightarrow n = 2^k \quad \boxed{\log_2 n = k}$$

$$7^{\log_2 n} + C(n^2) \left( \frac{7^{k-1}}{4^{k-1}} + \frac{7^{k-2}}{4^{k-2}} + \frac{7}{4} + 1 \right)$$

$$O(7^{\log_2 n}) + O(n^2 \cdot 7^{\log_2 n})$$

$$n^{\log_2 7} + O(n \sqrt{2 + \log_2 7}) \quad n^{\log_2 7}$$

$$\boxed{O(n^{2 + \log_2 7})} \quad \leftarrow$$



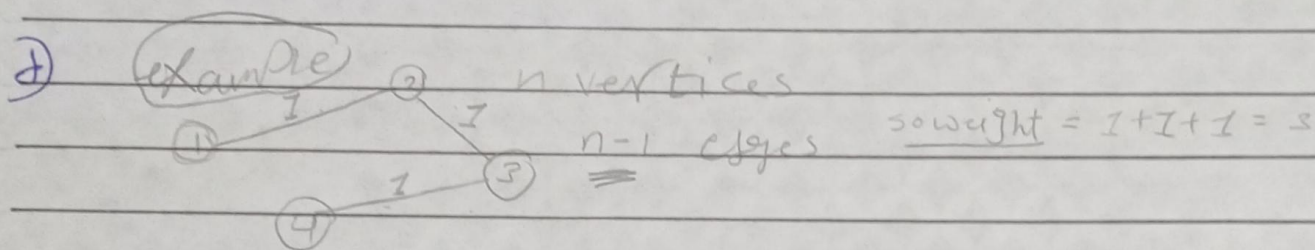
- ⑥  $\rightarrow$  inserting  $m$  elements into min-heap takes  $O(m \log m)$   
 $\rightarrow$  heap operations (extraction and insertion) take  $O(\log m)$   
 since there are  $m$  elements total this step takes  $O(n \log m)$

$\rightarrow$  ~~Complexity~~ <sup>total</sup>  $m \log m + n \log m = n \log m$

c) Huffman) Build heap  $\rightarrow O(n)$

2) Rebuild heap  $\rightarrow O(\log n)$  the time complexity  $\rightarrow O(n \log n)$

store and remove  $O(\log n)$



e) input: Array  $M$  of  $k$  integers

output: Pair  $(x, y)$  such that  $|x+y|$  is minimized

① sort array  $M$  in non-decreasing order  $\rightarrow O(k \log k)$

2) initialize two pointers: set  $i=0$  (left pointer)

set  $j=k-1$  (right pointer)

3) set  $\text{min\_sum} = \text{infinity}$

set  $\text{best\_pair} = (\text{None}, \text{None})$

4) while  $i < j$

$S = M[i] + M[j]$

5) update  $\text{min\_sum} = S$

update  $\text{best\_pair} = (M[i], M[j])$

6) if  $S > 0$

$j = j - 1$

if  $S < 0$   $i = i + 1$

ret.  $\text{best\_pair}$



$$\text{out-degree}(2) = 1$$

$$\text{in-degree}(2) = 2$$

$$\text{out-degree}(1) = 1$$

$$\text{in-degree}(1) = 0$$

$$\text{in-degree}(3) = 1$$

$$\text{out-degree}(3) = 0$$

$$\text{in-degree}(4) = 2$$

$$\text{out-degree}(4) = 1$$

$$\text{in-degree}(5) = 0$$

$$\text{out-degree}(5) = 0$$

$$\text{in-degree}(6) = 0$$

$$\text{out-degree}(6) = 2$$

