Investigating the optimal distance to take a photo with minimal distortion

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Introduction

As a member of the yearbook staff, I was drawn into the world of photography. It is truly rewarding when I am able to capture moments that visually tell a story. However, reviewing photos from past few school events, I’ve come to notice a recurring issue: image distortion. As someone who values the quality and storytelling of the image, I needed to address this problem. By investigating the patterns of distortion and shooting distance, I aim to find a practical method to minimize distortion as much as possible. While eliminating it entirely might not be possible, I intend to define an acceptable threshold for distortion and determine optimal shooting distance to achieve it.

Quantifying distortion

An image is considered distorted when an object supposedly straight appears curved. While it is visually observable, to quantify and compare different intensity of distortion, I decided to use the concept of curvature. For a function $f(x)$, the curvature K at a point is given by:

$$K = \frac{|f''(x)|}{\left(1 + f'(x)^2\right)^{\frac{3}{2}}}$$

Methodology

In the investigation, I aim to analyze the distortion patterns in an image and then quantify and compare them using the curvature formula. In order to do this, I first took a sample picture of grid lines printed on paper and observed how the distortion affected the lines. Then I mapped the picture on a plane to do a quadratic regression on chosen distorted lines and get their equation. Finally, using the obtained equations, I calculated the curvature to see the intensity of the distortion at different points on the picture.

Set up

The experiment was done using a Canon Rebel T5i camera with EF-S 18-55mm f/3.5-5.6 IS II lens. This is the usual setup I use for my yearbook work. Lens characteristics may vary from different lenses and focal length, so data from this setup might not apply to all cameras universally.

As explained in the methodology section, I printed a grid pattern on a piece of paper and put it on the wall. The camera was placed on a stationary tripod and its height was adjusted so that the camera would be perfectly level with paper. This ensured that the distortion is caused by the lens rather than the perspective distortion or other factors. Here are the specific settings of the camera:

- Focal length: $18cm$
- Shooting distance: $10cm$
- Aspect ratio: $3:2$

Calculations

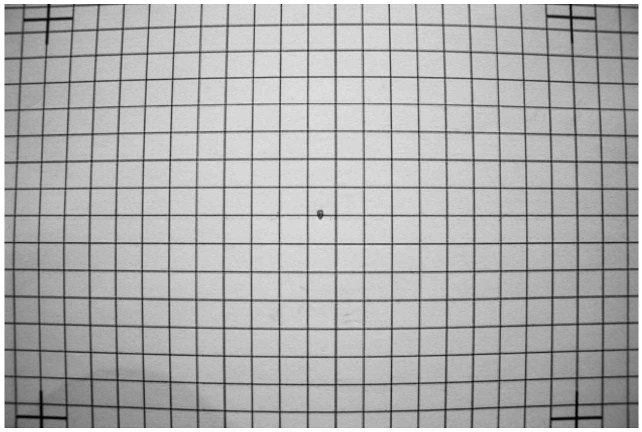


Figure 1: Grid line taken with focal length of 18mm and shooting distance of 10cm.

As shown clearly in figure 1, there is indeed a distortion. The effect is strongest at the edge where the curve of the line is more noticeable, while near the center, the effect is minimal.

Distortion in outermost line:

As a representative line, I picked the outermost line to quantify the curvature. After I mapped the image onto a plane with a size of 30 units by 20 units centered on the origin, I plotted and recorded some key points.

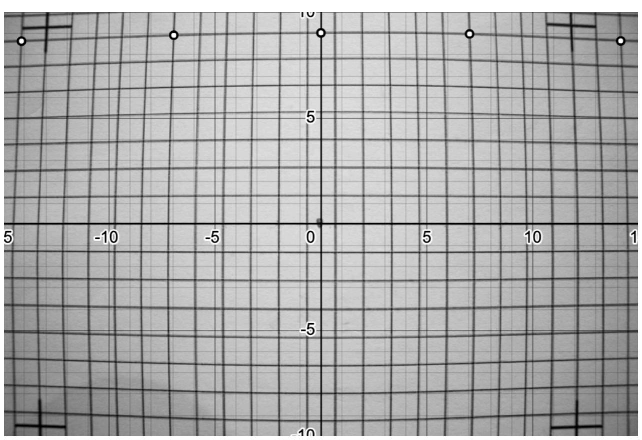


Figure 2: The image mapped on a plane and plotted for quadratic regression

x	y
-14.145	8.64
-6.95	8.92
0	9.04
7.02	8.98
14.16	8.65

Figure 3: Table of coordinates plotted on the line

Using quadratic regression, the line of best fit on these points was determined:

$$f(x) =- 0.00196x^2 + 0.00118x + 9.04$$

Calculating Curvature:

Curvature, as I mentioned earlier, is given by:

$$K = \frac{|f''(x)|}{\left(1 + f'(x)^2\right)^{\frac{3}{2}}}$$

Using the power rule, I can find the first and second derivative of $f(x)$:

$$f'(x) = -0.00392x + 0.00118$$

$$f''(x) = -0.00392$$

Now substituting the value into the formula,

$$K = \frac{|-0.00392|}{(1 + (-0.00392x+0.00118)^{\frac{3}{2}}}$$

When $x = 0$:

$$K = \frac{|-0.00392|}{(1+(-0.00392(0)+0.00118)^2)^{\frac{3}{2}}} = 0.00392$$

Thus, the curvature of the outermost curve at the Y intercept is 0.00392. Curvature helps assess the distortion by quantifying how much the curve deviates from being a straight line, or in other words, how sharp the curve bends at a specific point. The process of obtaining the curvature was repeated for several more grid lines to get more data for comparison.

Curvature of Different Lines:

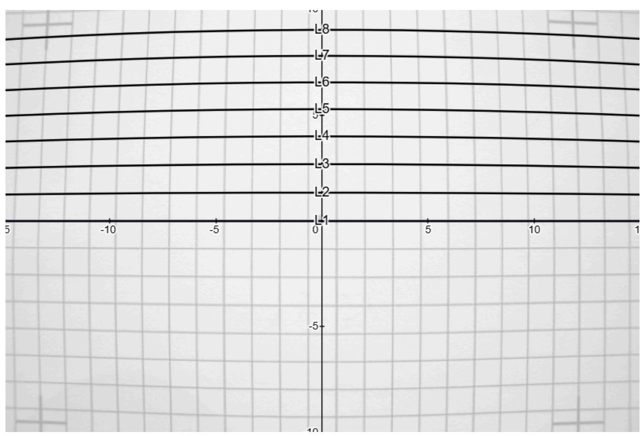


Figure 4: Lines of best fit graph for each horizontal distorted line with labels

The curvature for $L1$ to $L8$ was calculated and summarized in the table below. Then I plotted the data onto a graph to visually see the relationship between the Y intercept and the curvature.

Line name	Y intercept	Curvature
$L1$	0	0
$L2$	1.34	0.000751

$L3$	2.69	0.00157
$L4$	4.00	0.00221
$L5$	5.29	0.00276
$L6$	6.56	0.00317
$L7$	7.81	0.00364
$L8$	9.04	0.00392

Figure 6: Table of Y intercept and curvature of the lines

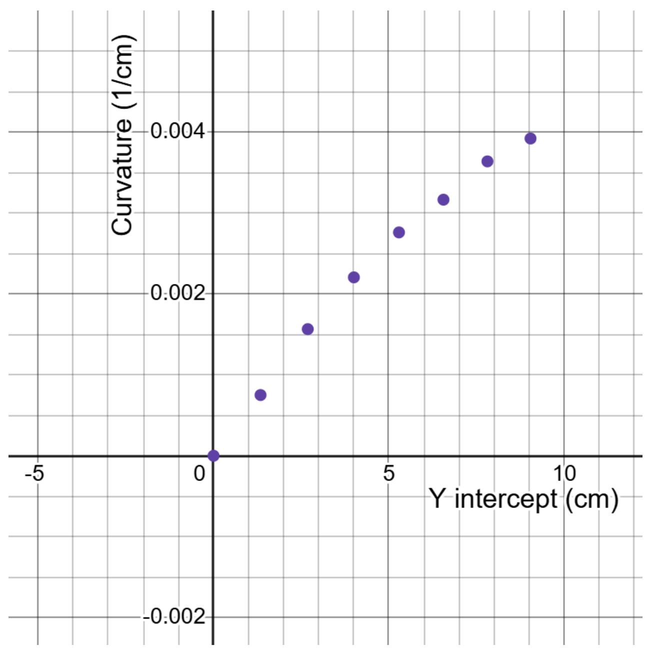


Figure 7: Data of Figure 6 plotted on a graph

The pattern of the graph confirms the curvature increases as distance from the center of the image increases, which means that the effect of distortion is the strongest at the edge. At the same time, it shows that it is impossible to take away the distortion completely from the picture since anything that takes up a larger area than a single point on the origin would have some amount of curvature. Thus, rather than attempting to eliminate distortion completely, it is more practical to set a permissible distortion and find an optimal shooting distance from there.

Defining permissible distortion

There is no universally acknowledged permissible distortion value, since it all depends on how every individual perceives it. For this investigation, I have determined the permissible distortion based on the visual evaluation. By analyzing the sample picture, grid lines and the lines of best fit, I've decided $L3$, with a curvature of 0.00157.

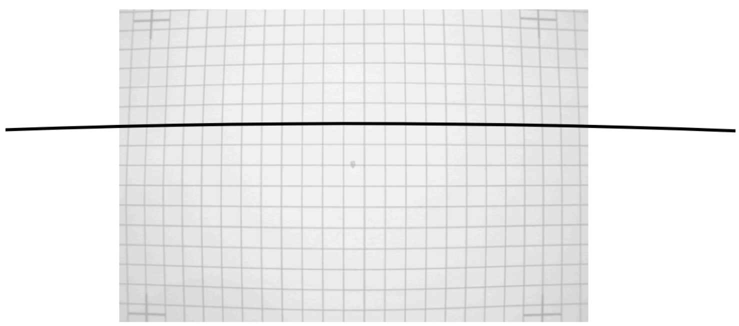


Figure 9: L3 with curvature of 0.00157 overlaid on sample picture

This is how $L3$ visually looks like. With the information of $L3$, the region of the so-called "safe zone" where an object in the picture gets the least effect of distortion can be found.

Finding "safe zone"

The highest point of this curve (the vertex) is located at $(0,\ 2.69)$. The curve is symmetrical on the bottom as well so the total height of safe zone would be:

$$2.69 \times 2 = 5.37\, units$$

By applying the aspect ratio of the picture, $3:2$, the width of the safe zone is:

$$5.37 \times \frac{3}{2} = 8.06\, units$$

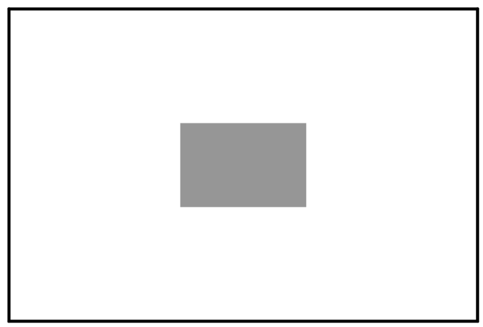


Figure 10: The visual comparison of original photo aspect to the safe zone

In Figure 10, the black border represents the original aspect, and the shaded area is the safe zone where the image distortion is minimal. By fitting the subject into the safe zone when taking the photo, the effect of distortion can be suppressed.

Optimal shooting distance

To determine the optimal shooting distance, I calculated the distance required to capture the grid line paper used for sample pictures within the safe zone. The approach I took is:

1. Calculate the angle of view for the safe zone
2. Find the shooting distance at which the A4 size paper would fit in the “safe zone”

The angle of view

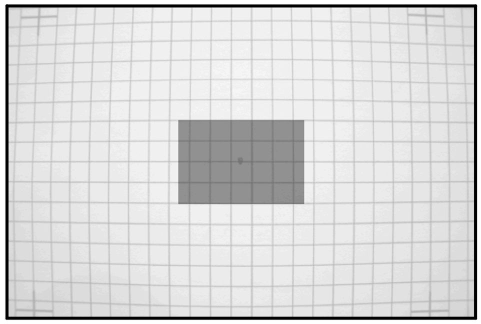
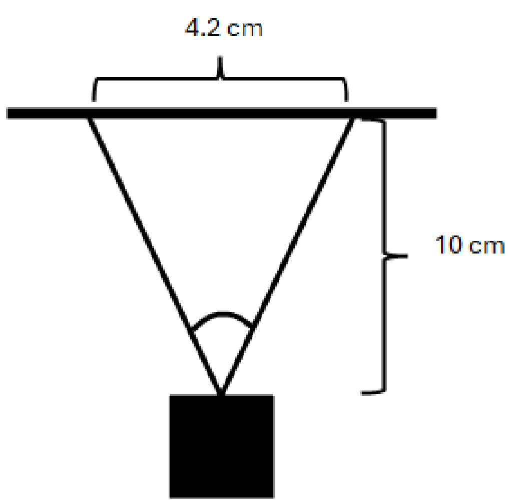


Figure 11: Figure 10 layover on figure 1

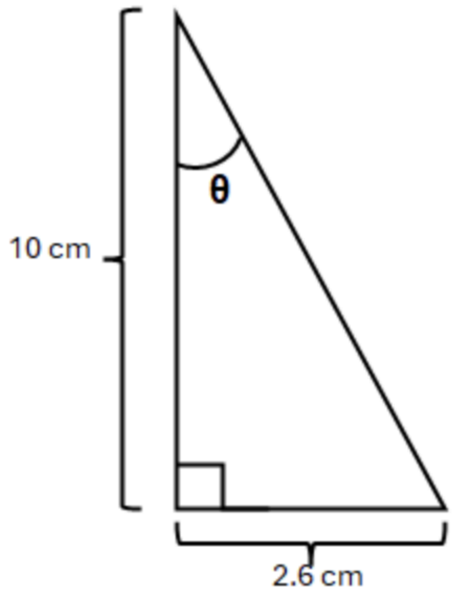
The real-life length of each side of the grid is $0.7cm$. By roughly estimating from the image, the width of the current safe zone is:

$$0.7 \times 6 = 4.2\ cm$$

With the information, a diagram of the arrangement of the camera and the paper can be drawn. The interior angle is the angle of view of the safe zone. The black box represents the camera and the wall as a thick black line.



To use the trigonometric function, I cut the triangle into half.



Angle θ is,

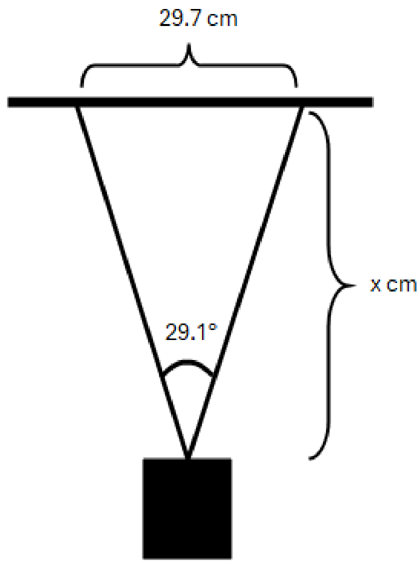
$$\theta = tan^{-1}\left(\frac{2.6}{10}\right) = 14.6°$$

Multiplying this by 2 gives the angle of view:

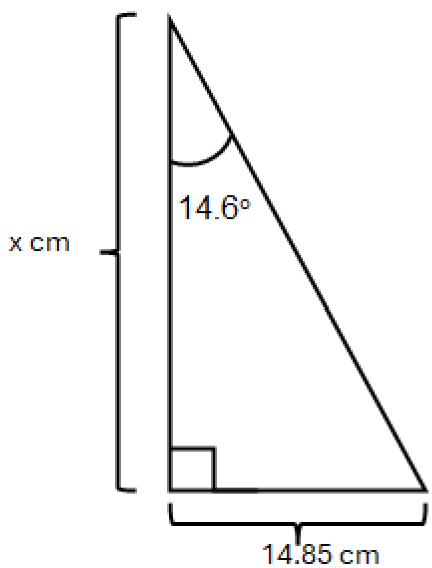
$$14.6° \times 2 = 29.1°$$

Calculate the distance

An A4 sheet of paper of paper that I used has a width of $29.7cm$ when placed horizontally. The diagram to represent the arrangement of camera so that the paper is within the angle of view of the safe zone is following:



Again, dividing the triangle in half in order to use the trigonometric function.



Using the tangent function, I solved for the optimal distance x.

$$tan\ (14.6)\ = \frac{14.85}{x}$$

$$x\ = \frac{14.85}{tan(14.6)} = 57.0cm$$

Thus, for an A4 sized paper, the optimal distance that minimizes the distortion is $57.0cm$.

Generalized equation

The distance I just solved for was only for a specific instance of A4 paper, but the concept can be applied to other objects with different dimensions. For any given object with a width w, the optimal distance D can be calculated with the following equation:

$$D = \frac{w}{tan(14.7)}$$

Conclusion

In this investigation, I mathematically established a relationship between the effect of distortion and distance from the center of the image, and then derived the relationship between the shooting distance and distortion. I also discovered that the distortion can't be eliminated completely but it is controllable by defining permissible distortion and setting a "safe zone."

For Canon Rebel T5i with EF-S 18-55mm lens, the optimal distance for shooting an landscape A4 paper was approximately $57.0\ cm$. The calculation framework can be used for other objects using the generalized formula. Though one issue that I encountered when actually putting my findings into practice is that when I fit my subject into the safe zone, it sometimes appears too small and those photos ended up requiring a substantial cropping. For the future, I'd like to explore more on how I can make the safe zone larger, whether it be comparing lenses from different manufacturers or experimenting with several focal lengths, so that I can utilize it in practice more often.

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