Control Theory Assignment № 5

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1. Variant selection

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Variant: e
$M = 4.2, m = 5.5, l = 2.1, g = 9.81$

A. Prove that it is possible to design state observer of the linearized system.

First of all, we should linearize our system. Let's refer to the Homework 4, where I referred to the material $Linearization.pdf$. But since now there are another values of constants, I changed them.

$$M = 4.2, m = 5.5, l = 2.1, g = 9.81$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(m+M)g}{Ml} & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{3597}{280} & 0 & 0 \\ 0 & \frac{10573}{980} & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{5}{21} \\ \frac{50}{441} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Notes

Friction coefficient is zero in the task, so $A_{3,3}, A_{4,3}$ both are 0.
Also equations are constructed differently in the material, and for this task correct thing will be positive signs at $A_{3,2}$ and $B_{4,1}$

Solution

Let $Q = [C^T \ \ A^T C^T]$

$$C^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, A^T C^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3597}{280} & \frac{10573}{980} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Q = I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determinant of Q is $1 \neq 0$, so our system is observable and we can design state observer for the system.

B. For open loop state observer, is the error dynamics stable?

To check open loop dynamics, we should make $B = 0$, to get rid of the feedback. Then, we should find eigenvalues of $A - Bu$, but because $B = 0$, we should only find eigenvalues of A.

$$\lambda I - A = \begin{bmatrix} \lambda & 0 & -1 & 0 \\ 0 & \lambda & 0 & -1 \\ 0 & -\frac{3597}{280} & \lambda & 0 \\ 0 & -\frac{10573}{980} & 0 & \lambda \end{bmatrix}$$

$$det(\lambda I - A) = 0$$

From this equation, we find eigenvalues:

$$\lambda_1 = 0, \lambda_2 = \frac{\sqrt{52865}}{70} \lambda_3 = \frac{-\sqrt{52865}}{70}$$

Because not all eigenvalues are negative, open-loop dynamics is unstable.

C. Design Luenberger observer for linearized system using both pole placement and LQR methods.

Let's find such matrix L, for which such statement holds: Real part of eigenvalues of $(A - LC)$ matrix should be negative.

$$A - LC = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{3597}{280} & 0 & 0 \\ 0 & \frac{10573}{980} & 0 & 0 \end{bmatrix} - \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \\ L_{31} & L_{32} \\ L_{41} & L_{42} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -L_{11} & -L_{12} & 1 & 0 \\ -L_{21} & -L_{22} & 0 & 1 \\ -L_{31} & \frac{3597}{280} - L_{32} & 0 & 0 \\ -L_{41} & \frac{10573}{980} - L_{42} & 0 & 0 \end{bmatrix}$$

$$\lambda I - (A - LC) = \begin{bmatrix} \lambda + L_{11} & -L_{12} & -1 & 0 \\ L_{21} & \lambda + L_{22} & 0 & -1 \\ L_{31} & L_{32} - \frac{3597}{280} & \lambda & 0 \\ L_{41} & L_{42} - \frac{10573}{980} & 0 & \lambda \end{bmatrix}$$

$$det(\lambda I - (A - LC)) = 0$$

I skipped the part with calculation of L matrix, because it is too long. At least now we know how to design Luenberger observer. After calculating L matrix, our Luenberger observer can be described as:

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x})$$

where $A\hat{x}$ is predictor and $L(y - C\hat{x})$ is a corrector.