Heap & Priority queues

Priority queues

Let *E* be a set mapped by a priority function *p*. We call a priority queue a data type that allows us to:

- represent *E*,
- add an element, with a given **priority**, to *E*,
- remove an element with the **lowest/highest** priority.

Implementations

Structure	Search max/min	Insertion	Deletion
Unsorted Array	`O(n)`	`O(1)`	`O(n)`
Unsorted List	`O(n)`	`O(1)`	`O(n)`
Sorted Array	`O(1)`	`O(n)`	`O(1)`
Sorted List	`O(1)`	`O(n)`	`O(1)`

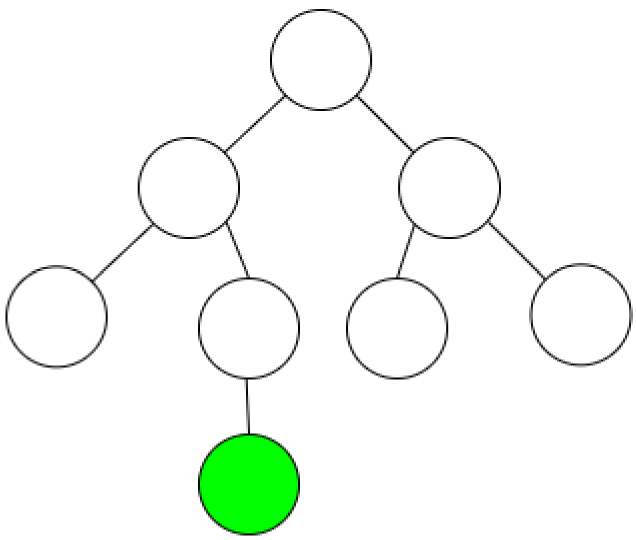
Optimised implementation

Structure	Search max/min	Insertion	Deletion
Heap	`O(1)`	`O(log(n))`	`O(log(n))`

Heap

Level of a node

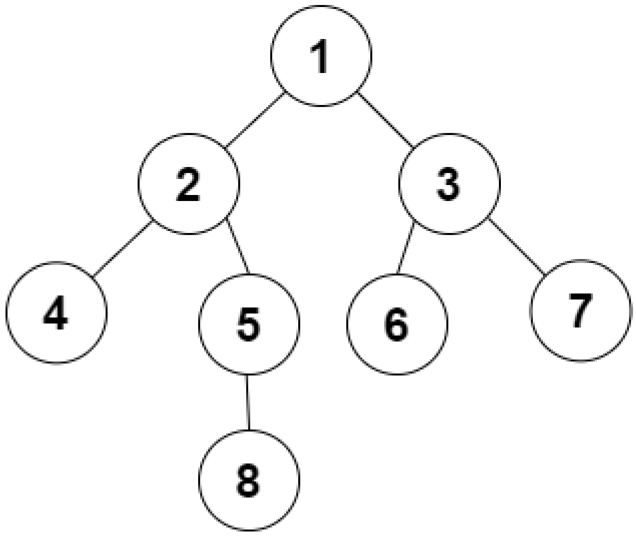
The level of a node X in a tree A is the **number of edges** on the path from the root node to X.



The level of the green node is **3**.

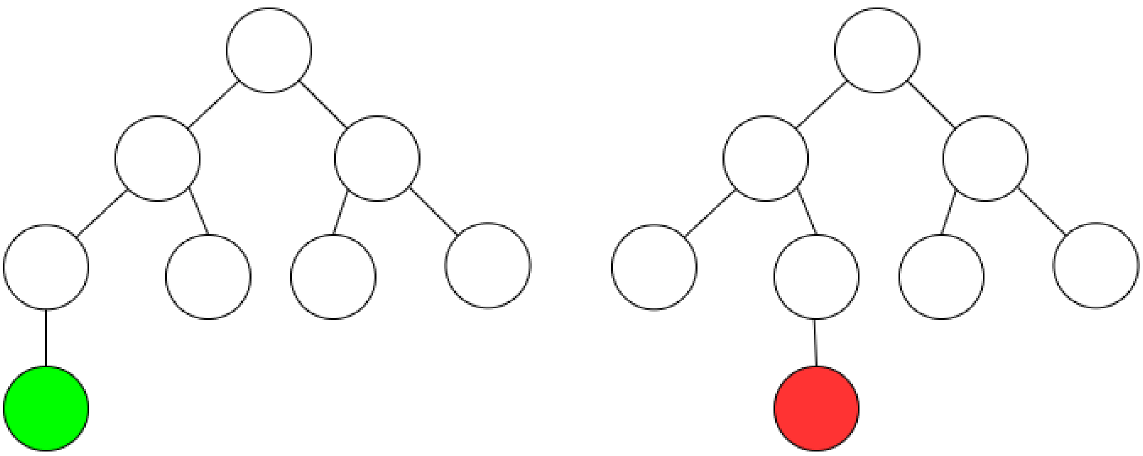
Hierarchical numbering

For a binary tree A, hierarchical numbering consists of numbering, starting from 1, the nodes from *top to bottom* and for each level from the *left to the right*.



Complete binary tree

A complete binary tree is a binary tree in which every level, **except possibly the last**, is completely filled, and all nodes are *as far left* as possible.



Complete Binary Tree Non-Complete Binary Tree

Heap

Let *E* be a set mapped by a priority function *p*. A heap representing *(E,p)* is a couple *T=(A,obj)* where *A* is a **complete tree** and *obj* is a **bijection** that maps for each node an element of *E*.

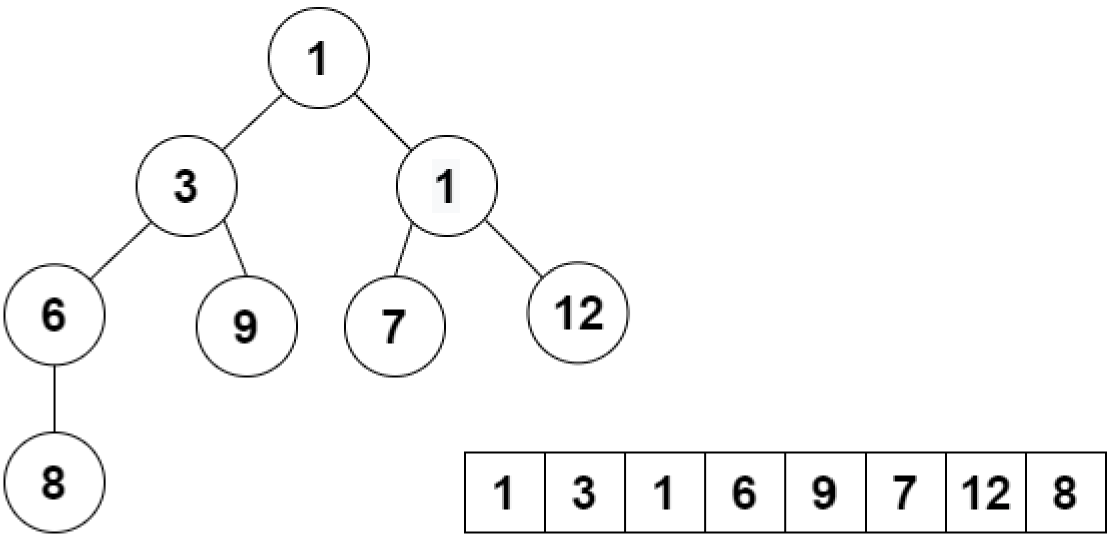
Thus, for all node *x* of *A* other than the root, *p(obj(x))>p(obj(Parent(x)))*

Structure

```
typedef struct{
    element e;
    int priority
} node;

typedef struct{
    int size;
    node* t;
} heap;
```

You may be wondering as to why we represent it as an array! Well let me explain.



The root is the node of index `0`. And for a node of index `i`, the **parent** is the node of index `i-1 div 2`, the **left child** is the node of index `2i+1` and the **right child** is the node of index `2i+2`.

Creation

```
heap create(int size) {
    node* t = (node*) malloc(size * sizeof(node));
    heap h;
    h.t = t;
    h.size = 0;
    return h;
}
```

Insertion

- Insert the element at the end of the table.
- Keep swapping withe the parent until the priority constraint is respected.

  ```
  heap insert(node o, heap h) {
    h.t[h.size] = o;
    h.size++;
    int current = h.size - 1;
  ```

```
  int parent = (current - 1) / 2;
  while (current > 0) {
      if (h.t[current].priority < h.t[parent].priority) {
          node temp = h.t[current];
          h.t[current] = h.t[parent];
          h.t[parent] = temp;
          current = parent;
          parent = (current - 1) / 2;
      }
      else
      {
          break;
      }
  }
  return h;
}
```

Deletion

IMPORTANT
In a **min heap**, we can only remove the node with **lowest priority**! In that case, it is the **root node**.

- Assign the value of the last node to the root.
- Delete the last node.
- Swap with the child node with lowest priority until the priority constraint is respected.

```
heap delete (heap h, node* o) {
  *o = h.t[0];
  h.t[0] = h.t[h.size - 1];
  h.size--;
  int current = 0;
  while (current < h.size)
  {
      int childMin = h.t[current * 2 + 1].priority > h.t[current * 2 +
2].priority ? current * 2 + 2 : current * 2 + 1;
      if (h.t[current].priority > heap.t[childMin].priority) {
          node temp = h.t[current];
          h.t[current] = h.t[childMin];
          h.t[childMin] = temp;
          current = childMin;
      }
      else {
          break;
      }
  }
  return h;
}
```