BME 318: Circuits and Systems for Biomedical Engineering

Practice Questions 2019/2020

1. Applications of first-order circuits

7.83 An *RC* circuit consists of a series connection of a 120-V source, a switch, a 34-MΩ resistor, and a 15-μF capacitor. The circuit is used in estimating the speed of a horse running a 4-km racetrack. The switch closes when the horse begins and opens when the horse crosses the finish line. Assuming that the capacitor charges to 85.6 V, calculate the speed of the horse.

7.69 For the op amp circuit in Fig. 7.134, find $v_o(t)$ for $t > 0$.

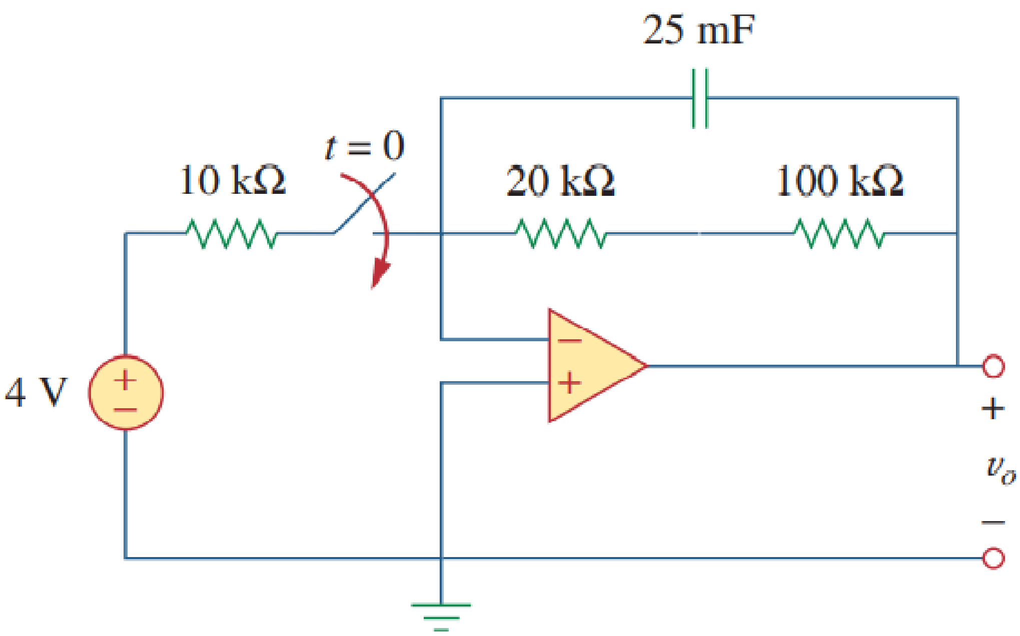

Figure 7.134
For Prob. 7.69.

7.70 Determine v_o for $t > 0$ when $v_s = 20$ mV in the op amp circuit of Fig. 7.135.

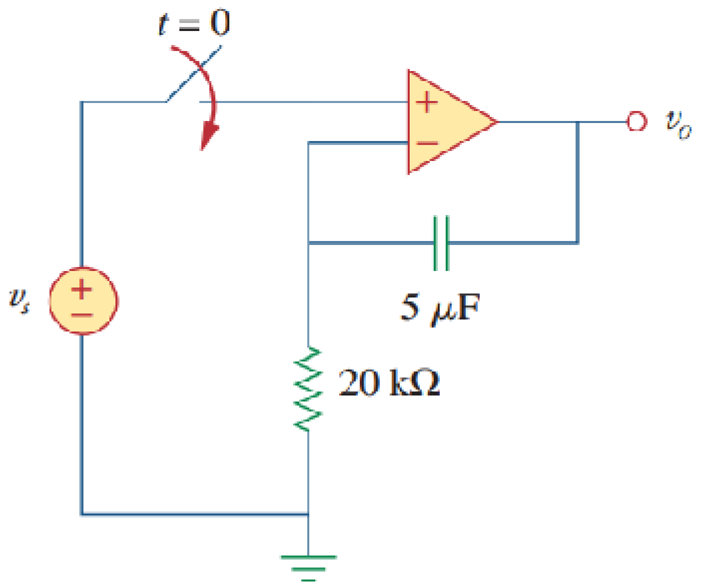

Figure 7.135
For Prob. 7.70.

7.56 For the network shown in Fig. 7.122, find $v(t)$ for $t > 0$.

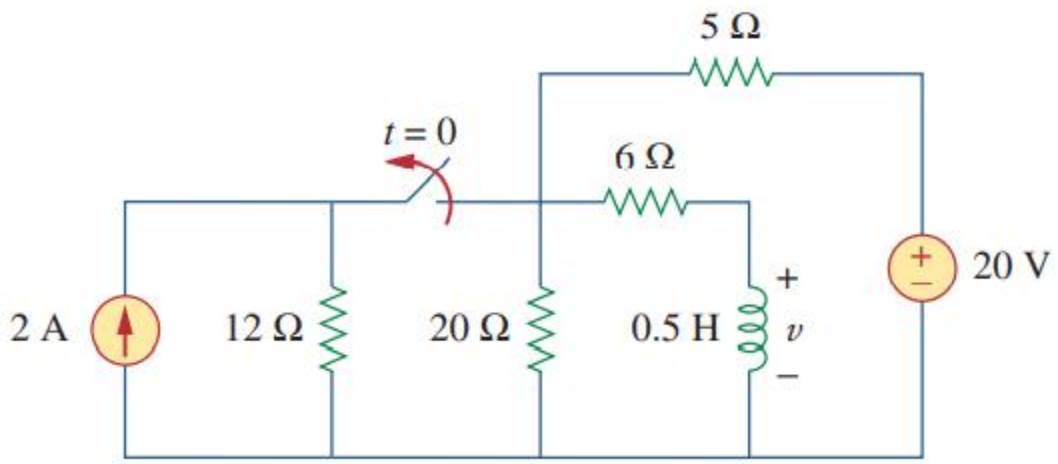

Figure 7.122
For Prob. 7.56.

***7.50** In the circuit of Fig. 7.117, find i_x for $t > 0$. Let $R_1 = R_2 = 1\ \mathrm{k}\Omega$, $R_3 = 2\ \mathrm{k}\Omega$, and $C = 0.25$ mF.

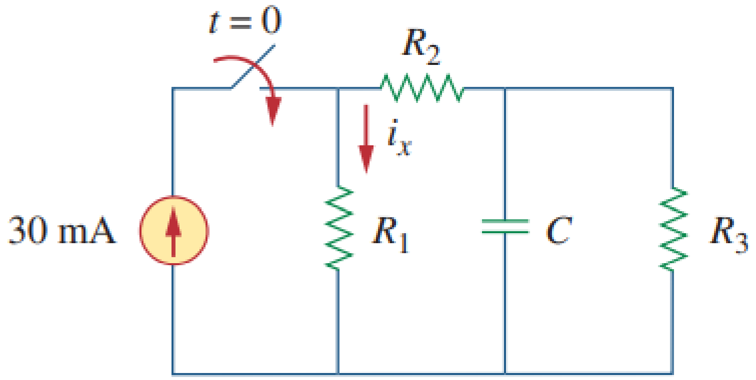

Figure 7.117
For Prob. 7.50.

7.49 If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.

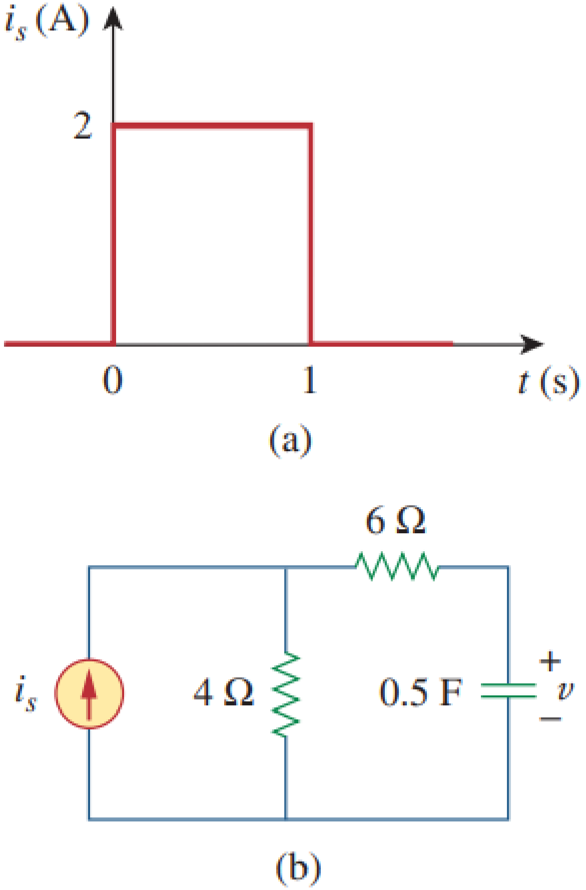

Figure 7.116

7.48 Find $v(t)$ and $i(t)$ in the circuit of Fig. 7.115.

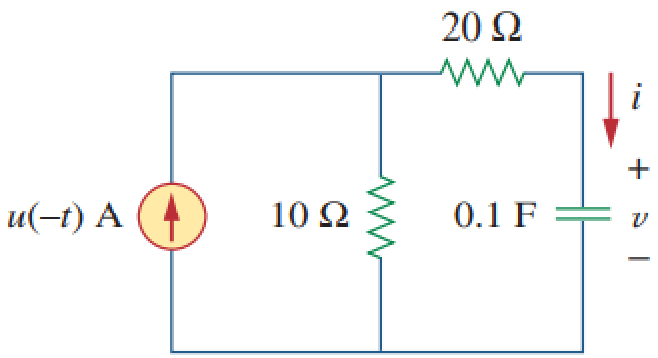

Figure 7.115

7.47 Determine $v(t)$ for $t > 0$ in the circuit of Fig. 7.114 if $v(0) = 0$.

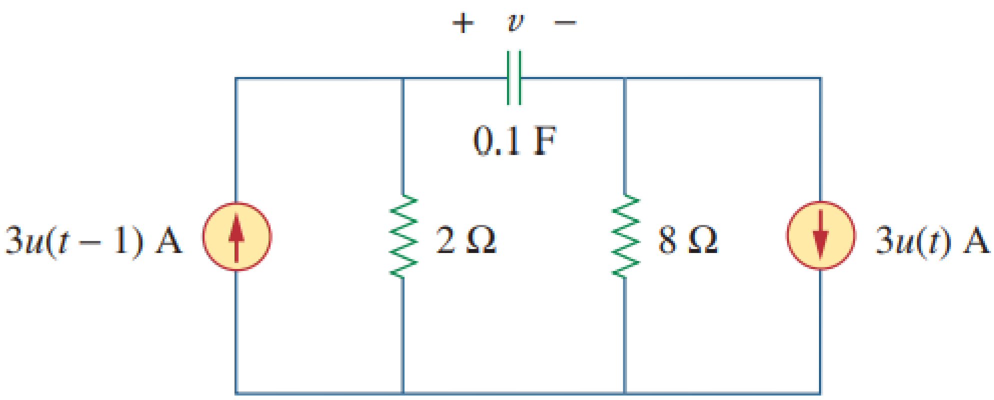

Figure 7.114

7.40 Find the capacitor voltage for $t < 0$ and $t > 0$ for each of the circuits in Fig. 7.107.

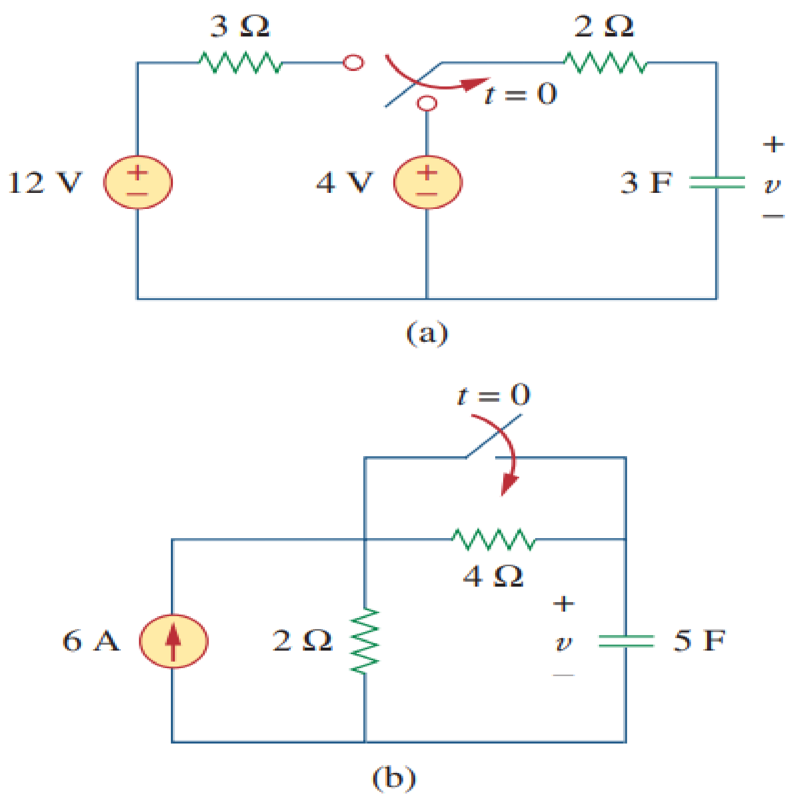

Figure 7.107

7.44 The switch in Fig. 7.111 has been in position a for a long time. At $t = 0$, it moves to position b. Calculate $i(t)$ for all $t > 0$.

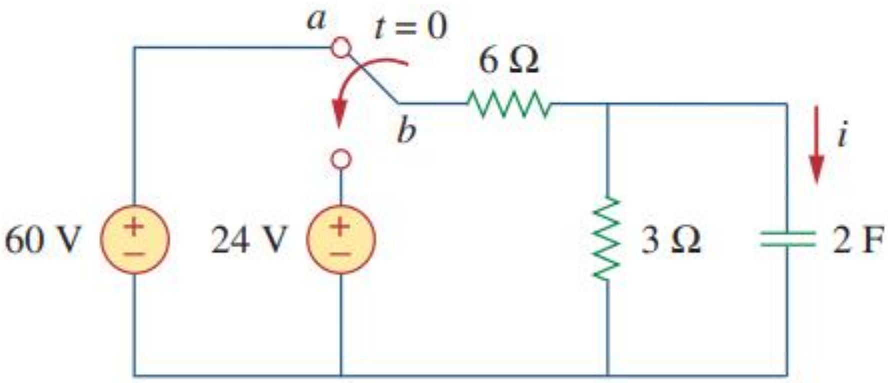

Figure 7.111

was designed to reduce the magnitude of the input voltage v_i by a factor of 10. As shown in Fig. 7.149, the oscilloscope has internal resistance R_s and capacitance C_s, while the probe has an internal resistance R_p. If R_p is fixed at 6 MΩ, find R_s and C_s for the circuit to have a time constant of 15 μs.

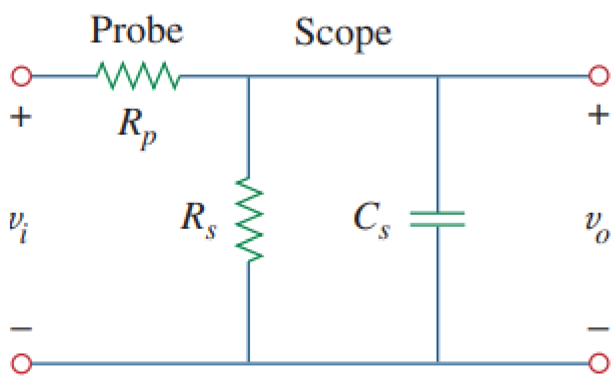

Figure 7.149

2. Second order circuits applications

Having been in position a for a long time, the switch in Fig. 8.21 is moved to position b at $t = 0$. Find $v(t)$ and $v_R(t)$ for $t > 0$.

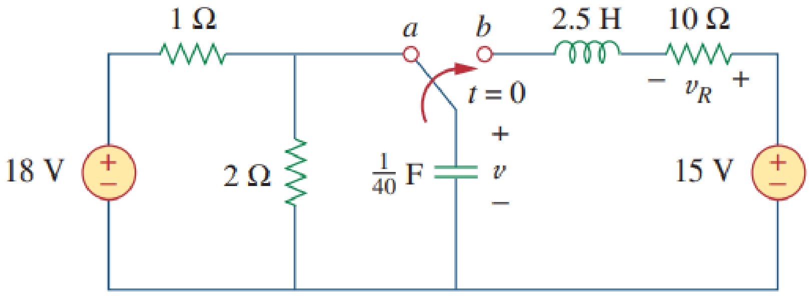

Figure 8.21
For Practice Prob. 8.7.

Answer: $15 - (1.7321 \sin 3.464t + 3 \cos 3.464t)e^{-2t}$ V, $3.464e^{-2t} \sin 3.464t$ V.

Practice Problem 8.11

In the op amp circuit shown in Fig. 8.34, $v_s = 10u(t)$ V, find $v_o(t)$ for $t > 0$. Assume that $R_1 = R_2 = 10$ kΩ, $C_1 = 20$ μF, and $C_2 = 100$ μF.

Answer: $(10 - 12.5e^{-t} + 2.5e^{-5t})$ V, $t > 0$.

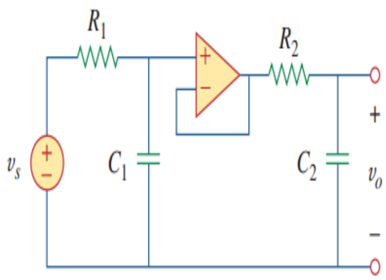

Figure 8.34
For Practice Prob. 8.11.

8.9 *PSpice* Analysis of *RLC* Circuits

RLC circuits can be analyzed with great ease using *PSpice*, just like students better understand finding initial and final values.

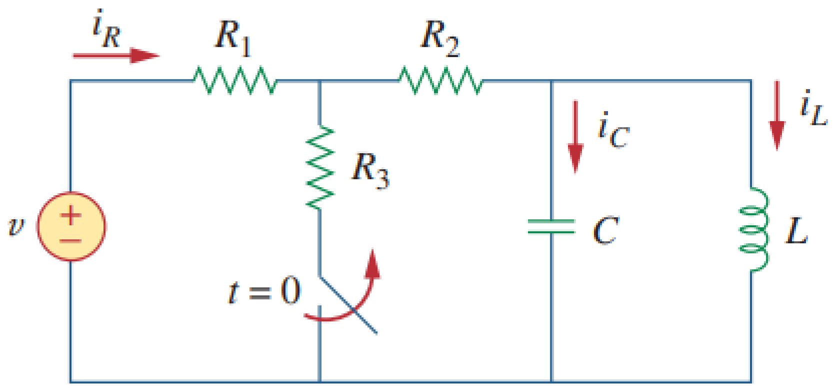

Figure 8.63

8.33 Find $v(t)$ for $t > 0$ in the circuit of Fig. 8.81.

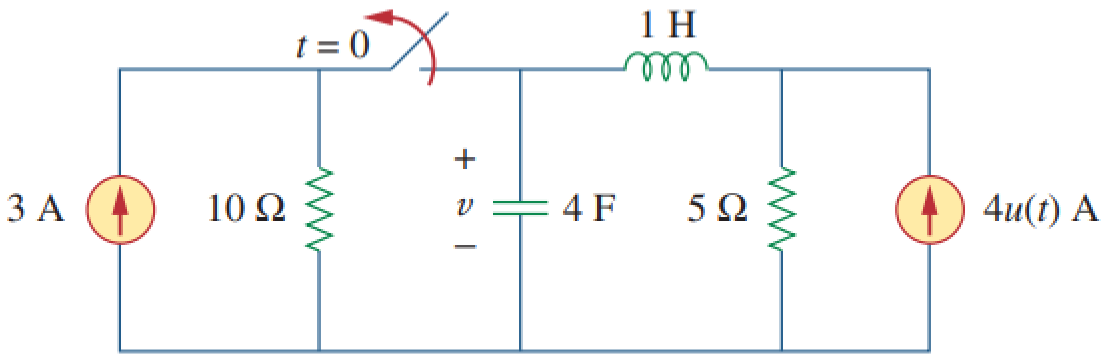

Figure 8.81
For Prob. 8.33.

8.35 Using Fig. 8.83, design a problem to help other students better understand the step response of series *RLC* circuits.

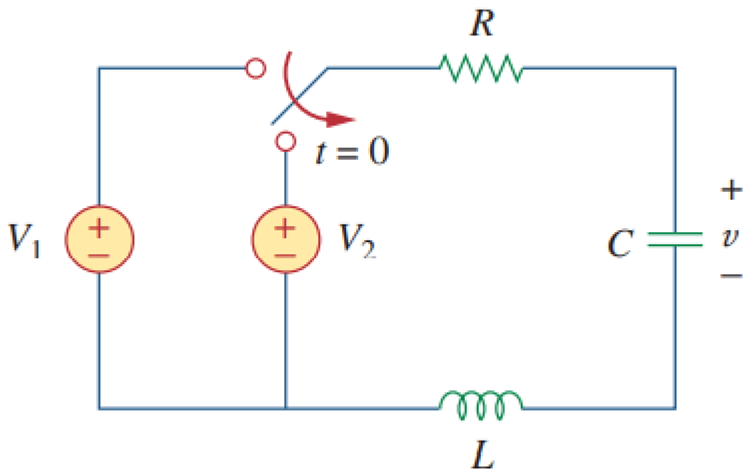

Figure 8.83
For Prob. 8.35.

8.40 The switch in the circuit of Fig. 8.88 is moved from position a to b at $t = 0$. Determine $i(t)$ for $t > 0$.

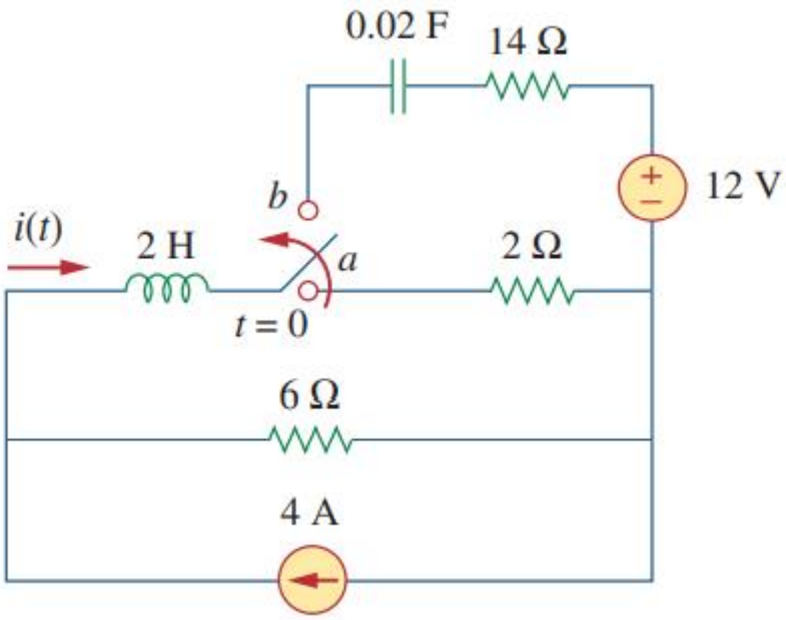

Figure 8.88

***8.41** For the network in Fig. 8.89, find $i(t)$ for $t > 0$.

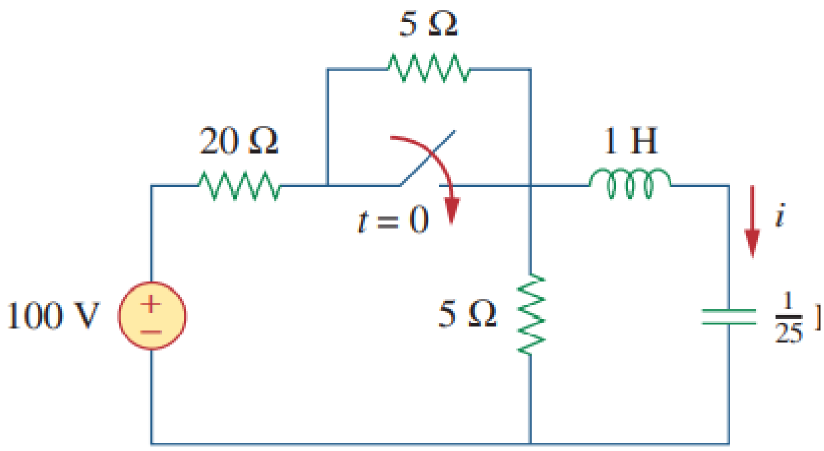

Figure 8.89
For Prob. 8.41.

***8.42** Given the network in Fig. 8.90, find $v(t)$ for $t > 0$.

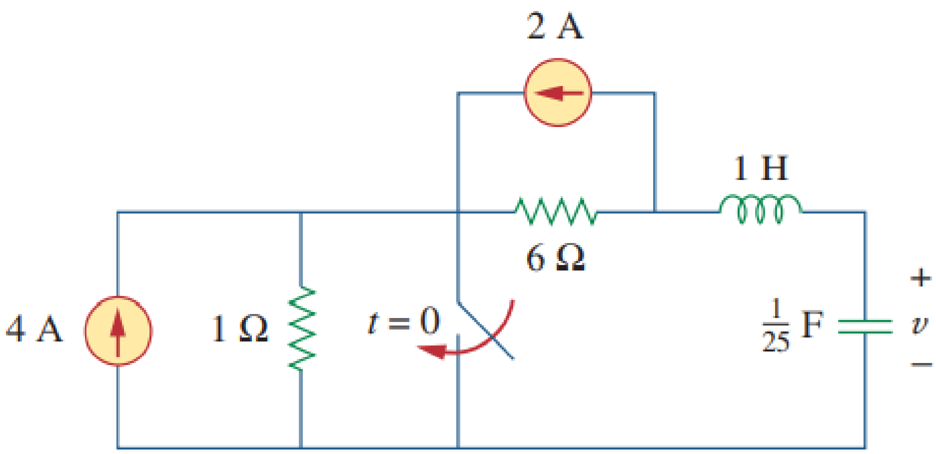

Figure 8.90
For Prob. 8.42.

8.43 The switch in Fig. 8.91 is opened at $t = 0$ after the circuit has reached steady state. Choose R and C such that $\alpha = 8$ Np/s and $\omega_d = 30$ rad/s.

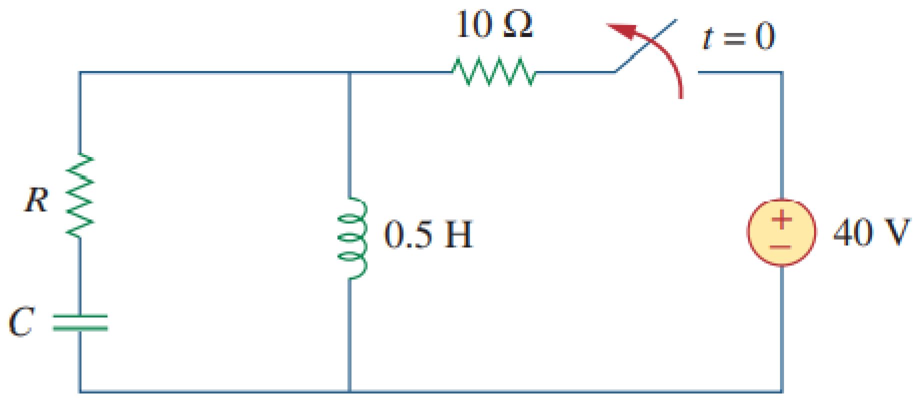

Figure 8.91
For Prob. 8.43.

8.80 A mechanical system is modeled by a series *RLC* circuit. It is desired to produce an overdamped response with time constants 0.1 ms and 0.5 ms. If a series 50-kΩ resistor is used, find the values of L and C.

8.81 An oscillogram can be adequately modeled by a second-order system in the form of a parallel *RLC* circuit. It is desired to give an underdamped voltage across a 200-Ω resistor. If the damping frequency is 4 kHz and the time constant of the envelope is 0.25 s, find the necessary values of L and C.

8.82 The circuit in Fig. 8.123 is the electrical analog of body functions used in medical schools to study convulsions. The analog is as follows:

C_1 = Volume of fluid in a drug

C_2 = Volume of blood stream in a specified region

R_1 = Resistance in the passage of the drug from the input to the blood stream

R_2 = Resistance of the excretion mechanism, such as kidney, etc.

v_0 = Initial concentration of the drug dosage

$v(t)$ = Percentage of the drug in the blood stream

Find $v(t)$ for $t > 0$ given that $C_1 = 0.5\ \mu\text{F}$, $C_2 = 5\ \mu\text{F}$, $R_1 = 5\ \text{M}\Omega$, $R_2 = 2.5\ \text{M}\Omega$, and $v_0 = 60u(t)$ V.

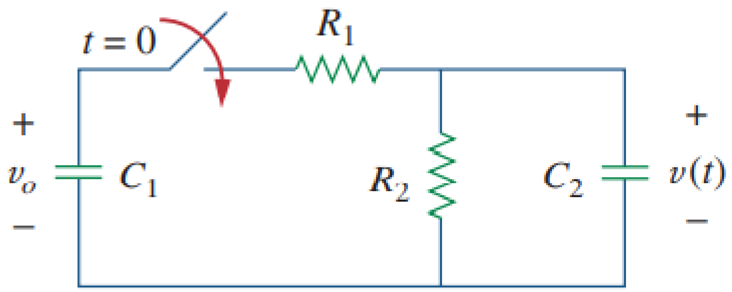

Figure 8.123
For Prob. 8.82.

3. Sinusoidal-Steady State Analysis

Find v_o and i_o in the op amp circuit of Fig. 10.32. Let $v_s = 12 \cos 5000t$ V.

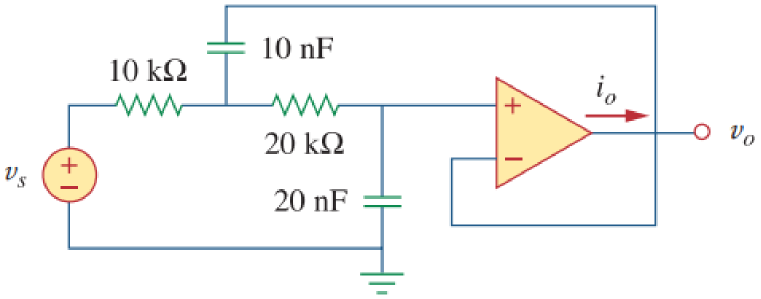

Figure 10.32
For Practice Prob. 10.11.

Answer: $4 \sin 5{,}000t$ V, $400 \sin 5{,}000t$ μA.

Example 10.12

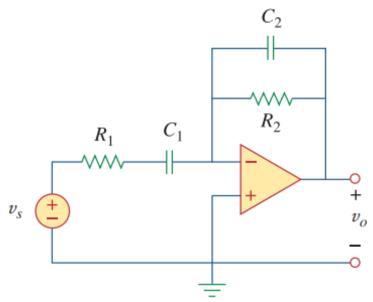

Figure 10.33
For Example 10.12.

Compute the closed-loop gain and phase shift for the circuit in Fig. 10.33. Assume that $R_1 = R_2 = 10$ kΩ, $C_1 = 2$ μF, $C_2 = 1$ μF, and $\omega = 200$ rad/s.

Solution:

The feedback and input impedances are calculated as

$$\mathbf{Z}_f = R_2 \left\| \frac{1}{j\omega C_2} \right. = \frac{R_2}{1 + j\omega R_2 C_2}$$

$$\mathbf{Z}_i = R_1 + \frac{1}{j\omega C_1} = \frac{1 + j\omega R_1 C_1}{j\omega C_1}$$

Since the circuit in Fig. 10.33 is an inverting amplifier, the closed-loop gain is given by

$$\mathbf{G} = \frac{\mathbf{V}_o}{\mathbf{V}_s} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} = \frac{-j\omega C_1 R_2}{(1 + j\omega R_1 C_1)(1 + j\omega R_2 C_2)}$$

Substituting the given values of R_1, R_2, C_1, C_2, and ω, we obtain

$$\mathbf{G} = \frac{-j4}{(1 + j4)(1 + j2)} = 0.434\underline{/130.6^\circ}$$

Thus, the closed-loop gain is 0.434 and the phase shift is 130.6°.

10.9 Use nodal analysis to find v_o in the circuit of Fig. 10.58.

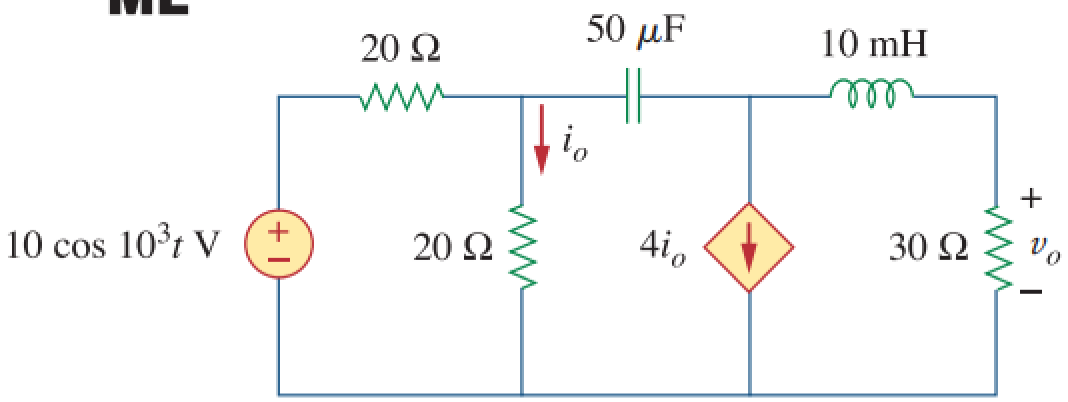

Figure 10.58
For Prob. 10.9.

10.20 Refer to Fig. 10.69. If $v_s(t) = V_m \sin\omega t$ and $v_o(t) = A \sin(\omega t + \phi)$, derive the expressions for A and ϕ.

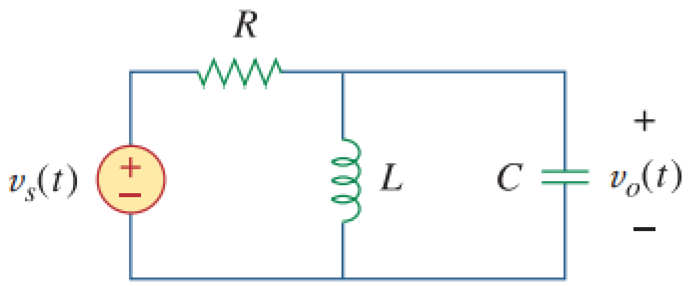

Figure 10.69
For Prob. 10.20.

10.30 Use mesh analysis to find v_o in the circuit of Fig. 10.78. Let $v_{s1} = 120 \cos(100t + 90°)$ V, $v_{s2} = 80 \cos 100t$ V.

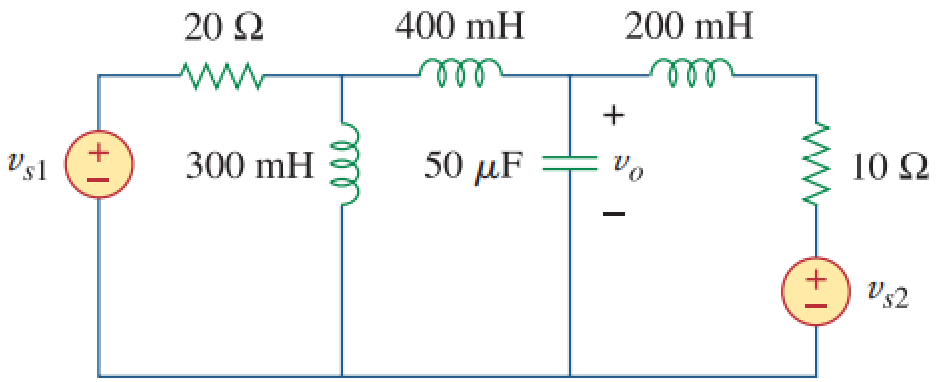

Figure 10.78
For Prob. 10.30.

10.28 In the circuit of Fig. 10.76, determine the mesh currents i_1 and i_2. Let $v_1 = 10 \cos 4t$ **V** and $v_2 = 20 \cos(4t - 30°)$ V.

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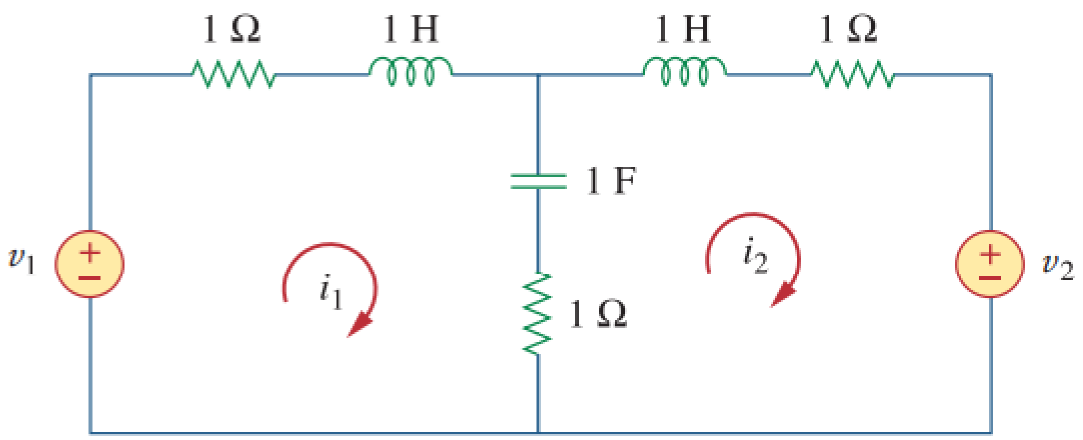

Figure 10.76
For Prob. 10.28.

10.39 Find $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{I}_3$, and $\mathbf{I}_x$ in the circuit of Fig. 10.84.

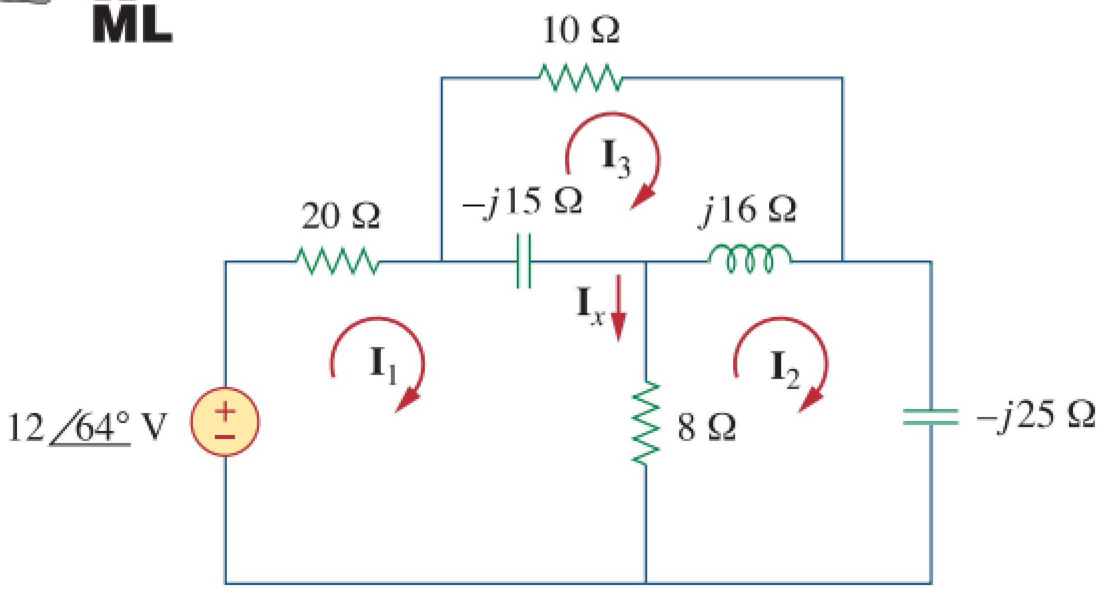

Figure 10.84
For Prob. 10.39.

10.74 Evaluate the voltage gain $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_s$ in the op amp circuit of Fig. 10.117. Find $\mathbf{A}_v$ at $\omega = 0$, $\omega \to \infty$, $\omega = 1/R_1C_1$, and $\omega = 1/R_2C_2$.

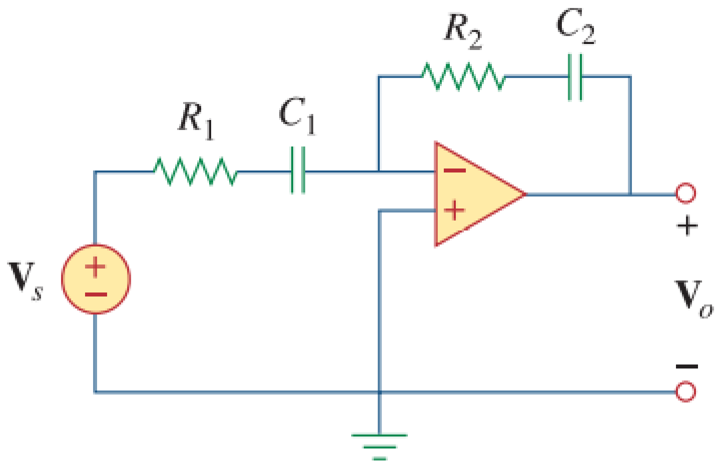

Figure 10.117
For Prob. 10.74.

4. Convolution

Practice Problem 15.13

Given $g(t)$ and $f(t)$ in Fig. 15.20, graphically find $y(t) = g(t) * f(t)$.

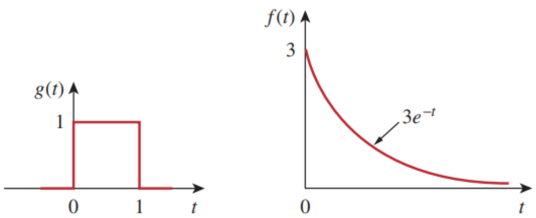

Figure 15.20
For Practice Prob. 15.13.

Answer: $y(t) = \begin{cases} 3(1 - e^{-t}), & 0 \leq t \leq 1 \\ 3(e - 1)e^{-t}, & t \geq 1 \\ 0, & \text{elsewhere.} \end{cases}$

5. Graph theory in circuit analysis
a. Definition of a Graph
b. Definition of Digraph
c. Definition of a Tree
d. Why is graph theory in circuit analysis important?
e. Incident matrices, cut-set

Refer the network shown in Fig. 2.15(a). Solve for branch currents and branch voltages.

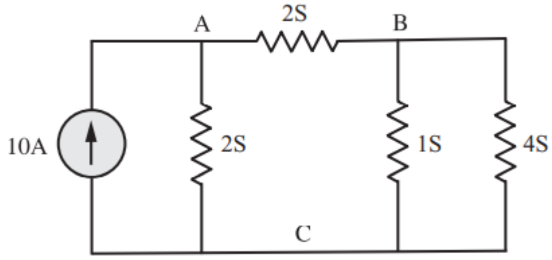

Figure 2.15(a)

SOLUTION

The oriented graph for the network is shown in Fig. 2.15(b). A possible *tree* and *cotree* with fundamental cut-sets are shown in Fig. 2.15(c).

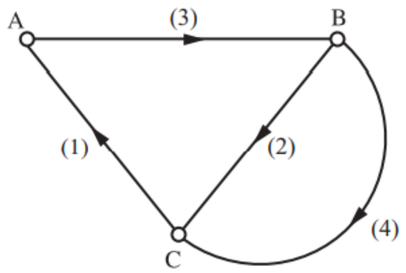

Figure 2.15(b) Directed graph for the network shown in Fig. 2.40

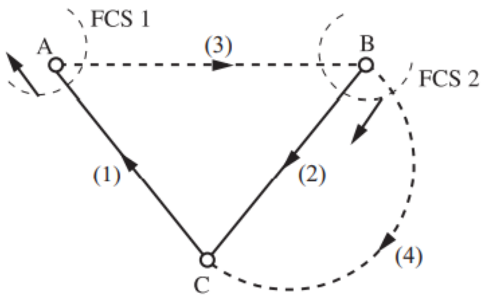

Figure 2.15(c) A possible tree (thick lines) and cotree (dotted lines)

Cut-set schedule:

Tree branch voltages	Branch Numbers			
	1	2	3	4
e_1	1	0	-1	0
e_2	0	1	-1	1

Cut-set matrix:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

Branch admittance matrix:

$$\mathbf{Y}_B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Cut-set admittance matrix:

$$\begin{aligned} \mathbf{Y}_N &= \mathbf{Q}\mathbf{Y}_B\mathbf{Q}^{\mathrm{T}} \\ &= \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 & -2 & 0 \\ 0 & 1 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$= \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix}$$

Equilibrium equations:

$$\mathbf{Y}_N \mathbf{E}_N = \mathbf{Q} \mathbf{I}_B$$

$$\Rightarrow \qquad \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = - \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \qquad \begin{bmatrix} 4 & 2 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

Solving using cramer's rule, we get

$$e_1 = \frac{-70}{24}\text{V}, \qquad e_2 = \frac{+20}{24}\text{V}$$

Branch voltage are found using the matrix equation:

$$\mathbf{V}_B = \mathbf{Q}^{\mathrm{T}}\mathbf{E}_N$$

$$\Rightarrow \qquad \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{-70}{24} \\ \frac{+20}{24} \end{bmatrix}$$

Hence,

$$V_1 = \frac{-70}{24}\mathrm{V} = -2.917\mathrm{V}$$

$$V_2 = \frac{+20}{24}\mathrm{V} = +0.833\mathrm{V}$$

$$V_3 = +\frac{70}{24} - \frac{20}{24} = +2.084\mathrm{V}$$

$$V_4 = \frac{20}{24} = +0.833\mathrm{V}$$

Branch currents are found using the matrix equation (2.8):

$$\mathbf{J}_B = \mathbf{Y}_B\mathbf{V}_B - \mathbf{I}_B$$

$$\Rightarrow \qquad \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} - \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \qquad I_1 = 2V_1 + 10 = 4.166 \text{ A}$$
$$I_2 = V_2 = 0.833 \text{ A}$$
$$I_3 = 2V_3 = 4.168 \text{ A}$$
$$I_4 = 4V_4 = 3.332 \text{ A}$$

Verification:
Refer Fig. 2.15(a).
KCL equations

$$I_1 = I_3 = 4.168 \text{ A}$$
$$I_3 = I_2 + I_4 = 0.833 + 3.332$$
$$= 4.166 \text{ A}$$

and *KVL equations:*

$$V_3 + V_2 + V_1 = 0$$
$$V_2 - V_4 = 0 \text{ are statisfied.}$$