Smart Beta 2.0

Noël Amenc and Felix Goltz

Noël Amenc
is a director at EDHEC-Risk Institute and CEO at ERI Scientific Beta in Nice, France.
noel.amenc@edhec.edu

Felix Goltz
is head of applied research at EDHEC-Risk Institute and research director at ERI Scientific Beta in Nice, France.
felix.goltz@edhec.edu

Everyone agrees that although capitalization-weighted equity indexes are the best representation of the market, they do not necessarily constitute an efficient benchmark that can be used as a reference for an informed investor's strategic allocation.[1] In other words, they do not constitute a starting point (for active investment) or an end point (for passive investment) that offers, through its diversification, a fair reward for the risks taken by the investor. "Smart beta"[2] is therefore a response from the market to a question that has formed the basis of modern portfolio theory since the work of the Nobel Prize winner Harry Markowitz: How is an optimally diversified portfolio constructed?

As with any technique or any model, implementation of these new forms of benchmarks is not risk free. Smart beta promoters highlight the risks inherent in the high concentration of cap-weighted indexes, and rightly so, but it is also necessary to grasp the risks to which investors are exposed when they adopt alternative benchmarks.

Talking about the superiority of smart beta indexes over the long term is totally legitimate, but it is also important to discuss the sources of this outperformance, its robustness, and the conditions that would lead to underperformance in the short or medium term.

This is one of the objectives of what EDHEC-Risk Institute calls the Smart Beta 2.0 approach. This new vision of smart beta investment ultimately aims to allow investors to understand and control the risks of investment in smart beta indexes so as to benefit fully from their performance.

Even though the majority of smart beta indexes have a strong probability of outperforming cap-weighted indexes over the long term, because of the high level of concentration of the latter, it should be noted that through their exposure to sources of risk that are different from those of cap-weighted indexes, these new benchmarks can sometimes significantly underperform market indexes for a considerable length of time. Exhibit 1 shows the magnitude and duration of the worst underperformance for a range of smart beta indexes.

The results show that smart beta strategies can encounter severe drawdowns relative to the cap-weighted reference index, which are, for most strategies, at least in the order of 10%. Moreover, as the time-under-water statistics suggest, underperformance can last for extended time periods.

CHOOSING AND CONTROLLING EXPOSURE TO SYSTEMATIC RISK FACTORS

Paying attention to the systematic risks of smart beta is today not only a genuine opportunity to create added value but also

EXHIBIT 1

Relative Risk of Various Alternative Beta Strategies, December 23, 2002–December 31, 2012

		Maximum Relative Drawdown	Time under Water (business days)
Indexes from Traditional Index Providers	FTSE RAFI U.S. 1000 Index	12.71%	439
	FTSE EDHEC Risk Efficient U.S. Index	8.72%	46
	MSCI USA Minimum Volatility Index	12.82%	371
	S&P 500 Equal Weight Index	13.72%	453
Scientific Beta USA Flagship Indexes	High Liquidity Max Decorrelation	10.17%	105
	High Liquidity Max Deconcentration	15.11%	110
	High Liquidity Efficient Max Sharpe	6.48%	113
	High Liquidity Efficient Min Volatility	6.31%	169

Notes: This exhibit summarizes the maximum relative drawdown numbers for four commercial smart beta indexes and 4 Scientific Beta USA flagship indexes with respect to the S&P 500 Index. Maximum relative drawdown is the maximum drawdown of the long–short index whose return is given by the fractional change in the ratio of strategy index to the benchmark index. Daily total return data for the December 23, 2002–December 31, 2012, period have been used; this is the earliest date that data for all indexes are available.

Source: Data from Datastream and www.scientificbeta.com.

a condition for the sustainability of smart beta. Although smart beta can play an important role in institutional investors' allocations, we think that this can only be at the price of implementing a genuine risk management process. The first approach to take into account the systematic risks of investing in smart beta that is very compatible with the idea that an index must remain a simple construction is the disentangling of the two ingredients that form the basis of any smart beta index construction scheme: the stock selection and weighting phases.

A clear separation of the selection and weighting phases enables investors to choose the risks to which they do or do not wish to be exposed. This choice of risk is expressed first by the definition of the investment universe. For example, an investor wishing to avail of a better-diversified benchmark than a cap-weighted index but disinclined to take on liquidity risk can decide to apply this scheme solely to a selection of very liquid stocks. In the same way, an investor who does not want the diversification of his benchmark to lead him to favor stocks with a value bias can decide that the selected weighting method is applied to a universe excluding growth stocks. In an article published recently in *The Journal of Portfolio Management* (Amenc, Goltz, and Lodh [2012]), we were able to show that the distinction between the selection and weighting phases (which can be made for most smart beta construction methods) could add value both in terms of performance and in controlling the investment risks. Exhibits 2 and 3 reproduce some of the results from the 2012 article.

Exhibit 2 shows the risk factor exposures that result when applying optimization-based weighting schemes to different stock selections. The results demonstrate that the three optimized strategies included in the analysis all have significant implicit small-cap exposure relative to the S&P 500 Index when no stock selection is made. However, when applying the weighting schemes only on the largest-cap stocks in the universe, none of the weighting schemes leads to pronounced small-cap exposure.

Amenc, Goltz, and Lodh [2012] showed that qualitatively similar results hold when selecting stocks based on dividend yield or low- and high-volatility characteristics. For these different stock characteristics, it is possible to reduce or cancel implicit factor tilts of a weighting scheme through an appropriate stock selection decision.

An additional question is whether improvements in risk–return properties of the weighting scheme relative to the broad cap-weighted index still hold after having corrected for the factor tilts. Exhibit 3 shows results that address this question for the case of size-based selection.

Overall, the results suggest that when reducing the small-cap exposure of these weighting schemes through an appropriate stock selection, each weighting scheme still improves its respective diversification objective relative to the standard cap-weighted index. We consider three optimization-based weighting schemes—minimum volatility (GMV), maximum Sharpe ratio

EXHIBIT 2

Size Exposure of Diversification Strategies Using Different Size-Based Stock Selections, July 5, 1963–December 31, 2010

Exposure of Excess Returns over CW	GMV Portfolio				MSR Portfolio				MDC Portfolio			
	All Stocks	**Small Size Universe**	**Medium Size Universe**	**Large Size Universe**	**All Stocks**	**Small Size Universe**	**Medium Size Universe**	**Large Size Universe**	**All Stocks**	**Small Size Universe**	**Medium Size Universe**	**Large Size Universe**
Market	**−26.20%**	**−25.41%**	**−28.03%**	**−23.92%**	**−21.92%**	**−23.69%**	**−23.94%**	**−20.09%**	**−8.60%**	**−7.93%**	**−10.57%**	**−6.59%**
Size	**−19.00%**	**−43.75%**	**−19.32%**	1.83%	**−21.13%**	**−46.28%**	**−21.40%**	0.29%	**−37.07%**	**−65.59%**	**−27.26%**	**−3.15%**

Notes: The exhibit shows the excess (over S&P 500) risk factor exposures of the GMV, MSR, and MDC portfolios based on broad S&P 500 stock universe and three size-based stock selections, using weekly return data. The stock selection is done at each rebalancing. We run the following regressions to identify factor exposures:

$$R_p - R_{CW} = \alpha + \beta_M \cdot R_{CW}$$

$$Res = \beta_s \cdot R_s$$

where R_p is the time series of test portfolio returns; R_{CW} is the S&P 500 time series returns; β_M is the market beta, β_S is the size (big − small) beta; R_S is the size factor, which is the return of a portfolio (cap-weighted) long in 1/5 largest-cap stocks and short in 1/5 smallest-cap stocks from the NYSE, AMEX, and NASDAQ universes; Res is the residual time series. This two-step process is used for each risk factor and for each test portfolio. The bold values indicate that the beta for the size factor tilt is significant at 1% confidence level. All averages reported are geometric averages, and all statistics are annualized.

Source: Data from CRSP.

(MSR), and maximum decorrelation (MDC)[3]—and analyze whether they achieve their explicit objective. For example, GMV portfolios explicitly aim at lowering portfolio volatility. GMV portfolios created for a stock selection of the largest stocks achieve a considerable reduction in volatility compared to the broad cap-weighted portfolio. MSR portfolios explicitly aim at increasing the return per unit of risk in the portfolio. Exhibit 3 suggests that applying a MSR weighting scheme to a stock selection allows the Sharpe ratio over the S&P 500 to be increased. In fact, such an increase is observed in all size-selected portfolios. The MDC weighting scheme aims at reducing portfolio risk through exploiting interaction effects between stocks. The success of this strategy can be measured by the ratio between the portfolio risk and the average constituent level risk. This ratio is referred to as the Goetzmann, Li, and Rouwenhorst [2005] (GLR) measure.[4] Exhibit 3 shows that building MDC portfolios for the medium-capitalization and large-capitalization stock selections leads to GLR measures that are higher than for the broad MDC portfolio, suggesting that they are less well diversified by this criterion, but their GLR measures are still considerably lower than that of the broad cap-weighted index, suggesting improved diversification.

Overall, the results show that, even after controlling for a specific risk factor through stock selection, the risk-return objectives of a diversification scheme can still be met and significant improvements over the cap-weighted reference index are possible.

Next, we tested whether liquidity risk can be controlled using a stock selection approach without compromising the performance of the smart beta strategy. In other words, we analyzed if the performance benefits of diversification strategies can be exploited in a highly liquid stock universe. For this, we construct four smart beta strategies on the top 50% of stocks in the U.S. universe in terms of liquidity and compare their performance with the strategies based on the full U.S. universe. The weighting schemes used are the same as the ones analyzed previously, but we add a simple diversification strategy based on equal weighting. In fact, the MDC strategy shown in Exhibit 4 provides the closest proxy for an equal-weighted strategy subject to implementation constraints (liquidity and turnover constraints). The results are reported in Exhibit 4.

The results show that high-liquidity stock selection does not have a large effect on the performance of

EXHIBIT 3
Attainment of Objective with Size-Based Stock Selection, January 2, 1959–December 31, 2010

Universe	All	Small	Medium	Large
A. Global Minimum Volatility				
Annual volatility	12.40%	13.67%	12.67%	12.59%
Reduction relative to S&P 500	19.8%	11.6%	18.0%	18.6%
B. Maximum Sharpe Ratio				
Sharpe ratio	0.51	0.65	0.51	0.35
Increase relative to S&P 500	85.6%	139.9%	85.9%	30.2%
C. Maximum Decorrelation				
GLR measure	0.139	0.134	0.167	0.208
Reduction relative to S&P 500	43.1%	45.3%	31.9%	15.0%

Notes: The exhibit compares the attainment of the investment objectives of the three optimized portfolios, each of which includes one third of the stocks in the universe (respectively, the smallest, medium, and largest capitalization stocks in the S&P 500 universe), using weekly return data. The risk-free rate is the return of three-month U.S. Treasury bill. All averages reported are geometric averages, and all statistics are annualized.

Source: Data from CRSP.

EXHIBIT 4
High-Liquidity Index Diversification Strategy Performance, June 21, 2002–December 31, 2012

	Efficient Max Sharpe	Efficient Min Volatility	Max Decorrelation	Max Deconcentration
A. Performance Based on All Stocks				
Annual returns	7.86%	8.58%	7.67%	7.79%
Annual volatility	20.12%	18.23%	21.09%	22.46%
Sharpe ratio	0.31	0.38	0.29	0.27
B. Performance Based on High-Liquidity Counterparts				
Annual returns	7.57%	7.91%	7.27%	7.33%
Annual volatility	21.34%	19.22%	22.44%	24.01%
Sharpe ratio	0.28	0.33	0.25	0.24

Notes: The exhibit shows the risk and return statistics of the Scientific Beta USA strategy indexes. The high-liquidity portfolios are constructed using the top 50% of stocks in the parent universe in terms of liquidity, using daily total return data. The risk-free rate is the return of the three-month U.S. Treasury bill. All averages reported are geometric averages, and all statistics are annualized.

Source: Data from www.scientificbeta.com.

the strategies. The annualized returns tend to decrease slightly relative to the indexes that do not restrict the stock universe to high-liquidity stocks. The Sharpe ratios also tend to be decreased, but not by a big amount. These results suggest that by a simple stock selection approach of selecting the most liquid stocks, one can avoid the liquidity problems of smart beta strategies while maintaining most of the potential for improved risk–reward properties.

Naturally, in addition to or as a substitution for this stock selection, the construction of smart beta indexes can use a second approach to take into account systematic risks linked to investing in smart beta. This approach consists of well-known constrained optimization techniques that allow maximal or minimal exposures to risk factors to be imposed. For example, sector exposures in smart beta indexes can be controlled to reduce the difference with respect to the cap-weighted reference index. Exhibit 5 shows a comparison of performance statistics for the S&P 500 with Scientific Beta USA Minimum Volatility indexes. Exhibit 5 contains results for the minimum volatility index with and without sector-neutrality constraints to examine the effect of such constraints on the strategy's ability to achieve its objective.

EXHIBIT 5

Controlling Sector Exposures of Minimum Volatility Indexes, June 21, 2002–December 31, 2012

	Efficient Min Volatility Index	Sector-Neutral Efficient Min Volatility Index	S&P 500
Annual returns	8.58%	8.10%	5.66%
Annual volatility	18.23%	19.27%	21.81%
Reduction in volatility compared to the S&P 500	16.4%	11.6%	—
Sharpe ratio	0.38	0.33	0.18

Notes: The exhibit shows the risk and return statistics for the Scientific Beta USA Min Volatility Index, the sector-neutral Scientific Beta USA Min Volatility Index, and the S&P 500 Index, using daily total return data. All averages reported are geometric averages, and all statistics are annualized.

Source: Data from Bloomberg and from www.scientificbeta.com.

The results show that imposing sector-neutrality constraints does not affect the performance of the Scientific Beta USA Min Volatility strategy by a large amount. The unconstrained portfolio achieves a 16.40% volatility reduction over the cap-weighted benchmark, whereas the sector-constrained portfolio still achieves an 11.8% reduction.

MANAGING THE SPECIFIC RISK OF SMART BETA INVESTING

In practice, the specific risks of smart beta strategies are not thoroughly analyzed or managed. A first approach is a negative perspective, stating that whatever fraction of the risk of a smart beta strategy relative to a reference cap-weighted index not explained by differences in systematic factor exposures (including the impact of omitted factors) is by definition due to specific risks. Turning to the analysis framework of modern portfolio theory actually provides a relevant conceptual structure for addressing this issue in more detail.

Modern portfolio theory has a straightforward prescription, namely that every investor should optimally combine risky assets so as to achieve the highest possible Sharpe ratio. Implementing this objective, however, is a complex task because of the presence of estimation risk for the required parameters, namely expected returns and covariance parameters. In practice, the costs of estimation error may entirely offset the benefits of optimal portfolio diversification (De Miguel, Garlappi, Nogales, and Uppal [2009] provided evidence that naively diversified portfolios have higher out-of-sample Sharpe ratios than scientifically diversified portfolios).

In this context, an investor interested in designing maximum Sharpe ratio (MSR) benchmarks may be better off investing in heuristic portfolio strategies, such as the equal-weighted (EW) portfolio, that can be done without having to estimate either expected return or covariance parameters, as opposed to trying to implement scientific diversification techniques that rely on imprecise parameter estimates. Similarly, an investor may be better off for example investing in a proxy for the global minimum variance (GMV) portfolio or the equal risk contribution (ERC) portfolio, which only require estimates for covariance parameters, as opposed to trying to estimate the maximum Sharpe ratio portfolio, which also requires expected returns estimates that are known to be noisier (see Merton [1980]).

In other words, the choice in risk and return parameter estimation for efficient diversification is between "trying," which has a cost related to estimation risk (i.e., the risk of a substantial difference between the estimated parameter value and the true parameter value), or "giving up," which has a cost related to optimality risk (i.e., the risk that the heuristic benchmark, such as GMV or EW, can be far from the optimal MSR benchmark). The trade-off between estimation risk and optimality risk occurs because using objectives that involve fewer parameters leads to a smaller amount of parameter risk but a higher amount of optimality risk, because one is using fewer dimensions for optimization.

In this sense, it can happen that a "good" proxy (i.e., a proxy based on parameters with little estimation risk) for a "bad" target (i.e., a target *a priori* far from the true MSR based on true population values) eventually dominates a "bad" proxy (i.e., a proxy based on parameters plagued with substantial estimation risk) for a "good" target (i.e., a target *a priori* close to the true MSR based on true population values).

Different portfolios are intuitively expected to incur more estimation risk or more optimality risk. For example, investing in cap-weighted (CW) or equal-weighted (EW) benchmarks involves no estimation risk (because no parameter estimates are required) but arguably a large amount of optimality risk (because these benchmarks are not expected in general to be close to the true MSR portfolio). Hence, holding EW or CW portfolios, which are not subject to estimation risk, involves an opportunity cost related to the fact that their Sharpe ratios may be dramatically inferior to the Sharpe ratio of the true MSR. On the flip side, investing in a GMV or ERC benchmark involves more estimation risk, because covariance parameter estimates are needed, and possibly less optimality risk if it turns out that these heuristic benchmarks are closer to the optimal MSR benchmarks. Finally, investing in MSR benchmarks involves even more estimation risk, because (possibly very noisy) expected return parameters are used in addition to covariance parameters. On the flip side, MSR benchmarks do not have any optimality risk because they would coincide with the true optimal portfolio in the absence of estimation risk.

Martellini, Milhau, and Tarelli [2013] provided a quantitative analysis of the trade-off between optimality risk and estimation risk. In this context, they first proposed an empirical analysis of optimality risk taken in isolation, that is, in the absence of any estimation risk. To conduct this analysis, they considered a large number of possible equity universes, defined in terms of many different possible reasonable true population values for risk and return parameters, and measure the difference for these parameter values (in terms of ex ante Sharpe ratios, i.e., based on true parameter values) between the true MSR portfolios and various heuristic portfolios, as well as various combinations of these portfolios. They then analyzed the distribution of this distance across all possible sets of parameter values so as to generate an absolute assessment of optimality risk for various heuristic portfolios, as well as a relative assessment of optimality risk among competing heuristic portfolios. For example, this analysis answered such questions as what is the probability (across all tested parameter values) that the GMV portfolio is closer than the EW portfolio to the (true) MSR portfolio, hence allowing the amount of optimality risk in EW versus GMV (or any other heuristic) benchmark to be compared.

In a second step of their analysis, estimation risk was introduced so as to help measure the distance of various heuristic benchmarks using imperfect estimates with respect to the true MSR portfolio. This analysis allows us to analyze the interaction between estimation risk and optimality risk and allows us to answer questions such as the following: Given realistic estimation errors in the covariance matrix and expected returns, what are the chances that an imperfectly estimated MSR, which suffers only from estimation risk (estimated MSR different from true MSR), will be closer to the true MSR portfolio in terms of ex ante Sharpe ratios compared to a GMV portfolio that is subject only to optimality risk (EW different from true MSR) but to a lower amount of estimation risk (because it does not require any expected return parameters)?

Overall, their analysis allows us to provide a detailed empirical assessment of total efficiency loss (in terms of differences in ex ante Sharpe ratios) between a given benchmark and the true MSR portfolio, by decomposing this efficiency loss in terms of optimality risk and estimation risk, according to the following: Total distance of a given benchmark with respect to the true MSR portfolio = Distance of the given target benchmark with respect to the true MSR portfolio assuming away estimation risk (a measure of optimality risk) + Distance between the imperfectly estimated target and the true target (a measure of estimation risk).

This analysis can also be used to manage specific risks of smart beta benchmarks. In particular, one may seek to have a strategic exposure to various smart beta benchmarks so as to diversify away these risks.

Interestingly, because the diversification strategies differ from each other in the assumptions they make and the objectives they aim to achieve, a combination of these different strategies will indeed allow the risks that are specific to each strategy to be diversified by exploiting the imperfect correlation between the different strategies' parameter estimation errors and the differences in their underlying optimality assumptions (see Tu and Zhou [2011], Kan and Zhou [2007], and Martellini, Milhau, and Tarelli [2013], among others, for theoretical and empirical evidence that portfolio of strategies dominate individual strategies in the presence of parameter estimation risk). Moreover, as the single strategies' performance will show different profiles of dependence on market conditions, a multi-strategy approach can help investors smooth the overall performance across market conditions. For instance, Amenc, Goltz, Lodh, and Martellini [2012] formed a combination of two diversification approaches that leads to a smoother conditional performance and higher probability of outperforming the cap-weighted index.

More generally, beyond a static diversification approach, one may implement dynamic diversification to various smart beta benchmarks based on such market conditions as average correlation levels, volatility levels, and so on.

MANAGING TRACKING ERROR RISK

If the goal of investment in smart beta is to outperform cap-weighted market indexes, we should note that this goal is exactly the same as that of a benchmarked active manager. It thus seems logical to take account of the tracking error of these new benchmarks with regard to cap-weighted indexes, not only to establish an equivalent comparison between them but also to control the risk of these new benchmarks underperforming the market indexes. Integrating the risk of underperforming cap-weighted indexes is all the more important in that, even if a chief investment officer (CIO) of a pension fund, for example, has chosen a new benchmark, there is no guarantee that the fund's stakeholders will agree with this choice. The popularity of cap-weighted indexes and the fact that they represent the average of the market mean that they will likely remain the ultimate reference for equity investing. As proof, even though all the promoters of smart beta indexes base their sales proposals on the necessity of no longer using cap-weighted indexes, which they rightfully consider to be poor-quality investment references, they systematically use these same cap-weighted indexes to benchmark the performance of their smart beta indexes. Ultimately, CIOs who replace a cap-weighted index with a smart beta index take considerable reputation risk. Whereas, in active investment management, the CIO can always remove the manager whose mandate is underperforming the benchmark, in the case of the choice of benchmark, only the CIO will be to blame.

We can, of course, argue that any management of a relative risk budget with respect to an inefficient benchmark such as a cap-weighted index can penalize the potential outperformance generated by a smart beta index over the long term. However, it is clear that short-term performance constraints do exist in asset management, private wealth management, or institutional investment management. It would be a shame if the good idea of tracking a smart benchmark were called into question due to short-term underperformance. In our view, the level of relative risk budget with respect to cap-weighted indexes should be proportional to the knowledge that the stakeholders have of the smart beta concept and to the confidence that the CIO has in his smart beta choices.

EXHIBIT 6
Relative Risk of Scientific Beta USA Strategy Indexes, June 21, 2002–December 31, 2012

	Efficient Max Sharpe	Efficient Min Volatility	Efficient Max Decorrelation	Efficient Max Deconcentration
A. Smart Beta without Relative Risk Control				
Excess returns over CW	1.88%	2.59%	1.69%	1.80%
Tracking error	3.18%	4.42%	3.33%	3.28%
95% Tracking error	5.09%	8.25%	4.90%	5.61%
B. Smart Beta with Relative Risk Control (3% tracking error)				
Excess returns over CW	0.84%	1.05%	1.05%	0.89%
Tracking error	1.69%	2.06%	1.91%	1.69%
95% Tracking error	2.69%	4.33%	3.51%	2.57%

Notes: This exhibit summarizes the relative risk and return statistics of four Scientific Beta USA strategy indexes without any relative risk control and with 3% tracking error control. The 95% tracking error is computed using a rolling one-year window and one-week step size over the analysis period. The benchmark used is the Scientific Beta USA cap-weighted index, and the analysis is based on daily total return data. All averages reported are geometric averages, and all statistics are annualized.

Source: Returns data from www.scientificbeta.com.

Amenc, Goltz, Lodh, and Martellini [2012] was able to show that, using a suitably designed relative-risk-control approach, it is possible to obtain reliable control of both the average tracking error and the extreme tracking error of smart beta strategies. This approach involves an alignment of the risks of the smart beta benchmark with those of the cap-weighted index. In addition to achieving effective control of the tracking error, such an approach maintains a significant potential for outperformance.

The results in Exhibit 6 show the effect of imposing a 3% tracking error constraints on Scientific Beta USA strategy indexes. The tracking error target is not exceeded substantially by any of the strategies even for the extreme observations of tracking error, as can be seen from the 95% tracking error observations. Despite the risk control, the relative-risk-controlled versions are still able to outperform their cap-weighted benchmarks, albeit by smaller margins. Investors thus face a clear trade-off between taking on relative risk and generating outperformance. However, in the relative-risk-controlled indexes, both overall tracking error and extreme tracking error figures are brought down substantially without eroding all of the potential for outperformance.

CONCLUSION: TOWARD MORE CHOICE FOR SMART BETA

Smart beta strategies offer interesting opportunities for investors, as such strategies recognize the importance of equity portfolio construction (or "beta") as a determinant for the risk and return of portfolios over the long run. However, the illustrations in this article show that smart beta strategies are also exposed to numerous risks, notably exposures to systematic risk factors, optimality and parameter estimation risks, and relative risk of periodic underperformance compared with cap-weighted indexes. When investing in smart beta benchmarks, investors should do so with full knowledge of the risks they are taking. Moreover, the illustrations show that it is possible to control some of the risks of smart beta strategies while maintaining considerable performance benefits. Ultimately, investors should not only analyze the risks of a given strategy but also be able to construct a benchmark that corresponds to their own choice of risks.

ENDNOTES

[1]See, for example, Amenc, Goltz, Tang, and Vaidyanathan [2012] for a literature review and evidence of investor opinions on cap-weighted indexes and their alternatives.

[2]We refer to systematic portfolio construction methods that deviate from using market capitalization as the key ingredient for constituent selection and/or weighting as smart beta strategies. Such strategies are also commonly referred to as advanced beta strategies, alternative equity indexes, non-cap-weighted indexes, or beta prime.

[3]GMV (global minimum variance) portfolios aim to minimize portfolio volatility. GMV optimization is performed in the presence of norm constraints (DeMiguel, Garlappi, Nogales, and Uppal [2009]) with a lower bound of N/3 on the effective number where N is the total number of stocks in the relevant universe. MSR (maximum Sharpe ratio) optimization aims to maximize the Sharpe ratio of the portfolio. The downside risks of stocks are used as a proxy for their expected returns (Amenc, Goltz, Martellini, and Retkowsky [2011]) and the covariance matrix is obtained using principal component analysis. MDC (maximum decorrelation) optimization is an approach that exploits low correlations across stocks to reduce portfolio risk rather than concentrating in low-volatility stocks, which is a limitation often underlined with GMV approaches. The MDC approach aims to minimize portfolio volatility under the assumption that volatility across all stocks is identical (Christoffersen et al. [2010]), hence focusing on exploiting differences in correlations rather than on exploiting differences in volatility across stocks.

[4]The GLR measure (Goetzmann, Li, and Rouwenhorst [2005]) is the ratio of the portfolio variance to the weighted variance of its constituents. A low GLR measure indicates that correlations have been well exploited and in this sense the portfolio is well diversified.

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