Tuning a Lamellophone Using Mathematical Approach

Tuning a Lamellophone Using Mathematical Methods

This is a basic instrument from Africa - a lamellophone - which is composed of metal plates attached to a sound box or a wooden board. Regrettably, the one at home was so badly out of tune that it was unplayable. Even though I know absolutely nothing about how to tune, I reasoned that because of the construction of the instrument, it should be possible to mathematically reason oneself into a proper setup to tune the lamellophone.

Section A. Understanding the Components of a Lamellophone

Let's look at the main parts of a lamellophone before going on with the methods of tuning. It consists of two main parts, the metal keys which are plucked and a box that serves as the drum. This box acts as a sound generator. It increases the strength of the sound without changing its frequency (the shape of the sound wave) vibrations. Thus, it is the metal keys which need to be plucked for each note and which need to be set during the tuning which determines the pitch of each note. These metal keys are attached to one end and at the other end they are free to vibrate if plucked. This particular configuration is similar to a physical model of a cantilever beam. A beam, when plucked, will vibrate at a particular frequency which can be determined by the following formula:

$$f = \frac{1.875^2}{2\pi} \cdot \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$$

Where:

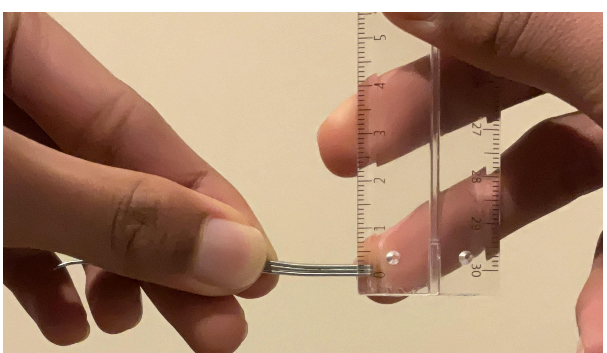
- f = frequency (Hz)
- L = key length (m)
- E = Young's modulus (Pa)
- I = second moment of area (m^4)
- ρ = density (kg/m^3)
- A = cross-sectional area (m^2)

Examining the variables that influence the frequency, the only adjustable factor in this case is the length of the metal key. This means that adjusting the key length is the primary method for tuning the instrument.

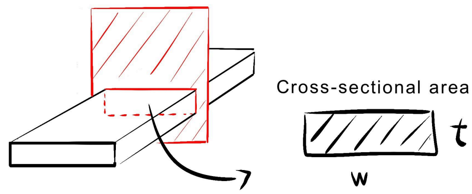
Section B. Methodology

To determine the correct key lengths for each note, I followed these steps:

1. **Measuring Key Dimensions**
   - I used a ruler to measure the width and length of each key.
   - Since the individual key thickness was too thin to measure accurately, I stacked multiple keys together and measured the total thickness. I then divided this value by the number of stacked keys to estimate the thickness of a single key.



- Using these measurements, I calculated the second moment of area and cross-sectional area needed for the frequency equation.



Equation for Second moment of area:

$$I_x = \frac{wt^3}{12}$$

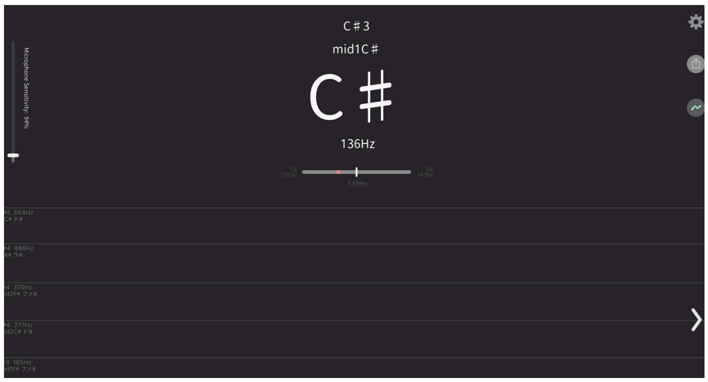
2. **Acquiring Other Necessary Variables**
   - Assuming that the material of the key is aluminum, the Young's modulus and density were obtained from a material specification database called MatLab.
   - The frequency for each of the notes has a specific value. For this experiment, numbers were referenced from a database [1].

Calculating the Required Key Lengths

- With all the necessary values for the formula, I calculated the required key length for each note, from C to B, using the mathematical model for a cantilever beam.

3. **Adjusting the Key Lengths**
   - Based on the calculated values, I physically adjusted each key's length using a ruler, ensuring that each key was set to the appropriate length according to the equation.

4. **Checking the Frequencies**
    - After tuning, I used a tuner application to measure the actual frequencies of each key and compare them to the target frequencies. The app is called PitchChecker and is available in Apple's App Store, and it takes in the sound from the device's microphone and displays the measured frequency of the sound and its note.



    - If discrepancies were found, I noted the differences for later analysis.

Section C. Calculating the Required Key Lengths for Each Note

Using the known relationship between frequency and key length, we rearrange the equation to solve for L:

$$L = \frac{1.875}{\sqrt{\frac{2\pi f}{\sqrt{\frac{EI}{\rho A}}}}}$$

For this experiment, the lamellophone keys were assumed to be made of aluminum, with the following material properties:

- **Young's modulus (E):** 68.3×10^{9} Pa
- **Second moment of area (I):** 2.5×10^{-13} m^4
- **Density (ρ):** 2700 kg/m^3

- **Cross-sectional area (A):** 3×10^{-6} m²

Using the frequency values of a standard musical scale, the calculated key lengths are as follows:

Note	Frequency (Hz)	Length (m)
C	261	0.0558
D	293	0.0525
E	329	0.0494
F	349	0.0478
G	391	0.0452
A	440	0.0428
B	493	0.0406

Section D. Comparison Between Calculated and Measured Frequencies

After I adjusted the key length based on the calculation, I measured the actual frequencies using a tuner app. However, I found that the measured values differed slightly from the calculated ones.

Note	Calculated (Hz)	Measured (Hz)	Error (Hz)
C	261	268	+7
D	293	300	+7
E	329	347	+18

Note	Calculated (Hz)	Measured (Hz)	Error (Hz)
F	349	370	+21
G	391	416	+25
A	440	468	+28
B	493	528	+35

Section E. Possible Causes of Error

Several factors may have contributed to these errors:

1. Measurement Errors

The values that require dimension of the key, such as the cross-sectional area and second moment of area, were measured using an analog ruler. Small errors in these measurements could have caused considerable difference in calculation, especially since the second moment of area depends on the cube of the thickness. The installation of the keys were also based on approximate measurements from the ruler, so that is another factor that could contribute to the error.

2. Variations in Material Properties

The Young's modulus (E) and density (ρ) values were based on theoretical data for aluminum. However, in reality, inconsistencies in manufacturing and aging of the metal would cause slight deviation from the theoretical values.

Section F. Verification of Causes of Error

To confirm that measurement inaccuracies, material property variations, and key installation errors contributed to the frequency deviations, hypothetical calculations were performed by adjusting each factor individually and observing its impact on the computed frequencies. For each simulation, only the parameter getting verified is modified, and other variables are assumed to be correct as they are listed or calculated in Section C.

1. Measurement Errors

Measurement inaccuracies, particularly in determining key thickness and width, directly affect the second moment of area and cross-sectional area. If the actual dimensions were 5% larger than initially recorded, the recalculated values for these parameters would be:

- **Second moment of area (I):** 3.04×10^{-13} m^4
- **Cross-sectional area (A):** 3.31×10^{-6} m^2

Reference frequency (Hz)	Simulated frequency (Hz)	Measured frequency (Hz)	Error of simulated to reference (Hz)
261	274	268	+13
293	309	300	+16
329	349	347	+20
349	373	370	+24
391	417	416	+26
440	466	468	+22
493	517	528	+24

Using these adjusted values, the newly calculated frequencies showed relatively smaller deviations from the measured values, confirming that minor measurement errors can significantly impact tuning accuracy.

2. Key Length Inaccuracy

Let's say that due to human error during the installation of keys using the ruler, the key lengths in reality were 1 mm shorter than what it was supposed to be.

- $L_{hypothetical} = L - 0.001$

Reference frequency (Hz)	Simulated frequency (Hz)	Measured frequency (Hz)	Error of simulated to reference (Hz)
261	271	268	+10
293	306	300	+13
329	347	347	+18
349	371	370	+22
391	416	416	+25
440	465	468	+25
493	518	528	+25

Slight misalignment during key installation could result in unintended variations in key length. If the actual lengths were 1 mm shorter than the intended values, the recalculated frequencies showed an increase compared to the reference frequency. The deviations observed closely matched the measured errors, indicating that even a small difference in length can have a profound effect on the final tuning.

3. Material Property Variations

Material inconsistencies, whether due to manufacturing differences or long-term deformation, also play a role in frequency discrepancies.

- A **5% increase in Young's modulus** 71.7×10^{9} Pa resulted in lower calculated frequencies, mirroring the downward shifts observed in the measured data.

Reference frequency (Hz)	Simulated frequency (Hz)	Measured frequency (Hz)	Error of simulated to reference (Hz)
261	267	268	+6
293	302	300	+9
329	341	347	+12
349	364	370	+15
391	407	416	+16
440	454	468	+14
493	504	528	+11

- A **5% decrease in density:** 2565 kg/m³ caused frequency changes as well, and the results became pretty close to the measured values

Reference frequency (Hz)	Simulated frequency (Hz)	Measured frequency (Hz)	Error of simulated to reference (Hz)
261	268	268	+7
293	302	300	+9
329	342	347	+13
349	365	370	+16
391	408	416	+17
440	455	468	+15
493	506	528	+13

The verification process confirms that measurement errors, material property variations, and key installation inaccuracies can impact the tuning of the lamellophone. Hypothetical adjustments to these factors resulted in frequency deviations that either closely matched or deviated from the measured discrepancies, showing different effects these values have on the resulting frequency and validating their role in the observed errors. This analysis highlights the necessity of precise measurements and material consistency for accurate tuning.

Section G. Possible Enhancements

In order to achieve better tuning, the following specifics need to be worked on:

Increasing Measuring Precision:

- Obtain precise measurements on the keys using more advanced equipment.

- If the manufacturer provides it, try to check the specific material characteristics.

Conclusion

I have managed to estimate the minimum key lengths for tuning the lamellophone by applying my mathematical skills. The real frequencies obtained were, however, somewhat different from the estimated frequencies due to inaccuracies associated with the measurement process, differing material properties and different styles of plucking. In fact, if the measurement was carried out with more precision and followed by some iterative tuning, the tuning could have been much better. This experiment showed that mathematical concepts can be employed in a more practical and applied manner for tuning musical instruments, which can then be improved by manual means in order to obtain the wanted musical scale.

Reference

[1] Byrd, Donald. “A Table of Musical Pitches.” *Luddy Homepage Server*, Indiana University, Aug. 2007, homes.luddy.indiana.edu/donbyrd/Teach/MusicalPitchesTable.htm.