Control Theory Assignment № 2

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1. Selecting variant:

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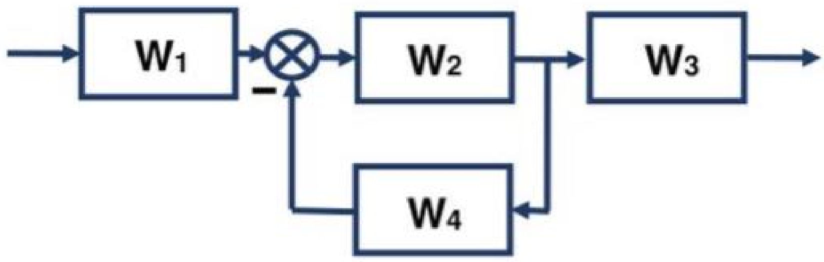
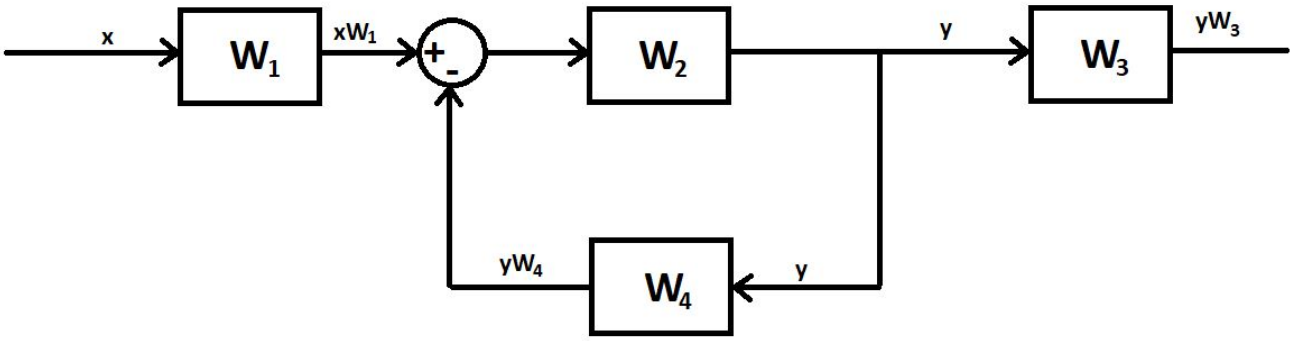
Variant: g

2. Transfer functions calculations:



$W_1 = \frac{2}{s+5}$
$W_2 = \frac{s+1}{s+0.5}$
$W_3 = \frac{1}{s+0.25}$
$W_4 = \frac{1}{2s+3}$

A. Calculate the total Transfer Function of the system:

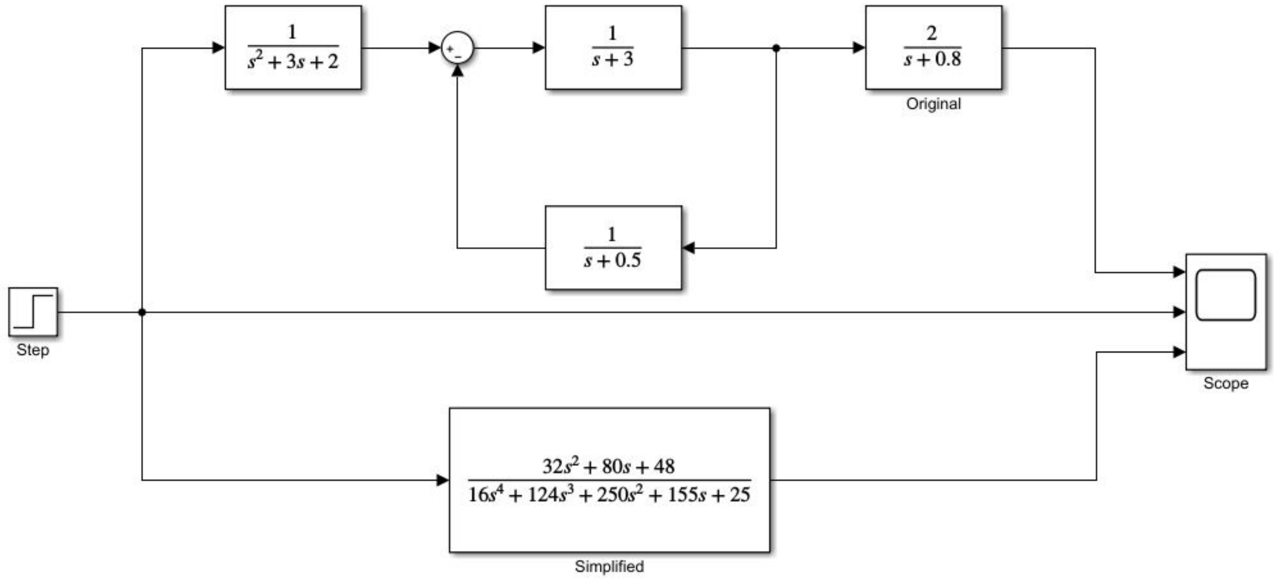


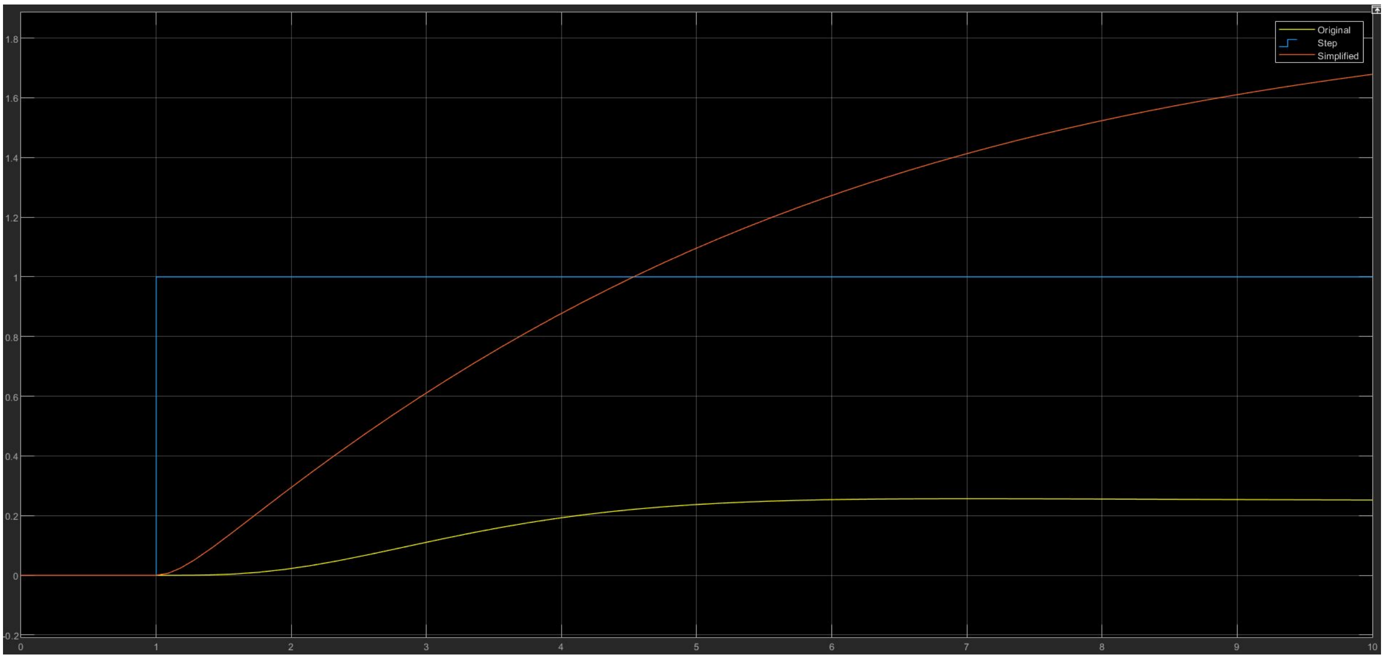
$(xW_1 - yW_4)W_2 = y$
$xW_1W_2 - yW_4W_2 = y$
$xW_1W_2 = y + yW_4W_2$
$xW_1W_2 = y(1 + W_4W_2)$
$y = \frac{xW_1W_2}{1+W_4W_2}$
$output = \frac{xW_1W_2W_3}{1+W_4W_2}$
W=Total Transfer Function

$W = \frac{W_1W_2W_3}{1+W_4W_2}$
$W_1W_2W_3 = \frac{2(s+1)}{(s+5)(s+0.5)(s+0.25)}$
$1 + W_4W_2 = \frac{s+1+(s+0.5)(2s+3)}{(s+0.5)(2s+3)}$
$W = \frac{2(s+1)}{(s+5)(s+0.5)(s+0.25)} * \frac{(s+0.5)(2s+3)}{s+1+(s+0.5)(2s+3)}$
$W = \frac{(2s+2)(2s+3)}{(s+5)(s+0.25)(s+1+(s+0.5)(2s+3))}$
$W = \frac{32s^2+80s+48}{16s^4+124s^3+250s^2+155s+25}$

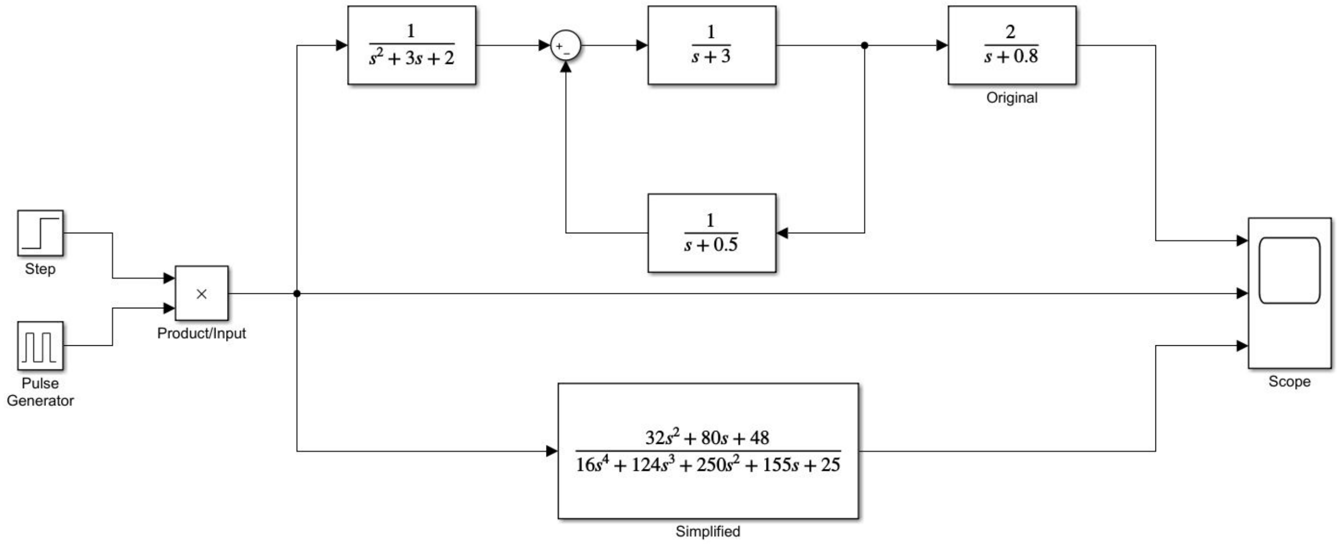
B. Build initial system shown in the block diagram and simplified in one Simulink schema and analyze its Step, Impulse and Frequency responses. Results should have a schema with both systems and 3 Scope plots(for each input). Each plot should have 3 lines - input signal, and two outputs from each system.

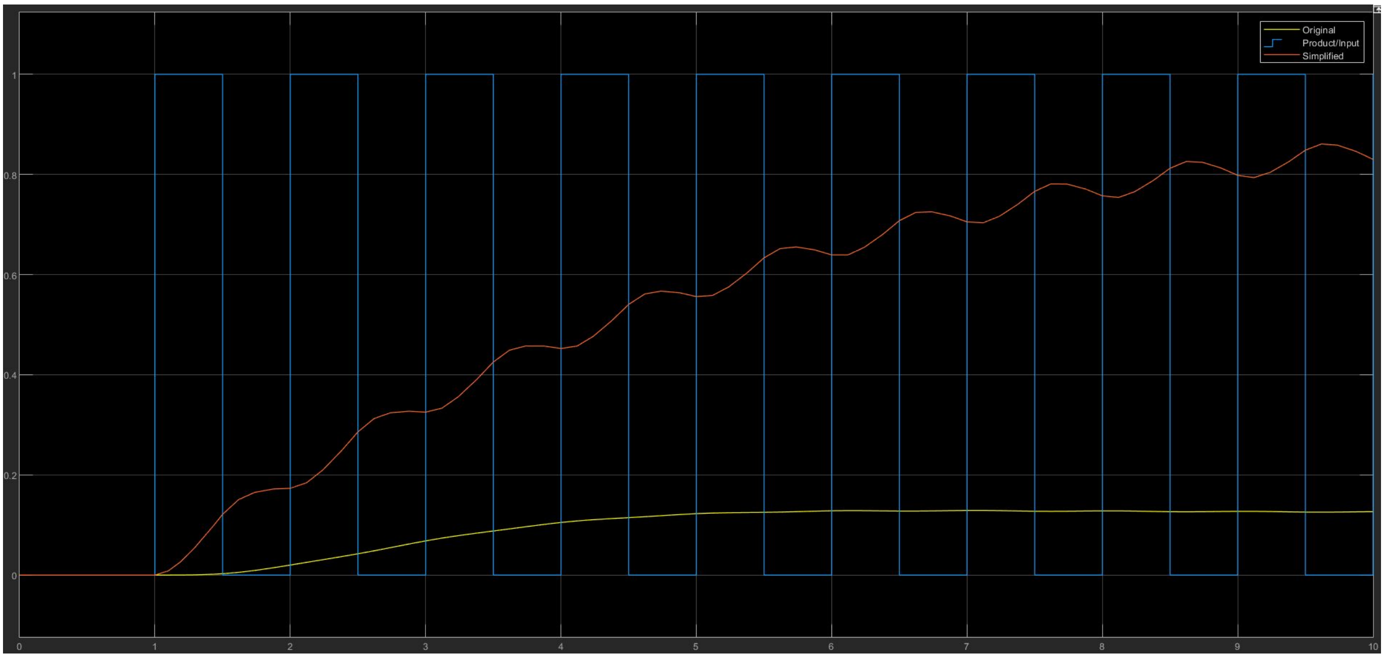
i. Schema for the step response:



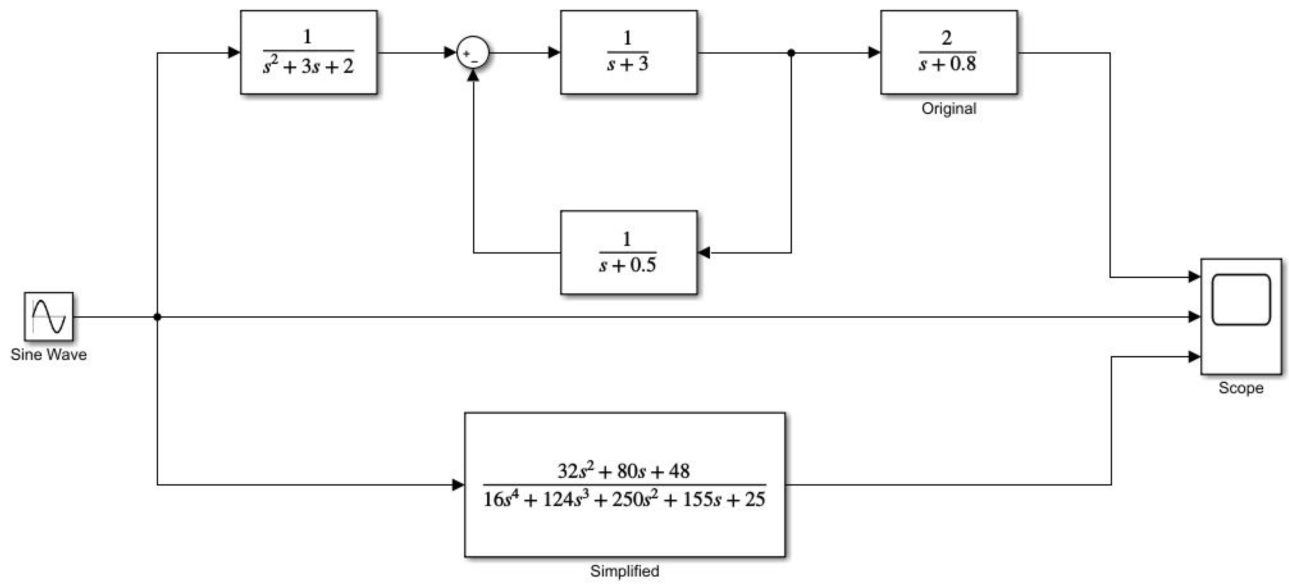


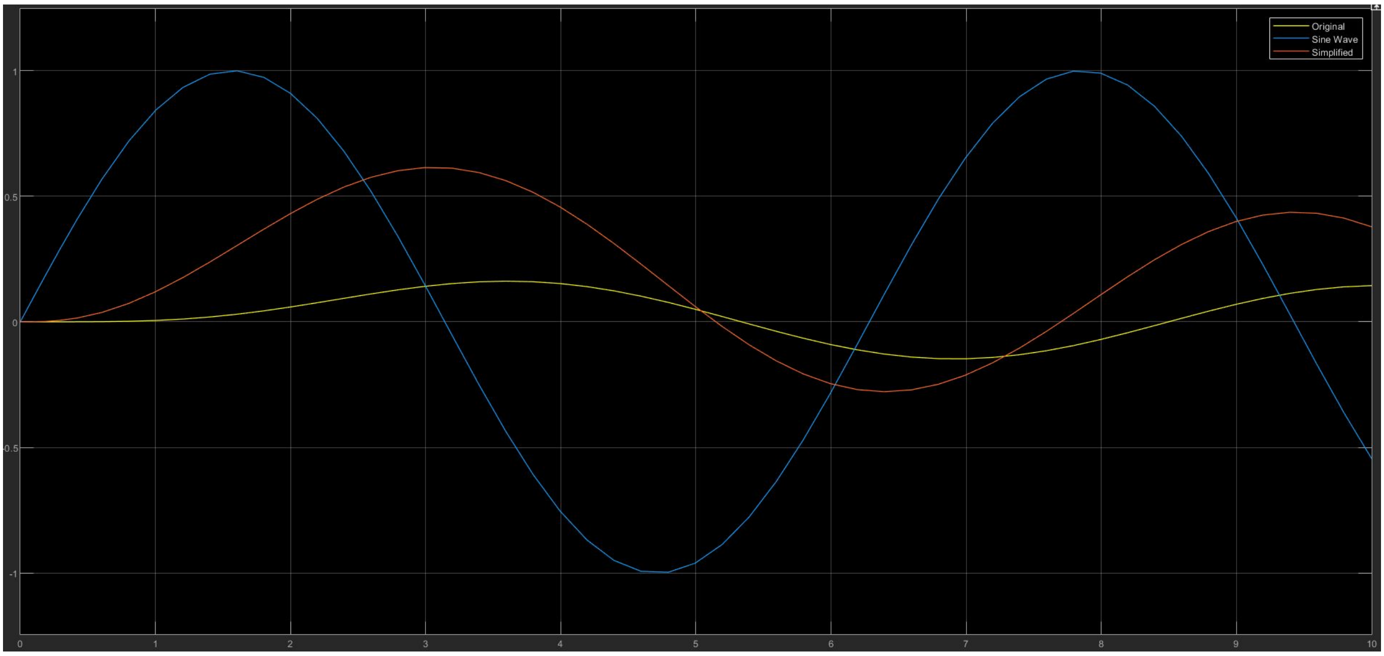
ii. Schema for the impulse response:





iii. Schema for the frequency response:



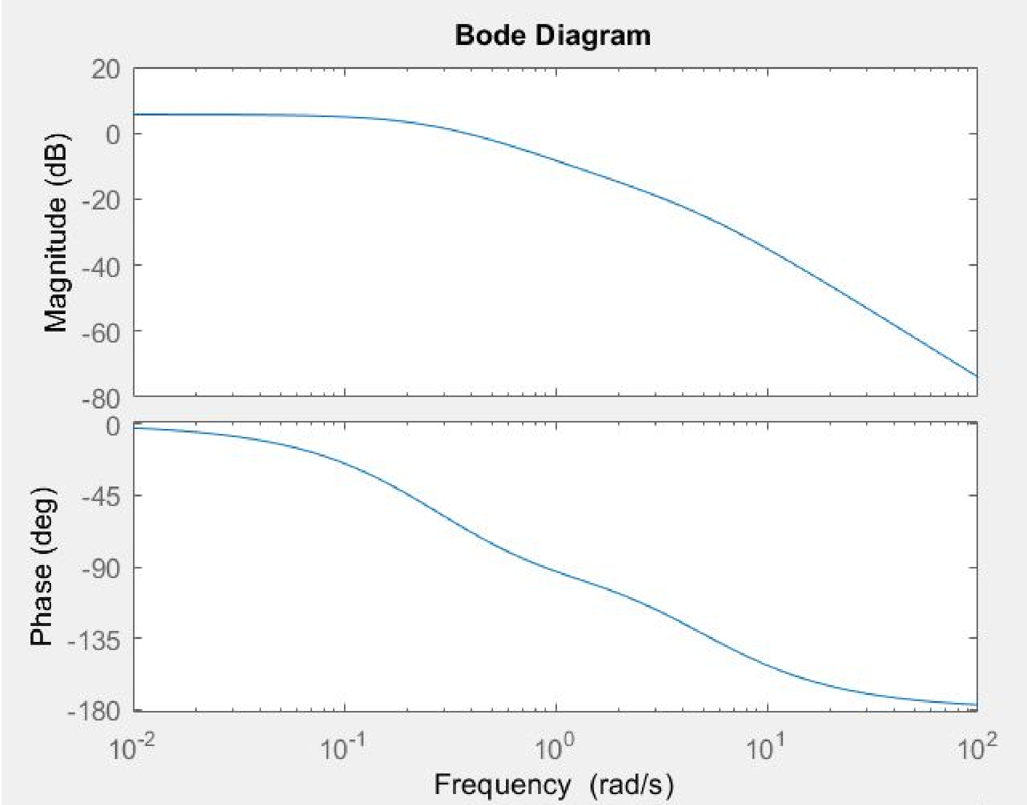


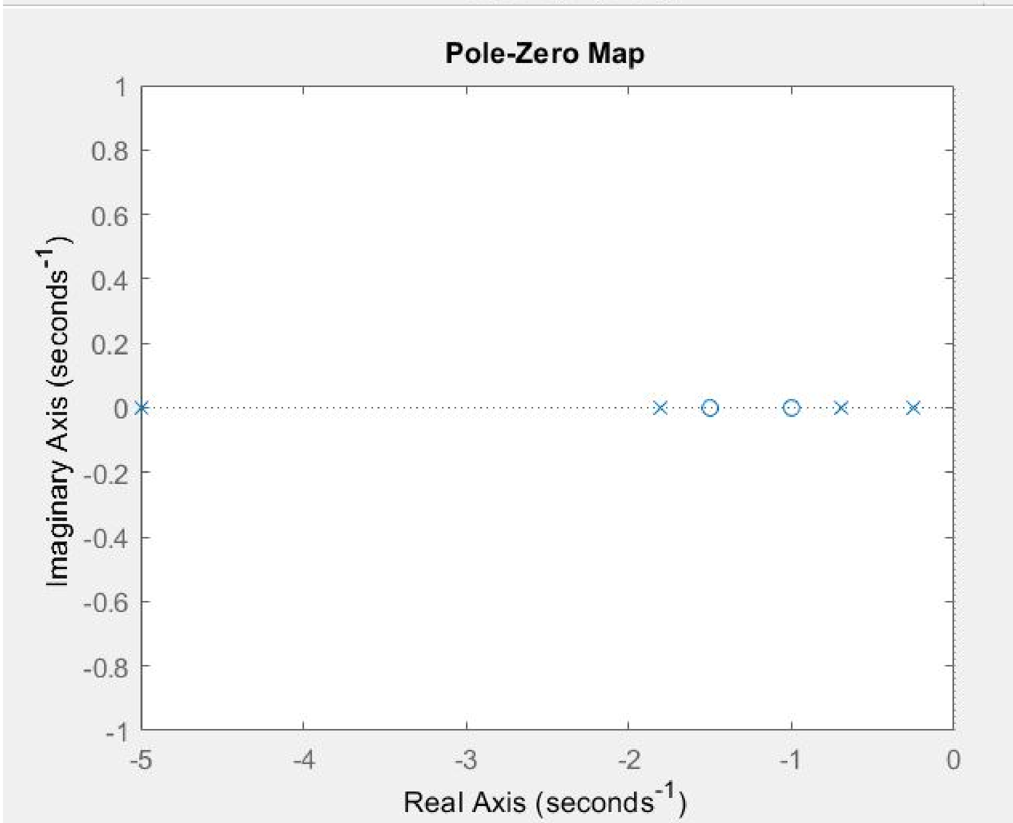
C. For one of the inputs (write down what you choose) generate a Bode and Pole-Zero map plots. Put plots and result - stable or unstable is system and why - in the report.

Chosen input: $Step$

Code in MATLAB:

```
G = tf([32, 80, 48], [16, 124, 250, 155, 25]);
bode(G);
figure();
pzplot(G);
```





Stability

System is stable, because all poles lie on negative x-axis

D. Analyze Bode plot - calculate asymptotes and frequence breaks and put calculations in report. Also calculate intersections of the plot with axes.

$\frac{32s^2+80s+48}{16s^4+124s^3+250s^2+155s+25}$

Using factorization, transform this transfer fucntion to following format:

$\frac{48(\frac{s}{1}+1)(\frac{s}{1.5}+1)}{25(\frac{2}{0.25}+1)(\frac{s}{5}+1)(\frac{s}{\frac{\sqrt{5}-5}{-4}}+1)(\frac{s}{\frac{\sqrt{5}+5}{4}}+1)}$

Note:

$\frac{\sqrt{5}-5}{-4} \approx 0.7$

$\frac{\sqrt{5}+5}{4} \approx 1.81$

So, we have:

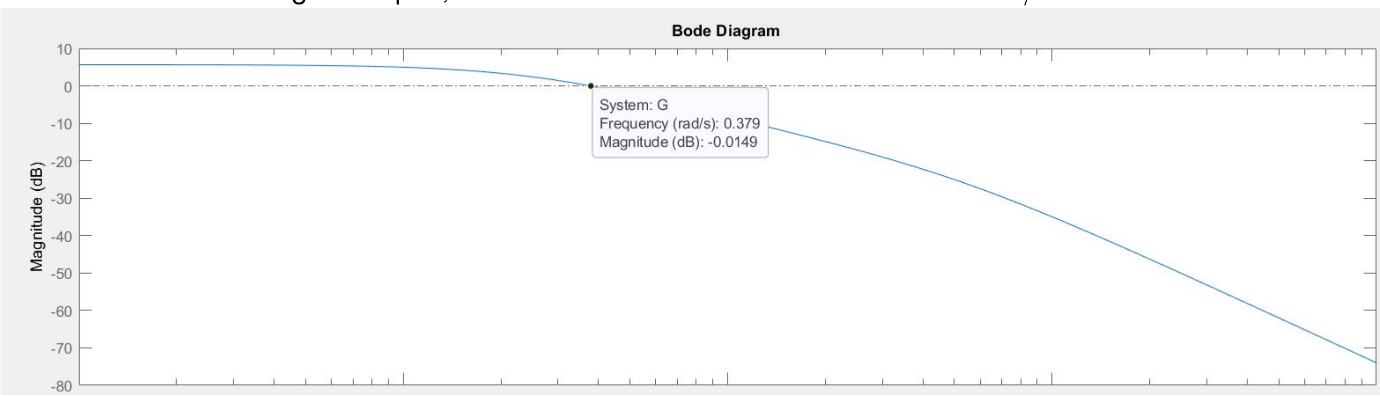
Constant: $\frac{48}{25} = 1.92 => 20log_{10}(1.92) = 5.666dB$

Frequency breaks at:(rad/s)

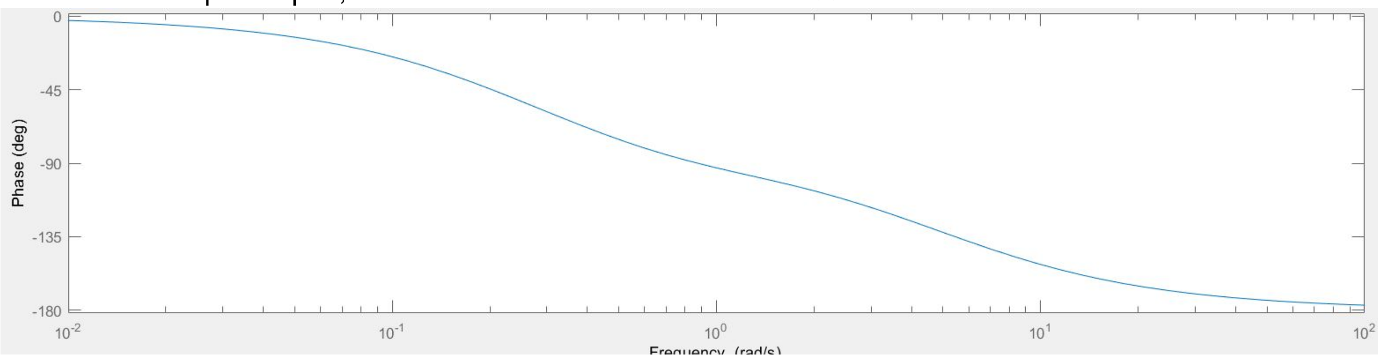
Zeros: $-1, -1.5$

Poles: $-0.25, -5, -0.7, -1.81$

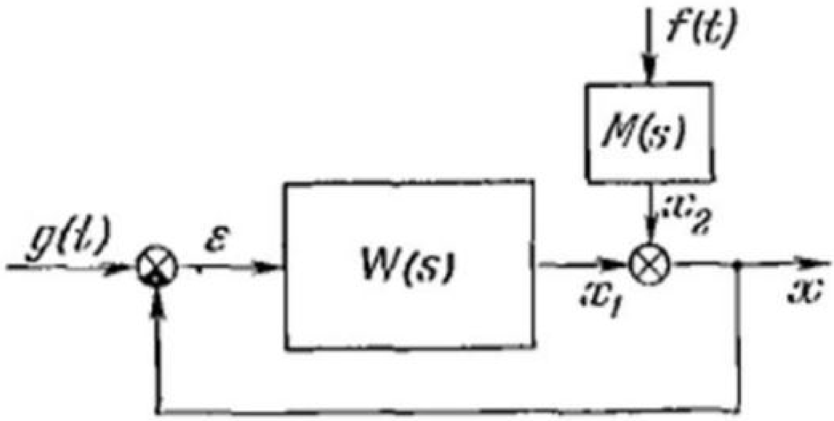
As we for the bode magnitude plot, we have intersection with axis at $\approx 0.379rad/s$



But in the bode phase plot, we do not have intersection with $-180°$



3. Find total transfer function for a closed-loop system:



$W(s) = \frac{2}{s^2+2}$
$M(s) = \frac{s+2}{2s+3}$
$\varepsilon = g(t) - x$
$x_1 = \varepsilon W(s)$
$x_2 = f(t)M(s)$
$x = x_1 + x_2 = \varepsilon W(s) + f(t)M(s)$
$x = (g(t) - x)W(s) + f(t)M(s) = g(t)W(s) - xW(s) + f(t)M(s)$
$x + xW(s) = g(t)W(s) + f(t)M(s)$
$x(1 + W(s)) = g(t)W(s) + f(t)M(s)$
$x = \frac{g(t)W(s)+f(t)M(s)}{1+W(s)}$
$x = \frac{g(t)\frac{2}{s^2+2}+f(t)\frac{s+2}{2s+3}}{1+\frac{2}{s^2+2}}$
$x = \frac{\frac{2g(t)(2s+3)+(sf(t)+2f(t))(s^2+2)}{(2s+3)(s^2+2)}}{\frac{s^2+4}{s^2+2}}$
$x = \frac{4sg(t)+6g(t)+s^3f(t)+2s^2f(t)+2sf(t)+4f(t)}{(2s+3)(s^2+2)}\frac{s^2+2}{s^2+4}$
$x = \frac{s^3f(t)+2s^2f(t)+s(4g(t)+2f(t))+6g(t)+4f(t)}{2s^3+3s^2+8s+12}$

4. Find transfer function of the system.

$$A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 1 \end{bmatrix}$$

Let's solve with the help of this formula: $W = C(sI - A)^{-1}B + D$

$$W = \begin{bmatrix} 1 & 3 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 3 \end{bmatrix} \left(\begin{bmatrix} s-3 & -1 \\ 2 & s-2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 3 \end{bmatrix} \left(\frac{1}{(s-3)(s-2)+2} \right) \left(\begin{bmatrix} s-2 & 1 \\ -2 & s-3 \end{bmatrix} \right) \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} \frac{s-2}{(s-3)(s-2)+2} & \frac{1}{(s-3)(s-2)+2} \\ \frac{-2}{(s-3)(s-2)+2} & \frac{s-3}{(s-3)(s-2)+2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{s-2}{(s-3)(s-2)+2} + \frac{-6}{(s-3)(s-2)+2} & \frac{1}{(s-3)(s-2)+2} + \frac{3s-9}{(s-3)(s-2)+2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{s-8}{(s-3)(s-2)+2} & \frac{3s-8}{(s-3)(s-2)+2} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2s-16}{(s-3)(s-2)+2} \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2s-16+(s-3)(s-2)+2}{(s-3)(s-2)+2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2s-16+s^2-5s+6+2}{s^2-5s+6+2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{s^2-3s-8}{s^2-5s+8} \end{bmatrix}$$

5. Find transfer functions of the system.

$$A = \begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 6 \end{bmatrix}$$

Let's solve with the help of this formula: $W = C(sI - A)^{-1}B + D$

$$W = \begin{bmatrix} 1 & 1 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 5 & 1 \\ 0 & -2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \left(\begin{bmatrix} s-5 & -1 \\ 0 & s+2 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \left(\frac{1}{(s-5)(s+2)} \right) \left(\begin{bmatrix} s+2 & 1 \\ 0 & s-5 \end{bmatrix} \right) \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

$$= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{s+2}{(s-5)(s+2)} & \frac{1}{(s-5)(s+2)} \\ 0 & \frac{s-5}{(s-5)(s+2)} \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{s+2}{(s-5)(s+2)} & \frac{s-4}{(s-5)(s+2)} \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

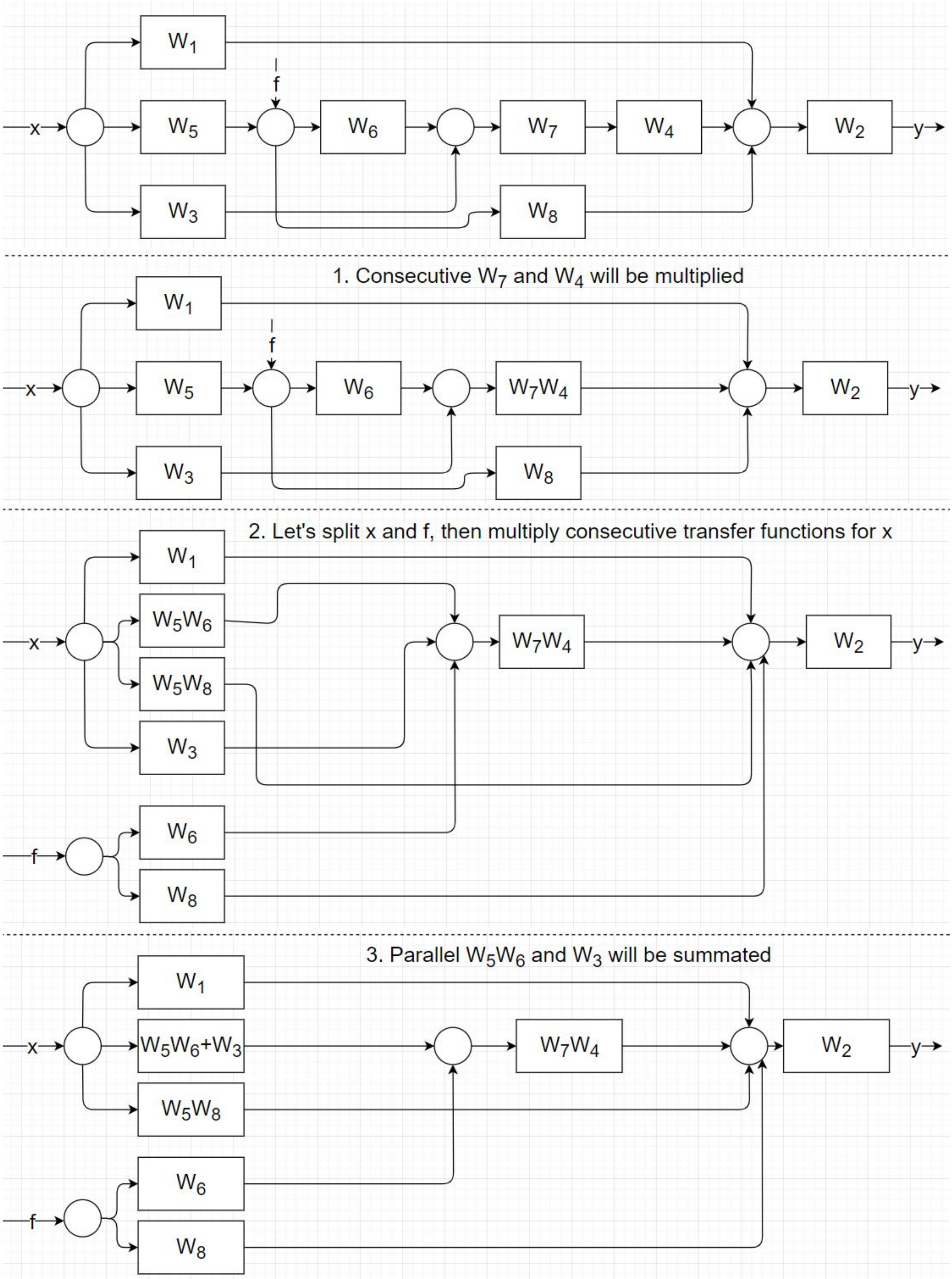
$$= \begin{bmatrix} \frac{2s-8}{(s-5)(s+2)} & \frac{2s+4+3s-12}{(s-5)(s+2)} \end{bmatrix} + \begin{bmatrix} 1 & 6 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2s-8+(s-5)(s+2)}{(s-5)(s+2)} & \frac{5s-8+6((s-5)(s+2))}{(s-5)(s+2)} \end{bmatrix} =$$

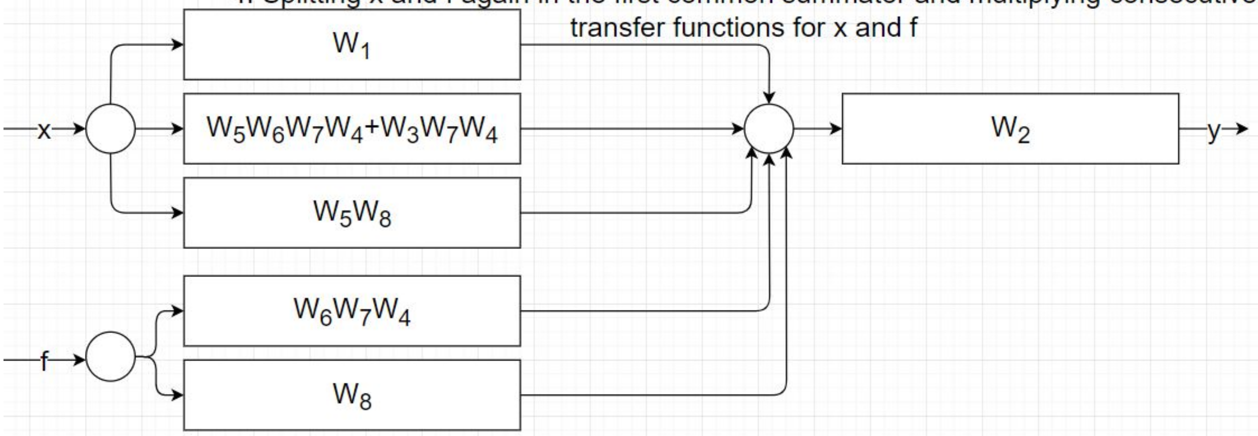
$$= \begin{bmatrix} \frac{2s-8+s^2-3s-10}{(s-5)(s+2)} & \frac{5s-8+6(s^2-3s-10)}{(s-5)(s+2)} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{s^2-s-18}{s^2-3s-10} & \frac{6s^2-13s-68}{s^2-3s-10} \end{bmatrix}$$

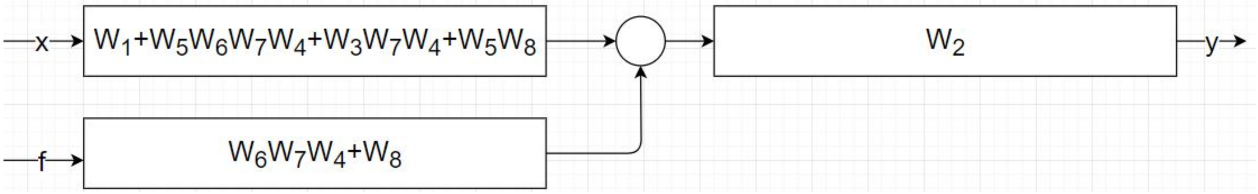
6. Simplify the system step by step and calculate total transfer function for both inputs x and f:



4. Splitting x and f again in the first common summator and multiplying consecutive transfer functions for x and f



5. Summating parallel transfer functions for x and f



6. Multiplying the last pairs of consecutive transfer functions for f and x